# Antikaon condensation and deconfinement phase transition in neutron stars

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Antikaon condensation and deconfinement phase transition in neutron stars are investigated in a chiral hadronic model (also referred as to the FST model) for the hadronic phase and in the MIT bag model for the deconfined quark matter phase. It is shown that the existence of quark matter phase makes antikaon condensation impossible in neutron stars. The properties of neutron stars are sensitive to the bag constant. For the small values of the bag constant, the pure quark matter core appears and hyperons are strongly suppressed in neutron stars, whereas for the large bag constant, the hadron-quark mixed phase exists in the center of neutron stars. The maximum masses of neutron stars with the quark matter phase are lower than those without the quark matter phase; meanwhile, the maximum masses of neutron stars with the quark matter phase increase with the bag constant.

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## I. INTRODUCTION

Hadronic matter is composed of hadrons with quarks and gluons confined within, and it may undergo the phase transition from hadronic matter to quark matter at high baryon density. The search for quark matter and its signals is one of the great challenges in nuclear physics. In addition to the experimental efforts neutron stars provide astrophysical laboratories for high-baryon-density physics. Theoretical investigations show that quark matter can be ruled out if neutron stars are considered as nucleon-only systems. However, the composition of neutron star interior is not so simple, it may be multiparticle mixture of mesons, nucleons, hyperons, and leptons. The baryon density in the neutron star interior can exceed a few times suturation density of nuclear matter, in this case quark matter possibly occurs in the cores of neutron stars [1,2]. The recent progress on quark matter in the cores of neutron stars is summarized in Ref. [3]. In fact, whether quark matter exists in the cores of neutron stars is a matter of controversy because of a lack of direct observation, intensive investigations are still expected.

Strong interaction plays a central role in neutron star matter, the more reliable hadronic model should be constructed for physics of neutron stars. Recently, a chiral hadronic model has been proposed by Furnstahl, Serot, and Tang (referred to in the following as the FST model) [4]. The features of the FST model are that the theorem of strong interaction predicted by quantum chromodynamics (QCD), i.e., a nonlinear realization of chiral symmetry, broken scale invariance, and the effect of vector dominance, is respected phenomenologically [4]. The purpose of this article extends the FST model to describe hadronic phase in neutron stars, and the well-known MIT bag model [5] is used for quark phase. The calculated results are also compared with those given by the nonlinear Walecka model [6,7]. The effects of the variation of MIT bag constant and hyperons on neutron stars and on the antikaon condensation [8] are emphasized.

The rest of this article is organized as follows. In Sec. II, the model and the basic formulas are given. The results and conclusions are presented in Sec. III.

## II. MODELS OF NEUTRON STAR MATTER

### A. The FST model for hadronic matter

The Lagrangian density of the FST model [4] with the inclusion of leptons and kaons is given by

$$\begin{aligned}\mathcal{L} =& \sum_B \bar{\psi}_B\big[i\gamma_\mu D_B^\mu + g_{AB}\gamma^\mu\gamma_5 a_\mu - M_B + g_{\sigma B}\phi \\ &- \frac{1}{2}g_{\rho B}\gamma_\mu \vec{\tau}\cdot\vec{b}^\mu\big]\psi_B + \frac{1}{2}\left[1+\eta\frac{\phi}{S_0}+\ldots\right] \\ &\times\left[\frac{1}{2}f_\pi^2 \mathrm{tr}(\partial_\mu U\partial^\mu U^+) + m_v^2 V_\mu V^\mu\right] - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &+ \frac{1}{4!}\xi\left(g_\omega^2 V_\mu V^\mu\right)^2 + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - H_q\left(\frac{S^2}{S_0^2}\right)^{\frac{2}{d}} \\ &\times\left(\frac{1}{2d}\ln\frac{S^2}{S_0^2} - \frac{1}{4}\right) - \frac{1}{4}\vec{G}_{\mu\nu}\cdot\vec{G}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{b}_\mu\cdot\vec{b}^\mu \\ &+ \sum_{L=e,\nu_e,\mu}\bar{\psi}_L(i\gamma_\mu\partial^\mu - m_L)\psi_L + L_K,\end{aligned} \tag{1}$$

where $\psi_B(B = n, p, \Lambda, \Sigma^+, \Sigma^-, \Sigma^0, \Xi^-, \Xi^0)$ denote the octet fields of baryons. $\mathcal{L}_K = \mathcal{D}_\mu^* \bar{K}\mathcal{D}^\mu K - m_K^{*2}\bar{K}K$ [9–11] is the Lagrangian density for kaons. Other notations in the above formulas are same as those used in Ref. [12]. In neutron stars, the critical conditions for $\bar{K} = (K^-, \bar{K}^0)$ condensation with an $s$ wave ($k = 0$) [10,12] read

$$\omega_{K^-} = \mu_{K^-} = \mu_e = \mu_n - \mu_p, \tag{2}$$

$$\omega_{\bar{K}^0} = \mu_{\bar{K}^0} = 0. \tag{3}$$

 

The closed nonlinear equations of motion can be derived from Eq. (1). Those equations plus Eqs. (2) and (3) under neutron star's conditions with (or without) quark matter can be solved by the self-consistently iterative approach in the framework of relativistic mean-field theory. The parameter set T3 of the FST model will be used to perform calculations in the present study because nuclear matter properties calculated by this parameter are very close to those of the relativistic Bruckner-Hartree-Fock (RBHF), especially in the high-density region [13], and other coupling constants are determined in the same way as in Ref. [12].

### B. The MIT bag model for quark matter

To describe the quark matter phase (QP) in neutron stars, we use the well-known MIT bag model [5] with the pressure and energy density of the QP given by

$$P_Q = \sum_{f=u,d,s} \frac{\nu_f}{6\pi^2} \int_0^{k_{Ff}} dk \frac{k^4}{\sqrt{k^2+m_f^2}} - B, \tag{4}$$

$$\epsilon_Q = \sum_{f=u,d,s} \frac{\nu_f}{2\pi^2} \int_0^{k_{Ff}} dk k^2 \sqrt{k^2+m_f^2} + B, \tag{5}$$

where the subscript $Q$ represents the quark matter phase and $B$ the bag constant. $\nu_f = 2_{\rm spin} \times 3_{\rm color}$ is for the quark degeneracy; $m_u = 5$ MeV, $m_d = 10$ MeV, and $m_s = 150$ MeV for current quark masses. As for the bag constant, if the value of $B^{1/4} <$ 170 MeV is taken, the deconfinement phase transition from the HP to the QP occurs at the density below the saturation density of nuclear matter; this result is not reasonable in physics. If $B^{1/4} > 200$ MeV, we find that the QP does not appear in neutron stars. Therefore, the values of the bag constant $B^{1/4}$ as a free parameter will be limited from 170 to 200 MeV for the deconfinement phase transition research in neutron stars.

### C. Conditions for the deconfinement phase transition

The deconfinement phase transition from the hadronic phase (HP) to the QP occurs when the Gibbs conditions are satisfied [1,14]. This result leads to the mixed phase (MP) to appear in neutron stars. Therefore, the MP lies in equilibrium:

$$P_H(\mu_n, \mu_e) = P_Q(\mu_n, \mu_e), \tag{6}$$

$$\mu_H = \mu_Q, \tag{7}$$

where quarks' chemical potentials are expressed as

$$\mu_u = (\mu_n - 2\mu_e)/3, \tag{8}$$

$$\mu_d = (\mu_n + \mu_e)/3, \tag{9}$$

$$\mu_s = \mu_d. \tag{10}$$

The global charge neutrality condition in neutron stars should be satisfied

$$(1-\chi)\rho_e^H + \chi\rho_e^Q = 0, \tag{11}$$

where $(1-\chi)$ and $\chi = V_Q/(V_H+V_Q)$ are the volume fractions of the hadronic and quark matter in the MP. The value of $\chi$ changes from 0 to 1, which can be fixed by solving the coupled equations consistently. If $\chi = 1$, the pure quark matter core exists, whereas $\chi = 0$, the pure hadronic matter in neutron stars.

The baryon density $\rho_B$ and the energy density are given as

$$\rho_B = (1-\chi)\rho_B^H + \chi\rho_B^Q, \tag{12}$$

$$\varepsilon = (1-\chi)\varepsilon_H + \chi\varepsilon_Q. \tag{13}$$

## III. RESULTS AND CONCLUSIONS

### A. Results

Figure 1 shows the fractions of particles as functions of baryon density in neutron stars in the FST model and in the nonlinear Walecka model. In the FST model [see Fig. 1(a)], the $u$, $d$, and $s$ quarks appear at the critical density $\rho_c = 1.06\rho_0$, here $\rho_0 = 0.15\,{\rm fm}^{-3}$ is the saturation density of nuclear matter. Once the QP emerges, the fraction of quarks increases rapidly and the fractions of leptons ($e$, $\mu$) and hadrons ($n$, $p$) drop dramatically. Meanwhile, only a few $\Lambda$ hyperons appear and other hyperons are strongly suppressed in neutron stars. The MP ends at $4.56\rho_0$, i.e., the pure quark core exists in neutron stars if $\rho \geqslant 4.56\rho_0$. It is also shown that the QP makes $\bar{K}$ condensation impossible in neutron stars because antikaons as a type of particle species do not emerge in Fig. 1.

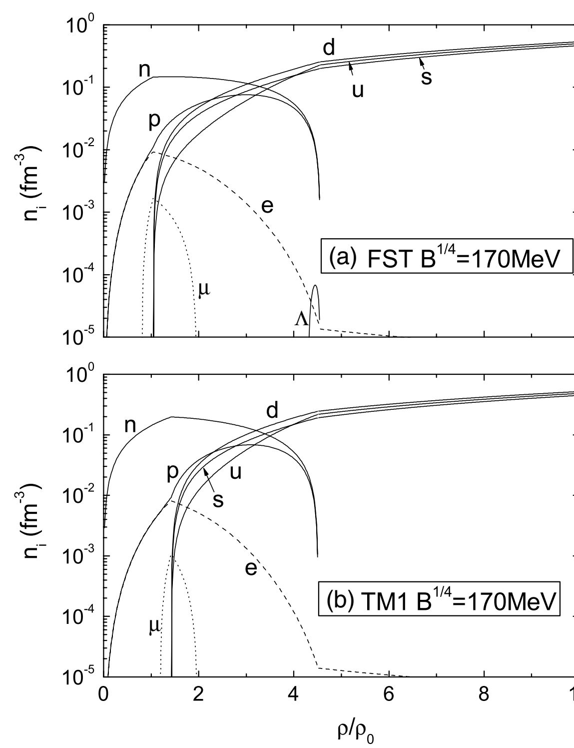


FIG. 1. The number densities $n_i$ of various particles in neutron star matter in the FST model with the parameter set T3 and in the nonlinear Walecka model with the parameter set Tm1. The bag constant $B^{1/4} = 170$ MeV is taken.

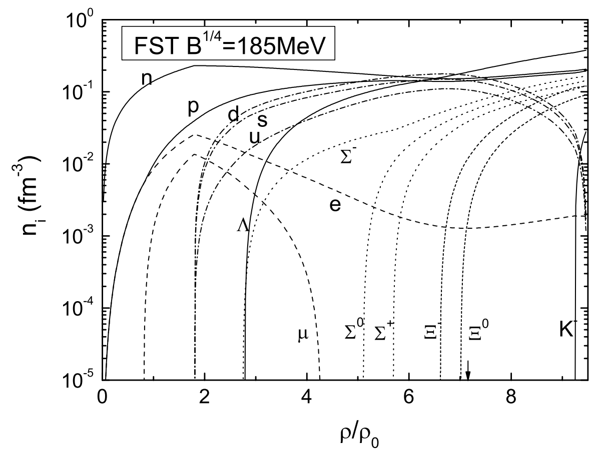


FIG. 2. Same as Fig. 1 but for $B^{1/4} = 185$ MeV in the FST model. The arrow at the transverse axis indicates the central density of maximum neutron stars.

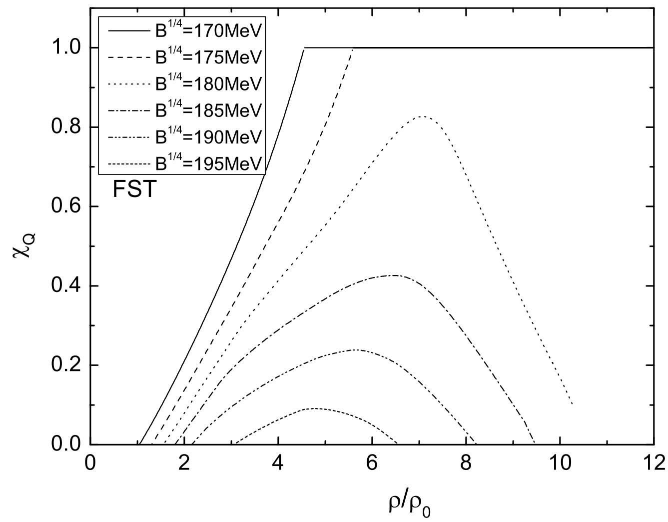


FIG. 3. The volume fraction of the quark matter phase versus the baryon density in neutron stars in the FST model with the parameter set T3.

As for the results calculated in the nonlinear Walecka model, it shows in Fig. 1(b) that the critical density of quarks is 1.44 $\rho_0$ and the pure quark core appears if $\rho \geqslant 4.52\rho_0$. Except for no $\Lambda$ hyperons appearing in neutron stars in the nonlinear Walecka model, the behaviors of Fig. 1(b) are similar to those of Fig. 1(a).

The fractions of particles in neutron stars are plotted in Fig. 2 in the FST model with the large bag constant $B^{1/4} = 185$ MeV. It is seen that hyperons ($\Lambda$, $\Sigma$, $\Xi$) can appear in neutron stars. $\bar{K}$ condensation cannot exist in neutron stars because the critical density ($\rho_c = 9.26\rho_0$) of $K^-$ condensation is higher than the central density ($\rho_m = 7.16\rho_0$) of the maximum neutron stars. The critical density of quarks is shifted to 1.82 $\rho_0$ compared with the small value of the bag constant (i.e., $\rho_c = 1.06\rho_0$ for $B^{1/4} = 170$ MeV). In the high-density region the fractions of the HP increase and the fractions of the QP drop correspondingly, which is different from the case of $B^{1/4} = 170$ MeV. Therefore, these results imply that the compositions of neutron stars are sensitive to the bag constant.

The volume fraction of the QP as a function of baryon density in neutron stars is displayed in Fig. 3. The obtained results illustrate that the critical density for the QP increases with the increasing bag constant. If the value $B^{1/4} = 170$ or 175 MeV is taken, the pure quark core can appear in neutron stars. If the value of $B^{1/4}$ varies from 180 to 195 MeV, the curve for the volume fraction $\chi_Q$ of the QP first increases and then bends down and finally decreases with baryon density in the high-density region. As a result, the QP ceases at high densities and purely hadronic matter dominates. From Fig. 3, one can also observe that in the cases for the low values of the bag constant ($B^{1/4} \leqslant 175$MeV) the pure quark core is formed in neutron stars, whereas the MP may exist in the center of neutron stars for the large bag constant (i.e., 180 MeV $\leqslant B^{1/4} < 200$ MeV). In the latter case the region of the MP turns out to shrink further with the increasing of bag constant. If $B^{1/4} > 200$ MeV, no QP can be found in neutron stars. It should be noted that the surface properties and screening Coulomb effects of the QP are not included in the present study. The more realistic and complicated calculations indicate that those effects restrict the MP to a narrow density region [15,16].

Antikaons can condensate when Eqs. (2) and (3) are satisfied, this issue can also be discussed in detail by analyzing the behavior of the effective kaon energy in neutron stars. Figure 4 depicts the effective kaon energy as a function of

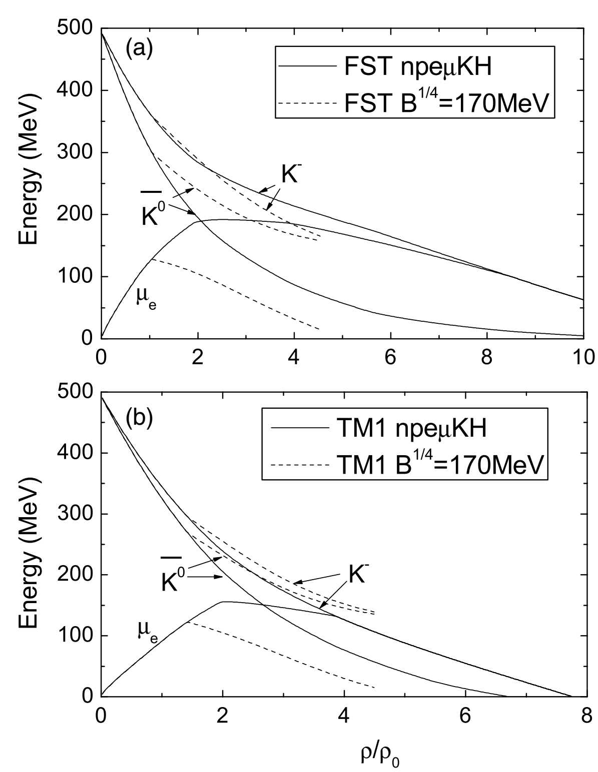


FIG. 4. The effective energy of antikaon versus the baryon density in neutron stars in the FST model with the parameter set T3 and in the nonlinear Walecka model with the parameter set Tm1.

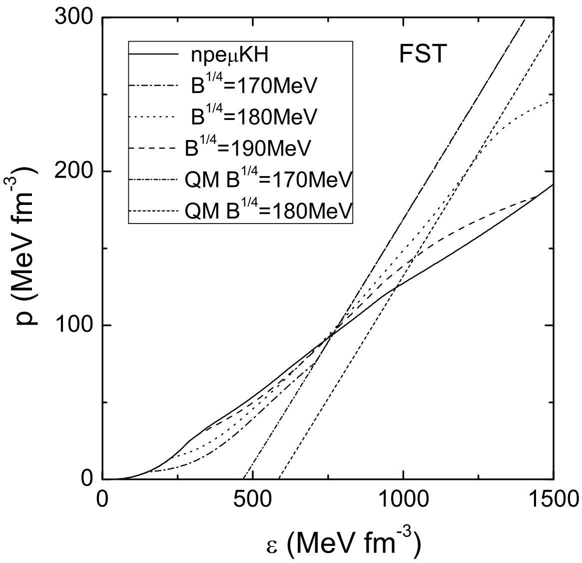


FIG. 5. Pressure $P$ as a function of energy density $\varepsilon$ in the FST model with the parameter set T3. The curves for the EOS with pure quark matter (QM) are also plotted.

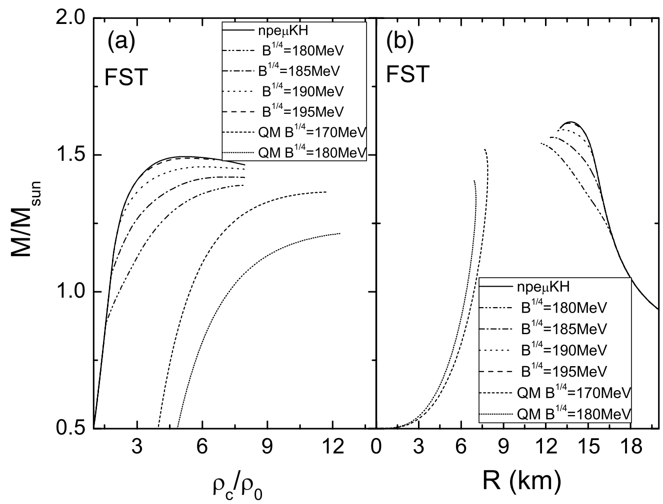


FIG. 6. The masses of neutron stars versus the baryon density (a), and the mass-radius relation (b) calculated in the FST model with the parameter set T3. The results given by the EOS with QM are presented for comparison.

baryon density in the FST model and in the nonlinear Walecka model. In the FST model without the QP [see Fig. 4(a)], the critical density of $K^-$ condensation is 8.5 $\rho_0$ (i.e., transverse axis value of the cross point for effective $K^-$ energy and electron chemical potential). However, $K^-$ condensation can not appear in neutron stars because its critical density is higher than the central density of neutron stars. It is seen that the inclusion of the QP makes $\bar{K}$ condensation impossible in neutron stars. The similar results are obtained in the nonlinear Walecka model [see Fig. 4(b)]. The critical densities of $K^-$ and $\bar{K}^0$ condensation are 3.88 $\rho_0$ and 6.72 $\rho_0$ in the case without the QP, respectively. Because the critical density of $\bar{K}^0$ condensation is higher than the central density of neutron stars, the $\bar{K}^0$ condensation does not occur in neutron stars either. It can be concluded that the inclusion of the QP makes $\bar{K}$ condensation impossible both in the nonlinear Walecka model and in the FST model.

The pressure as a function of energy density in neutron stars in the FST model is presented in Fig. 5. The kinks at the lower energy densities correspond to the appearance of quarks in neutron stars. These results make the corresponding curves lower than those without the QP (see solid line) in the lower-energy-density region. With the increase of energy densities those curves turn out to be higher than those without the QP in the higher-energy-density region. For comparison we also present the results given by the pure quark matter (QM). It is seen that the pressure increases linearly as the energy densities increase for the QM case. For small value of bag constant, i.e., $B^{1/4} = 170$ MeV, the curve of the equation of state (EOS) with the QP coincides with that of QM in the higher-energy-density region; this result means that the EOS of QM becomes dominant in this case (i.e., the appearance of pure quark core). However, for large value of bag constant, i.e., $B^{1/4} = 180$ MeV, the behaviors of the curves for the EOS with the QP and QM are quite different in the higher-energy-density region, this result implies that pure quark core does not occur in neutron stars for large bag constant. These results are consistent with those predicted by Fig. 3.

By integrating the Tolman-Oppenheimer-Vokoff equations together with the EOS of the FST model, the neutron star masses versus the baryon density and the mass-radius relation for neutron stars are shown in Fig. 6. It is found that the maximum masses of neutron stars with the QP are smaller than those without the QP. The maximum masses of neutron stars increase with increasing the bag constant. Whereas the maximum mass of quark stars decrease with the increase of bag constants. The results in the FST model predict that the maximum masses of neutron stars with the QP are lower than 1.5 $M_{\text{sun}}$.

### B. Conclusions

We have investigated antikaon condensation and the deconfinement phase transition from the HP to the QP in neutron stars. To describe the HP we use a chiral hadronic model called the FST model. The MIT bag model is adopt to describe the QP. The results obtained in the FST model are compared with those in the nonlinear Walecka model. The conclusions are that the QP makes antikaon condensation impossible in neutron stars, and the properties of neutron stars are sensitive to the bag constant. For the small bag constants (i.e., smaller than 180 MeV), the pure quark core exists in neutron stars and hyperons are suppressed by the presence of the QP in neutron stars. For the large bag constants (i.e., from 180 to 195 MeV), hyperons can appear in neutron stars. Moreover the volume fractions of the QP decrease with the increasing bag constant in the high-density region, in this case only the MP can exist in the center of neutron stars (i.e., no pure quark core). Moreover, the critical density of the QP is shifted to higher densities when the bag constant increases. The QP makes the maximum masses of neutron stars lower than those without QP. The maximum masses of neutron stars increase with the increasing bag constant, and the corresponding results are different from those for pure quark stars. It should be noted that the surface properties and screening Coulomb effects are neglected in the present calculations; those effects should be studied in our future program.


## ACKNOWLEDGMENTS

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