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Effects of non-causal artifacts in a hadronic rescattering model for Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV

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It has been shown that calculations based on our hadronic rescattering model agree reasonably well with experimental results from RHIC Au+Au collisions. Because of the large particle densities intrinsically present at the early time steps of Monte Carlo calculations attempting to model RHIC collisions undesirable artifacts resulting in non-causality may be present. The effects of such artifacts on observables calculated from the rescattering model are studied in the present work in two ways: (1) varying the time step and (2) using the subdivision method. It is shown that although non-causal artifacts are present in the rescattering model they have no appreciable effects on the calculated observables, thus strengthing the confidence in the results of this rescattering model for RHIC energies.

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I. INTRODUCTION

Calculations from our hadronic rescattering model have been shown to agree reasonably well with experimental results obtained from the Relativistic Heavy Ion Collider (RHIC) with $\sqrt{s} = 130$ GeV and 200 GeV Au+Au collisions [1–3]. More specifically, these calculations provide good representations of the data for (1) the particle mass dependence of the m_T distribution slope parameters (i.e., radial flow), (2) the p_T, particle mass, and pseudorapidity dependences of the elliptic flow, and (3) the p_T, centrality, and azimuthal angle dependences of two-pion Hanbury-Brown–Twiss (HBT) measurements. Although the agreement between the model and data is sometimes more qualitative than quantitative, this agreement with such a broad range of RHIC experimental observables is still a noteworthy accomplishment for a single, simple model.

Many hadrons are initially produced in a relatively small volume in RHIC-type collisions, particularly at early times in the interaction. It is a challenging task for a Monte Carlo calculation of this type to deal accurately with binary collisions between particles in such a large particle density environment without introducing numerical artifacts which may affect the results. One such undesirable artifact that can occur when the interaction range between two particles is much greater than the scattering mean-free-path is non-causality of the collisions resulting in superluminal particle velocities [4,5]. The method of subdivision can be used to minimize these artifacts associated with high particle density [6].

The effects of such artifacts on observables calculated from the rescattering model are studied in the present work in two ways: (1) varying the time step and (2) using the subdivision method. The goal will be to determine whether non-causal artifacts are present in the rescattering model and, if so, whether they have appreciable effects on the calculated observables. This will provide a test of whether or not one can have confidence in the results of this rescattering model for RHIC energies. Section II will describe the calculational methods used and Sec. III with give the results of the study.

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II. CALCULATIONAL METHODS

A. Hadronic rescattering calculation

A brief description of the rescattering model calculational method is given below. The method used is similar to that used in previous calculations for lower CERN Super Proton Synchrotron (SPS) energies [7]. Rescattering is simulated with a semiclassical Monte Carlo calculation which assumes strong binary collisions between hadrons. The Monte Carlo calculation is carried out in three stages: (1) initialization and hadronization, (2) rescattering and freeze out, and (3) calculation of experimental observables. Relativistic kinematics is used throughout. All calculations are made to simulate RHIC-energy Au+Au collisions.

The hadronization model employs simple parametrizations to describe the initial momenta and space-time of the hadrons similar to that used by Herrmann and Bertsch [8]. The initial momenta are assumed to follow a thermal transverse (perpendicular to the beam direction) momentum distribution for all particles,

$$\frac{1}{m_T}\frac{dN}{dm_T} = C\frac{m_T}{\exp\left(\frac{m_T}{T}\right) \pm 1}, \tag{1}$$

where $m_T = \sqrt{p_T{}^2 + m_0{}^2}$ is the transverse mass, p_T is the transverse momentum, m_0 is the particle rest mass, C is a normalization constant, and T is the initial "temperature parameter" of the system, and a Gaussian rapidity distribution for mesons,

$$\frac{dN}{dy} = D\exp\left[-\frac{(y-y_0)^2}{2\sigma_y{}^2}\right], \tag{2}$$

where $y = 0.5\ln[(E+p_z)/(E-p_z)]$ is the rapidity, E is the particle energy, p_z is the longitudinal (along the beam direction) momentum, D is a normalization constant, y_0 is the central rapidity value (midrapidity), and σ_y is the rapidity width. Two rapidity distributions for baryons have been tried: (1) flat and then falling off near beam rapidity and (2) peaked at central rapidity and falling off until beam rapidity. Both baryon distributions give about the same results. The initial space-time of the hadrons for $b = 0$ fm (i.e., zero impact parameter or central collisions) is parameterized as having cylindrical symmetry with respect to the beam axis. The transverse particle density dependence is assumed to be that of a projected uniform sphere of radius equal to the projectile radius, $R(R = r_0 A^{1/3}$, where $r_0 = 1.12$ fm and A is the atomic mass number of the projectile). For $b > 0$ (noncentral collisions) the transverse particle density is that of overlapping projected spheres whose centers are separated by a distance b. The particle multiplicities for $b > 0$ are scaled from the $b = 0$ values by the ratio of the overlap volume to the volume of the projectile. The longitudinal particle hadronization position (z_{had}) and time (t_{had}) are determined by the relativistic expressions [9],

$$\begin{aligned} z_{\text{had}} &= \tau_{\text{had}} \sinh y, \\ t_{\text{had}} &= \tau_{\text{had}} \cosh y, \end{aligned} \tag{3}$$

where y is the particle rapidity and τ_{had} is the hadronization proper time. Thus, apart from particle multiplicities, the hadronization model has three free parameters to extract from experiment: σ_y, T and τ_{had}. The hadrons included in the calculation are pions, kaons, nucleons and lambdas (π, K, N, and Λ), and the $\rho, \omega, \eta, \eta', \phi, \Delta$, and K^* resonances. For simplicity, the calculation is isospin averaged (e.g., no distinction is made among a π^+, π^0, and π^-). Resonances are present at hadronization and also can be produced as a result of rescattering. Initial resonance multiplicity fractions are taken from Herrmann and Bertsch [8], who extracted results from the HELIOS experiment [10]. The initial resonance fractions used in the present calculations are: $\eta/\pi = 0.05$, $\rho/\pi = 0.1$, $\rho/\omega = 3$, $\phi/(\rho + \omega) = 0.12$, $\eta'/\eta = K^*/\omega = 1$ and, for simplicity, $\Delta/N = 0$.

The second stage in the calculation is rescattering which finishes with the freeze-out and decay of all particles. Starting from the initial stage ($t = 0$ fm/c), the positions of all particles are allowed to evolve in time in small time steps (normally $\Delta t = 0.1$ fm/c is used) according to their initial momenta. At each time step each particle is checked to see (a) if it has hadronized yet, i.e., if $t \geqslant t_{\text{had}}$ [see Eq. (3)] and can begin participating in rescattering, (b) if it decays, and (c) if it is sufficiently close to another particle to scatter with it. The determination of (c) is carried out in the c.m. frame of the two particles in order to avoid the frame dependence of the result [11]. The conditions on the spacial and time separations of the particles i and j in the pair c.m. frame, $|\Delta \mathbf{r}_{\text{cm}}|_{ij}$ and $|\Delta t_{\text{cm}}|$ respectively, for a scattering between them to take place are

$$\begin{aligned} |\Delta \mathbf{r}_{\text{c.m.}}|_{ij} &< d_{ij}, \\ |\Delta t_{\text{c.m.}}| &< \Delta t_0, \end{aligned} \tag{4}$$

where $d_{ij} = \sqrt{\sigma_{ij}/\pi}$ and σ_{ij} is the scattering cross section for the particles, and $\Delta t_0 = 1$ fm/c is used to enforce some measure of simultaneity of the particles in the pair c.m. frame. Isospin-averaged s-wave and p-wave cross sections for meson scattering are obtained from Prakash *et al.* [12]. The calculation is carried out to 100 fm/c, although most of the rescattering finishes by about 30 fm/c. The rescattering calculation is described in more detail elsewhere [7].

Calculations are carried out assuming initial parameter values and particle multiplicities for each type of particle. In the last stage of the calculation, the freeze-out and decay momenta and space-times are used to produce observables such as pion, kaon, and nucleon multiplicities and transverse momentum and rapidity distributions. The values of the initial parameters of the calculation and multiplicities are constrained to give observables which agree with available measured hadronic observables. As a cross-check on this, the total kinetic energy from the calculation is determined and compared with the RHIC center of mass energy to see that they are in reasonable agreement. Particle multiplicities were estimated from the charged hadron multiplicity measurements of the RHIC PHOBOS experiment [13]. Calculations were carried out using isospin-summed events containing at freezeout for central collisions ($b = 0$ fm) about 5000 pions, 600 kaons, and 750 nucleons (Λ's were decayed). The hadronization model parameters used were $T = 300$ MeV, $\sigma_y = 2.4$, and $\tau_{\text{had}} = 1$ fm/c. It is interesing to note that the same value of τ_{had} was required in a previous rescattering calculation to successfully describe results from SPS Pb+Pb collisions [7].

As an indicator of the presence of superluminal artifacts, the transverse signal propagation velocity, v_{sT}, is calculated for all particle pairs in the rescattering calculation for each time step from [5]

$$\begin{aligned} \mathbf{v}_s &= \frac{\mathbf{r}_1(t_{\text{pres}}) - \mathbf{r}_2(t_{\text{prev}})}{t_{\text{pres}} - t_{\text{prev}}} \\ v_{sT} &= \sqrt{v_{sx}^2 + v_{sy}^2}, \end{aligned} \tag{5}$$

where $\mathbf{v}_s$ is the signal propagation velocity vector, $\mathbf{r}_1(t_{\text{pres}})$ is the position of particle 1 for the present collision with particle 2 at time t_{pres}, $\mathbf{r}_2(t_{\text{prev}})$ is the position of particle 2 for its previous collision at time t_{prev}, and v_{sx} and v_{sy} are the perpendicular components of $\mathbf{v}_s$ in the transverse plane. Note that even though the rescattering calculations are carried out with relativistic kinematics, it is still possible for v_{sT} to be greater than c for some particle pairs due to the details of how the two-particle collisions are implemented.

B. Subdivision method

The method of subdivision is based on the invariance of Monte Carlo particle-scattering calculations for a simultaneous decrease of the scattering cross sections by some factor, l, and increase of the particle density by l, where l is called the subdivision [4]. As l becomes sufficiently large, non-causal artifacts become insignificant. The present rescattering calculation will be tested comparing pion observables from the "nominal" calculation, i.e., $l = 1$, with subdivisions of $l = 5$ and $l = 8$. Note that a subdivision of $l = 5$ has been shown to significantly suppress non-causal artifacts present in some Monte Carlo calculations [4,5]. Since the particle density increase is accomplished by increasing the particle number by a factor l, the computer CPU time taken per event increases by a factor l^2. For the present study, 1640, 124,

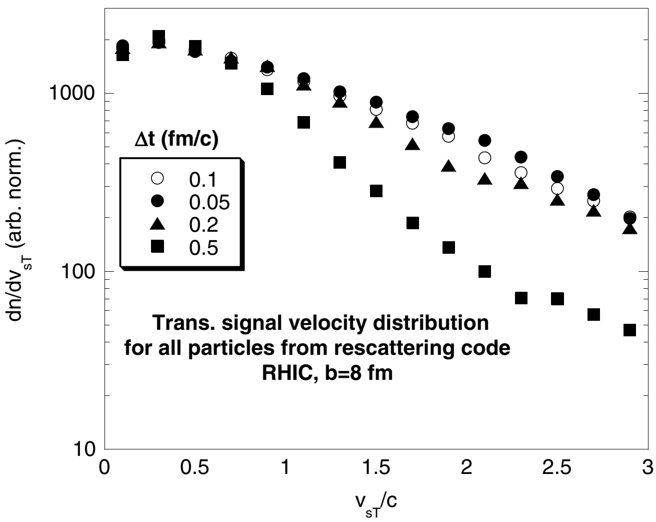

FIG. 1. Transverse signal velocity distributions for all particles for various time steps.

and 120 events were generated for the $l = 1$, $l = 5$, and $l = 8$ samples, respectively. Fortunately, the statistical value of the $l > 1$ events is l times greater per event than for $l = 1$. As an example, the CPU time taken to generate the $l = 8$ sample of events was 600 CPU-hours on 2.8 GHz PC processors at the Ohio Supercomputing Center.

III. RESULTS

Figures 1–9 present results from the hadronic rescattering model simulating hadrons produced in RHIC-energy collisions for various time steps and subdivisions. Quantities plotted are v_{sT} histograms including all particles in the calculation and several hadronic observables, i.e., transverse momentum distributions, elliptic flow, and HBT, to study the effects

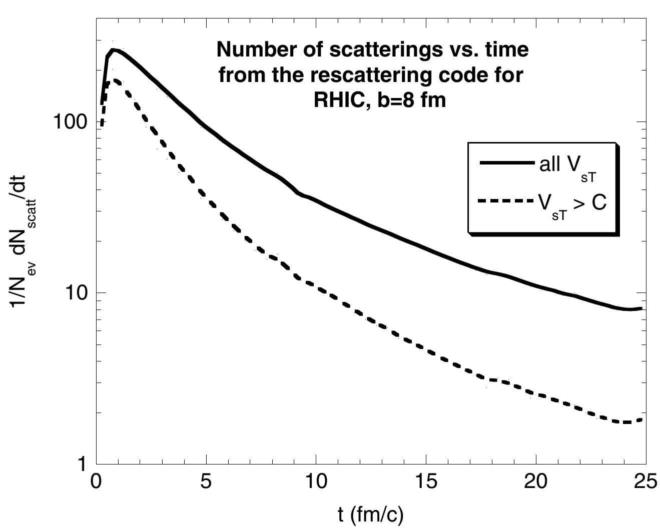

FIG. 2. Time dependence of the average v_{sT} from the rescattering model for the time step $\Delta t = 0.1$ fm/c for the cases (a) all scatterings and (b) superluminal scatterings, i.e., $v_{sT} > c$.

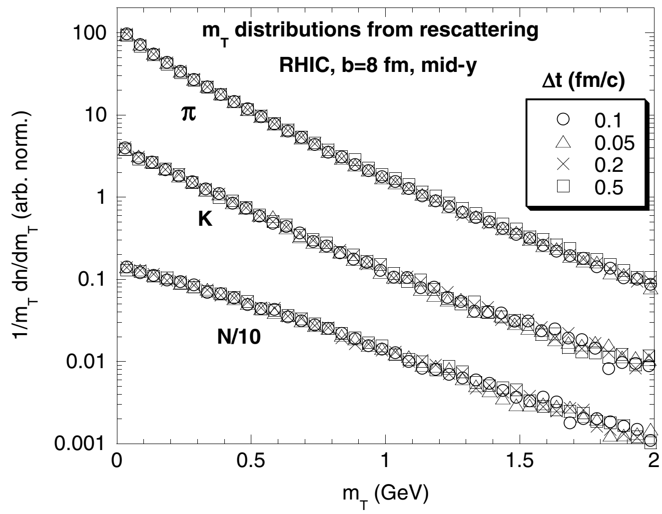

FIG. 3. Pion, kaon, and nucleon m_T distributions for various time steps.

of the different time steps and subdivisions. Descriptions of how the observables are extracted from the rescattering calculation are given elsewhere [2]. All calculations are carried out for an impact parameter of 8 fm to simulate a medium noncentral collision which should result in significant elliptic flow for the purposes of the present test. Observables are calculated in midrapidity bins of either $-2 < y < 2$ or, for HBT, $-1 < y < 1$. Statistical errors are shown either as error bars or are of the order of the marker size when error bars are not shown.

A. Time step study

Figures 1–5 show comparisons of various quantities for four different time steps in the rescattering calculations, $\Delta t = 0.1, 0.05, 0.2$ and 0.5 fm/c. The time step used in calculating observables which have been compared with RHIC data is

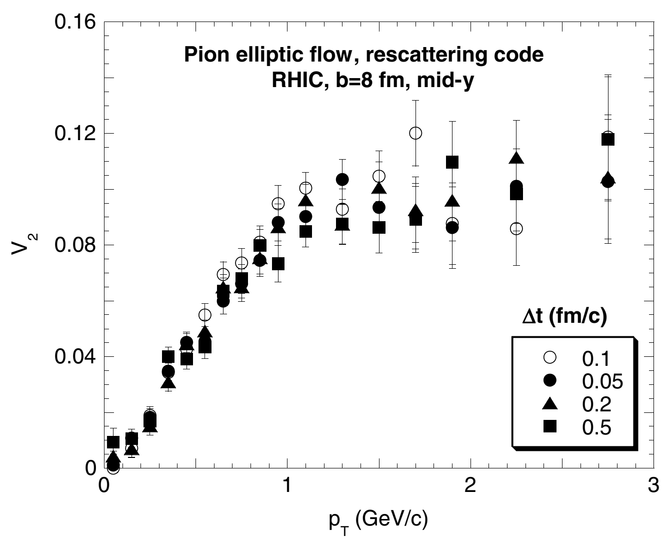

FIG. 4. Pion elliptic flow vs. p_T for various time steps.

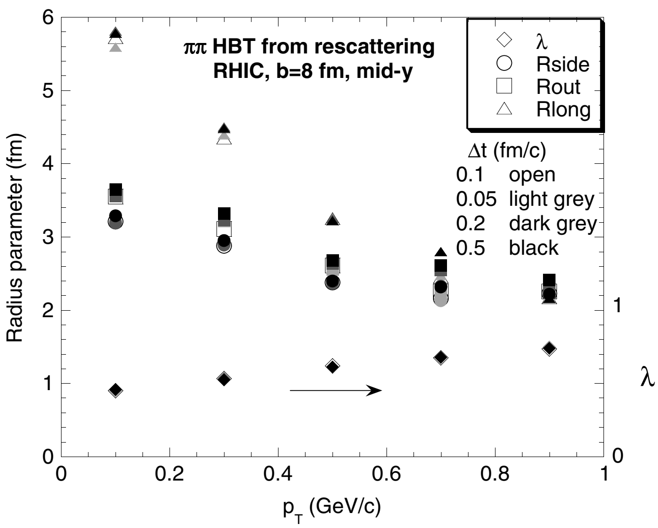

FIG. 5. Two-pion HBT source parameters vs. p_T for various time steps. The ordinate scale for λ is shown to the right.

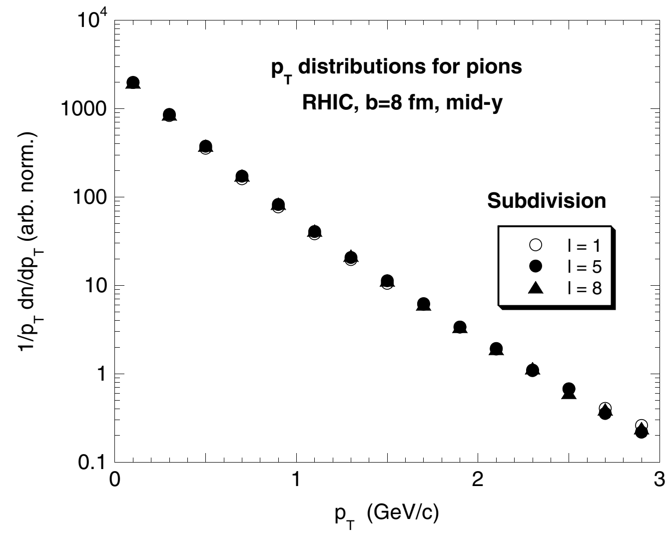

FIG. 7. Pion p_T distributions for $l = 1$, 5, and 8.

$\Delta t = 0.1$ fm/c [1–3], so two of the other time steps used represent half of the usual step and twice the usual step. The $\Delta t = 0.5$ fm/c time step is also interesting to study since when multiplied by c it represents the value of the average interaction range calculated from the rescattering code. Use of this time step might minimize superluminal effects [4,5], but since it is a factor of five larger than the nominal time step it potentially could also result in significant changes in the calculated observables. Figure 1 shows histograms of the quantity v_{sT}. It is seen that v_{sT} extends beyond c with an exponential tail for all time steps, indicating the presence of superluminal artifacts. Figure 2 shows the time dependence in the first 25 fm/c of the average v_{sT} from the rescattering model for the time step $\Delta t = 0.1$ fm/c for the cases (a) all scatterings and (b) superluminal scatterings, i.e., $v_{sT} > c$. Clearly the fraction of superluminal scatterings is greatest at earlier times, falling off dramatically for later times [note that the increase in the number of scatterings seen at early times is due to hadronization taking place via Eq. (3)]. Considering again Fig. 1, for $v_{sT} < c$ all time steps lie on top of each other whereas for $v_{sT} > c$ they differ according to what would be expected for the different time steps, i.e., smaller and thus more frequent time steps should enhance the superluminal artifacts, whereas larger and thus less frequent time steps should reduce the artifacts. For $\Delta t = 0.05$ fm/c an enhancement of $\sim 10-20\%$ in the artifacts over the nominal case is seen, while in the case of $\Delta t = 0.2$ fm/c a reduction in the artifacts by about 30% over the nominal time step is seen. For $\Delta t = 0.5$ fm/c a larger reduction, by a factor of four, occurs as expected.

The effects of these time steps on various hadronic observables are shown in Figs. 3–5. Figure 3 shows m_T

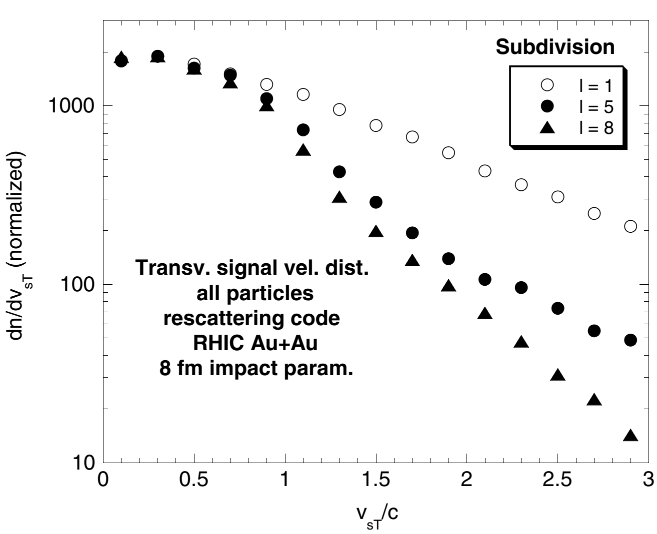

FIG. 6. Transverse signal velocity distributions for all particles for $l = 1$, 5, and 8.

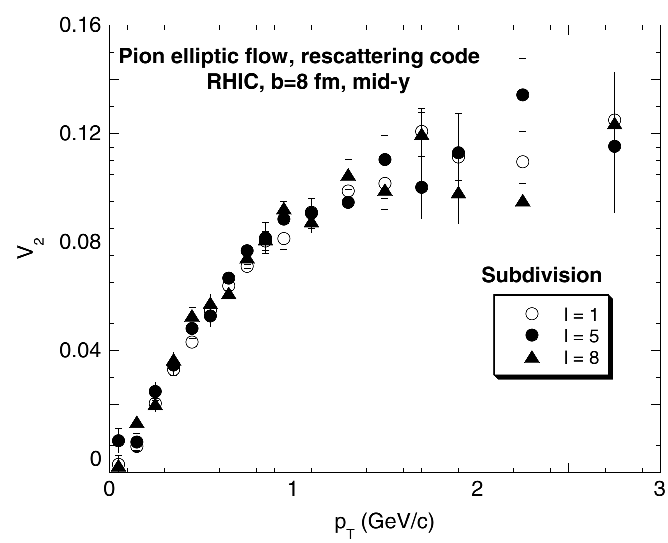

FIG. 8. Pion elliptic flow vs. p_T for $l = 1$, 5, and 8.

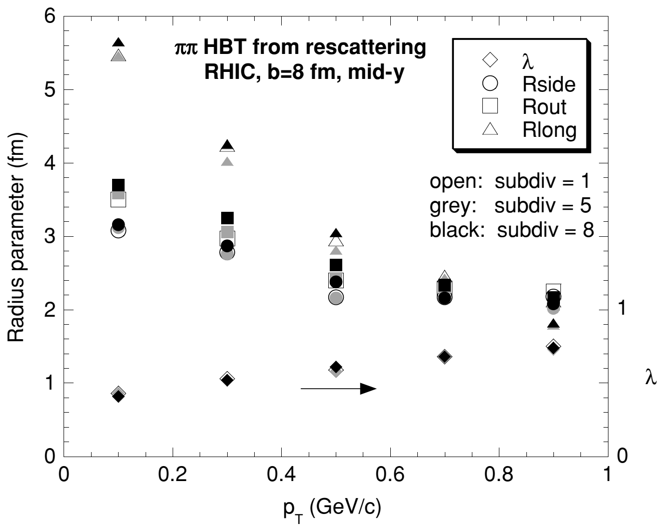

FIG. 9. Two-pion HBT source parameters vs. p_T for $l = 1, 5$, and 8. The ordinate scale for λ is shown to the right.

distributions for pions, kaons, and nucleons, Fig. 4 shows the elliptic flow, V_2, vs. p_T for pions and Fig. 5 shows two-pion HBT source parameters [3] vs. p_T. As seen in all three figures, even though the different time steps have noticeable effects on the artifacts, especially $\Delta t = 0.5$ fm/c (as seen in Fig. 1), they do not significantly effect the hadronic observables, the observables for all four time steps agreeing within the statistical errors of the calculations.

B. Subdivision study

A subdivision study is also carried out to complement the time step study and since it may provide a greater ability to reduce superluminal artifacts. This is seen in Fig. 6 which shows v_{sT} histograms for the three subdivision used: $l = 1, 5$, and 8. While all three subdivisions mostly lie on top of each other for $v_{sT} < c$, one sees factors of 5–10 reductions in the superluminal artifacts for $l = 5$ and 8. If these artifacts have a large influence on the values of the hadronic observables extracted from the rescattering model, it should surely be evident for the $l = 5$ and 8 cases. Figures 7–9 show the effects of these different subdivisions on pion p_T distributions, elliptic flow and HBT, respectively.

As was the case in the time step study, the hadronic observables shown in Figs. 7–9 are not significantly effected by the different subdivisions used in spite of the fact that superluminal artifacts are greatly suppressed in the subdivision study.

IV. DISCUSSION AND CONCLUSIONS

It can be concluded from Figs. 1–9 that non-causal (superluminal) artifacts while being present in the hadronic rescattering code used in this study do not significantly effect the hadronic observables calculated from this code. It is possible to speculate on why this is the case. There are three main features of the code which might tend to reduce the effects of these artifacts: (1) individual particles are allowed to scatter only once per time step, (2) a "scattering time" of two nominal time steps is defined during which particles that have scattered are not allowed to rescatter, and (3) once two particles have scattered with each other, they are not allowed to scatter with each other again in the calculation. The present study was performed for noncentral collisions ($b = 8$ fm), but it is expected that the same conclusion, i.e., that superluminal artifacts do not significantly effect the results of calculations with the rescattering code, would also be obtained for central collisions. This is because the particle density is not very different for central collisions in the model than for mid-peripheral collisions since the particle multiplicities are scaled by the overlap volume for $b > 0$, as mentioned earlier.

In summary, two methods have been used to test the validity of the present rescattering model calculations in the presence of non-causal (superluminal) artifacts. It is found that the results of this model are not appreciably effected by such artifacts, thus strengthing the confidence in the results presented previously from this rescattering model for RHIC.

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