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Global properties of the Skyrme-force-induced nuclear symmetry energy

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A novel concept for the nuclear symmetry energy (NSE) is corroborated by large scale calculations. The paper firmly demonstrates that within the local density approximation, the value of the NSE coefficient, $a_{\rm sym}(A)$, depends on two basic ingredients: the mean-level spacing, $\varepsilon(A)$, and the effective strength of the isovector mean-potential, $\kappa(A)$. Surprisingly, our results reveal that these two basic ingredients of $a_{\rm sym}$ are almost equal after rescaling them linearly by the isoscalar and the isovector effective masses, respectively. This result points towards a hidden and hitherto unresolved fundamental property of the effective nuclear interaction. In addition, our analysis yields naturally the ratio of the surface-to-volume contributions to $a_{\rm sym}$ with a value of ∼1.6, consistent with hydrodynamical estimates for the static dipole polarizability as well as the neutron-skin. Although the present study is restricted to energy density functionals obtained from Skyrme forces the method is general and can easily be applied to more general local energy density functionals and nonnuclear bifermionic systems.

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The knowledge of the nuclear equation of state (EOS) for neutron-rich systems is of fundamental importance for nuclear physics and nuclear astrophysics. The stability of neutron-rich nuclei, the r-process nucleosynthesis, the structure of neutron star, and the simulations of supernovae-collapse depend sensitively on the EOS and, in particular, on the nuclear symmetry energy (NSE). In all standard text books of nuclear physics, the NSE strength is divided into a kinetic and potential term, see, e.g., Ref. [1]. For modern formulations of the nuclear many body problem based on Hohenberg-Kohn-Sham local energy density functional (LEDF) formalism this division becomes artificial and obscures the physical origin of the NSE. Indeed, a direct microscopic calculations show that the kinetic energy, $\Delta E_{\rm kin}(N-Z) = E_{\rm kin}(N-Z) - E_{\rm kin}(N-Z=0)$, calculated along various isobaric $A = $ const chains does by no means correlate with $N-Z$ as depicted in Fig. 1, see also Ref. [2]. In addition, the division of the NSE into volume and surface contribution (and eventually higher order) are difficult to reconcile microscopically with a kinetic energy dependence, whereas they are naturally embedded in our approach.

Leaving stereotype expressions behind, we define two basic ingredients for the NSE strength $a_{\rm sym}(A)$: the mean-level spacing, $\varepsilon(A)$, and the effective strength of the isovector mean-potential, $\kappa(A)$, see Ref. [3]. Since $\varepsilon(A)$ is a well defined quantity with well known asymptotic limit, having volume (A^{-1}) and surface ($A^{-4/3}$) parts, our large scale study of the A-dependence offers a natural series of cross-checks for this novel interpretation of the origin of the NSE and, in turn, provides the basic constraints for $\kappa(A)$ and the LEDF. In particular, our analysis reveals the striking property $\frac{m_0^*}{m}\varepsilon(A) \approx \frac{m_1^*}{m}\kappa(A)$, that holds for velocity-dependent interactions having isoscalar m_0^* and isovector m_1^* effective masses, respectively. Since this result emerges from entirely different Skyrme forces (SF) with respect to parameters and constraints, it indicates an underlying fundamental property of the effective NN forces, that remains to be explained.

The main objective of this work is to study the symmetry energy, notwithstanding, that a proper understanding is crucial in order to reach a consensus on the existing variety of SF parameterizations, or to constrain the coupling constants of a more general LEDF. Our results point toward a deeper relation between the average level spacing and the strength of the mean isovector potential which has not been addressed hitherto. We focus our investigation on the nuclear case and study Skyrme forces only because it belongs to the best studied functionals in nuclear physics. However, the concept can easily be applied to any EDF, see e.g., Ref. [4], and is not restricted to nuclear phenomena alone but can be employed to any bifermionic system. In particular, it may be of relevance for the fast developing field of ultracold Fermi gases.

In asymmetric infinite nuclear matter (INM), in the vicinity of the saturation density, ρ_0, the EOS (the energy-density per particle) is conveniently parametrized using the following Taylor expansion:

$$\frac{\mathcal{E}_I(\rho)}{A} \approx -a_V + \frac{K_\infty}{18\rho_0^2}(\rho-\rho_0)^2 + \ldots + \left[a_{\rm sym} + \frac{p}{\rho_0^2} \times (\rho-\rho_0) + \frac{\Delta K_\infty}{18\rho_0^2}(\rho-\rho_0)^2 + \ldots\right] I^2 + \ldots, \tag{1}$$

where $I \equiv |N-Z|/A$. The isoscalar INM saturation density ρ_0, and the values of the volume binding energy a_V, the incompressibility parameter K_∞, and the asymmetry energy $a_{\rm sym}$ serve as primary constraints for microscopic nuclear models. For modern Skyrme force parameterizations which are subject of the present work $\rho_0 \approx 0.16$ fm^{-3}, $a_V \approx -15.9 \pm 0.2$ MeV, $a_{\rm sym} \approx 32 \pm 2$ MeV, and $K_\infty \approx 225 \pm 25$ MeV. Higher-order curvature corrections to the NSE, p, ΔK_∞, are rather poorly constrained.

Integrating out the $\boldsymbol{r}$-dependence from the energy-density leads then to the semiempirical mass formula (LD) which is

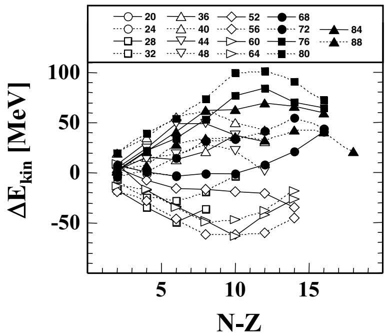

FIG. 1. The kinetic energy $\Delta E_{\mathrm{kin}}(N-Z) = E_{\mathrm{kin}}(N-Z) - E_{\mathrm{kin}}(N-Z=0)$ versus $N-Z$ calculated along isotopic chains ranging from $A = 20$ till 88, see legend. The calculations have been done using the Skyrme-Hartree-Fock approach. Note, that $\Delta E_{\mathrm{kin}}(N-Z)$ does not show the parabolic behavior $\Delta E_{\mathrm{kin}}(N-Z) \sim (N-Z)^2$ characteristic for the NSE . This clearly proves that for microscopic calculations that reproduce the binding energy, the kinetic energy is not the proper source for the symmetry energy. This is also in contradiction to the Fermi gas model.

conventionally written as

$$\frac{E}{A} = -a_V + \frac{a_S}{A^{1/3}} + \left[a_{\mathrm{sym}}^{(V)} - \frac{a_{\mathrm{sym}}^{(S)}}{A^{1/3}} + \ldots\right]\left(I^2 + \lambda\frac{I}{A}\right) + \ldots, \tag{2}$$

where a_S and $a_{\mathrm{sym}}^{(S)}$ are coefficients defining contributions from the surface energy and the surface part of the symmetry energy, respectively. There is at present no consensus concerning the magnitude, λ, as well as origin of the term linear in $\sim I$, which is often called the Wigner energy. Another controversy exists concerning the surface contribution to the NSE. The values of the surface-to-volume ratio $r_{S/V} = a_{\mathrm{sym}}^{(S)}/a_{\mathrm{sym}}^{(V)}$ quoted in the literature vary strongly. For example, Danielewicz [5] estimates it to be $2.0 \leqslant r_{S/V} \leqslant 2.8$, the mass formula of Ref. [6] yields $r_{S/V} \approx 1.6$ while the hydrodynamical-type models that include properly polarization of the isovector density predict $r_{S/V} \approx 2$ [7].

In our previous letter [3] we have demonstrated that the Skyrme-Hartree-Fock (SHF) symmetry energy behaves rather unexpectedly according to the formula:

$$E_{\mathrm{sym}}^{(\mathrm{SHF})} = \tfrac{1}{2}\varepsilon(A,T_z)T^2 + \tfrac{1}{2}\kappa(A,T_z)T(T+1), \tag{3}$$

where $\varepsilon(A,T_z) \approx \varepsilon(A)$ and $\kappa(A,T_z) \approx \kappa(A)$ denote the mean-level spacing at the Fermi energy in isosymmetric nuclei and effective strength of the isovector mean-potential, respectively. More precisely, κ is related to the isovector part of the SF induced LEDF (S-LEDF) $\mathcal{H}(\boldsymbol{r}) = \sum_{t=0,1}\mathcal{H}_t(\boldsymbol{r})$:

$$\begin{aligned}\mathcal{H}_t &= C_t^{\rho}\rho_t^2 + C_t^{\Delta\rho}\rho_t\Delta\rho_t + C_t^{\tau}\rho_t\tau_t \\ &\quad + C_t^{J}\overleftrightarrow{J}_t^2 + C_t^{\nabla J}\rho_t\nabla\cdot\boldsymbol{J}_t.\end{aligned} \tag{4}$$

Definitions of all local densities and currents $\rho, \tau, \overleftrightarrow{J}$ as well as the explicit expressions for coupling constants C_t can be found in numerous references and we follow the notation used in Ref. [8]. Due to the isoscalar-density dependence of the SF, the coupling constants $C_t^{\rho}[\rho_0]$ of the S-LEDF are functionals of ρ_0, giving rise to the isoscalar rearrangement mean-potential $U_0 = \sum_{t=0,1}\frac{\partial C_t^{\rho}}{\partial\rho_0}\rho_t^2$. Since our procedure of extracting ε and κ involves setting the $C_1 \equiv 0$, see Ref. [3], part of the U_0 related to the C_1^{ρ} is formally treated as being related to the isovector degrees of freedom. Note that this separation is consistent with the way the symmetry energy constraint is superimposed on the SF.

We determine the global mass dependence of the SHF values of $\varepsilon(A)$ and $\kappa(A)$ by means of a systematic calculation covering all even-even nuclei having $20 \leqslant A \leqslant 128$ from $N = Z$ to almost the neutron drip line. Coulomb and pairing effects are disregarded, i.e., the emphasis is on the strong interaction acting in the particle-hole channel. The calculations are performed for a set of different SF parametrizations as the SkP [9], SkXc [10], Sly4 [11], SkO [12], SkM* [13], and SIII [14], using the SHF code HFODD of Dobaczewski *et al.* [8].

The procedure used to extract $\varepsilon(A,T_z)$ and $\kappa(A,T_z)$ follows exactly the one outlined in Ref. [3]. First, we set all the isovector coupling constants $C_1 \equiv 0$ in the S-LEDF (4) and extract $\varepsilon(A,T_z)$ by comparing calculated excitation energy $\Delta E_{\mathrm{SHF}}^{(t=0)}(A,T_z) \equiv E_{\mathrm{SHF}}^{(t=0)}(A,T_z) - E_{\mathrm{SHF}}^{(t=0)}(A,0)$ to

$$\Delta E_{\mathrm{SHF}}^{(t=0)}(A,T_z) = \tfrac{1}{2}\varepsilon(A,T_z)T^2. \tag{5}$$

In the next step, we compute the total SHF binding energy $E_{\mathrm{SHF}}(A,T_z)$ and compare

$$\Delta E_{\mathrm{SHF}}(A,T_z) - \Delta E_{\mathrm{SHF}}^{(t=0)}(A,T_z) = \tfrac{1}{2}\kappa(A,T_z)T(T+1), \tag{6}$$

in order to determine $\kappa(A,T_z)$.

For each A and small T_z, the values of $\varepsilon(A,T_z)$ oscillate quite rapidly. However, they clearly tend to stabilize for $T_z \geqslant 8$ where $\varepsilon(A,T_z) \approx \varepsilon(A)$. The values of $\kappa(A,T_z)$ appear to stabilize faster and $\kappa(A,T_z) \approx \kappa(A)$ essentially already for $T_z \geqslant 4$. It should be mentioned that in the case of the SkO parametrization the formula (6) does work only approximately. For this force we observe a clear enhancement in the linear term, $\sim T$. This effect is, however, much weaker than the analogous effect found recently within relativistic mean field [4], where it restores the $E_{\mathrm{sym}} \sim T(T+1)$ dependence of the total NSE.

For further quantitative analysis of the mass dependence of the NSE we use the mean values of $\bar{\varepsilon}(A)$ and $\bar{\kappa}(A)$. These averages over T_z at fixed A are calculated using the following restricted set of nuclei: $T_z \geqslant 4$ for $A = 20$; $T_z \geqslant 6$ for $A = 24$; and $T_z \geqslant 8$ for $A \geqslant 28$. By using a restricted set of nuclei we smooth out both $\bar{\varepsilon}(A)$ and $\bar{\kappa}(A)$ curves in order to diminish the possible influence of shell structure.

The $\bar{\varepsilon}$ and $\bar{\kappa}$ curves are presented in Fig. 2. The figure shows several universal features which appear to be independent of the type of the SF parametrization: (*i*) strong dependence of $\bar{\varepsilon}(A)$ on kinematics (shell effects); (*ii*) almost no dependence of $\bar{\kappa}(A)$ on kinematics; (*iii*) clear surface ($\sim\frac{1}{A^{4/3}}$) dependence reducing the dominant volume term ($\sim\frac{1}{A}$) in both $\bar{\varepsilon}(A)$ and $\bar{\kappa}(A)$.

Indeed, the values of $\bar{\varepsilon}(A)$ show characteristic kinks close to double-(semi)magic A-numbers. These kinks are magnified when all the calculated nuclei are used (no smoothing) to compute $\bar{\varepsilon}(A)$, but without affecting qualitatively the overall profile of the curve. On the other hand, $\bar{\kappa}(A)$ is almost perfectly smooth with barely visible traces of shell structure. It confirms

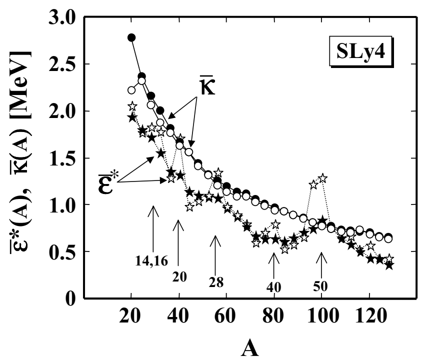

FIG. 2. The isoscalar effective mass scaled values of $\bar{\varepsilon}^*(A) \equiv \frac{m_0^*}{m}\bar{\varepsilon}(A)$ (stars) and $\bar{\kappa}(A)$ (circles) calculated using the SHF method with SLy4 parametrization. Open symbols denote $\bar{\varepsilon}^*(A)$ and $\bar{\kappa}(A)$ averaged over all the calculated nuclei. Filled symbols mark smoothed values of $\bar{\varepsilon}^*(A)$ and $\bar{\kappa}(A)$ calculated using a restricted set of data. Vertical arrows indicate major shell gaps. Note the strong influence of shell structure on $\bar{\varepsilon}^*(A)$ and the smooth behavior of $\bar{\kappa}(A)$.

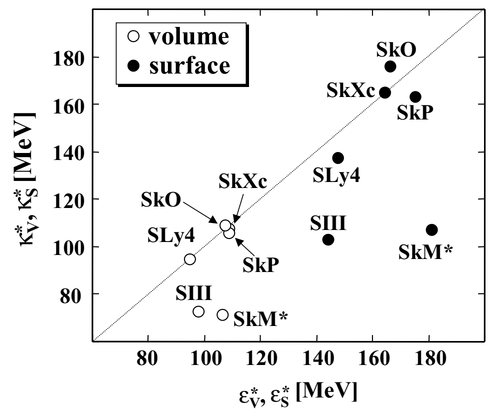

FIG. 3. The correlation between the effective mass scaled volume ε_V^* and κ_V^* (open dots) and the surface ε_S^* and κ_S^* expansion coefficients. Note, that except for the SIII and SkM* interactions, which are not constrained properly, the expansion coefficients are equal, see also Table I.

our earlier conclusion [3] that the gross features of the Skyrme isovector mean potential can be almost perfectly quantified by a smooth curve parametrized by a small number of global parameters.

In the analysis of a leptodermous expansion of $\bar{\varepsilon}(A)$ and $\bar{\kappa}(A)$ we consider volume (V) and surface (S) terms assuming that: $\bar{\varepsilon}(A) = \varepsilon_V/A - \varepsilon_S/A^{4/3}$ and $\bar{\kappa}(A) = \kappa_V/A - \kappa_S/A^{4/3}$. The values of the isoscalar-effective-mass-scaled expansion coefficients $\varepsilon_V^*, \varepsilon_S^*$ as well as the values of the isovector-effective-mass-scaled expansion coefficients κ_V^*, κ_S^* are collected in Table I. First of all, let us observe that the calculated value of $\varepsilon_V^* \approx 100$ MeV corresponds to the pure Fermi gas estimate $\varepsilon_{\rm FG}$. This result can be understood based on the analytical expression for the Skyrme force NSE coefficient in the limit of symmetric INM, $a_{\rm sym}^{(\infty)}$, provided that the standard textbook formula is rewritten in the following way:

$$a_{\rm sym}^{(\infty)} = \frac{1}{8}\varepsilon_{\rm FG}\left(\frac{m}{m_0^*}\right) + \left[\left(\frac{3\pi^2}{2}\right)^{2/3} C_1^\tau \rho^{5/3} + C_1^\rho \rho\right] \equiv \frac{1}{8}[\varepsilon_{(\infty)} + \kappa_{(\infty)}], \quad (7)$$

where C_1^τ and C_1^ρ define the isovector part of the S-LEDF, see Eq. (4). Equation (7) clearly separates the contributions from the isovector and isoscalar part and relates the latter to the single particle energies in INM, $\varepsilon_p = \frac{p^2}{2m} + \Sigma(\boldsymbol{p}, \varepsilon_p) = \frac{p^2}{2m_0^*}$, with a self-energy term, $\Sigma(\boldsymbol{p}, \varepsilon_p)$, that describes the interaction with the nuclear medium incorporated into the isoscalar effective mass. Hence, Eq. (7) further supports our interpretation of the NSE strength.

The most striking result of our analysis is the **near-equality** of $\bar{\varepsilon}^* \approx \bar{\kappa}^*$ occurring for all modern parametrizations, see Table I and Fig. 3. Indeed, $\bar{\varepsilon}^*$ differs from $\bar{\kappa}^*$ only for old parametrizations like the SIII and SkM*. This result confirms the rather loose claims often appearing in textbooks that "*the kinetic energy* [$\varepsilon_{\rm FG}$] *and the isovector mean-potential contribute to the* $a_{\rm sym}$ *in a similar way*" is indeed correct but only after disregarding non-local effects. To our knowledge, it has never been discussed why these apparently independent quantities should be similar.

The most important and challenging question is whether the relation $\bar{\varepsilon}^* \approx \bar{\kappa}^*$ is purely accidental, reflects a certain symmetry of the SHF solution or pertains to a fundamental property of the effective *NN* interactions. It is probably not

TABLE I. The table includes: the isoscalar, m_0^*/m, and the isovector, m_1^*/m, effective masses; the volume, $\varepsilon_V^*(\kappa_V^*)$, the surface $\varepsilon_S^*(\kappa_S^*)$, and the ratios $r_\varepsilon = \varepsilon_S^*/\varepsilon_V^*(r_\kappa = \kappa_S^*/\kappa_V^*)$ of the expansion coefficients scaled by the isoscalar $\bar{\varepsilon}^*(A)$ and the isovector $\bar{\kappa}^*(A) \equiv \frac{m_1^*}{m}\bar{\kappa}(A)$ effective masses, respectively; the ratios of volume $r_V^* = \varepsilon_V^*/\kappa_V^*$ and surface $r_S^* = \varepsilon_S^*/\kappa_S^*$ expansion coefficients and the INM estimate $r_{(\infty)}^* = \varepsilon_{(\infty)}^*/\kappa_{(\infty)}^*$; the INM estimate $a_{\rm sym}^{(\infty)}$ and the calculated values $a_{\rm sym}^{(V)}$, $a_{\rm sym}^{(S)}$ and $r_{S/V} = a_{\rm sym}^{(S)}/a_{\rm sym}^{(V)}$ of the symmetry energy coefficient as defined in Eq. (2). The values of $\varepsilon_{V(S)}^*$, $\kappa_{V(S)}^*$, $a_{\rm sym}^{(\infty)}$, $a_{\rm sym}^{(V)}$, and $a_{\rm sym}^{(S)}$ are given in MeV.

	m_0^*/m	m_1^*/m	ε_V^*	ε_S^*	r_ε	κ_V^*	κ_S^*	r_κ	r_V^*	r_S^*	$r_{(\infty)}^*$	$a_{\rm sym}^{(\infty)}$	$a_{\rm sym}^{(V)}$	$a_{\rm sym}^{(S)}$	$r_{S/V}$
SLy4	0.695	0.800	94.5	147.5	1.56	94.7	137.5	1.45	1.00	1.07	1.07	32.0	31.8	48.0	1.51
SkXc	1.006	0.752	108.6	164.3	1.51	107.6	165.2	1.54	1.01	0.99	0.88	30.1	31.4	47.9	1.53
SkP	1.000	0.741	108.8	175.1	1.61	106.0	163.1	1.54	1.03	1.07	0.95	30.0	31.5	49.4	1.57
SkO	0.896	0.852	107.2	166.2	1.55	110.6	176.1	1.59	0.97	0.94	0.79	32.0	31.2	49.0	1.57
SkM*	0.789	0.653	106.3	180.9	1.70	71.4	107.3	1.50	1.49	1.69	1.37	30.0	30.5	49.2	1.61
SIII	0.763	0.655	97.5	143.8	1.47	75.2	103.2	1.37	1.30	1.39	1.34	28.2	30.3	43.3	1.43

surprising that it does not hold explicitely in the INM limit, since the analytical formula (7) does not contain any scaling of the isovector effective mass, m_1^*. Indeed, Eq. (7) relates to the translationally invariant medium while m_1^* defines the enhancement of the energy weighted sum rule for the translational symmetry violating (finite nucleus) dipole mode.

On the other hand, it is rather hard to believe that by complete accident four modern parametrizations would behave synchronously in the isovector channel, correlating $\bar{\varepsilon}^*$ and $\bar{\kappa}^*$, particularly that these forces have been fitted to the data in a truely different manner. Looking into the key isovector coupling constants C_1^ρ and C_1^τ one immediately observes that they vary in an entirely irregular fashion from force to force. The C_1^ρ increases by a factor of five when going from SLy4 to SkP or SkXc while for the momentum term C_1^τ even the sign appears to be not established, see Fig. 4. In spite of that C_1^ρ and C_1^τ appear to be linearly correlated as shown in Fig. 4. This result and the near-equality $\bar{\varepsilon}^* \approx \bar{\kappa}^*$ indicates that isovector part of the LEDF is neither well understood nor well constrained. To find an explanation of, in particular, the later result will be therefore a challenge for further studies especially that, in our opinion, such an explanation cannot be done at the level of effective theory operating exclusively with coupling constants fitted to the data. Indeed, violation of $\bar{\varepsilon}^* \approx \bar{\kappa}^*$ correlation by older parametrizations like SkM* or SIII suggests that this near equality reflects the nature of the effective interaction in nuclei rather than some hidden symmetry of the SHF solution.

Our interpretation of $\varepsilon(A)$ can be further tested by evaluating $r_\varepsilon = \varepsilon_S/\varepsilon_V$ using the semi-classical approximation [15]. The appropriate formula which takes into account the diffuseness of the potential, see Ref. [16]:

$$\varepsilon(A) \sim g(\epsilon_F)^{-1} \sim \frac{1}{A}\left(1 - \frac{\pi}{4k_F}\frac{S_M}{V_M} + \dots\right), \tag{8}$$

where $g(\epsilon_F)$ is the level density at the Fermi energy, $k_F \approx 1.36$ fm^{-1} while V_M and S_M denote volume and surface matter-distribution, respectively. Assuming spherical geometry $\frac{S_M}{V_M} \approx \frac{3}{r_o A^{1/3}}$ and adopting for $r_o \approx 1.14$ fm, i.e. the value consistent with the standard Skyrme force saturation density $\rho_0 \approx 0.16$ fm^{-1}, one obtains $r_\varepsilon \approx \frac{3\pi}{4k_F r_o} \approx 1.52$ which is indeed very close to the calculated ratios, see Table I.

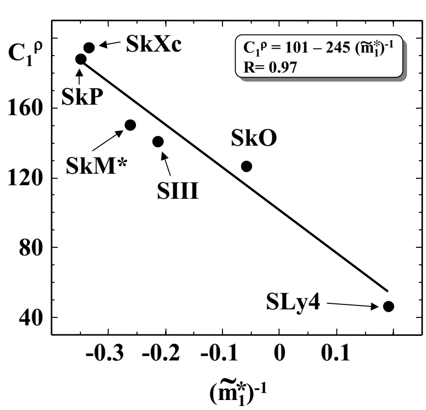

FIG. 4. The values of C_1^ρ versus the kinematic effective mass defined as $(\tilde{m}_1^*)^{-1} \equiv \frac{2m}{\hbar^2} C_1^\tau \rho$ for the SF used in this work. Solid line is a result of a linear fit which is shown in the legend.

The SHF models yield $r_{S/V} \sim 1.6$ in accordance with the LD ratio [6]. The static dipole polarizability (SDP) α_D [$\sigma(\omega)$ denotes the photoabsorption cross-section],

$$\sigma_{-2} \equiv \int \frac{\sigma(\omega)}{\omega^2} d\omega \equiv 2\pi^2 \frac{e^2}{\hbar c}\alpha_D, \tag{9}$$

provides an independent cross-check of the ratio $r_{S/V}$. Indeed, using the so-called hydrodynamical model a simple estimate for α_D can be derived [7]:

$$\alpha_D \approx \alpha_D^{(\mathrm{M})}\left(1 + \frac{5}{3}\frac{r_{S/V}}{A^{1/3}} + \dots\right), \tag{10}$$

where $\alpha_D^{(\mathrm{M})} = \frac{1}{24}\frac{\langle r^2\rangle}{a_{\mathrm{sym}}^{(V)}}$ is the so-called Migdal SDP value [17] which is valid for large systems with negligible surface contribution. Using $\langle r^2\rangle = \frac{3}{5}R^2 A$ where $R = 1.2A^{1/3}$ fm and $a_{\mathrm{sym}}^{(V)} = 30$ MeV one obtains, in the Migdal's limit, $\sigma_{-2}^{(\mathrm{M})} A^{-5/3} \approx 1.73$ μb/MeV. Using this estimate and the experimental value of $\sigma_{-2}^{(\mathrm{exp})} A^{-5/3} \approx 2.7 \pm 0.2$ μb/MeV which is almost constant for $A \geqslant 100$ [18], one obtains $r_{S/V} \sim 1.65$, consistent with our estimates based on the SHF calculations.

The neutron skin thickness

$$\delta r^2 \equiv \int r^2 \rho_1 d\tau = \langle r_n^2\rangle - \langle r_p^2\rangle$$
$$= 12\frac{N-Z}{A}\alpha_D a_{\mathrm{sym}}^{(V)}\left[1 - \frac{r_{S/V}}{A^{1/3}}\right] \tag{11}$$

is yet another quantity which sensitively depends on isovector properties. Using an explicit expression for α_D one obtains the hydrodynamical model formula for the relative neutron skin thickness,

$$\delta r_R^2 \equiv \frac{\delta r^2}{\langle r^2\rangle} \approx \frac{N-Z}{A}\left\{1 + \frac{2}{3}\frac{r_{S/V}}{A^{1/3}} - \dots\right\}, \tag{12}$$

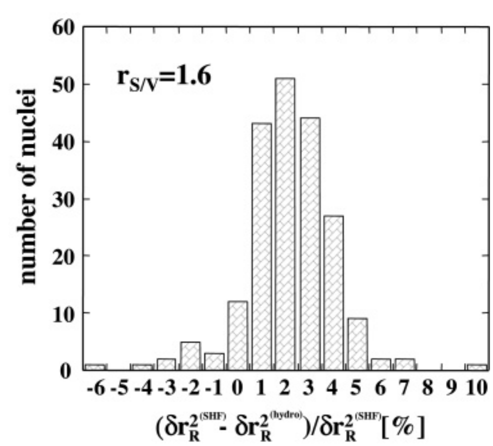

FIG. 5. The relative difference between the values of neutron skin thickness calculated using SHF model, $\delta r_R^{2\,(\mathrm{SHF})}$, and the values, $\delta r_R^{2\,(\mathrm{hydro})}$, estimated from the hydrodynamical expression (12) taken at $r_{S/V} = 1.6$. Note, that the relative difference (width) does not exceeds $\pm 2\%$ supporting the consitency of our approach.

which does not depend on a_{sym} but only on the ratio $r_{S/V}$. The estimate (12) taken for $r_{S/V} = 1.6$ provides again a very consistent results with our microscopic SHF calculations as shown in Fig. 5. Unfortunately, neutron rms radii are even today rather poorly known and can therefore not be used to constrain the LEDF or the EOS, see however Ref. [19]. It is worth noticing that the expansion (12) differs qualitatively from the semiempirical formulas used in the literature, see Refs. [20,21].

In summary, the global mass dependence of the NSE strength $a_{\text{sym}}(A)$ and its two basic ingredients related to the mean-level spacing, $\varepsilon(A)$, and to the mean-isovector potential, $\kappa(A)$ is studied in detail within the SHF theory. Our interpretation of the symmetry energy enables us to unambiguously establish the surface-to-volume ratio of $a_{\text{sym}}(A)$, $r_{S/V} \approx 1.6$ in agreement with the LD value of Ref. [6]. This ratio is consistent with simple hydrodynamical estimates for the SDP and neutron skin thickness. The most striking results of our calculations is the near-equality of $\bar{\varepsilon}^* \approx \bar{\kappa}^*$ revealing that *contribution to* a_{sym} *due to the mean-level spacing and due to the mean-isovector potential are similar* but only after disregarding nonlocal effects. This indicates a fundamental property of the nuclear mean field that requires further studies.

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