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Nuclear electromagnetic current in the relativistic approach with the momentum-dependent self-energies

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We define the new description of the electromagnetic current to hold the current conservation in the momentum-dependent Dirac fields from the Ward identity. To describe the momentum dependence we solve the relativistic Hartree-Fock approximation by using the one-pion exchange. In addition we discuss on contribution from the one-pion exchange current and the core polarization. It is shown that the one-pion exchange current can reduce the convection current in the isovector case, whose value has been too large because of the small effective mass in the usual relativistic Hartree approximation.

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I. INTRODUCTION

The past decades have seen many successes in the relativistic treatment of the nuclear many-body problem. The relativistic framework has big advantages in several aspects [1,2]: a useful Dirac phenomenology for the description of nucleon-nucleus scattering [3,4], the natural incorporation of the spin-orbit force [1] and the saturation properties of nuclear matter in the microscopic treatment with the Dirac Brueckner Hartree-Fock (DBHF) approach [5].

These results conclude that there are large attractive scalar and repulsive vector fields and that the nucleon effective mass is very small in the medium. However, this small effective mass leads to small Fermi velocity, which makes some troubles in the nuclear properties: too-large magnetic moment [6] and too-large excitation energy of the isoscalar giant quadrupole resonance (ISGQR) state [7]. As for the isoscalar magnetic moment, this enhancement is canceled by the ring-diagram contribution [8]; this relation is completely realized by the gauge invariance [9]. As for the isovector one, however, this contribution does not plays a significant role because the symmetry force is not sufficiently large.

In this subject most of people believed that the momentum dependence of the Dirac fields is negligible in the low-energy region, particularly below the Fermi level. A momentum dependence of the Schrödinger equivalent potential automatically emerges as a consequence of the Lorentz transformation properties of the vector fields without any explicit momentum dependence of the scalar and vector fields. In fact, only very small momentum dependence has appeared in the relativistic Hartree-Fock (RHF) calculation [10,11].

In the high-energy region, however, the vector fields must become very small to explain the optical potential of the proton-nucleus elastic scattering [3,12] and the transverse flow in the heavy-ion collisions [13]. The momentum-dependent part is not actually small though it has not been clearly seen in the low energy phenomena. Furthermore, S. Typel [14] introduce the nonlocal parts and succeeded in improving nuclear properties.

In the previous article [15] we showed that the momentum dependence of the Dirac fields is very sensitive to the Fermi velocity though it hardly affects the nuclear equation of state. In that work we introduced the one-pion exchange force, which produces the dominant contribution of the momentum dependence and suppresses the Fermi velocity, and explain the ISGQR energy.

We can easily imagine that the one-pion exchange force largely produce the momentum dependence because the interaction range is largest. Because the momentum-dependent fields break the current conservation, we have to define the new current caused by the vertex correction.

In this article, thus, we investigate the nuclear current using the momentum-dependent Dirac fields. For this purpose we define a new current to hold the current conservation in the momentum-dependent Dirac fields and discuss its effect on the the nuclear static current. In this work we focus only on the convection current that is sensitive to the Fermi velocity omitting the spin current.

In the next section we explain our formalism to make a conserved current under the momentum-dependent self-energies. In Sec. III we show our numerical results for the static current in our formulation. Then we summarize our work in Sec. IV.

II. FORMALISM

A. Nucleon propagator

Now we describe the propagator of nucleon with momentum p in the isospin space as follows:

$$S(p) = \begin{bmatrix} S_p(p) & 0 \\ 0 & S_n(p) \end{bmatrix}. \tag{1}$$

Here we assume the spin isospin-saturated nuclear matter, and define the proton and neutron propagator as

$$S_N(p) = S_p(p) = S_n(p). \tag{2}$$

The nucleon propagator in the self-energy Σ is given by

$$S_N^{-1}(p) = \not{p} - M - \Sigma(p), \tag{3}$$

where $\Sigma(p)$ has a Lorentz scalar part U_s and a Lorentz vector part $U_\mu(p)$ as

$$\Sigma(p) = -U_s(p) + \gamma^\mu U_\mu(p). \tag{4}$$

For future convenience we define the effective mass and the kinetic momentum as

$$\begin{aligned} M^*(p) &= M - U_s(p), \\ \Pi_\mu(p) &= p_\mu - U_\mu(p). \end{aligned} \tag{5}$$

The single-particle energy with momentum $\boldsymbol{p}$ is obtained as

$$\begin{aligned} \varepsilon(\boldsymbol{p}) &= p_0|_{\text{on-mass-shell}} \\ &= \sqrt{\Pi^2(\boldsymbol{p}) + M^{*2}} + U_0(\boldsymbol{p}). \end{aligned} \tag{6}$$

Then the detailed form of the nucleon propagator Eq. (3) is represented by

$$S_N(p) = S_F(p) + S_D(p) \tag{7}$$

with

$$S_F(p) = [\not{\Pi}(p) + M^*(p)] \frac{1}{\Pi^2 - M^{*2} + i\delta} \tag{8}$$

$$S_D(p) = 2i\pi[\not{\Pi}(p) + M^*(p)] n(\boldsymbol{p}) \theta(p_0) \delta[V(p)], \tag{9}$$

where $n(\boldsymbol{p})$ is the momentum distribution and

$$V(p) \equiv \tfrac{1}{2}[\Pi^2(p) - M^{*2}(p)]. \tag{10}$$

B. Momentum-dependent self-energies

We can easily suppose that it is the one-pion exchange force that produces the major momentum dependence because the interaction range is largest. In this work, thus, we introduce the momentum dependence to the Dirac fields arising from the one-pion exchange, and discuss how the Fock parts affects the nuclear current.

Along this line we define a Lagrangian density in the system as

$$\begin{aligned} \mathcal{L} = &\bar{\psi}(i\not{\partial} - M)\psi + \tfrac{1}{2}\partial_\mu\phi_a\partial^\mu\phi_a - \tfrac{1}{2}m_\pi^2\phi_a\phi_a - \widetilde{U}[\sigma] \\ &+ \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \rho_{\mu a}\rho_a^\mu + i\frac{f_\pi}{m_\pi}\bar{\psi}\gamma_5\gamma^\mu\tau_a\psi\partial_\mu\phi_a \\ &+ g_\sigma\bar{\psi}\psi\sigma - g_\omega\bar{\psi}\gamma_\mu\psi\omega^\mu - \frac{C_v^{IV}}{2M^2}\{\bar{\psi}\gamma_\mu\tau\psi\}^2, \end{aligned} \tag{11}$$

where $\psi, \phi, \sigma, \omega$, and ρ are the nucleon, pion, σ-meson, ω-meson, ρ-meson fields, respectively, and the suffix a indicates the isospin component. In the above expression we use the pseudovector (PV) coupling form as an interaction between nucleon and pion. The self-energy potential of the σ field $\widetilde{U}[\sigma]$ is given as Ref. [13,16].

$$\widetilde{U}[\sigma] = \frac{\frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}B_\sigma\sigma^3 + \frac{1}{4}C_\sigma\sigma^4}{1 + \frac{1}{2}A_\sigma\sigma^2}. \tag{12}$$

The symbols m_π, m_σ, and m_ω are the masses of π, σ, and ω mesons, respectively. In addition, we also introduce the isovector nucleon-nucleon interaction into the Lagrangian (11) so as to discuss on the isovector current later.

Next we calculate the nucleon self-energies. The nucleon self-energies are separated into the local part and the momentum-dependent part as $U_\alpha(p) = U_\alpha^L + U_\alpha^F(p)$, where $\alpha = s, \mu$. The σ- and ω-meson exchange parts produce only very small momentum dependence of nucleon self-energies [10,11] as their masses are large. In fact, the relativistic Hartree (RH) and RHF approximations yield no different results in nuclear matter properties after fitting parameters of σ- and ω-exchanges [10]. However, the one-pion exchange force is a long-range one and makes a large momentum dependence although it does not contribute to the local part in the spin-saturated system. Subsequently we make the local part by RH of the σ- and ω-meson exchanges, and the momentum-dependent part by RHF of the pion exchange, and thus we omit the kinetic-energy part of mesons except pion in Eq. (11). This method is shown in Ref. [12] to keep the self-consistency within the RHF framework.

In this model the local part of the self-energies are given as

$$U_s^L = g_\sigma\langle\sigma\rangle \tag{13}$$

$$U_\mu^L = \delta_{0\mu}\frac{g_\omega{}^2}{m_\omega^2}\rho_H, \tag{14}$$

where $\langle\sigma\rangle$ is the scalar mean field obtained as

$$\frac{\partial}{\partial\langle\sigma\rangle}\tilde{U}[\langle\sigma\rangle] = g_\sigma\rho_s. \tag{15}$$

In the above equations the scalar density ρ_s and the vector Hartree density ρ_H are given by

$$\rho_s = 4\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}n(\boldsymbol{p})\frac{M_\alpha^*(p)}{\tilde{\Pi}_0(p)}, \tag{16}$$

$$\rho_H = 4\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}n(\boldsymbol{p})\frac{\Pi_0(p)}{\tilde{\Pi}_0(p)}, \tag{17}$$

where $n(\boldsymbol{p})$ is the momentum distribution and $\tilde{\Pi}_\mu(p)$ is defined by

$$\tilde{\Pi}_\mu(p) = \frac{1}{2}\frac{\partial}{\partial p^\mu}[\Pi^2(p) - M^{*2}(p)]. \tag{18}$$

As a next step we define the momentum-dependent part of the self-energies as the Fock term arising from the one-pion exchange, which is given as

$$\begin{aligned} \Sigma_F(p) &\equiv U_s^F(p) - \gamma^\mu U_\mu^F \\ &= \frac{if_\pi^2}{m_\pi^2}\sum_a\int\frac{d^4k}{(2\pi)^4}(\not{p} - \not{k})\gamma_5\tau_a S(k)\tau_a\gamma_5 \\ &\quad\times(\not{p} - \not{k})\Delta_\pi(p-k) \\ &= \frac{1+\tau_3}{2}\frac{if_\pi^2}{m_\pi^2}\int\frac{d^4k}{(2\pi)^4}(\not{p} - \not{k})\gamma_5[S_p(k) + 2S_n(k)] \\ &\quad\times\gamma_5(\not{p} - \not{k})\Delta_\pi(p-k) \\ &\quad+\frac{1-\tau_3}{2}\frac{if_\pi^2}{m_\pi^2}\int\frac{d^4k}{(2\pi)^4}(\not{p} - \not{k})\gamma_5[2S_p(k) + S_n(k)] \\ &\quad\times\gamma_5(\not{p} - \not{k})\Delta_\pi(p-k) \\ &= \frac{1+\tau_3}{2}\Sigma_F^{(p)}(p) + \frac{1-\tau_3}{2}\Sigma_F^{(n)}(p), \end{aligned} \tag{19}$$

where the superscripts (p) and (n) indicate the self-energies for proton and neutron, respectively, and $\Delta_\pi(q)$ is the pion propagator defined as

$$\Delta_\pi(q) = \frac{1}{q^2 - m_\pi^2}. \tag{20}$$

As mentioned, only the isospin symmetric system is considered here. Hence, the proton and neutron propagators are equal to each other, $S_p = S_n = S_N$. Taking the density-dependent part S_D instead of the full propagator S_N, we can obtain the scalar and vector parts as

$$U_s^F(p) = \frac{3f_\pi^2}{2m_\pi^2} \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} n(\boldsymbol{k}) \frac{M^*(k)}{\tilde{\Pi}_0(k)} (p-k)^2 \Delta_\pi(p-k), \tag{21}$$

$$U_\mu^F(p) = U_\mu^{(1)}(p) + U_\mu^{(2)}(p) \tag{22}$$

with

$$U_\mu^{(1)}(p) = -\frac{3f_\pi^2}{2m_\pi^2} \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} n(\boldsymbol{k}) \frac{\Pi_\mu(k)}{\tilde{\Pi}_0(k)} (p-k)^2 \Delta_\pi(p-k), \tag{23}$$

$$U_\mu^{(2)}(p) = -\frac{3f_\pi^2}{m_\pi^2} \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} n(\boldsymbol{k}) \times \frac{\Pi(k)\cdot(p-k)}{\tilde{\Pi}_0(k)} (p_\mu - k_\mu) \Delta_\pi(p-k). \tag{24}$$

The tensor-coupling part of the vector self-energy, $U_\mu^{(2)}$, is very small if the self-energy is independent of momentum [10]. By substituting the RH results, indeed, we numerically check that its contribution is less than 0.1 % of $U_\mu^{(1)}$. Then we can neglect this term in this work.

Furthermore, the Fock parts do not become zero at the infinite limit of the momentum $|\boldsymbol{p}|$. One usually disregards these contributions by introducing the cutoff parameter. In this work, instead of that, we subtract these contributions from the momentum-dependent parts (these contributions can be renormalized into the Hartree parts): $U_\alpha \to U_\alpha - U_\alpha$ $(p \to \infty)$.

Thus we obtain the momentum-dependent parts of the self-energies as

$$U_s^F(p) = \frac{3f_\pi^2}{2} \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} n(\boldsymbol{k}) \frac{M^*(k)}{\tilde{\Pi}_0(k)} \Delta_\pi(p-k), \tag{25}$$

$$U_\mu^F(p) = -\frac{3f_\pi^2}{2} \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} n(\boldsymbol{k}) \frac{\Pi_\mu(k)}{\tilde{\Pi}_0(k)} \Delta_\pi(p-k). \tag{26}$$

C. One-body current operator

If the self-energy has a momentum dependence, the current operator must be also changed to satisfy the current conservation. We then define the current vertex $\Gamma^\mu(p+q,p)$ as

$$\Gamma^\mu(p+q,p) = \frac{1+\tau_3}{2}\gamma^\mu + \Lambda^\mu(p+q,p). \tag{27}$$

The Ward-Takahashi (WT) identity gives the following relation about the current vertex

$$S(p+q)q^\mu\Gamma_\mu S(p) = -S(p+q) + S(p). \tag{28}$$

This expression is rewritten as

$$q^\mu\Gamma_\mu = S^{-1}(p+q) - S^{-1}(p). \tag{29}$$

Substituting Eq. (3) into Eq. (29), the density-dependent vertex correction Λ^μ is obtained as

$$q_\mu\Lambda^\mu(p+q,p) = -\Sigma(p+q) + \Sigma(p). \tag{30}$$

In general the vertex correction is very complicated, and its exact expression cannot be easily derived in the RHF framework. However, this vertex correction can be obtained in the zero-momentum limit $q \to 0$. Here we assume that the current operator in the zero-momentum limit $q \to 0$ has a contribution only from the proton as

$$\lim_{q\to 0}\Gamma_\mu(p+q,p) \equiv \Gamma_\mu(p) = \frac{1+\tau_3}{2}\tilde{\Gamma}_\mu(p), \tag{31}$$

where $\tilde{\Gamma}_\mu$ is the static proton current operator. In this case the current operator, Γ_μ, and the vertex correction, Λ_μ, become

$$S_p(p)\tilde{\Gamma}_\mu(p)S_p(p) = -\frac{1+\tau_3}{2}\frac{\partial}{\partial p_\mu}S_p(p), \tag{32}$$

$$\Lambda_\mu(p) = \lim_{q\to 0}\Lambda_\mu(p+q,p) = -\frac{1+\tau_3}{2}\frac{\partial}{\partial p^\mu}\Sigma^{(p)}(p). \tag{33}$$

As for isovector meson exchanges, the current operator includes a diagram of the photon that connects with the exchange meson (the mesonic current) and those of the photon that contacts with the meson-nucleon vertex (the contact current). Generally the electromagnetic current is contributed not only from the proton current but also from the neutron current. In the following we confirm the above assumption of the vertex correction at $q \to 0$ limit (33).

When we use the one-pion exchange force, the electromagnetic interaction Lagrangian density is written as

$$\mathcal{L}_{\rm em}(x) = \mathcal{L}_{\rm em}^v(x) + \mathcal{L}_{\rm em}^m(x) + \mathcal{L}_{\rm em}^c(x) \tag{34}$$

with

$$\mathcal{L}_{\rm em}^v = -e\bar{\psi}(x)\gamma_\mu\frac{1+\tau_3}{2}\psi, \tag{35}$$

$$\mathcal{L}_{\rm em}^m = -ie[\phi_1(x)\partial_\mu\phi_2(x) - \phi_2(x)\partial_\mu\phi_1(x)], \tag{36}$$

$$\mathcal{L}_{\rm em}^c = -\frac{ief_\pi}{m_\pi^2}[\bar{\psi}(x)\gamma_\mu\gamma_5\tau_1\psi(x)\phi_2(x) - \bar{\psi}(x)\gamma_\mu\gamma_5\tau_2\psi(x)\phi_1(x)]. \tag{37}$$

Then the vertex corrections can be separated into three parts as

$$\Lambda_\mu(p+q,p) = \Lambda_\mu^m(p+q,p) + \Lambda_\mu^c(p+q,p) + \Lambda_\mu^v(p+q,p), \tag{38}$$

where the first and second terms correspond to pionic and contact currents derived from Eqs. (36) and (37), respectively, and the third term is the vertex correction in a narrow sense.

The pionic current, Λ^m, where the photon connects with the pion exchanged between nucleons, is given as

$$\begin{aligned}\Lambda_\mu^m &= -\int \frac{d^4k}{(2\pi)^4}\left(\frac{if_\pi}{m_\pi}\right)(\not p - \not k + \not q)\gamma_5\tau_i S(k)\tau_j\gamma_5(\not p - \not k)\\ &\times \left(\frac{if_\pi}{m_\pi}\right)(\delta_{1i}\delta_{2j} - \delta_{2i}\delta_{1j})i\Delta_\pi(p-k-q)(-i)\\ &\times (2p-2k-q)_\mu i\Delta(p-k)\\ &= -\frac{2if_\pi^2}{m_\pi^2}\int\frac{d^4k}{(2\pi)^4}(\not p - \not k + \not q)\gamma_5\\ &\times\left[\frac{1+\tau_3}{2}S_n(k) - \frac{1-\tau_3}{2}S_p(k)\right]\gamma_5(\not p - \not k)\\ &\times \Delta_\pi(p-k+q)(2p-2k+q)_\mu\Delta_\pi(p-k). \end{aligned} \tag{39}$$

The contact current, Λ^c, is obtained as

$$\begin{aligned}\Lambda_\mu^c &= -\int\frac{d^4k}{(2\pi)^4}\Bigg[\left(\frac{if_\pi}{m_\pi}\right)(\not p - \not k + \not q)\gamma_5\tau_i S(k)\left(\frac{if_\pi}{m_\pi}\right)\\ &\times \tau_j\gamma_5\gamma_\mu\Delta_\pi(p-k+q)\\ &+\left(\frac{if_\pi}{m_\pi}\right)\gamma_\mu\gamma_5\tau_j S(p)\tau_i\left(\frac{if_\pi}{m_\pi}\right)\\ &\times(\not p - \not k)i\Delta_\pi(p-k+q)\Bigg](\delta_{1i}\delta_{2j}-\delta_{2i}\delta_{1j})\\ &= \frac{-2if_\pi^2}{m_\pi^2}\int\frac{d^4k}{(2\pi)^4}\Bigg\{(\not p - \not k + \not q)\gamma_5\Bigg[\frac{1+\tau_3}{2}S_n(k)\\ &- \frac{1-\tau_3}{2}S_p(k)\Bigg]\gamma_5\gamma_\mu\Delta_\pi(p-k+q)\\ &+\gamma_5\gamma_\mu\left[\frac{1+\tau_3}{2}S_n(k) - \frac{1-\tau_3}{2}S_p(k)\right]\\ &\times\gamma_5(\not p - \not k)\Delta_\pi(p-k)\Bigg\}. \end{aligned}\tag{40}$$

Then the sum of the above two terms, Eqs. (39) and (40), becomes

$$\begin{aligned}\Lambda_\mu^m + \Lambda_\mu^c = \frac{2if_\pi^2}{m_\pi^2}\int\frac{d^4k}{(2\pi)^4}\Bigg\{&(\not p - \not k + \not q)\gamma_5\\ &\times\left[\frac{1+\tau_3}{2}S_n(k) - \frac{1-\tau_3}{2}S_p(k)\right]\gamma_5(\not p - \not k)\\ &\times\Delta_\pi(p-k+q)(2p-2k+q)_\mu\Delta_\pi(p-k)\\ &-(\not p - \not k + \not q)\gamma_5\left[\frac{1+\tau_3}{2}S_n(k) - \frac{1-\tau_3}{2}S_p(k)\right]\\ &\times\gamma_5\gamma_\mu\Delta_\pi(p-k+q) - \gamma_5\gamma_\mu\Bigg[\frac{1+\tau_3}{2}S_n(k)\\ &-\frac{1-\tau_3}{2}S_p(k)\Bigg]\gamma_5(\not p - \not k)\Delta_\pi(p-k)\Bigg\}. \end{aligned}\tag{41}$$

By taking the zero-momentum transfer limit, we can obtain the contribution from the pionic and contact current as

$$\begin{aligned}\Lambda_\mu^m(p) + \Lambda_\mu^c(p) &\equiv \lim_{q\to 0}\left[\Lambda_\mu^m(p+q,p) + \Lambda_\mu^c(p+q,p)\right]\\ &= -\frac{2if_\pi^2}{m_\pi^2}\frac{\partial}{\partial p^\mu}\int\frac{d^4k}{(2\pi)^4}(\not p - \not k)\gamma_5\\ &\times\left[\frac{1+\tau_3}{2}S_n(k) - \frac{1-\tau_3}{2}S_p(k)\right]\\ &\times\gamma_5(\not p - \not k)\Delta_\pi(p-k)\\ &= -\frac{3+\tau_3}{3}\frac{\partial}{\partial p^\mu}\Sigma_F^{(p)}(p)\\ &+\frac{3-\tau_3}{3}\frac{\partial}{\partial p^\mu}\Sigma_F^{(n)}(p). \end{aligned}\tag{42}$$

Next we consider the contribution form the vertex correction in the narrow sense, Λ_μ^v. It is given as

$$\begin{aligned}\Lambda_\mu^v(p+q,p) = -\frac{if_\pi^2}{m_\pi^2}\sum_a\int\frac{d^4k}{(2\pi)^4}(\not p - \not k)\gamma_5\tau_a S(k+q)\\ \times\Gamma_\mu S(k)\tau_a\gamma_5(\not p - \not k)\Delta(p-k). \end{aligned}\tag{43}$$

The $q\to 0$ limit of it becomes

$$\begin{aligned}\Lambda_\mu^v(p) &\equiv \lim_{q\to 0}\Lambda_\mu^v(p+q,p)\\ &= -\frac{if_\pi^2}{m_\pi^2}\frac{3-\tau_3}{2}\\ &\times\int\frac{d^4k}{(2\pi)^4}\not k\gamma_5 S_p(p-k)\tilde\Gamma_\mu S_p(p-k)\gamma_5\not k\Delta_\pi(k)\\ &= \frac{if_\pi^2}{m_\pi^2}\frac{3-\tau_3}{2}\frac{\partial}{\partial p^\mu}\int\frac{d^4k}{(2\pi)^4}\not k\gamma_5 S_p(p-k)\gamma_5\not k\Delta(k)\\ &= \frac{3-\tau_3}{6}\frac{\partial}{\partial p^\mu}\Sigma_F^{(p)}(p) - \frac{3-\tau_3}{3}\frac{\partial}{\partial p^\mu}\Sigma_F^{(n)}(p). \end{aligned}\tag{44}$$

Finally, the the vertex correction is given as the summation of the above three contributions, which becomes

$$\begin{aligned}\Lambda_\mu(p) &= \Lambda_\mu^v(p) + \Lambda_\mu^m(p) + \Lambda_\mu^c(p)\\ &= -\frac{(1+\tau_3)}{2}\frac{\partial}{\partial p^\mu}\Sigma_F(p). \end{aligned}\tag{45}$$

This equation imply that the ansatz shown in Eq. (31) is consistent in the present calculation.

In the above we first introduce the ansatz [Eq. (31)] for the isospin dependence of the current operator, and we then show its consistency in the whole expressions. The vertex corrections given in Eq. (45) can be exactly derived in the perturbative way, namely using the RH propagator as $S(p)$ and the current operator as $\tilde\Gamma_\mu = \gamma_\mu$.

In this work we restrict our discussion on the convection current, not on the spin current. The spin current is contributed from the anomalous current, which is proportional to $\sigma_{\mu\nu}q^\nu$ and disappear in the zero-momentum transfer limit.

Using the above vertex correction, the current of the whole system is given as

$$\begin{aligned}j_\mu &= \int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}\left\{\frac{1+\tau_3}{2}\left[\gamma_\mu - \frac{\partial\Sigma(p)}{\partial p_\mu}\right]S(p)\right\}\\ &= \int\frac{d^4p}{(2\pi)^3}n^{(p)}(\boldsymbol{p})\left[\Pi_\mu(p) - \Pi^\nu(p)\frac{\partial U_\nu(p)}{\partial p_\mu}\right.\end{aligned}$$

$$+ M^*(p)\frac{\partial U_s}{\partial p_\mu}\bigg]\delta[V(p)] = \int \frac{d^3p}{(2\pi)^3} n^{(p)}(\boldsymbol{p}) \left.\frac{\tilde{\Pi}_\mu(p)}{\tilde{\Pi}_0(p)}\right|_{p_0=\varepsilon(\boldsymbol{p})}, \tag{46}$$

$n^{(p)}$ is the Fermi distribution function for proton, and $\tilde{\Pi}_\mu$ is defined by

$$\tilde{\Pi}_\mu(p) \equiv \frac{\partial}{\partial p^\mu} V(p) = \frac{1}{2}\frac{\partial}{\partial p^\mu}[\Pi^2(p) - M^{*2}(p)]. \tag{47}$$

Let us consider a one-particle state on the Fermi surface. If the Dirac field is fixed to be those at the ground state, the space current contributed from this nucleon can be written as

$$\boldsymbol{j} = \frac{\tilde{\boldsymbol{\Pi}}(p)}{\tilde{\Pi}_0(p)}|_{|\boldsymbol{p}|=p_F} = \boldsymbol{D}_{\boldsymbol{p}}\varepsilon(\boldsymbol{p})|_{|\boldsymbol{p}|=p_F}, \tag{48}$$

where the total derivative D_p is defined on the on-mass-shell condition: $p_0 = \varepsilon(\boldsymbol{p})$. The above equation is equivalent to that derived by the semiclassical way [12].

In the nonrelativistic framework the effective mass is defined by

$$M_L^* = \left[2\frac{d}{d\boldsymbol{p}^2}\varepsilon(\boldsymbol{p})\right]^{-1}|_{|\boldsymbol{p}|=p_F}, \tag{49}$$

which is the so-called *Landau mass*. As a result, the above spatial current is

$$\boldsymbol{j} = \frac{\boldsymbol{p}_F}{M_L^*}. \tag{50}$$

In our case including the momentum-dependent Dirac fields, the value of the Landau mass M_L^* cannot be uniquely determined from the relativistic effective mass M^*, whereas in the Hartree approximation the Landau mass becomes $M_L^* = \Pi_0(\boldsymbol{p}_F) = \sqrt{p_F^2 + M^{*2}}$.

This current is correct only when the Dirac fields, U_s and U_μ, do not vary from those at the ground state. In actual case, however, the one particle on the Fermi surface interacts with nucleons in Fermi sea and cause another current, which is called the core-polarization current. This current appears even in the RH approximation and must be taken into account to satisfy the Ward-Takahashi identity completely [8,9]. In the next subsection we discuss this effect in our approach.

D. Core-polarization current

The current operator given in the previous subsection is correct only if the Dirac fields, U_s and U_μ, are fixed independently of the configuration. In actual these fields are self-energies and vary with the configuration. To satisfy the WT identity, we need to introduce a contribution from the ring diagram by using the random-phase approximation (RPA) [8,9], whereas our vertex correction corresponds to a contribution from the exchange diagram in RPA. This ring-diagram contribution plays a role to cancel the enhancement of the isoscalar current because of the small Dirac effective mass M^*. However, it is not so easy to solve the full RPA, including the ring and exchange diagrams, by keeping the consistency between the current operator and the RHF self-energies.

However, the ring diagram contribution in the particle-hole basis is equivalent to the core polarization in the zero-momentum transfer limit $\boldsymbol{q} \to 0$ [17–19]. Indeed this effect cancels contribution of the effective mass in the isoscalar case in the nonrelativistic framework [20]. In this subsection, then, we explain our method to introduce the core-polarization current in our framework.

Here we should consider a system where one valence nucleon populates a state on the Fermi surface of the saturated nuclear matter. In this system the momentum distribution can be described as

$$n(\boldsymbol{p}, \tau) = n_0(\boldsymbol{p}) + \tfrac{1}{4}\Delta n(\boldsymbol{p}, \tau), \tag{51}$$

where $n_0(\boldsymbol{p}) = \theta(p_F - |\boldsymbol{p}|)$ shows the usual Fermi distribution with the Fermi momentum p_F and $\Delta n(\boldsymbol{p}) \propto \delta(|\boldsymbol{p}| - p_F)$ indicates the valence nucleon part. The suffix τ indicates the isospin for the valence nucleon,

The valence nucleon varies the self-energies of nucleons below Fermi surface from that at the saturated matter as

$$U_\alpha(p) \to U_\alpha(p) + \Delta U_\alpha(p). \tag{52}$$

In addition the function $V(p)$ is also varied as

$$V(p) = V_0(p) + \Delta V(p) \tag{53}$$

with

$$\Delta V(p) = -\Pi^\mu \Delta U_\mu + M^*\Delta U_s. \tag{54}$$

The currents density is described with the following expression.

$$j^\mu_{\rm tot} = -\sum_{\tau=\pm1}\int \frac{d^4p}{(2\pi)^3} f(p,\tau)\frac{\partial V(p)}{\partial p^\mu}, \tag{55}$$

where $f(p,\tau)$ is the four-dimensional momentum distribution for nucleon with isospin τ, which is given as

$$f(p,\tau) = n(\boldsymbol{p},\tau)\delta(p_0 - \varepsilon_{\boldsymbol{p}}) \tag{56}$$

$$= \frac{1}{\tilde{\Pi}_0(p)} n(\boldsymbol{p},\tau)\delta[V(p)]\theta(p_0). \tag{57}$$

The variation along Eqs. (51)–(54) leads to the above four-dimensional momentum distribution $f(p,\tau)$ as

$$f(p,\tau) = f_0(p,\tau) + \Delta f(p,\tau) \tag{58}$$

with

$$f_0(p,\tau) = n_0(\boldsymbol{p})\delta[V_0(p)]\theta(p_0), \tag{59}$$

$$\Delta f(\boldsymbol{p},\tau) = \Delta n(\boldsymbol{p},\tau)\delta[V_0(p)]\theta(p_0) + n_0(\boldsymbol{p}) \times \left\{\frac{\partial\delta[V(p)]}{\partial V}\right\}_{V=V_0} \Delta V(p,\tau)\theta(p_0). \tag{60}$$

The first term comes from the valence nucleon, and the second one from the core polarization.

The total current density is given as

$$j_{\rm tot}^{\mu} = \int \frac{d^4p}{(2\pi)^3} f(p)\delta[V(p)]\frac{\partial V(p)}{\partial p_\mu} \quad (61)$$

$$= \delta_0^\mu \rho_B + j_{\rm val}^\mu + j_{\rm cor}^\mu. \quad (62)$$

The first term is the current density of the saturated matter, and the second current density $j_{\rm val}^\mu$ shows the contribution from the valence nucleon as

$$j_{\rm val}^\mu = \int \frac{d^4p}{(2\pi)^3}\Delta n(\boldsymbol{p})\delta(p_0-\varepsilon_{\boldsymbol{p}})\frac{\tilde{\Pi}^\mu(p)}{\tilde{\Pi}_0(p)}. \quad (63)$$

The third current density $j_{\rm cor}^\mu$ is so called the core-polarization current, which is caused by the variation of the self-energies of nucleons in Fermi sea and given by

$$j_{\rm cor}^\mu = -2\sum_{\tau=\pm1}\int\frac{d^4p}{(2\pi)^3}n_0(\boldsymbol{p},\tau)\left(\frac{\partial\Delta V(p)}{\partial p_\mu}\delta[V_0(p)] + \frac{\partial V_0(p)}{\partial p_\mu}\left\{\frac{\partial\delta[V(p)]}{\partial V}\right\}_{V=V_0}\Delta V(p)\right) \quad (64)$$

$$= -2\sum_{\tau=\pm1}\int\frac{d^4p}{(2\pi)^3}n_0(\boldsymbol{p},\tau)\frac{\partial}{\partial p^\mu}\{\Delta V(p)\delta[V_0(p)]\}. \quad (65)$$

Here it should be noted that the time component of the core-polarization current density does not change the nucleon density:

$$j_{\rm cor}^0 = -2\sum_{\tau=\pm1}\int\frac{d^4p}{(2\pi)^3}n_0(\boldsymbol{p},\tau)\frac{\partial}{\partial p^0}\{\Delta V(p)\delta[V_0(p)]\} = 0. \quad (66)$$

Now we define the z axis as the direction of the current at the matter. First we calculate the isoscalar current density by taking the valence nucleon part of the momentum distribution to be

$$\Delta n(\boldsymbol{p},\tau) = \frac{(2\pi)^3}{\Omega}\delta(\boldsymbol{p}-\boldsymbol{a}) \quad (67)$$

with

$$\boldsymbol{a} = p_F\hat{z}, \quad (68)$$

where Ω is the volume of the system, which should be finally taken to be infinite. The core-polarization current becomes

$$j_{\rm cor}^3 = -4\int\frac{d^4p}{(2\pi)^3}\theta(p_F-|\boldsymbol{p}|)\frac{\partial}{\partial p_z}\{\Delta V(p)\delta[V_0(p)]\} \quad (69)$$

$$= -4\int\frac{d^4p}{(2\pi)^3}\delta(p_F-|\boldsymbol{p}|)\frac{p_z}{p_F}\{\Delta V(p)\delta[V_0(p)]\} \quad (70)$$

$$= -\frac{1}{2\pi^3}\int d\Omega_p p_F^2\cos\theta_{\boldsymbol{p}}\frac{\Delta V(p)}{\tilde{\Pi}_0(p)}. \quad (71)$$

Then we separate it to several parts as

$$j_{\rm cor}^3 = j_{\rm cor}^3(H) + j_{\rm cor}^3(F) \quad (72)$$

with

$$j_{\rm cor}^3(H) = -\frac{1}{2\pi^3}\int d\Omega_p p_F^2\cos\theta_{\boldsymbol{p}}\frac{-\Pi^\mu\Delta U_\mu^H + M^*\Delta U_s^H}{\tilde{\Pi}_0(p)} \quad (73)$$

$$j_{\rm cor}^3(F) = -\frac{1}{2\pi^3}\int d\Omega_p p_F^2\cos\theta_{\boldsymbol{p}}\frac{-\Pi^\mu\Delta U_\mu^F + M^*\Delta U_s^F}{\tilde{\Pi}_0(p)}, \quad (74)$$

where $\Delta U_{\mu(s)}^H$ and $\Delta U_{\mu(s)}^F$ are shown to be contributions of $\Delta U_{\mu(s)}$ from Hartree and Fock parts of self-energies, respectively.

It is not so easy to solve the above equation exactly in the RHF case though it is possible in the RH case. However, we have known that the actual momentum dependence is very small, at least below the Fermi momentum. Then we can suppose that a perturbative way is possible with the respect to the momentum dependence.

Before explaining the actual method, first, we explain the relativistic Hartree (RH) case. There the self-energies are momentum independent, and the valence current becomes

$$j_{\rm var}^3 = \frac{1}{\Omega}\frac{p_F}{E_F^*}. \quad (75)$$

In this case the core-polarization current is calculated in the following way:

$$j_{cor}^3 = -\frac{1}{2\pi^3}\int d\Omega_p p_F^2\cos\theta_{\boldsymbol{p}}\frac{\Pi_z\Delta U_z^H}{E_F^*}$$

$$= -\frac{4}{3\pi^2}\frac{p_F^3}{E_F^*}\Delta U_z^H. \quad (76)$$

In the RH calculation

$$\Delta U_z^H = \frac{g_v^2}{m_v^2}\int\frac{d^3p}{(2\pi)^3}n(\boldsymbol{p})\frac{p_z}{E_p^*}$$

$$= \frac{g_v^2}{m_v^2}j^3. \quad (77)$$

Substituting Eq. (77) into Eq. (76), we can get

$$j^3 = \frac{1}{\Omega}\frac{p_F}{E_F^*} - \left\{\frac{g_v^2}{m_v^2}\frac{4}{3\pi^2}\frac{p_F^3}{E_F^*}\right\}j^3$$

$$= \frac{1}{\Omega}\frac{p_F}{E_F^*}\left(1+\frac{g_v^2}{m_v^2}\rho_B\frac{1}{E_F^*}\right)^{-1}. \quad (78)$$

This renormalized current given in the above equation is exactly same as that in the RPA [8]. In the RH case the Fermi energy is obtained as

$$\varepsilon_F = E_F^* + \frac{g_v^2}{m_v^2}\rho_B \quad (79)$$

and then

$$j^3 = \frac{1}{\Omega}\frac{p_F}{\varepsilon_F}. \quad (80)$$

In the low-density region below about the saturation, $\varepsilon_F \approx M$, so that we can see that the core polarization plays a role to cancel the effect of the effective mass in the valence current.

In the RHF case the contribution from the Hartree part is large, and we cannot use the perturbative way. Because momentum dependence of the self-energies is not so large, however, the difference between $\tilde{\Pi}_0$ and Π_0 is small and then the Hartree part of the total current $j^3(H)$ can be approximately gotten with the following equation.

$$\begin{aligned} j^3(H) &\approx \int \frac{d^3p}{(2\pi)^3} n(\boldsymbol{p}) \frac{\Pi_z(p)}{\tilde{\Pi}_0(p)} \\ &\approx \int \frac{d^3p}{(2\pi)^3} n(\boldsymbol{p}) \frac{\Pi_z(p)}{\Pi_0(p)} \\ &\approx j^3_{\text{var}}(H) - \Delta U_z \int \frac{d^3p}{(2\pi)^3} n_0(\boldsymbol{p}) \left[\frac{\partial}{\partial U_z} \frac{\Pi_z(p)}{\Pi_0(p)} \right]_{\Delta U_z=0} \\ &\approx j^3_{\text{var}}(H) - \Delta U_z \int \frac{d^3p}{(2\pi)^3} n_0(\boldsymbol{p}) \frac{1}{\Pi_0(p)} \left[1 - \frac{\Pi_z^2(p)}{\Pi_0^2(p)} \right], \end{aligned} \tag{81}$$

where $j^3_{\text{var}}(H)$ is the valence part of the Hartree current as

$$j^3_{\text{var}}(H) \approx \frac{1}{\Omega} \frac{\Pi_z(p_F)}{\tilde{\Pi}_0(p_F)}. \tag{82}$$

The space component of the vector self-energy, which is caused only by the Fock contribution in the saturation matter, is very small, and then ΔU_z is thought to be contributed from the Hartree parts as

$$\Delta U_z \approx \Delta U_z^H = \frac{g_v^2}{m_v^2} j^3(H). \tag{83}$$

Then the Hartree contribution of the core-polarization current is approximately given as

$$j^3_{\text{cor}}(H) = j^3(H) - j^3_{\text{var}}(H) \approx \frac{-V_C^H(\text{IS})}{1 + V_C^H(\text{IS})} j^3_{\text{var}}(H) \tag{84}$$

with

$$V_C^H(\text{IS}) = \frac{g_v^2}{m_v^2} \int \frac{d^3p}{(2\pi)^3} n_0(\boldsymbol{p}) \frac{1}{\tilde{\Pi}_0} \left[1 - \frac{\Pi_z^2(p)}{\Pi_0^2(p)} \right]. \tag{85}$$

As for the Fock part, the momentum dependence of self-energies are not so large, and its contribution is not so big in the total current. Instead of getting it exactly, thus, we can use the perturbative way for the Fock part of the core-polarization current. Along this line the variation of the self-energies are taken to be only the contribution from the valence nucleon as

$$\Delta U_s^F(p) \approx \frac{3f_\pi^2}{2} \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \Delta n(\boldsymbol{k}) \frac{M^*(k)}{\tilde{\Pi}_0(k)} \Delta_\pi(p-k), \tag{86}$$

$$\Delta U_\mu^F(p) \approx -\frac{3f_\pi^2}{2} \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \Delta n(\boldsymbol{k}) \frac{\Pi_\mu(k)}{\tilde{\Pi}_0(k)} \Delta_\pi(p-k). \tag{87}$$

Then we substitute them into the Eq. (74) and obtain

$$\begin{aligned} j^3_{\text{cor}}(F) = \frac{3f_\pi^2}{4\pi^3} \tau_3 \int d\Omega_p p_F^2 \cos\theta_{\boldsymbol{p}} \\ \times \frac{\Pi_0^2(p_F) - \Pi_v^2(p_F) + M^{*2}(p_F)}{\tilde{\Pi}_0^2(p_F)} \Delta(0; \boldsymbol{p}-\boldsymbol{a}). \end{aligned} \tag{88}$$

Next we consider the isovector current. In the similar way we can calculate the isovector current by taking the variation part of the momentum distribution as

$$\Delta n(\boldsymbol{p}, \tau) = \frac{(2\pi)^3}{\Omega} \delta(\boldsymbol{p}-\boldsymbol{a}). \tag{89}$$

In this work the nuclear system is taken to be the isospin symmetric saturated matter plus valence nucleon. Thus, the isovector properties can be treated in the perturbative way. In this case, the Dirac fields of the valence nucleon isoscalar one and those of the nucleon in Fermi sea have a very small isovector part coming from the valence nucleon.

As for the Hartree part we substitute the following $V_C^H(\text{IV})$ instead of $V_C^H(\text{IS})$ into Eq. (84):

$$V_C^H(\text{IV}) = \frac{C_v^{IV}}{M^2} \int \frac{d^3p}{(2\pi)^3} n_0(\boldsymbol{p}) \frac{1}{\tilde{\Pi}_0} \left[1 - \frac{\Pi_z^2(p)}{\Pi_0^2(p)} \right]. \tag{90}$$

As for the Fock part, furthermore, the variations of the self-energies become

$$\Delta U_s^F(p) \approx \tau_3 \frac{f_\pi^2}{2} \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \Delta n(\boldsymbol{k}) \frac{M^*(k)}{\tilde{\Pi}_0(k)} \Delta_\pi(p-k), \tag{91}$$

$$\Delta U_\mu^F(p) \approx -\tau_3 \frac{f_\pi^2}{2} \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \Delta n(\boldsymbol{k}) \frac{\Pi_\mu(k)}{\tilde{\Pi}_0(k)} \Delta_\pi(p-k). \tag{92}$$

Then the Fock contribution of the isovector core polarization current is obtained as

$$\begin{aligned} j^3_{\text{cor}}(F) = \frac{f_\pi^2}{4\pi^3} \tau_3 \int d\Omega_p p_F^2 \cos\theta_{\boldsymbol{p}} \\ \times \frac{\Pi_0^2(p_F) - \Pi_v^2(p_F) + M^{*2}(p_F)}{\tilde{\Pi}_0^2(p_F)} \Delta(0; \boldsymbol{p}-\boldsymbol{a}). \end{aligned} \tag{93}$$

In the next section we calculate the actual current using the above formulation. Because we cannot solve the RPA with the ring and exchange diagrams, we need to introduce some approximations explained in this subsection. First, to examine the approximations, we compare the isoscalar total current, j^3_{tot} to the normal current, $j^3_0 = p_F/\varepsilon_F$, because the two current must agree with each other in exact calculations [9]. Next, we discuss the isovector current.

III. RESULTS

In this section we show results calculated with the above formulation. In this calculation we use the parameters (PF1) [15] for the σ and ω exchanges to reproduce the saturation properties of nuclear matter: the binding energy BE = 16 MeV, the incompressibility $K = 200$ MeV, and the effective mass $M^*/M = 0.7$ at the saturation density $\rho_0 = 0.17\ \text{fm}^{-3}$. For comparison we give results with momentum-independent self-energies obtained by the parameter set PM1 [16] that gives the same saturation properties. As for the isovector nucleon-nucleon interaction, C_v^{IV}, we take the value of PM1. These values are written in Table I.

In Fig. 1 we draw the momentum dependence of the scalar self-energy $U_s(p)$ and that of the time component of the

TABLE I. Parameter sets in this article. In all cases have used $m_\pi = 138$ MeV, $m_\sigma = 550$ MeV, $m_\omega = 783$ MeV, and $C_\sigma = 0$.

	g_σ	g_ω	B_σ	A_σ	f_π	C_v^{IV}
PF1	9.699	9.880	27.61	6.134	1.008	20.32
PM1	9.408	9.993	23.52	5.651	0.0	20.32

vector self-energy $U_0(p)$. It can be seen that the variation of the momentum-dependent self-energies is only 2.5% at most below Fermi level.

In Fig. 2 we show the density-dependence of the Dirac self-energies U_s and U_0 on the Fermi surface (a) and the Landau mass (b) with the parameter sets, PF1 and PM1. Though two results of U_s and U_0 almost agree with each other, we can see a rather large difference in the Landau mass: the value at $\rho_B = \rho_0$ is $M_L^*/M = 0.85$ in PF1, which is consistent with the value expected by the analysis of ISGQR as shown previously [15]. On the contrary, the momentum-independent calculation (PM1) gives $M_L^*/M = 0.74$, which overestimates the excitation energy of ISGQR.

Hence it is shown that the very small momentum dependence in the nucleon self-energies enhances the Fermi velocity about 15%, and gives a significant difference in the Landau mass. Furthermore, we can also see an interesting behavior of M_L^* in PF1, namely that its value agrees with the bare mass at $\rho_B \approx 0.5\rho_0$ and becomes larger with the decrease of the density. Effects of small Dirac effective mass are largely canceled at low density by the momentum dependence caused by the one-pion exchange.

In Fig. 3 we show the density dependence of the isoscalar current density. In the upper panel [Fig. 3(a)] the solid and chain-dotted lines indicate the total current and the valence current, respectively. For comparison the current for the RH approximation are also drawn there with the dashed line. From that we can know that the Fock contribution suppresses the RH current, and the core polarization further suppresses it.

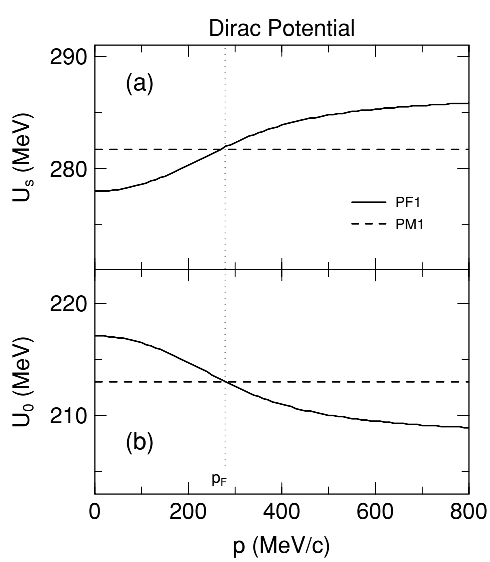

FIG. 1. Momentum dependence of the scalar (a) and vector (b) self-energies. The solid and dashed lines indicate the results with PF1 and PM1, respectively. The dotted line denotes the position of the Fermi momentum at $\rho_B = \rho_0$.

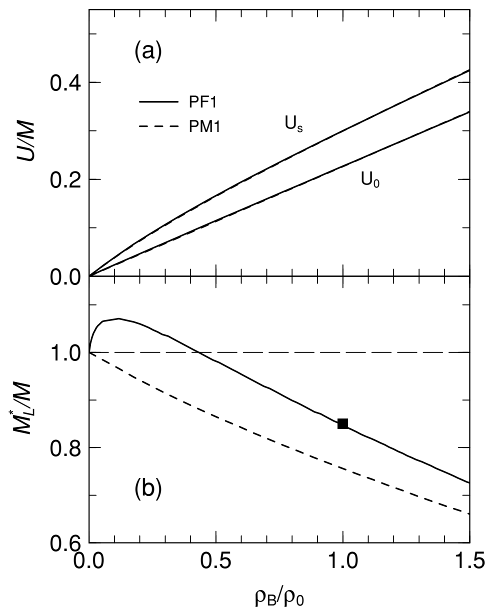

FIG. 2. Density dependence of the Dirac self-energies U_s and U_0 on the Fermi surface (a) and the Landau mass (b). The solid and dashed lines indicate the results for PF1 and PM1, respectively, and the full square in (b) denotes the value expected empirically from ISGQR.

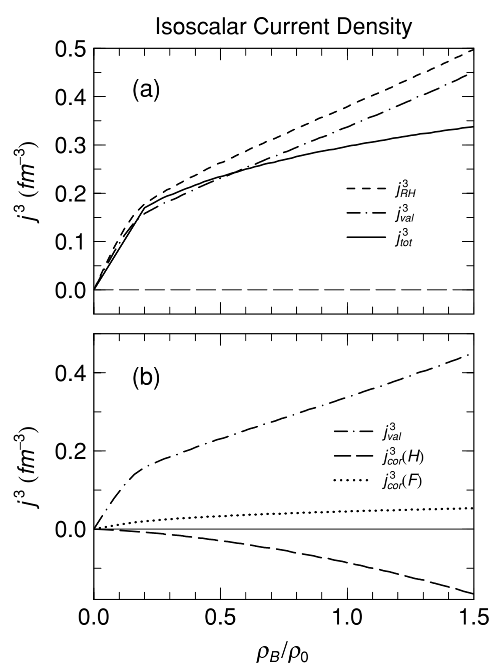

FIG. 3. Density dependence of the isoscalar nuclear current (a) and parts of the core-polarization current (b). The dot-dotted, dashed, and solid lines represent the RH current, the valence current, and the total current, respectively. The long-dashed and dotted lines denote contributions in the core polarization current from the Hartree part and Fock part, respectively.

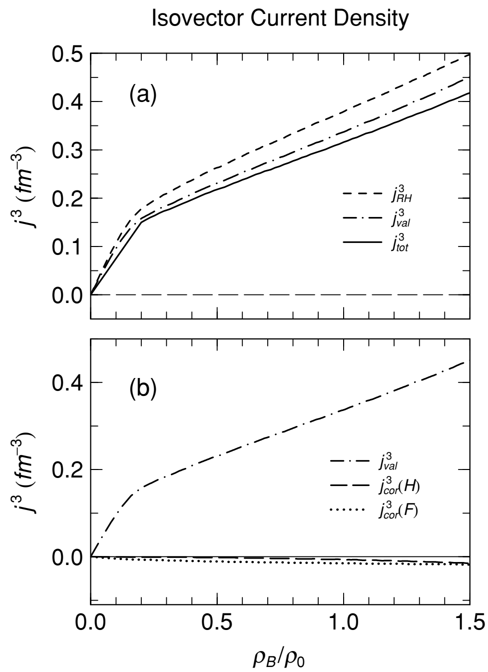

FIG. 4. Density dependence of the isovector nuclear current (a) and parts of the core-polarization current (b).

In the lower panel [Fig. 3(b)] we show the contribution from the core polarization. The long dashed and dotted lines indicate the core-polarization current contributed from the Hartree and Fock parts, respectively. The Hartree contribution reduces the current, whereas the Fock contribution enhances it.

In Fig. 4 we show the isovector currents; the meaning of each line is the same as that in Fig. 3. We notice that the Fock contribution suppresses also the isovector current. However, the core polarization hardly affects the total isovector current.

The most direct observable for the nuclear static current must be the magnetic moment; the nuclear medium effect is examined as the discrepancy from the Schmidt value. Thus we should compare our results to the normal current, which is a current with no medium effect and given as

$$j_0^3 = v_F = \frac{p_F}{\varepsilon_F}. \tag{94}$$

Here we define the following quantity as

$$\Delta j_r^3 = \frac{j_{\rm tot}^3 - j_0^3}{j_o^3}. \tag{95}$$

In Fig. 5 we show the density dependence of Δj_r^3. The Ward-Takahashi identity requires that the total isoscalar current agrees with the normal current, $\Delta j_r^3 = 0$, at all densities [9]. Indeed the total isoscalar current almost agrees with the normal current, within a few percentages of error. Particularly the disagreement, Δj_r^3, is less than 0.02 below the saturation density, $\rho_B/\rho_0 \leqslant 1$.

Here we give comments on the relation between the core-polarization calculation and our approach. It was shown in Ref. [18] that the core-polarization contribution becomes equivalent to that of the ring diagram only if the Lorentz

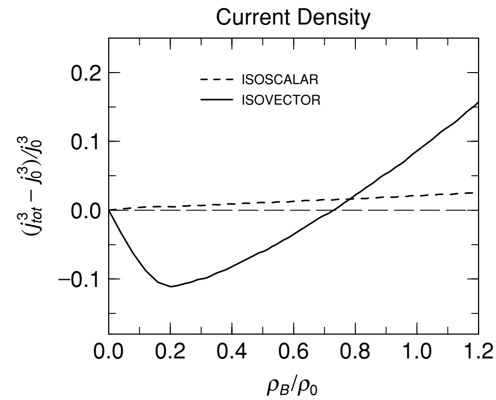

FIG. 5. The density dependence of the difference between the normal current and the total current in our model, normalized by the normal current. The solid and dashed lines represent the isoscalar and isovector currents, respectively.

covariance and the first law of the thermodynamics are satisfied. Although the usual RHF approach does not satisfy this criterion [21], Weber *et al.* have given the energy-momentum tensor satisfying the Lorentz covariance and the thermodynamical consistency in a semiclassical way [12]. Our current given in this article is completely equivalent to theirs. However, the formalism of Weber *et al.* has not been derived with the field theoretical way. In addition, we use some approximations to derive our expression for the core-polarization contribution. Thus we cannot perfectly certify our approach in the analytical way. However, we already know that the isoscalar current in full calculations must agree with the normal current. The discrepancy in our approach is only about 2% at the saturation density and less in lower density as shown above. Therefore we can confirm that our approach contain no serious problem at lease in numerical results.

The total isovector current is 10% less than the normal current in the density region around $\rho_B = \rho_0/4$ (Fig. 5). This result is consistent with the experimental fact that the isovector magnetic moment is 10% less than the Schmidt value; here we should note that the magnetic moment indicates the medium effect in surface region. However, our calculation is performed for the infinite matter and does not include the contribution from the spin current. Therefore the present result does not directly correspond to the experimental observables. Nevertheless our results suggest the importance of the momentum dependence of the self-energies in studies of the magnetic moment.

Here we should give a further comment. Bentz *et al.* have shown in Ref. [22] that the Landau mass is reduced by the one-pion exchange, which is opposite to ours. Their result is consistent with the nonrelativistic analysis on the magnetic moment with the exchange current [23], in which it was shown that the exchange current enhances the convection part and reduces the spin part, and as a combined effect the isovector magnetic moment is reduced. Bentz *et al.* [22] used pseudoscalar (PS) coupling, and the sign of U_μ^{MD} was taken to be opposite to ours. The full HF calculation with PS coupling makes too large a contribution to the Dirac self-energies [4], whereas Bentz *et al.* calculated the Fock term with a perturbative method. Thus a calculation with PV coupling must be more reliable than that with PS coupling.

In addition the large discrepancy between the PS and PV coupling methods comes from relativistic effects in the one-pion exchange. Because the pion mass is smaller than the nucleon Fermi energy, relativistic effects must be larger in the one-pion exchange. Miyazawa treated the one-pion exchange in the nonrelativistic way [23]. Thus it is not strange that our results qualitatively disagree with Miyazawa's.

IV. CONCLUDING REMARKS

In this article we have studied the static current in the system with one valence nucleon on the Fermi surface of the saturated nuclear matter. We employ the RHF framework with the momentum-dependent self-energies. It was shown that this current is determined by the Landau mass M_L^* independently of the effective mass M^*.

As shown in the Ref. [15], the very small momentum dependence in the nucleon self-energies enhances the Fermi velocity, even if this momentum dependence is negligibly small for the nuclear equation of state. In the present calculation the Fermi velocity is enhanced 15% by the momentum dependence caused by the one-pion exchange. Moreover, We succeed in reducing the isovector current in low-density region; the value of the current is almost equivalent to the current without effective mass at $\rho_B \approx 0.5\rho_0$, and 10% suppressed around $\rho_B \approx 0.25\rho_0$. The latter result is consistent with the result that the observed isovector magnetic moment is 10% smaller than the Schmidt value. However, the quantitative conclusion has not been so clear.

As seen in this article the momentum-dependent parts, which are nonlocal in the finite nuclei, are very effective in observables related with Fermi velocity even if these parts are small. In future we need to discuss effects of the nonlocal parts of Dirac fields to study nuclear structure and reactions.

The typical value of effective mass is empirically known as $M_N^*/M_N = 0.55 - 0.7$ [3,5,24–27]. If we use other parameter sets that yield smaller effective masses than ours, the momentum-dependent part created by the one-pion exchange do not have a sufficient effect to explain the Fermi velocity expected from experimental analysis. Exchange forces of other mesons, σ, ω, η, and δ, also contribute to suppressing the Fermi velocity if we choose the PV coupling for π and η nucleon coupling.

In this work we focus on the momentum dependence of the Dirac self-energies, and use the most simple expressions of the mean-field part in the RMF approach. Then we do not take into account some important aspects of this approach. Particularly it has been pointed out that the chiral symmetry plays a very important role in constructing models in the RMF theory [2, 28]. When comparing calculational results with experimental ones, we need to improve our model including these effects. In addition we ignore antinucleon degrees of freedom [20,29]. They are matters of future work.

In this work we calculate and discuss only the convection current, but not spin current. The spin current includes the contribution from terms proportional to q^μ, but it is not easy to get the vertex correction at finite momentum transfer. Without this work, however, we cannot make a final conclusion about the nuclear electromagnetic current in the RMF approach. In the future we would also like to try it and to discuss effects of this current in high-momentum transfer phenomena such as quasielastic electron scattering [30].

APPENDIX: ONE-BODY CURRENT OPERATOR

To satisfy the current conservation, the current operator Γ^μ and the density-dependent vertex correction Λ^μ must have the following relations:

$$q_\mu S(p+q)\Gamma^\mu(p+q,p)S(p) = S(p+q) - S(p), \quad \text{(A1)}$$

$$q_\mu \Lambda^\mu(p+q,p) = -\Sigma_F(p+q) + \Sigma_F(p). \quad \text{(A2)}$$

Within the one-boson exchange force, the Fock part of the self-energy is generally written in the following way:

$$\begin{aligned}\Sigma_F(p) = {}& i\sum_a C_a \int \frac{d^4k}{(2\pi)^4}\gamma^a S(k)\gamma_a \Delta^{(a)}(p-k) \\ & + i\sum_b \tilde{C}_a \int \frac{d^4k}{(2\pi)^4}[(\not{p}-\not{k}),\gamma^b] \\ & \times S(k)[\gamma_b,(\not{p}-\not{k})]\Delta^{(b)}(p-k), \end{aligned} \quad \text{(A3)}$$

where $\gamma_{a(b)}$ is the γ matrix with the suffix $a(b)$ indicating the scalar, pseudoscalar, vector, axial-vector, and tensor, and $\Delta^{(a)}$ is the propagator of meson with the quantum number indicated with the suffix a.

Substituting Eq. (A3) into Eq. (A2), we get

$$\begin{aligned}q_\mu \Lambda^\mu(p+q,p) = {}& -\Sigma_F(p+q) + \Sigma_F(p) \\ = {}& -i\sum_a C_a \int \frac{d^4k}{(2\pi)^4}\gamma^a S(k)\gamma_a \\ & \times [\Delta^{(a)}(p-k+q) - \Delta^{(a)}(p-k)] \\ & - i\sum_b \tilde{C}_b \int \frac{d^4k}{(2\pi)^4}\{[(\not{p}-\not{k}+\not{q}),\gamma^b] \\ & \times S(k)[\gamma_b,(\not{p}-\not{k}+\not{q})]\Delta^{(b)}(p-k+q) \\ & - [(\not{p}-\not{k}),\gamma^b]S(k) \\ & \times [\gamma_b,(\not{p}-\not{k})]\Delta^{(b)}(p-k)\}. \end{aligned} \quad \text{(A4)}$$

When we omit the vertex form factor of the meson-nucleon coupling, the meson propagator is given as

$$\Delta_a(k) = \frac{1}{k^2 - m_a^2}, \quad \text{(A5)}$$

and

$$\Delta^{(a)}(k+q) - \Delta^{(a)}(k) = -\Delta^{(a)}(k+q)q(2k+q)\Delta^{(a)}(k). \quad \text{(A6)}$$

Then the above equation can be rewritten as the following expression:

$$q^{\mu}\Lambda_{\mu} = -i\sum_a C_a \int \frac{d^4k}{(2\pi)^4}\gamma^a S(k)\gamma_a[\Delta^{(a)}(p-k+q) \times (2p-2k+q)_{\mu}\Delta^{(a)}(p-k)] - i\sum_b \tilde{C}_b \times \int \frac{d^4k}{(2\pi)^4}\{[\gamma^b, (\not{p}-\not{k}+\not{q})]S(k)[\gamma_b, (\not{p}-\not{k})] \times \Delta^{(b)}(p-k+q)(2p-2k+q)_{\mu}\Delta^{(b)}(p-k) - [(\not{p}-\not{k}+\not{q}), \gamma^b]S(k)[\gamma_b, \gamma_{\mu}]\Delta^{(b)}(p-k+q) - [\gamma_{\mu}, \gamma^b]S(k)[\gamma_b, (\not{p}-\not{k})]\Delta^{(b)}(p-k)\}. \quad (A7)$$

From the above equation we can get the following vertex correction, which we call $\Lambda^{(1)}$.

$$\Lambda^{(1)}_{\mu} = i\sum_a C_a \int \frac{d^4k}{(2\pi)^4}\gamma^a S(k)\gamma_a \times \{\Delta^{(a)}(p-k+q)(2p-2k+q)_{\mu}\Delta^{(a)}(p-k)\} + i\sum_b C_b \int \frac{d^4k}{(2\pi)^4}\{[(\not{p}-\not{k}+\not{q}), \gamma^b]S(k)[\gamma_b, (\not{p}-\not{k})] \times \Delta^{(b)}(p-k+q)(2p-2k+q)_{\mu}\Delta^{(b)}(p-k) - [(\not{p}-\not{k}+\not{q}), \gamma^b]S(k)[\gamma_b, \gamma_{\mu}]\Delta^{(b)}(p-k+q) - [\gamma^b, \gamma_{\mu}]S(k)[\gamma_b, (\not{p}-\not{k})]\Delta^{(b)}(p-k)\}. \quad (A8)$$

However, Eq. (A3) can be rewritten as

$$\Sigma_F(p) = i\sum_a C_a \int \frac{d^4k}{(2\pi)^4}\gamma^a S(p-k)\gamma_a\Delta^{(a)}(k) + i\sum_b \tilde{C}_b \int \frac{d^4k}{(2\pi)^4}[\not{k}, \gamma^b]S(p-k)[\gamma_b, \not{k}]\Delta^{(b)}(k). \quad (A9)$$

Equation (A9) is obtained only by a variable transformation $(k \to p-k)$ from Eq. (A3).

Substituting Eq. (A9) into Eq. (A2), we obtain the following:

$$q_{\mu}\Lambda^{\mu}(p+q, p) = -\Sigma(p+q) + \Sigma(p) = i\sum_a C_a \int \frac{d^4k}{(2\pi)^4}\gamma^a\{S(p-k+q) - S(p-k)\}\gamma_a\Delta^{(b)}(k) + i\sum_b \tilde{C}_b \int \frac{d^4k}{(2\pi)^4}[\not{k}, \gamma^b][S(p-k+q) - S(p-k)][\gamma_b, \not{k}]\Delta^{(b)}(k). \quad (A10)$$

Using Eq. (A1) the above equation reduces to the following expression:

$$q^{\mu}\Lambda_{\mu} = iq^{\mu}\sum_a C_a \int \frac{d^4k}{(2\pi)^4}\gamma^a S(k+q)\Gamma_{\mu}S(k)\gamma_a\Delta^{(a)}(p-k) + iq^{\mu}\sum_b \tilde{C}_b \int \frac{d^4k}{(2\pi)^4}\{\gamma^b(\not{p}-\not{k})S(k+q)\Gamma_{\mu}S(k) \times (\not{p}-\not{k})\gamma_b\Delta^{(b)}(p-k). \quad (A11)$$

From the above equation we can get another expression of the vertex correction, which we call $\Lambda^{(2)}$, as

$$\Lambda^{(2)}_{\mu} = i\sum_a C_a \int \frac{d^4k}{(2\pi)^4}\gamma^a S(k+q)\Gamma_{\mu}S(k)\gamma_a\Delta^{(a)}(p-k) + i\sum_b \tilde{C}_b \int \frac{d^4k}{(2\pi)^4}\{[(\not{p}-\not{k}), \gamma^b]S(k+q) \times \Gamma_{\mu}S(k)[\gamma_b, (\not{p}-\not{k})]\Delta^{(b)}(p-k)\}. \quad (A12)$$

The two expressions of the vertex correction, $\Lambda^{(1)}$ and $\Lambda^{(2)}$, are not the same because there are ambiguous terms proprotional to $\sigma_{\mu\nu}q^{\nu}$, $q_{\mu} - (p\cdot q)p_{\mu}/p^2$, and so on.

Now we take the limit of $q \to 0$.

$$\lim_{q\to 0}\Lambda^{(1)}_{\mu} = -i\sum_a C_a \int \frac{d^4k}{(2\pi)^4}\gamma^a S(k)\gamma_a\frac{\partial}{\partial p^{\mu}}\Delta^{(a)}(p-k) + i\sum_b \tilde{C}_a \int \frac{d^4k}{(2\pi)^4}[\gamma^{\nu}, \gamma^b]S(k)[\gamma_b, \gamma^{\kappa}] \times \frac{\partial}{\partial p^{\mu}}D^{(b)}_{\nu\kappa}(p-k)$$
$$= -i\sum_a C_a\frac{\partial}{\partial p^{\mu}} \int \frac{d^4k}{(2\pi)^4}\gamma^a S(k)\gamma_a\Delta^{(a)}(p-k) + i\sum_b \tilde{C}_a\frac{\partial}{\partial p^{\mu}} \int \frac{d^4k}{(2\pi)^4}[\gamma^{\nu}, \gamma^b]S(k) \times [\gamma_b, \gamma^{\kappa}]D^{(b)}_{\nu\kappa}(p-k)$$
$$= -\frac{\partial}{\partial p^{\mu}}\Sigma_F(p) \quad (A13)$$

with

$$D^{(a)}_{\mu\nu}(q) = \Delta^{(a)}(q)q_{\mu}q_n u. \quad (A14)$$

Similarly

$$\lim_{q\to 0}\Lambda^{(2)}_{\mu} = -i\sum_a C_a \int \frac{d^4k}{(2\pi)^4}\gamma^a\frac{\partial}{\partial p^{\mu}}S(p-k)\gamma_a\Delta^{(a)}(k) + i\sum_b \tilde{C}_a \int \frac{d^4k}{(2\pi)^4}[\gamma^{\nu}, \gamma^b]\frac{\partial}{\partial p^{\mu}}S(p-k) \times [\gamma_b, \gamma^{\kappa}]D^{(b)}_{\nu\kappa}(p-k)$$
$$= -i\sum_a C_a\frac{\partial}{\partial p^{\mu}} \int \frac{d^4k}{(2\pi)^4}\gamma^a S(k)\gamma_a\Delta^{(a)}(p-k) + i\sum_b \tilde{C}_a\frac{\partial}{\partial p^{\mu}} \int \frac{d^4k}{(2\pi)^4}[\gamma^{\nu}, \gamma^b]S(k)$$

$$\times [\gamma_b, \gamma^\kappa] D^{(b)}_{\nu\kappa}(p-k) = -\frac{\partial}{\partial p^\mu}\Sigma_F(p) \quad \text{(A15)}$$

In the zero-momentum limit $q \to 0$, thus, the two expressions of the vertex correction, $\Lambda^{(1)}$ and $\Lambda^{(2)}$, agree with each other.

Here we give a comment. If we substitute the full propagator, $S(k)$, into Eq. (A8), we have to solve the vacuum polarization, which is also very difficult. In the usual RMF approach we usually calculate observables contributed from the nucleon in the Fermi sea by using only the density-dependent part, $S_D(k)$, instead of the full propagator, $S(k)$. In the case of the RH, where the self-energies are momentum independent, the following equation is satisfied,

$$i\int \frac{d^4k}{(2\pi)^4}[S_F(k+q)\not{q} S_D(k)\Delta(p-k) + S_D(k)\not{q} S_F(k-q)\Delta(p-k+q)] = i\int \frac{d^4k}{(2\pi)^4} S_D(k)[\Delta(p-k+q) - \Delta(p-k)]. \quad \text{(A16)}$$

This equation implies us that we can describe particle-hole excitations with the usual approximation that the density-dependent part of the nucleon propagator S_D (9) is used in $\Lambda^{(1)}$ instead of the full propagator. The actual momentum dependence is very small, and the Eq. (A16) is approximately satisfied in the RHF case, too.

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