Constraints on energy of ^{9}B($1/2^+$) and ^{10}C(0_2^+)

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(Received 17 April 2006; published 15 June 2006)

The second 0^+ state in ^{10}Be and ^{10}B is shown to be almost pure $(sd)^2$ in character. Its energy puts a severe constraint on the possible location of the first $1/2^+$ state in ^{9}B, and on the location of ^{10}C (0_2^+).

DOI: 10.1103/PhysRevC.73.064302 PACS number(s): 21.10.Dr, 21.10.Jx, 21.10.Sf, 27.20.+n

The first $1/2^+$ state in ^{9}B remains elusive. This state must be present, as it is the mirror of a well-known $1/2^+$ level in ^{9}Be [1], just at neutron threshold. Its energy has been the subject of a large number of theoretical [2–8] and experimental [9–17] papers. One investigation of the reaction ^{9}Be(^{6}Li, ^{6}He) reported [12] populating the $1/2^+$ state at an excitation energy of 1.32 ± 0.08 MeV, with a width of 0.86 ± 0.26 MeV. Another study [17] of the same reaction reported not finding it. Some of those same workers used the reaction ^{6}Li(^{6}Li, t)^{1}H ^{8}Be to look for it, and reported [14] finding it at an energy above 0.6 MeV and below 1.7 MeV. They state "0.6 MeV is the lowest possible energy" and the "state is below 1.67 MeV." Two specific fits of their spectra gave values of 1.6 ± 0.1 MeV (two states, plus their interference) and 0.73 ± 0.05 MeV (three states, no interference). Theoretical energies cover most of this range, and more.

The sensitivity of Coulomb energies to occupancy of an $\ell = 0$ orbital is well known. In light nuclei, the shift in excitation energy between mirror pairs increases with the $2s1/2$ occupancy. This effect is large and is sometimes called the Thomas-Ehrman shift. The effect has been used by us, and others, to estimate the energy of a mirror state whose dominant configuration is known, or to estimate the dominant configuration admixtures for a pair of mirror states with known energies. Here, we use the energies of the 0_2^+ state in ^{10}Be and its analog in ^{10}B to compute the energy of the core ^{9}B($1/2^+$) state as a function of the d^2/s^2 ratio in the 0_2^+ state. We first discuss the properties of this 0_2^+ level and then its connection to the energy of the $1/2^+$ state.

The relevant 0^+ states of ^{10}B are listed in Table I. Another $0^+(sd)^2$ state should lie a few MeV above the lowest one, but it is of no interest here. The lowest p-shell 0^+ state (the g.s. of ^{10}Be) has a large spectroscopic factor for ^{9}Be(d,p) in the calculations of Cohen-Kurath, [18] who find $S = 2.36$. The experimental value is 2.1 [19]. Of course, in the simplest model, the $(sd)^2$ 0^+ state would have no single-neutron strength. The second CK 0^+ state is above 12 MeV and has a small computed S. So, any appreciable single-particle strength for 0_2^+ would likely come from mixing with 0_1^+, not with higher states. This mixing turns out to be quite small, as we now demonstrate.

The spectroscopic factor for 0_2^+ in ^{9}Be(d,p) is very small, but not reliably determined. In ^{10}B, its analog is unbound (by 0.974 MeV) to proton emission and hence has some natural width. We use the expression $C^2S = \Gamma_{\rm exp}/\Gamma_{\rm sp}$, where C is an isospin Clebsch-Gordan coefficient ($C^2 = 1/2$ here) and $\Gamma_{\rm sp}$ is computed in a simple potential model. For $\Gamma_{\rm sp}$ we obtain 235 keV with standard parameters, which, together with the experimental width of 2.65 ± 0.18 keV, results in $S = (2.26 \pm 0.15) \times 10^{-2}$, i.e., only about 1% of the value for the first 0^+ state. We thus are confident that 0_2^+ is primarily of $(sd)^2$ character.

The dominant structure is $\alpha(2s1/2)^2 + \beta(1d5/2)^2$ coupled to ^{8}Be(g.s.). The $(d3/2)^2$ configuration is certainly present at some level, but the bulk of its strength lies high enough to be ignored. States based on excited states of ^{8}Be require the $(sd)^2 J$ value to be nonzero, making their contribution small, but not zero. We return to this point below. Our analysis is complicated by the fact that the lowest $1/2^+$ and $5/2^+$ states are not single-particle, but are of mixed parentage, the largest components being

$$1/2^+ = A[(0^+) \times (2s1/2)] + B[(2^+) \times (1d5/2)], \quad \text{and}$$
$$5/2^+ = C[(0^+) \times (1d5/2)] + D[(2^+) \times (2s1/2)] + E[(2^+) \times (1d5/2)].$$

Theoretical estimates [20] of these coefficients are $A^2 = 0.69$, $B^2 = 0.27$; $C^2 = 0.50$, $D^2 = 0.28$, and $E^2 = 0.19$. It is simpler to couple $2s1/2$ and $1d5/2$ to these two physical states than to attempt to locate the centroid of the $2s1/2$ and $1d5/2$ strengths in ^{9}Be and ^{9}B. Such coupling will of necessity bring about some $(2^+) \times (sd)_2^2$ in the 0_2^+ state. And some reasonable amounts of those components are likely present [21].

Our approach is then straightforward. (Energies of the relevant states are depicted in Fig. 1.) We couple a $2s1/2$ neutron to the $1/2^+$ level of ^{9}Be and vary the potential well depth to get the energy equal to that of ^{10}Be(0_2^+)($E_x = 6.179$ MeV). We then repeat by coupling a $1d5/2$ neutron to the $5/2^+$ level of ^{9}Be. Thus, we have potentials that produce $(1/2) \times (2s)$ and $(5/2) \times (1d)$ states, both with the energy of ^{10}Be(0_2^+). We then use this potential unchanged except for the addition of a Coulomb term to compute the energy of the analog 0_2^+ state in ^{10}B, which is 50% ^{9}Be + p and 50% ^{9}B + n. The computed energy of this 0_2^+ state in ^{10}B [for the $(1/2) \times (2s)$ component] depends on the assumed energy of the $1/2^+$ level in ^{9}B. (The $5/2^+$ state is well known.) Thus, if the admixture of $(1/2) \times (2s)$ and $(5/2) \times (1d)$ in this 0_2^+ state were known, it would be a simple matter to use the known

TABLE I. Properties of relevant 0^+, $T = 1$ states in ^{10}B.

Structure	E_x (MeV)		S	
	Calc	Exp[b]	Calc	Exp
p shell	1.418[a]	1.740	2.36[a]	2.1[c]
p shell	12.47[a]	Unknown	0.39[a]	—
^{8}Be(g.s.)× $(sd)^2$	See text	7.560	0	$(2.26 \pm 0.15)\times 10^{-2d}$

[a]Cohen-Kurath [18].
[b]Reference [1].
[c]Reference [19].
[d]From measured p width [1] of 2.65 ± 0.18 keV and our sp width of 235 keV.

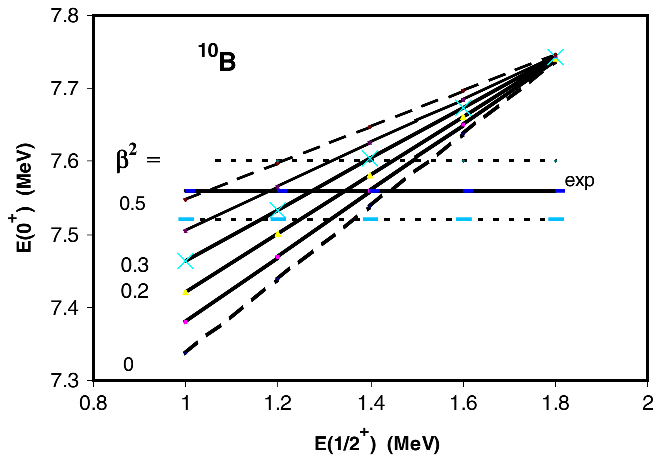

FIG. 2. (Color online) The computed energy of the second 0^+ state in ^{10}B is plotted vs the assumed energy of the $1/2^+$ state in ^{9}B, for various values of β^2 [the percentage of $(5/2) \times (1d)$ in the 0^+ state], ranging from 0 to 50% (upward sloping lines). Dark solid sloping lines are for $\beta^2 = 0.20$ and 0.30. The horizontal line is at the known 0^+ energy of 7.560 MeV, and the dashed horizontal lines represent ± 40 keV model uncertainty. If $\beta^2 = 0.25 \pm 0.05$, this calculation provides $E_x(1/2^+) = 1.31 \pm 0.11$ MeV in ^{9}B.

0_2^+ energy to find the $1/2^+$ energy in ^{9}B. This admixture is not precisely known, so we can investigate the results as a function of this mixing.

In Fig. 2, the computed energy of the second 0^+ state in ^{10}B is plotted vs the assumed energy of the $1/2^+$ state in ^{9}B, for various values of β^2 [the percentage of $(5/2) \times (1d)$ in the 0^+ state], ranging from 0 to 50% (upward sloping lines). Solid sloping lines are for $\beta^2 = 0.20$ and 0.30. The horizontal line is at the known 0^+ energy of 7.560 MeV, and the dashed horizontal lines represent ± 40 keV model uncertainty.

It seems reasonable to expect that the d^2/s^2 ratio in the first $(sd)^2$ 0^+ state in ^{10}Be is about the same as in the $(sd)^2$ part of ^{12}Be(g.s.). This expectation certainly holds for ^{14,16}C [22]. Results of ^{10}Be$(t,p)^{12}$Be(g.s.) [23], e.g., require a substantial $(sd)^2$ component in ^{12}Be(g.s.). The energy difference between ^{12}Be and ^{12}O was used [24] to estimate the amount of $(2s1/2)^2$ in the g.s. as 53%, with the remainder split among $(1d)^2$ and $1p$ shell. One deficiency of that calculation was the omission of excited core states for the p-shell component.

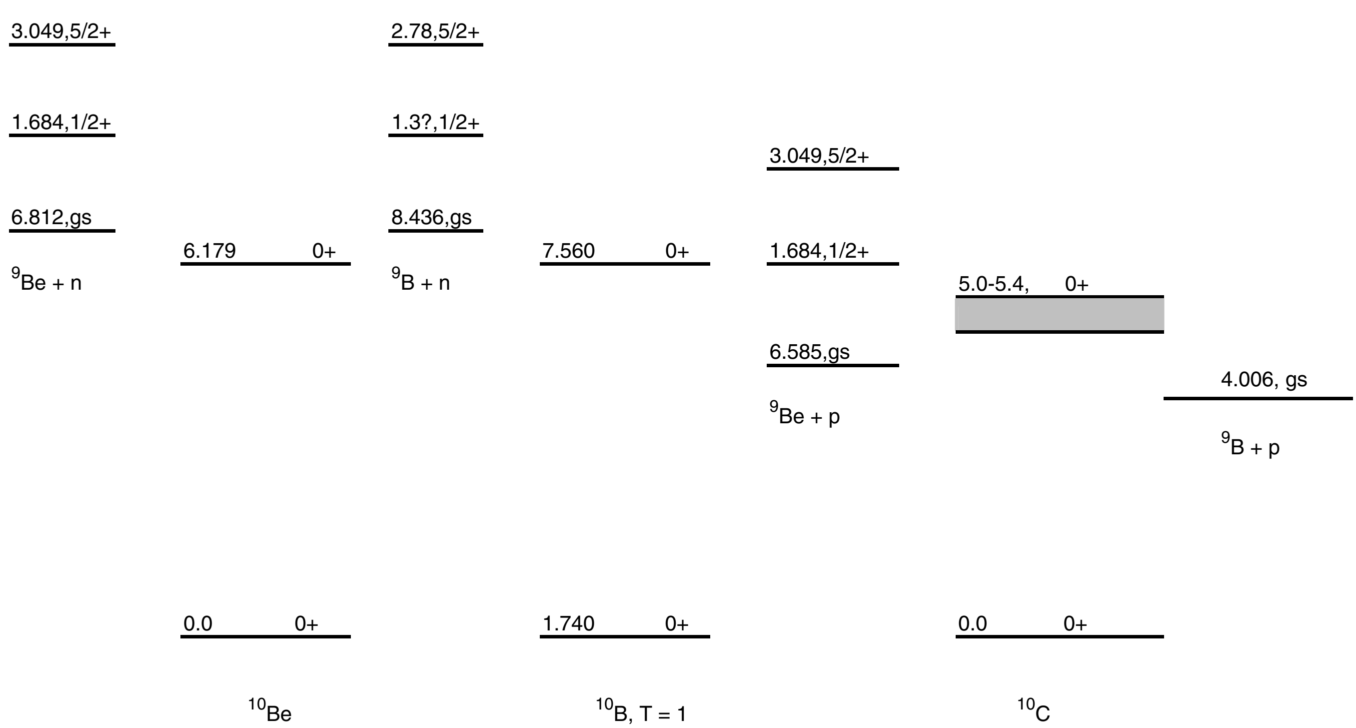

FIG. 1. Energies of the relevant states in $A = 9, 10$ nuclei.

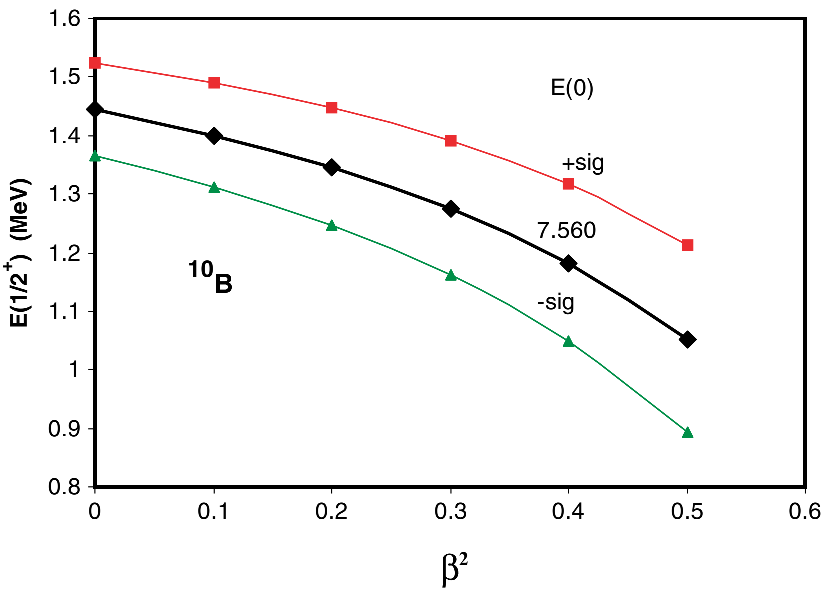

FIG. 3. (Color online) Vertical axis is the assumed $1/2^+$ energy in ^{9}B. Horizontal axis is the assumed $(5/2) \times (1d)$ percentage in ^{10}B(0_2^+). Dark solid curve is for $E_x = 7.560$ MeV, lighter curves for sig = 40 keV on either side of that value.

However, that omission has very little effect on the amount of s^2 present, because the excitation-energy shifts for p and d are both small and similar. One estimate of the d^2/s^2 ratio in ^{12}Be was 0.22/0.78. We thus expect something similar in ^{10}Be.

In Fig. 3, we plot vs β^2 the energy of ^{9}B($1/2^+$) that gives a particular energy for the 0_2^+ state in ^{10}B. The dark solid curve corresponds to the actual excitation energy of 7.560 MeV for the 0^+ state. A recent experiment [25] on ^{12}Be breakup determined the spectroscopic factor for ^{12}Be(g.s.) $\rightarrow$ ^{11}Be($5/2^+$) to be 0.48 ± 0.06, out of a maximum possible value of 2.0. Thus, in the simplest models, this value would correspond to $\beta^2 = 0.24 \pm 0.03$—very close to our value. However, this β^2value seems slightly large for ^{12}Be(g.s.), which we know contains some p-shell component. If we combine this value of $\beta^2 = 0.24 \pm 0.03$ with our earlier $\alpha^2 = 0.53$ for ^{12}Be(g.s.) (with the remainder being p shell), and if the $(sd)^2$ part of ^{12}Be(g.s.) is about the same as for ^{10}Be(0_2^+), we would have $\beta^2(^{10}\mathrm{Be}) = 0.31$, very close to the end-point of our uncertainty band of 0.25 ± 0.05.

In the past we have used configuration-mixed wave functions to compute Coulomb energies for several levels of a number of nuclei. Generally, the average deviation of our calculations from experiment is a few keV, with a spread of 30–40 keV. In the present case, an uncertainty of ± 40 keV in the calculated position of ^{10}B(0_2^+) translates into an 80 keV uncertainty in the energy of ^{9}B($1/2^+$). If we add an additional uncertainty of 35 keV for uncertainty in the d^2/s^2 ratio ($\Delta\ \ \beta^2 = 0.05$), we get 87 keV if added in quadrature, 115 keV if added linearly. Thus, we adopt an uncertainty of $\pm$ 110 keV in our predicted position of ^{9}B($1/2^+$). Our value of 1.31 ± 0.11 MeV for the excitation energy is well within the range of most experimental results, and is only slightly higher than the midpoint of the two energies suggested by Ref. [14]. It is quite close to the value reported by Burlein *et al.* [12] and to a recent theoretical value [7].

With the $1/2^+$ state of ^{9}B at $E_x = 1.31 \pm 0.11$ MeV and our choice of $\beta^2 = 0.25 \pm 0.05$, we can compute the expected position in ^{10}C of the mirror of ^{10}Be(0_2^+). The result is $E_x = 5.18 \pm 0.11$ MeV. For a state at this excitation energy, the $\ell = 1$ sp p width is 190 keV, with about a factor of 2 uncertainty because of the uncertainty in excitation energy. If isospin is conserved, we expect $S(^{10}\mathrm{C}) = S(^{10}\mathrm{B})$, i.e., about 2×10^{-2}, so the experimental proton width of this 0_2^+ state in ^{10}C should be about 4^{+4}_{-2} keV. Two broad states are known [1] at $E_x =$ 5.22 and 5.38 MeV, but we expect that neither of them is the 0_2^+ state. Rather, they are probably mirrors of the 1^-, 2^- states at 5.96 and 6.26 MeV in ^{10}Be. The second p-shell 2^+ state should also lie near here. The 0^+ state of ^{10}C will be very difficult to populate. It should have very little parentage for ^{10}B(p,n) [26] or ^{10}B(^{3}He,t) [27] because it has two sd-shell protons and ^{10}B has none. Of course, if ^{8}Be had been stable, it would have been very strong in $2p$ transfer [e.g., ^{8}Be(^{3}He,n)]. There are no obvious heavy-ion radioactive beam paths to populate it. Perhaps the best bet is ^{20}Ne(α,^{14}C)^{10}C. The target has two sd-shell protons, and any direct ten-nucleon pickup (where two neutrons come from the sd-shell and all the rest from the $1p$ shell) should leave behind the ^{10}C(0_2^+) state. If the reaction has measurable cross section, the yield for ^{20}Ne[α, ^{14}O(g.s.)]^{10}Be(0_2^+) should give an excellent idea of the expected cross section for ^{14}C(g.s.) + ^{10}C(0_2^+). Also worthy of consideration are the reactions ^{9}Be(^{16}O, ^{15}C)^{10}C (a sequential process involving picking up one neutron from the p shell, and adding two protons to the sd shell), and/or ^{7}Be(α,n)^{10}C (in which a neutron enters the p shell, and two protons enter the sd shell).

In conclusion, the second 0^+ state at 6.179 MeV in ^{10}Be is shown to have nearly pure $(sd)^2$ character. The position of the analog in ^{10}B allows the energy of ^{9}B($1/2^+$) to be computed as a function of β^2, the amount of $(5/2) \times (1d)$ in the 0^+ state. For values near $\beta^2 = 0.25$, which we favor, we get the ^{9}B($1/2^+$) state at an excitation energy of 1.31 ± 0.11 MeV, where the uncertainty comes from the assumptions of the model and the estimated uncertainty in β^2. As a bonus, we expect the 0_2^+ state in ^{10}C to be at 5.18 ± 0.11 MeV and to be narrow.

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