Sum rules for the spin dependent structure functions g_1 in the isovector reaction

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In the isovector reaction, the sum rule for the spin dependent function g_1 which is related to the cross section of the photoproduction is derived. In the small Q^2 region, the sum rule is dominated by the low energy contribution and it tightly connects the resonance, the elastic, and the nonresonant contributions.

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It has been known that some sum rules derived from the canonical quantization on the null-plane get the contribution from the nonlocal quantity corresponding to the matrix element of the bilocal current which is absent in the equal-time formalism [1]. The sum rule for the spin dependent function g_1^{ab} corresponding to the moment at $n = 0$ is one example,where a, b denotes the flavor suffix of the currents. This sum rule is for the antisymmetric combination with respect to a, b. The corresponding sum rule in the equal-time formalism had been considered peculiar since it was invalid in the free field model. This fact was discussed in Ref. [2], and also in Ref. [3]. The null-plane method circumvented this defect.

Recently, there has been great experimental interest in the behavior of the polarized structure functions in the low Q^2 region [4]. Motivated by this, the sum rule for the g_1^{ab} derived from the connected hadronic matrix element of the current anticommutation relation on the null-plane has been transformed to the one which is sensitive to the behavior in this region [5]. Here, we report that the same method can be applied to the sum rule known in the null-plane formalism based on the current commutation relation, and transform it to the experimentally testable form. It should be noted that the current commutation relation on the null-plane is an operator relation while the current anticommutation relation on the null-plane exists only as a stable hadronic matrix element. We can derive the latter from the former but we cannot do the converse. Further, the sum rule derived here is a nonsinglet quantity and that in Ref. [5] include a singlet quantity. This difference is reflected in the high energy behavior, i.e., the superconvergence relation in the derivation of the sum rule.

According to Ref. [1], we obtain

$$\int_0^1 \frac{dx}{x} g_1^{[\mathrm{ab}]}(x, Q^2) = -\frac{1}{16} f_{\mathrm{abc}} \int_{-\infty}^{\infty} d\alpha \big[A_c^5(\alpha, 0) + \alpha \bar{A}_c^5(\alpha, 0)\big], \tag{1}$$

where $A_c^{5\beta}(x|0)$ is the antisymmetric bilocal current, and its matrix element is defined as

$$\langle p, s|A_c^{5\beta}(x|0)|p, s\rangle_c = s^\mu A_c^5(p\cdot x, x^2) + p^\mu(x\cdot s) \times \bar{A}_c^5(p\cdot x, x^2) + x^\mu(x\cdot s) \times \tilde{A}_c^5(p\cdot x, x^2). \tag{2}$$

Since the right-hand side of the sum rule is Q^2 independent, we obtain for the antisymmetric combination with respect to a, b

$$\int_0^1 \frac{dx}{x} g_1^{[\mathrm{ab}]}(x, Q^2) = \int_0^1 \frac{dx}{x} g_1^{[\mathrm{ab}]}\big(x, Q_0^2\big). \tag{3}$$

Now, we take $Q_0^2 = 0$ and use the relation

$$G_1^{\mathrm{ab}}(\nu, 0) = -\frac{1}{8\pi^2\alpha_{\mathrm{em}}}\big\{\sigma_{3/2}^{\mathrm{ab}}(\nu) - \sigma_{1/2}^{\mathrm{ab}}(\nu)\big\} = -\frac{1}{8\pi^2\alpha_{\mathrm{em}}}\Delta\sigma^{\mathrm{ab}}(\nu). \tag{4}$$

By setting $a = (1 + i2)/\sqrt{2}$, $b = a\dagger$, and separating out the elastic contribution, we obtain the sum rule which relates the g_1 and the cross section of the photoproduction in the isovector reactions.

Now the Regge theory predicts as $g_1^{[ab]} \sim \beta x^{-\alpha(0)}$ with $\alpha(0) \leqslant 0$, and hence the sum rule is convergent. However, the perturbative behavior like the DGLAP fit to the unmeasured small x region has large ambiguity [6] and the sum rule is possibly divergent. The double logarithmic $(\log(1/x))^2$ resummation give more singular behavior than the Regge theory [7] and the sum rule (3) is also divergent. Though, whether the sum rule diverges or not cannot be judged rigorously by these discussions, it is desirable to discuss the regularization of the sum rule and give it a physical meaning even when the sum rule is divergent. Now, the regularization of the divergent sum rule has been known to be done by the analytical continuation from the nonforward direction [8]. We first derive the finite sum rule in the small but sufficiently large $|t|$ region by assuming the moving pole or cut. Then we subtract the singular pieces which we meet as we go to the smaller $|t|$ from both hand sides of the sum rule by obtaining the condition for the coefficient of the singular piece. After taking out all singular pieces we take the limit $|t| \to 0$. The sum rule obtained in this way can be transformed to the form where the high energy behavior from both hand sides of the sum rule is subtracted away. Practically, if the cancellation at high energy is effective, since the condition is needed only in the high energy limit, we can consider the sum rule irrespective of the condition. The sum rule of this type can be obtained as follows.

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The hadronic tensor is defined as

$$W_{\rm ab}^{\mu\nu}\big|_{\rm spin\,dependent} = \frac{1}{4\pi m_N}\int d^4x \exp(iq\cdot x)\langle p,s| \times \left[J_a^\mu(x)J_b^\nu(0)\right]|p,s\rangle_c|_{\rm spin\,dependent}. \quad (5)$$

Since we take $a=(1+i2)/\sqrt{2}$, $b=a^\dagger$ which means to take J_a^μ as $J_{1+i2}^\mu/\sqrt{2}$ and the state $|p\rangle$ as the proton, the Born term is given as

$$W_{\rm ab}^{\mu\nu}\big|_{\rm Born} = \frac{1}{4\pi m_N}\int d^4x \exp(iq\cdot x)\sum_{s',n}\langle p,s|J_a^\mu(x)|n,s'\rangle \times \langle n,s'|J_b^\nu(0)|p,s\rangle_c, \quad (6)$$

where n in the intermediate state specifies both the neutron and its momentum and the n in the sum means to take the momentum integral. Then we define

$$\langle p,s|J_{1+i2}^\mu(0)|n,s'\rangle = \bar u_s(p)\left(\gamma^\mu g_V^+ + \tfrac{1}{2}(p+n)^\mu f_V^+\right)u_{s'}(n). \quad (7)$$

where the form factors g_V^+ and f_V^+ are related to the usual Dirac and Pauli form factors or Sachs form factors as $g_V^+ = F_1^+ + F_2^+ = G_M^+$ and $m_N f_V^+ = -F_2^+ = -(G_M^+ - G_E^+)/(1+Q^2/4m_N^2)$. It should be noted that the positive component of the form factor is connected to the difference between the form factor of the proton and that of the neutron. This is because $J_{1+i2}^\mu(0) = [J_3^\mu(0), I_+] = [J_{\rm em}^\mu(0), I_+]$ since the hypercharge current commutes with I_+, where $J_{\rm em}^\mu(0)$ is the electromagnetic current and I_+ satisfies $I_+|n\rangle = |p\rangle$ and $\langle p|I_+ = \langle n|$. Then it is straightforward to take out the Born term contribution to the spin dependent function $g_1^{\rm ab}$. Now we take $\nu_c^Q = m_p E_Q$ where E_Q is given as $E_Q = E_c + Q^2/2m_p$ with E_c being the cut off energy of the photon in the laboratory frame. By separating out the Born term we rewrite the regularized sum rule as

$$B(Q^2)+K(E_c,Q^2) = \int_{E_0}^{E_Q}\frac{dE}{E}\left[2g_1^{1/2}(x,Q^2) - g_1^{3/2}(x,Q^2)\right] + \frac{m_p}{8\pi^2\alpha_{\rm em}}\int_{E_0}^{E_c} dE[2\Delta\sigma^{1/2} - \Delta\sigma^{3/2}], \quad (8)$$

by using the isospin rotation as in the Cabibbo-Radicati sum rule [9], where $B(Q^2)$ is given as

$$B(Q^2) = \frac{1}{4}\left\{(\mu_p-\mu_n) - \frac{1}{1+Q^2/4m_p^2}G_M^+ \times \left[\left(1-\frac{Q^2}{4m_p^2}\right)G_E^+(Q^2) + \frac{Q^2}{2m_p^2}G_M^+(Q^2)\right]\right\}, \quad (9)$$

with

$$G_E^+(Q^2) = G_E^p(Q^2) - G_E^n(Q^2),$$
$$G_M^+(Q^2) = G_M^p(Q^2) - G_M^n(Q^2), \quad (10)$$

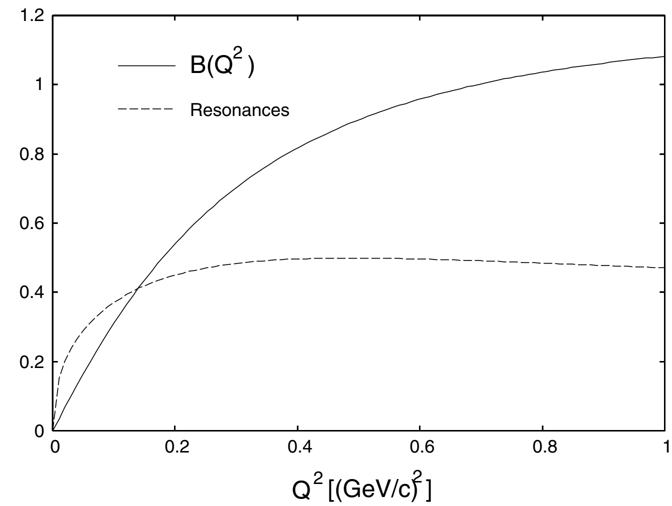

FIG. 1. The contributions from the Born terms as given by the $B(Q^2)$ and those from the resonances.

and

$$K(E_c,Q^2) = -\int_{E_Q}^\infty \frac{dE}{E}\left[2g_1^{1/2}(x,Q^2) - g_1^{3/2}(x,Q^2)\right] - \frac{m_p}{8\pi^2\alpha_{\rm em}}\int_{E_c}^\infty dE[2\Delta\sigma^{1/2} - \Delta\sigma^{3/2}]. \quad (11)$$

Here, the suffix 1/2 or 3/2 in g_1 and $\Delta\sigma$ means the quantity in the reaction (isovector photon) + (proton) → (states of isospin I) where $I = 1/2, 3/2$. Then, $g_1(x,Q^2)$ in the virtual charged photon reaction ($g_1^{\rm ab}(x,Q^2) - g_1^{ba}(x,Q^2)$) is transformed to the quantities in the real neutral isovector photon corresponding to the vector current J_3^μ as ($2g_1^{1/2}(x,Q^2) - g_1^{3/2}(x,Q^2)$) by a simple isotopic analysis. Similar fact applies to $\Delta\sigma$. As discussed in Ref. [5], if we take $E_c = 2({\rm GeV}^2)$ and a small Q^2, the contribution from $K(E_c,Q^2)$ is expected to be small and almost negligible. We can expect the same kind of things happens also in this case. The contributions from the Born terms $B(Q^2)$ can be estimated by using the standard dipole fit, where Galster parametrization is used for the G_E^n [10]. The resonance contributions on the right-hand side of the sum rule (8) can be estimated by the parameters given in Ref. [11] if we neglect the isoscalar photon contribution. The results are given in Fig. 1. From it, we see that, to satisfy the sum rule, the difference of the nonresonant contribution between $\int_{E_0}^{E_Q}\frac{dE}{E}[2g_1^{1/2}(x,Q^2) - g_1^{3/2}(x,Q^2)]$ and $-\frac{m_p}{8\pi^2\alpha_{\rm em}}\int_{E_0}^{E_c} dE[2\Delta\sigma^{1/2} - \Delta\sigma^{3/2}]$ is negative in the very small Q^2 region and becomes positive above some value near $Q^2 \sim 0.15({\rm GeV}/c)^2$. This sign change occurs in the region where the change of the difference between the resonances becomes small while that between the Born terms is rapid.

In summary, in the isovector reaction, the sum rule for the spin dependent function $g_1^{\rm ab}$ which is related to the cross section of the photoproduction is given. By taking the parameter in the sum rule appropriately, the sum rule is expected to be dominated by the low energy contributions. Then, the sum rule shows that the resonance, the elastic, and the nonresonant contributions are tightly connected in the small Q^2 region.

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