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Positive-energy one-particle levels in quadrupole-deformed Woods-Saxon potentials

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Positive-energy one-particle levels for neutrons in Y_{20} deformed Woods-Saxon potentials are examined using the eigenphase representation. Taking the example of $\Omega^\pi = 1/2^+$ levels, not only one-particle resonant levels but also all solutions in the eigenphase representation within a model space are studied. It is shown that a particular eigenphase solution among an infinite number of eigenphase solutions at a given energy plays a crucial role in producing low-lying one-particle resonance, whereas for the excitation energy lower than a few MeV the eigenphase sum is almost equal to the particular eigenphase when the sum is expressed by the value *mod* $n\pi$. Some one-particle resonant levels defined in terms of eigenphase, which have no correspondence to any bound one-particle levels, are found and discussed. It is shown that the relative probability of the $s_{1/2}$ component estimated using the probabilities inside the Woods-Saxon potentials is a decisive factor for obtaining one-particle resonant levels as a continuation of weakly bound $\Omega^\pi = 1/2^+$ levels.

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I. INTRODUCTION

Recent experimental data on the nuclei far from the β stability line provides a challenge to the conventional theory of nuclear structure. Because the Fermi level of drip-line nuclei in the mean-field approximation is very close to the continuum, both weakly bound and positive-energy one-particle levels play a crucial role in the many-body correlation of those nuclei. The traditional harmonic-oscillator model is no longer properly applicable to the study of drip-line nuclei. Thus, to understand the structure of deformed drip-line nuclei, positive-energy one-particle levels in deformed finite-well potentials must be first understood. One-particle resonant levels are well-defined in spherical potentials, whereas a systematic study of one-particle resonant levels in deformed potentials is scarcely found in the available literature.

In the study of positive-energy one-particle levels it is of essential importance to solve the Schrödinger equation in coordinate space with the appropriate asymptotic boundary conditions, as is done in the present work. If one makes an approximation of limiting the system to a finite box with a boundary condition at the surface of the box, the numerical results obtained depend inevitably on the size of the box. We study positive-energy one-particle levels for neutrons using the eigenphase representation, which is a natural extension of the bound one-particle levels in deformed potentials. For a given positive-energy, a given potential and a given quantum number Ω^π, where Ω expresses the component of one-particle angular-momentum along the symmetry axis, there are an infinite number of levels corresponding to different eigenphases. At low excitation energies most eigenphases are very close to $n\pi$, where n is an integer. Limiting the model space so as to exclude those uninteresting eigenphases ($\approx n\pi$), the dependence of remaining interesting eigenphases on energies is examined.

While weakly bound $s_{1/2}$ levels play a unique role in spherical drip-line nuclei, weakly bound $\Omega^\pi = 1/2^+$ levels, which always contain some amount of $s_{1/2}$ component, exhibit a unique and important role in axially symmetric quadrupole-deformed drip-line nuclei. Thus, as numerical examples in the present article, positive-energy one-particle levels for neutrons with $\Omega^\pi = 1/2^+$ are examined in detail, in which the $s_{1/2}$ component may play a unique role especially when the levels become unbound. It is noted that in spherical potentials there is no one-particle resonant level for the $\ell = 0$ channel, because of the lack of the centrifugal potential. One-particle resonance in spherical potentials is defined so that at the resonance energy the phase shift for a given angular momentum ℓ increases through $\frac{1}{2}\pi$ as the one-particle energy increases [1].

A part of the present problem is studied in Ref. [2], in which the possible continuation of weakly bound one-particle (Nilsson) levels to the positive-energy region as one-particle resonant levels is examined, depending on the values of the Ω^π quantum number. In Ref. [2] one-particle resonant levels in deformed potentials are defined so that one of the eigenphases increases through $\frac{1}{2}\pi$ as the one-particle energy increases. The definition has a meaning analogous to that of one-particle resonances in spherical potentials. We note that one-particle resonant levels defined in a similar way are used also in the study of proton emission in deformed nuclei outside the proton drip line [3]. Whereas in the formulation of Ref. [3] one-particle energy in the continuum is complex number, we work on real variables irrespective of the sign of one-particle energy, because it is favorable to have one-particle wave functions expressed in real numbers, especially when various observables are to be calculated.

In Sec. II our model is briefly described and the eigenphase representation is explained, whereas numerical results and discussions are given in Sec. III. Conclusions and perspectives are given in Sec. IV.

II. MODEL AND EIGENPHASE REPRESENTATION

The structure of one-particle levels in axially symmetric quadrupole-deformed Woods-Saxon potentials is studied, solving the Schrödinger equation in coordinate space with appropriate asymptotic boundary conditions. All positive-energy one-particle levels within a model space are calculated in the eigenphase representation, whereas one-particle bound levels in deformed potentials are obtained by solving the well-known eigenvalue problem.

Because the same model Hamiltonian as that employed in Ref. [2] is used, here we write only the minimum amount of necessary formulas. Except the radius we employ the standard parameters of the Woods-Saxon potentials used in β stable nuclei [2,4]. The radius parameter R is varied so as to vary the strength of our one-body potential. Writing the single-particle wave function as

$$\Psi_\Omega(\vec{r}) = \frac{1}{r}\sum_{\ell j} R_{\ell j\Omega}(r)\mathbf{Y}_{\ell j\Omega}(\hat{r}), \tag{1}$$

which satisfies

$$H\Psi_\Omega = \varepsilon_\Omega \Psi_\Omega, \tag{2}$$

where Ω expresses the component of one-particle angular momentum $\vec{j}$ along the symmetry axis, which is a good quantum number. The coupled equations for the radial wave functions are written as

$$\left\{\frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} + \frac{2m}{\hbar^2}[\varepsilon_\Omega - V(r) - V_{so}(r)]\right\} R_{\ell j\Omega}(r) = \frac{2m}{\hbar^2}\sum_{\ell' j'}\langle \mathbf{Y}_{\ell j\Omega}|V_{\rm coupl}|\mathbf{Y}_{\ell' j'\Omega}\rangle R_{\ell' j'\Omega}(r), \tag{3}$$

where

$$\begin{aligned}
V(r) &= V_{\rm WS} f(r)\\
f(r) &= \frac{1}{1+\exp[(r-R)/a]}\\
k(r) &= rV_{\rm WS}\frac{df(r)}{dr}\\
\langle \mathbf{Y}_{\ell j\Omega}|V_{\rm coupl}|\mathbf{Y}_{\ell' j'\Omega}\rangle &= -\beta k(r)\langle \mathbf{Y}_{\ell j\Omega}|Y_{20}(\hat{r})|\mathbf{Y}_{\ell' j'\Omega}\rangle\\
&= -\beta k(r)(-1)^{\Omega-1/2}\\
&\quad\times\sqrt{\frac{(2j+1)(2j'+1)}{20\pi}}\\
&\quad\times C(j,j',2;\Omega,-\Omega,0)\\
&\quad\times C\left(j,j',2;\frac{1}{2},-\frac{1}{2},0\right).
\end{aligned} \tag{4}$$

The eigenvalues $\varepsilon_\Omega(<0)$ of the coupled Eq. (3) for a given value of Ω, which is equivalent to Ω appearing in the asymptotic quantum numbers $[Nn_z\Lambda\Omega]$, are obtained by solving the equations in coordinate space for a given set of potential parameters, with both the condition, $R_{\ell j\Omega}(r)=0$ at $r=0$, and the asymptotic behavior of $R_{\ell j\Omega}(r)$ for $r\to\infty$, as

$$R_{\ell j\Omega} \propto r h_\ell(\alpha_b r), \tag{5}$$

where $h_\ell(-iz) \equiv j_\ell(z) + i n_\ell(z)$, in which j_ℓ and n_ℓ are spherical Bessel and Neumann functions, respectively, and

$$\alpha_b^2 \equiv -\frac{2m\varepsilon_\Omega}{\hbar^2}. \tag{6}$$

The normalization of bound one particle wave functions is obtained from the condition

$$\sum_{\ell,j}\int_0^\infty |R_{\ell j\Omega}(r)|^2 dr = 1. \tag{7}$$

For positive-energy ($\varepsilon_\Omega > 0$) one-particle levels we solve the coupled Eq. (3) in coordinate space for a given set of potential parameters, requiring

$$R_{\ell j\Omega}(r) = 0 \qquad \text{at} \quad r = 0 \tag{8}$$

and the asymptotic behavior of $R_{\ell j\Omega}(r)$ for $r\to\infty$ as

$$\begin{aligned}
R_{\ell j\Omega}(r) &\propto \cos(\delta_\Omega)\alpha_c r j_\ell(\alpha_c r) - \sin(\delta_\Omega)\alpha_c r n_\ell(\alpha_c r)\\
&\to \sin\left(\alpha_c r + \delta_\Omega - \ell\frac{\pi}{2}\right),
\end{aligned} \tag{9}$$

where

$$\alpha_c^2 \equiv \frac{2m}{\hbar^2}\varepsilon_\Omega. \tag{10}$$

Eigenphases δ_Ω as well as the structure of respective positive-energy one-particle wave functions, which are obtained in the present way, are totally independent of the upper limit of radial integration, $R_{\rm max}$, if $f(r)$ and $k(r)$ in (4) are already negligible at $r = R_{\rm max} \gg R$.

For a given potential and at a given energy ε_Ω we look for all eigenphases which are common to all open channels (ℓ, j) for a given Ω. The way of solving the coupled-channel Eq. (3) is taken from Ref. [5]. For a given potential and a given ε_Ω we have in principle an infinite number of eigenphase solutions δ_Ω. When we limit the model space (ℓ, j) for a given Ω to be finite, the number of eigenphase solutions becomes equal to that of wave function components with different (ℓ, j) values. The value of δ_Ω determines the relative amplitudes of different (ℓ, j) components. The total normalization of the positive-energy one-particle wave functions can be left arbitrary in the present work.

The asymptotic behavior Eq. (9) defined using only one of the eigenphases, is a natural choice so that the asymptotic radial wave-functions behave in the same way for all angular momentum components. This is analogous to the requirement of the asymptotic behavior of bound-state wave functions, Eq. (5). As one-particle resonant energies we look for the values of ε_Ω, for which one of the eigenphases δ_Ω increases through $\frac{1}{2}\pi$ as ε_Ω increases. When we find one-particle resonant levels thus defined, we calculate the width of the resonance using the formula

$$\Gamma \equiv \frac{2}{d\delta_\Omega/d\varepsilon_\Omega|_{\varepsilon_\Omega=\varepsilon_{\rm res}}}, \tag{11}$$

where the denominator is calculated at the resonance energy. When we are interested in the low-energy one-particle resonant levels, of which the energies are smoothly connected to eigen energies of weakly bound one-particle levels for slightly stronger potential strengths, inside the nuclear potential the relative amplitudes of (ℓ, j) components of the radial wave function are very similar to those of the weakly bound one-particle level. In contrast, the relative amplitudes of (ℓ, j) components, which belong to other eigenphases ($\neq\frac{1}{2}\pi$) for the same potential and the same one-particle energy, are different from those of weakly bound one-particle levels.

In Ref. [6] a more general aspect of the eigenphase representation, for example the role of eigenphase in scattering problems, is considered and the usefulness of the sum of eigenphases under certain circumstances is described. In some literature [6,7] one finds the statement that the sum of eigenphases

$$\Delta(\varepsilon_\Omega) \equiv \sum_n \delta_\Omega^{(n)}(\varepsilon_\Omega) \tag{12}$$

is the proper quantity to be used for the definition of resonances. However, in the present context the sum of eigenphases corresponds to a mixing of deformed one-particle wave functions, which are orthogonal to each other and have different asymptotic behaviors of radial wave functions. Such kind of mixing might be expected when some broad and/or overlapped resonances are present. As will be seen later, when the low-energy one-particle resonant levels with $\Omega^\pi = 1/2^+$ are looked for, it hardly makes a difference to use the eigenphase sum or a particular one of eigenphases. This is because eigenphases other than the particular one vary very slowly and smoothly as the energy increases from zero and, consequently, the sum of those other eigenphases is nearly equal to $n\pi$ where n expresses an integer. Nevertheless, as a principle problem it seems meaningful to define one-particle resonance by the condition that one of eigenphases increases through $\frac{1}{2}\pi$. Furthermore, in Ref. [7] it is stated that for multichannel resonances the sum of eigenphases increases by π as the energy passes through the resonance energy, showing a numerical example of the $\ell = 1$ and $\ell = 3$ channels. As shown in Sec.III, this statement is not applicable to the case of $\Omega^\pi = 1/2^+$ levels, of which the wave functions necessarily contain some amount of $s_{1/2}$ component.

III. NUMERICAL CALCULATIONS AND DISCUSSIONS

Because we have presented the numerical results of one-particle resonant levels [2] as well as weakly bound one-particle orbits [8] in the Y_{20}-deformed Woods-Saxon potentials taking examples of *sd*-shell nuclei, in the present article, for convenience, we show numerical results for the same mass-number region.

In Fig. 1 three $\Omega^\pi = 1/2^+$ Nilsson levels in the *sd*-shell are shown by thick solid curves with filled circles, which are the same as those in Fig. 1 of Ref. [2]. When the three $\Omega^\pi = 1/2^+$ Nilsson levels are well bound, they are assigned [9], from left to right, by the asymptotic quantum numbers $[Nn_z\Lambda\Omega]$ = [220 1/2], [211 1/2], and [200 1/2]. Among those three bound levels, for the deformation parameter $\beta = 0.5$ and the diffuseness $a = 0.67$ fm only the weakly bound [2001/2] level will continue to the region of $\varepsilon_\Omega > 0$ as a one-particle resonant levels, when the potential strength becomes slightly weaker. In the numerical calculations $s_{1/2}, d_{3/2}$, and $d_{5/2}$ channels are included, because the inclusion of higher (ℓ, j) channels does not change the following discussion in the range of $(R/r_0)^3$ values chosen in Fig. 1. Then, for a given potential and at a given energy ε_Ω (>0) we obtain three (and not an

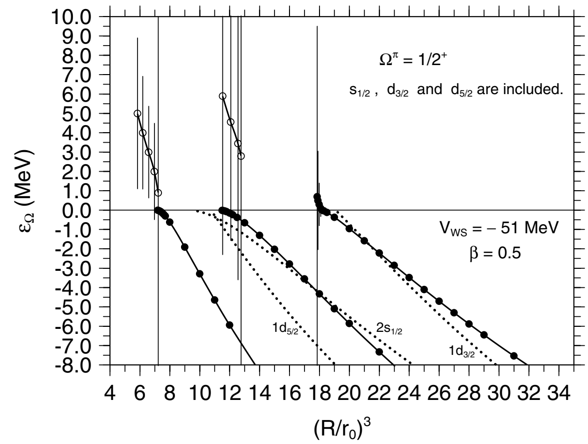

FIG. 1. Neutron one-particle resonant and bound levels with $\Omega^\pi = 1/2^+$ in axially symmetric quadrupole-deformed Woods-Saxon potentials as a function of the potential strength, $(R/r_0)^3$. The radius of the Woods-Saxon potential is expressed by R, whereas $r_0 = 1.27$ fm is used. The asymptotic quantum numbers $[Nn_z\Lambda\Omega]$ assigned traditionally to those three levels plotted by filled circles connected with thick solid curves are [220 1/2], [211 1/2], and [200 1/2], from left to right. The solid curves with open circles are one-particle resonant levels obtained from the definition using one of eigenphases, but no weakly bound Nilsson levels related to them are present. The width of one-particle resonant levels with $\varepsilon_\Omega > 0$ denoted by thin vertical lines is defined by Eq. (11). For reference, the energies of the bound Nilsson $1d_{5/2}, 2s_{1/2}$, and $1d_{3/2}$ levels in the spherical Woods-Saxon potential as a function of the potential strength, $(R/r_0)^3$, are denoted by dotted curves. See the text for details.

infinite number of) eigenphases for $\Omega^\pi = 1/2^+$ in our model space. The width of one-particle resonant levels defined by Eq. (11) is denoted by thin vertical lines. For reference, the eigenenergies of bound $1d_{5/2}, 2s_{1/2}$, and $1d_{3/2}$ orbits calculated for the spherical Woods-Saxon potential, $\beta = 0$, are also plotted by dotted curves as a function of the potential strength, $(R/r_0)^3$. The open circles connected by the solid curve express also one-particle resonant levels, following the definition that at respective ε_Ω values one of eigenphases increases through $\frac{1}{2}\pi$ as ε_Ω increases. Those one-particle resonant levels, all of which have very large width, indeed lack the weakly bound Nilsson levels that are smoothly connected to them.

To illustrate the difference between the one-particle resonant levels, which are denoted by the filled and open circles in Fig. 1, in Figs. 2(a) and 2(b) the calculated eigenphases expressed by the values *mod* $n\pi$ as a function of $\varepsilon_\Omega > 0$ are shown choosing the potential strength $(R/r_0)^3 = 18$ and 12, respectively. Because we have included three channels, $s_{1/2}$, $d_{3/2}$, and $d_{5/2}$, for the given potential and the given value of ε_Ω, we obtain three solutions of eigenphase, which are shown by the thick solid, dotted, and dashed curves. The filled and open circles in Figs. 2(a) and 2(b), respectively, denote the one-particle resonances that correspond to the the same symbols at respective ε_Ω and $(R/r_0)^3$ values in Fig. 1. For the potential with $(R/r_0)^3 = 18$ there is no weakly bound $\Omega^\pi = 1/2^+$ level, and the increase of the eigenphase expressed by the dotted curve at very small values of $\varepsilon_\Omega > 0$ in Fig. 2(a)

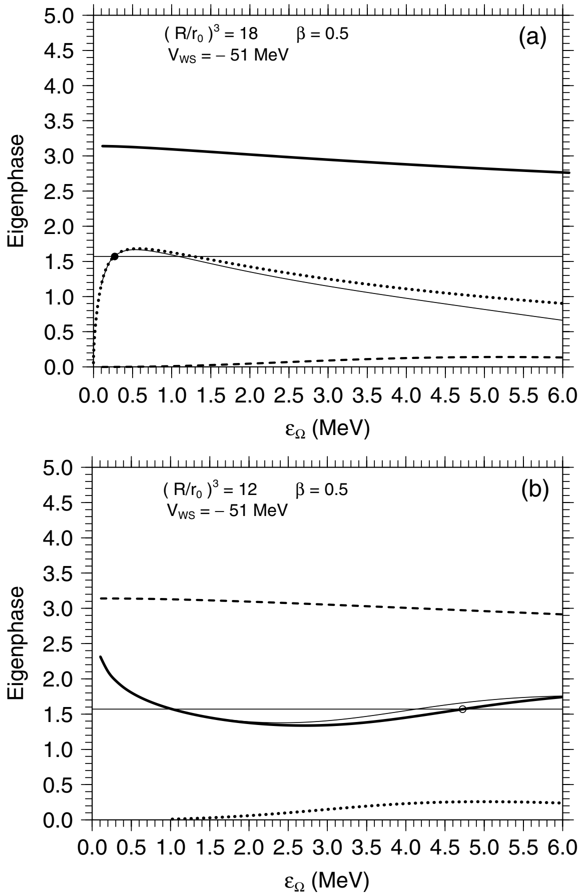

FIG. 2. (a) Eigenphases expressed by the values *mod nπ* as a function of one-particle energy, for the potential strength $(R/r_0)^3 = 18$, for which no weakly bound $\Omega^\pi = 1/2^+$ level is present. One-particle resonance is indicated by the filled circle. The value of $\pi/2$ is denoted by the thin horizontal line, whereas the sum of three eigenphases is expressed by the thin solid curve. (b) The same as in Fig. 2(a), but for the potential strength $(R/r_0)^3 = 12$, for which a weakly bound $\Omega^\pi = 1/2^+$ level is present. One-particle resonance is indicated by the open circle. The value of $\pi/2$ is denoted by the thin horizontal line, whereas the sum of three eigenphases is expressed by the thin solid curve.

is proportional to $\varepsilon_\Omega^{1/2}$, which is known as the ε_Ω dependence of the phase shift in the *s* channel for spherical potentials. In contrast, for the potential with $(R/r_0)^3 = 12$ a weakly bound $\Omega^\pi = 1/2^+$ level is present (see Fig. 1), and the solid curve in Fig. 2(b) starts from $\delta_\Omega = \pi$ at $\varepsilon_\Omega = 0$ and steeply decreases as ε_Ω increases from zero. From the behavior of the solid curve as a function of ε_Ω it is also seen that there is no weakly bound Nilsson level related to the "one-particle resonance" indicated by the open circle. Indeed, the value of $\delta_\Omega = \frac{1}{2}\pi$ is realized at a larger value of ε_Ω, and the energy of the resonance indicated by the open circle in Fig. 2(b) cannot continuously decrease to zero as the potential strength increases. Namely, as the value of $(R/r_0)^3$ becomes larger, there is a minimum value of the resonance energy at which the resonance width becomes infinitely large (see also Fig. 4). In both Figs. 2(a) and 2(b) the sum of three eigenphases, which is expressed by the value *mod nπ* where *n* is an integer, is plotted by

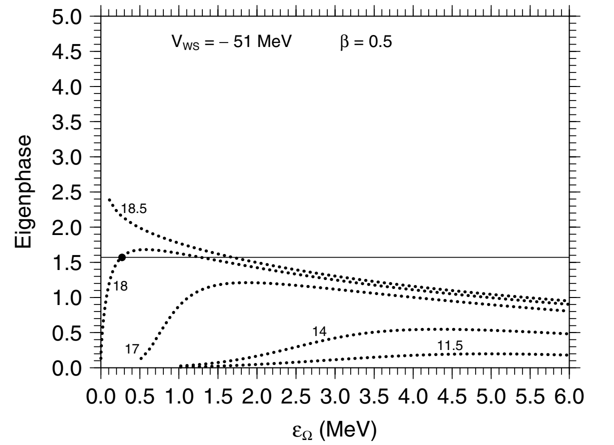

FIG. 3. One of the three eigenphases is plotted for various strengths of the potential, $(R/r_0)^3$, as a function of one-particle energy, ε_Ω. The values of $(R/r_0)^3$ are written close to respective dotted curves. A weakly bound level is present for $(R/r_0)^3 = 18.5$, but not for 18. The curve for $(R/r_0)^3 = 18$ is the same as the dotted curve in Fig. 2(a).

the thin curve. It is seen that for small values of $\varepsilon_\Omega > 0$ the sum of eigenphases almost coincides with a particular one of them that varies considerably, whereas other eigenphases vary very slowly and smoothly as ε_Ω starts to increase from 0. In the literature [7] one finds a statement that, for multichannel resonances, the eigenphase sum increases by π as the energy passes through the resonance energy. This statement is not applicable for the resonances with $\Omega^\pi = 1/2^+$, in which the $s_{1/2}$ component plays a crucial role.

In Figs. 3 and 4 two of the three eigenphases are plotted for various strengths of the potential as a function of ε_Ω. The curve denoted by the potential strength, $(R/r_0)^3 = 18$, in Fig. 3 is the same as the dotted curve in Fig. 2(a), whereas the one marked by $(R/r_0)^3 = 18$ in Fig. 4 is the same as the solid

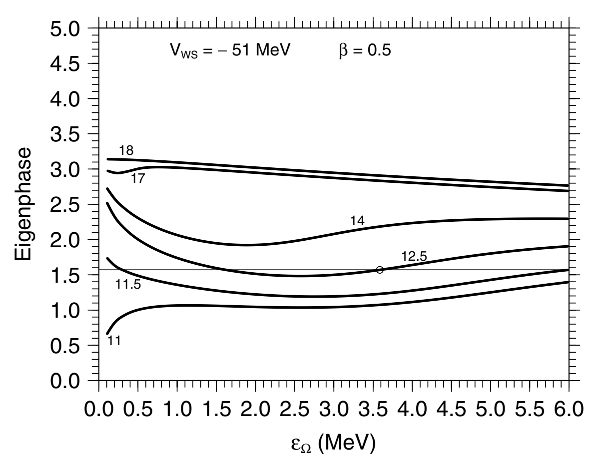

FIG. 4. One of the three eigenphases, which is different from that shown in Fig. 3, is plotted for various strengths of the potential, $(R/r_0)^3$, as a function of one-particle energy, ε_Ω. The values of $(R/r_0)^3$ are written close to respective solid curves. A weakly bound level is present for $(R/r_0)^3 = 11.5$, but not for 11. The curve for $(R/r_0)^3 = 18$ is the same as the solid curve in Fig. 2(a).

curve in Fig. 2(a). We find that for a given potential strength only one of the three eigenphases varies considerably as a function of ε_Ω and may eventually cross the line of $\frac{1}{2}\pi$. This behavior of eigenphases for $\Omega^\pi = 1/2^+$ remains the same when channels with higher (ℓ, j) values are included in the present region of $(R/r_0)^3$ values.

In Ref. [2] the low-energy one-particle resonance with $\Omega^\pi = 1/2^+$, which can be regarded as a continuation of some weakly bound Nilsson level for a slightly stronger strength of the potential, is studied. The aim of the study is to find out which one-particle levels in the continuum are important in the many-body correlation of the ground or low-lying excited states. In Ref. [2] it is also shown that for larger deformations and more diffuse potentials it is more difficult to obtain $\Omega^\pi = 1/2^+$ one-particle resonant levels. Thus, for a given diffuseness of the Woods-Saxon potential, we examine what is the element crucial for the presence of low-energy $\Omega^\pi = 1/2^+$ one-particle resonant levels. If Nilsson levels are sufficiently bound such as those well approximated by the modified oscillator model, it is known that the [200 1/2] level among the three $\Omega^\pi = 1/2^+$ Nilsson levels in the *sd* shell has the largest $s_{1/2}$ component. For example, see Tables 5–9 in Ref. [9]. Since one-particle resonant levels with $\Omega^\pi = 1/2^+$ would decay preferably through the $\ell = 0$ channel because of the absence of centrifugal barrier, one expects that a smaller $s_{1/2}$ component is preferred to realize one-particle resonant levels. In contrast, as shown in Fig. 1, the [200 1/2] level is the only $\Omega^\pi = 1/2^+$ level that can continue to the region of positive energy as a one-particle resonant level, if we use the parameters chosen in Fig. 1. Because the structure of the one-particle wave function inside the potential varies continuously when a given weakly bound Nilsson level becomes a one-particle resonant level for a slightly weaker strength of the potential, we estimate the relative probability of the $s_{1/2}$ component inside the potential defined by

$$P(s_{1/2}) = \frac{\langle s_{1/2}|V(r)|s_{1/2}\rangle}{\langle d_{5/2}|V(r)|d_{5/2}\rangle + \langle d_{3/2}|V(r)|d_{3/2}\rangle + \langle s_{1/2}|V(r)|s_{1/2}\rangle}. \tag{13}$$

In Fig. 5 the quantity in Eq. (13) is plotted for three $\Omega^\pi = 1/2^+$ Nilsson levels as a function of ε_Ω. It is seen that in the case of deeply bound levels the [200 1/2] level has indeed the largest $P(s_{1/2})$ value, which approaches the total $s_{1/2}$ probability tabulated in Table 5–9 of Ref. [9]. In contrast, the [200 1/2] level has the smallest $P(s_{1/2})$ value as $\varepsilon_\Omega(< 0)$ approaches 0. We note that the quantity defined in Eq. (13) is indeed continuous when the sign of ε_Ω changes from negative to positive in the case that one-particle resonant level can be present as in the [200 1/2] level. The maximum value of $P(s_{1/2})$ at $\varepsilon_\Omega \to 0$, for which the continuation as a one-particle resonant level can be found, depends on the value of diffuseness of the potential and is larger for smaller diffuseness. For the potential parameters used in Fig. 5 the [200 1/2] level is the only one that continues to the positive energy region as one-particle resonant level. Then, as ε_Ω increases from zero, the relative probability of the $s_{1/2}$ component inside the potential starts to steeply increase as shown in Fig. 5 and, at the same time, the width of the resonance soon becomes extremely large as seen in Fig. 1. Consequently, before ε_Ω becomes several hundred keV, the meaning of the resonance is essentially lost.

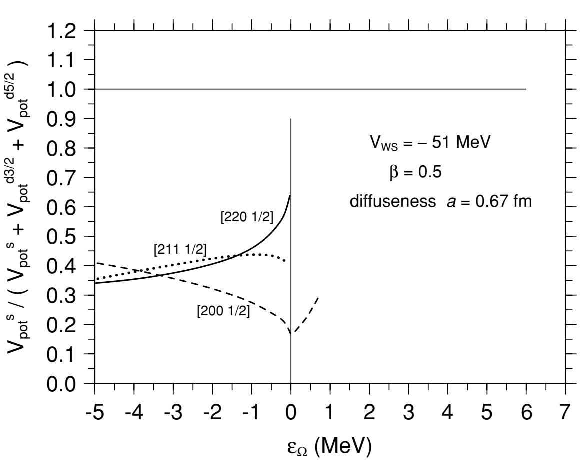

FIG. 5. The relative probability of the $s_{1/2}$ component inside the Woods-Saxon potential, (13), for the three $\Omega^\pi = 1/2^+$ Nilsson levels in the *sd* shell, as a function of ε_Ω. The dashed curve for $\varepsilon_\Omega > 0$ is obtained from the wave functions of the one-particle resonant levels, which are related to the weakly bound [200 1/2] level.

To understand the variation of $P(s_{1/2})$ values of three Nilsson levels as a function of $\varepsilon_\Omega < 0$, which is exhibited in Fig. 5, the eigenenergies of $1d_{5/2}$, $2s_{1/2}$, and $1d_{3/2}$ levels for spherical potentials are shown in Fig. 1, for reference. In the spherical Woods-Saxon potential, in which all three levels are deeply bound, or in the case of a modified oscillator potential [9] the $2s_{1/2}$ level lies around the middle of the $1d_{5/2}$ and $1d_{3/2}$ levels. However, the diagonal deformation energy for the $1d$ orbits with $\Omega^\pi = 1/2^+$ is negative for prolate deformation, whereas the deformation energy is zero for the $s_{1/2}$ orbit. Consequently, for a sufficiently large prolate deformation the [200 1/2] level, which is the highest lying $\Omega^\pi = 1/2^+$ level in the *sd* shell, obtains the largest $s_{1/2}$ component. In contrast, when the potential strength becomes weaker, the eigen energy of the $2s_{1/2}$ level does not increase so much as that of $1d$ levels and, consequently, the $2s_{1/2}$ level approaches the $1d_{5/2}$ level. As a result of it, for $\varepsilon_\Omega(< 0) \to 0$ the [220 1/2] level obtains a large component of $s_{1/2}$ as exhibited in Fig. 5. It is remarked that in Fig. 5 the potential strengths for three Nilsson levels at a given value of ε_Ω are different and, thus, the sum of the relative $s_{1/2}$ probability over three Nilsson levels at a given value of ε_Ω is different from unity.

IV. CONCLUSIONS AND PERSPECTIVES

Positive-energy one-particle levels for neutrons in axially symmetric quadrupole-deformed Woods-Saxon potentials are studied by solving the Schrödinger equation in coordinate space with the appropriate asymptotic boundary conditions. Limiting the model space to the *sd* shell, all three eigenphases for $\Omega^\pi = 1/2^+$ are studied for a given strength of the deformed potential and at a given one-particle energy. It is found that in the region of small one-particle energy $\varepsilon_\Omega(>0)$ only one of the eigenphases varies considerably, whereas the others start from $n\pi$ at $\varepsilon_\Omega = 0$ and vary very slowly and smoothly.

There is no resonance in the $s_{1/2}$ channel in spherical potentials, whereas some amount of $s_{1/2}$ component is always admixed into all $\Omega^\pi = 1/2^+$ levels in quadrupole-deformed potentials. The role of those admixed $s_{1/2}$ components in possible one-particle resonances with $\Omega^\pi = 1/2^+$ is one of the subjects of present interest because the decay of $\Omega^\pi = 1/2^+$ resonances may most easily go through the $s_{1/2}$ channel because the absence of centrifugal barrier. When one-particle resonant levels defined in terms of eigenphase are obtained in the low-energy region, some one-particle resonant levels are found to be a smooth continuation of specified weakly bound Nilsson levels, whereas the others have no related bound Nilsson levels. The latter has usually a large width and is hardly expected to play a unique role as a resonance, but its possible role in any observable is interesting to study. To obtain one-particle resonant levels as the continuation of weakly bound $\Omega^\pi = 1/2^+$ levels when the potential strength becomes slightly weaker, the relative probability of the $s_{1/2}$ component of weakly bound one-particle levels inside the potential has to be smaller than some critical value. The critical value depends on the diffuseness of the potential.

It is preferable to develop a simple analytical model finding some crucial parameters to understand the physics obtained in the present article.

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