Integer ratios in E_x/S_n observed in the resonances of light nuclei

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The ratio of excitation energies E_x and neutron separation energy S_n for neutron resonances of $^{16}O + n$ are observed to be ratios of simple integers. Similar analyses are made for the resonances of light target nuclei, ^{3}He, $^{6,7}Li$, $^{8,9}Be$, $^{10,11}B$, $^{12,13}C$, $^{14,15}N$, $^{16,18}O$ showing that $E_x/S_n = 4/3$ or 5/3 are values that appear in many nuclei. A statistical test shows that these integer ratios do not exist by accident but that they have a physical origin. A classical model of neutron resonance reaction is developed relating these integer ratios, based on the breathing of the compound nucleus that is coherent with the incident neutron wave. Origin of integer ratios among E_x, S_n of different nuclei are discussed based on a constant G value and integer splitting for many nuclei.

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I. INTRODUCTION

The complicated and strong interactions among nucleons in a nucleus make the description of excitation levels of the nucleus a controversial problem. In the lower excitation region, E_x, $J\pi$ are measured with sufficient accuracy, and they can be predicted by the shell model assuming interaction potentials. In the region above the particle separation energy, where the level density is quite high, E_x, Γ, $J\pi$ are measured by resonance reactions using particle beams. In particular in neutron reactions, these resonances correspond to quasistable states of the compound nucleus (CN) above the neutron separation energy $E_x \sim 8$ MeV, where many degrees of freedom will be excited and mixed to form very complicated structures. Therefore, neutron resonances are surmised to form a quantum chaos. In fact, statistical properties of the observed neutron resonance data are in good agreement with the predictions of random matrix theory (RMT): Wigner (GOE) distribution for nearest-neighbor-level spacings, Porter-Thomas distribution for strengths, and Δ_3 statistics for long-range correlations.

However, several properties are observed that contradict the predictions of RMT. One of them is the crystalline-like structure of resonance positions. Using a Fourier-like analysis and D_{ij} (spacings between two arbitrary levels) distributions, several special level spacings can be found (hereafter called dominant spacings) that appear more frequently than expected from GOE in the energy region considered. The ratio of several of the dominant spacings of many nuclei are integer ratios, which suggests the existence of some constant value widespread among the nuclei. These position/spacing correlations are widely found in neutron resonances of many nuclei over a wide mass region, with large deviations from GOE distributions of the levels [1–7]. These periodic positioning of the neutron resonance levels will be deeply related to the resonance reaction mechanism, and we have developed a classical model of CN [8,9].

Moreover, integer ratios between E_x and S_n are found for many of $^{16}O + n$ resonances up to $E_x \leqslant 10$ MeV, and a statistical test shows that this is not a coincidence [10]. Similar integer ratios are observed for the resonance energy levels of $^{3}He + n$, $^{6,7}Li + n$, $^{8,9}Be + n$, $^{10,11}B + n$, $^{12,13}C + n$, $^{14,15}N + n$, and $^{18}O + n$. Interestingly, specific values of $E_x/S_n = 4/3$ or 5/3 appear in many of these nuclei. A statistical test shows that this is not a coincidence but that they have a physical origin.

To propose an interpretation of these integer rations, a classical model of resonance reaction is discussed that considers a time periodic breathing of the compound nucleus that is coherent with the incident neutron wave. Observed integer ratios in E_x, S_n among different nuclei are considered to be because of a constant origin coupled with integer splitting in many nuclei.

II. INTEGER RATIOS IN E_x/S_n FOR $^{16}O + n$ RESONANCES

In the course of our level position/spacing investigation, we have found simple integer ratios between E_x and S_n and E_n (neutron energy in center-of-mass system) for many of $^{16}O + n$ resonances, where $S_n = 4143.3$ keV. For examples, for the first resonance at $E_x = 4551.9$ keV, $E_x/S_n = (11/10) \times 0.9987$, for the second resonances at 5084.2 keV, $E_x/S_n = (16/13) \times 0.9970$, for the third resonance at 5375.1 keV, $E_x/S_n = (13/10) \times 0.9979$, and so on, as shown in Table I. The original parameters of 37 resonances with $E_x \leqslant 10$ MeV are from Sayer *et al.* [11]. When E_x/S_n is close to an integer ratio n/m, the deviation Δ from 1 is defined as $\Delta = [(E_x/S_n)/(n/m)] - 1$. Logically, by using arbitrary large values of m and n, Δ can be made to take an arbitrary small value. However, it must be stressed that even for relatively small values of m (for example $m \leqslant 13$), the values of Δ are appreciably small for these resonances; the values of Δ gather around 0 within a width of 0.52%, as shown in Table I and Fig. 1.

The question of the origin of these integer ratios arises as their occurrence could be purely coincidental or be because of some regular feature. To answer this question, we have calculated the probability of appearance of integer ratios $x = n/m$ ($m \leqslant 13$) for these resonances assuming a random distribution of excitation energies $E_x = S_n + E_n$ in the region.

To minimize end effects, the region near S_n and $2S_n$ are excluded. The probability calculation is made for resonances in the range of $\sim 5.0 \leqslant E_x \leqslant 7.5$ MeV ($E_x/S_n 1.20 \sim 1.81$), in which 14 resonances are observed. We regard

TABLE I. Integer ratios E_x/S_n for resonances of $^{16}O + n$ up to $E_x \leqslant 8$ MeV. $S_n = 4143.3$ keV. Deviation Δ is defined as $\Delta = \{(E_x/S_n)/(n/m)\} - 1$, expressed in percentages. The resonances marked A fulfill the criteria: $5.0 \leqslant E_x \leqslant 7.5$ MeV, ($E_x/S_n = n/m$, $m \leqslant 13$) and Δ in a width 0.52%.

j	$J\pi$	E_x (keV)	E_n (keV)	E_n^* (keV)	E_x/S_n	Δ (%)	
1	3/2−	4551.9[a]	434.3	408.5	11/10	−0.13	
2	3/2+	5084.2	1000.2	940.8	16/13	−0.30	A
3	3/2−	5375.1	1309.4	1231.7	13/10	−0.21	A
4	7/2−	5696.7	1651.4	1553.4	11/8	−0.01	A
5	5/2−	5732.3	1689.1	1588.8	18/13	−0.08	A
6	3/2+	5868.7	1834.1	1725.2	17/12	−0.02	A
7	1/2−	5932.0	1901.4	1788.5	10/7	0.22	A
8	1/2+	6380.2	2377.9	2236.7	20/13	0.09	A
9	5/2+	6860.7[b]	2888.7	2717.2	5/3	−0.65	
10	7/2−	6971.9	3006.9	2828.4	32/19	−0.09	
11	5/2−	7164.6	3211.8	3021.1	19/11	0.11	A
12	3/2+	7239.1	3291.0	3095.6	7/4	−0.16	A
13	5/2+	7378.2	3438.8	3234.7	16/9	0.17	A
14	5/2−	7380.8[c]	3441.5	3237.3			
15	3/2−	7446.9	3511.9	3303.5	9/5	−0.15	A
16	7/2−	7686.9[a]	3767.0	3543.4	13/7	−0.10	
17	1/2−	7896.3[a]	3989.6	3752.8	40/21	0.05	
18	1/2+	7963.3[a]	4060.8	3819.8	25/13	−0.06	

[a]Excluded to minimize end effects in probability calculations.
[b]Excluded because of a large deviation of E_x/S_n from 1.0.
[c]Excluded because of a too small difference of E_x with the previous one.

two resonances with very small energy difference (at $E_x =$ 7378.2 and 7380.8 keV) as one resonance for the probability calculation. Integer ratios with a denominator less than 13($E_x/S_n = n/m$, $m \leqslant 13$) are found in 11 resonances of 13, within a width 0.52%, as shown in Table I and Fig. 1. In the region there are 36 rational points with an average width of 32.5 keV. For a random sampling in this region, the probability p to be on a rational point is estimated as $p = 36 \times 32.5/(7500 - 5000) = 0.468$. The expected number of occurrences is $13 \times 0.468 = 6.1$. The probability of 11 occurrences in 13 trials is estimated by the binomial distribution $B(11; 13, p) = {}_{13}C_{11}\,p^{11}(1 - p)^2 = 0.51 \times 10^{-2}$, and $\sum_{k=11}^{13} B(k; 13, p) = 0.6 \times 10^{-2}$.

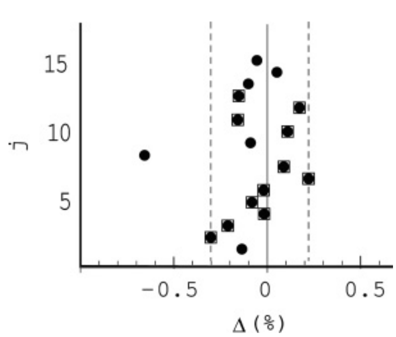

FIG. 1. Deviation from integer ratios $\Delta = \{(E_x/S_n)/(n/m)\}$-1, expressed in percentages for the resonances in Table I. (Abscissa) Δ. (Ordinate) Order of resonance j in Table I, where $j = 14$ is skipped. The squares correspond to the resonances labeled A in Table I. The dash lines show the boundary of the width 0.52%.

Though the sample number is small, the hypothesis of a random distribution of level dispositions is questionable with a statistical significant level of 1%. It is inferred that the resonance level position is not at random but that it occurs at preferred points where E_x/S_n or E_x/E_n are integer ratios. A possible mechanism for this process is discussed in Sec. IV.

III. INTEGER RATIOS IN E_x/S_n FOR THE RESONANCES OF LIGHT NUCLEI

Similarly to what has been done for $^{16}O + n$, integer ratios are examined for the resonance levels of light nuclei where the level density is not so high. These levels include levels measured not only by neutron reactions but also by other particle reactions or by the decay of γ rays. The target nuclei are ^{3}He, ^{6,7}Li, ^{8,9}Be, ^{11,12}B, ^{12,13}C, ^{14,15}N, ^{16,18}O and excitation energies are taken from [12,13]. Several of the calculated ratios of E_x/S_n represented by n/m are simple integer ratios, where m and n are small integers. Large m and n will be less important. For more clarity, we plotted in Fig. 2 a histogram on the (m,n) plane representing the sum of the nuclei for which a given ratio occurs in the energy region $S_n \leqslant E_x \leqslant 2S_n$. It is worth noting that $n/m = 4/3$, 5/3, 8/7, 11/8, 10/9, etc., appear in many nuclei. The simple cases of $n/m = 4/3$ and 5/3, which are the most frequently appearing ones are given in Table II, with compound nuclei, $J\pi$, E_x, S_n, n/m, Δ, N, 1-$B(0, N, p)$, and their category. Deviations $|\Delta|$ are almost within 0.55% except for ^{17}O which belong to category B.

Again, the question arises of the origin of the existence of these peaks at 4/3 or 5/3 and whether it is accidential. To answer this question, probability calculations are made assuming a simple model, in which N resonance levels are randomly distributed in the energy region from S_n to $2S_n$. For one resonance level, the probability p to occupy the channels at $(4/3)S_n$ or $(5/3)S_n$ within $\pm 0.55\%$ accuracy, is $p = [(4/3) + (5/3)] \times 0.011 = 0.033$. For N resonance levels, the probability to occupy k levels at these channels is calculated by the Binomial distribution: $B(k; N, p) = {}_N C_k\, p^k (1 - p)^{N-k}$, ($k = 0, 1, 2 \ldots N$). For our discussion of the probability, $\sum_k^N B(k, N, p)$ is always used, which is 1-$B(0, N, p)$ for $k = 1$. For ^{8}Li, ^{10}Be, and ^{16}N, no levels are observed for which $E_x/S_n = 4/3$ or 5/3, whereas for ^{11}B

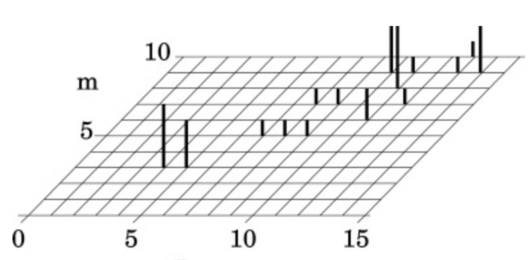

FIG. 2. Two-dimensional histogram of E_x/S_n ratios represented by integer ratios (n/m), within errors $|\Delta| \leqslant 0.0055$ for nuclei of category A and C in Table II.

TABLE II. Light nuclei examined. For resonances at $E_x/S_n = 4/3$ or 5/3, E_x, E_X/S_n, Δ, the number of levels N between $S_n \sim 2S_n$, the probability of occurrence 1-$B(0, N, p)$, and the category are shown. Category A: level observed at 4/3 or 5/3; category B: excluded because of large N; category C: level not observed at 4/3 or 5/3.

j	Compound Nuclei	$J\pi$	E_x (keV)	S_n (keV)	E_x/S_n 4/3	5/3	Δ (%)	N	Probability 1-$B(0, N, p)$	Category
1	^{4}He		27500	20577	•		0.23	7	0.215	A
2	^{7}Li	7/2−	9670	7250	•		0.03	5	0.154	A
3	^{8}Li			2032				4	0.126	C
4	^{9}Be	1/2−	2780	1665		•	0.15	4	0.126	A
5	^{10}Be			6812				6	0.182	C
6	^{11}B		15290	11454	•		0.12	19	(0.471)	B
			19130	11454		•	0.21	19	(0.471)	B
7	^{12}B	4−	4518	3370	•		0.53	9	0.261	A
		3+	5612	3370		•	−0.08	9	0.261	A
8	^{13}C	3/2+	8200	4946		•	−0.53	7	0.209	A
9	^{14}C	2−	13700	8176		•	0.53	24	(0.553)	B
10	^{15}N		14400	10833	•		−0.31	75	(0.919)	B
11	^{16}N			2489				7	0.209	C
12	^{17}O	5/2+	6860	4143		•	−0.65	21	(0.505)	B
13	^{19}O	3/2−	5300	3955	•		0.49	12	0.331	A

and ^{12}B, levels are observed for both values, $E_x/S_n = 4/3$ and 5/3.

The probabilities of occurrence of more than one level in these two channels 1-$B(0, N, p)$ are shown in Table II. For nuclides for which N is more than about 20, the probability of occupation of these channels by a level is very high ($N \times 0.03 \sim 1$), therefore no judgment can be made on the problem of their accidental or predictable origin. So we exclude four nuclides with large N (category B; ^{10}B, ^{13}C, ^{14}N, and ^{16}O) in further discussions. Among the remaining ones (category A and C), ^{11}B levels are observed at both points $E_x/S_n = 4/3$ and 5/3 on the histogram. To make an equal treatment of ^{11}B in the probability calculation, we treat ^{11}B as two virtual nuclei, one with a level such that $E_x/S_n = 4/3$, and the other one for 5/3 with the same probability 1-$B(0, N, p) = 0.261$. Thus the total number of nuclide increases to 10 and the probabilities ranges from a minimum of 0.126 to a maximum of 0.331 with an arithmetic average of 0.207.

For the sake of simplicity, we assume that the probability is equal to the average value 0.207 for all 10 nuclei. If the occurrence is accidental, it will be expected in $10 \times 0.207 \sim 2$ nuclides. In facts, it occurs in 7 of 10 nuclides. The probability of occurrence of such a fact is calculated by the Binomial distribution to be 1.1×10^{-3}. This value is sufficiently small to rule out the assumption of random occurrence of these levels. Though the number of samples is small for statistics, we can still deduce that the resonance levles distibute not at random but at preferred positions corresponding to integer ratios $E_x/S_n = 4/3$ or 5/3 because of some physical reason.

However, these resonances seem to have no fixed $J\pi$, as shown in Table II, therefore common excitation mode such as collective type will not be expected for these resonances. It is infered that E_x are generally determined by nuclear potential interactions preferentially locate at nearby points where E_x/S_n are simple integer ratios.

A possible explanations of these integer ratios, breathing of the CN coherent with the incident wave is described in the following section.

IV. BREATHING OF THE COMPOUND NUCLEUS

Integer ratios in E_x/S_n or E_x/E_n described above will be related to a simple mechanism of the resonance reactions, the breathing of the compound nucleus (CN) that is time coherent with the incident neutron wave [10]. We assume that time periodic behaviors of CN can be decomposed into several normal modes with harmonic frequencies.

An S-matrix $S(E)$ is defined for the neutron-nucleus reaction from which the cross section $\sigma_s(E) = (\pi/k^2)(2l + 1)|1 - S(E)|^2$, etc., is determined. A relation between $S(E)$ and the response function is discussed after Sitenko [14].

For an s-wave resonance, the incident wave $\psi^-(r, t)$ and outgoing wave $\psi^+(r, t)$ around the interaction region of radius R are,

$$\psi^-(r, t) = \int_0^\infty dE' a(E')(1/r) \exp[-ik'r - (i/\hbar)E't], \quad (1)$$

$$\psi^+(r, t) = \int_0^\infty dE' a(E') S(E')(1/r) \exp[ik'r - (i/\hbar)E't]. \quad (2)$$

The response function $F(\tau)$ is defined by the causality principle, as,

$$\psi^+(r, t) = \int_0^\infty d\tau F(\tau)\psi^-(r, t - \tau). \quad (3)$$

Multiplying Eqs. (1)–(3) by $\exp(-iEt/\hbar)$ and integrating over t from $-\infty$ to ∞, $S(E)$ can be expressed as a Fourier transform of the response function $F(\tau)$,

$$S(E)e^{2ikR} = \int_0^\infty d\tau F(\tau) e^{i\frac{E\tau}{\hbar}}, \tag{4}$$

where τ is the time for the response to come back.

Equation (4) is a basic relation on which the following discussions stand. In the continuum region, $S(E)$ has no peak, and $F(\tau)$ is expected to be a nonperiodic or stochastic function of infinitely long time period. In contrast for an isolated resonance at E_0, $S(E)$ has a peak at E_0 (recoil corrected). That is, the scattered wave $\psi^+(r,t)$ is significant if $F(\tau)$ and the incident wave $\psi^-(r,t)$ are time coherent with each other, during the lifetime $\sim h/\Gamma$, where Γ is the total width of the resonance. Off resonance, only potential scattering remains. At the resonance, $F(\tau)$ must be a periodic function with a period $\tau_{\rm rec} = 2\pi\hbar/E_0$, or more generally $\tau_{\rm rec} = (l/m)(2\pi\hbar/E_0)$, where l and m: are small integers.

At the resonance, the response function $F(\tau)$ can be expressed by a Fourier series with higher harmonics of periods $\tau_j = \tau_{\rm rec}/k_j$ and frequencies $\omega_j = k_j(m/l)E_0/\hbar$, where k_j are integers $(j = 1, 2, \ldots M)$. M is the number of harmonics which are considered to be equivalent to the degrees of freedom excited at the resonance. Frequency ratios as well as time periods of these higher harmonics are commensurable (forming integer ratios) with each other. A unit time τ_0 exists as the greatest common divisor (GCD) of $\tau_j (j : 1, 2, \ldots M)$, and τ_j is written as $n_j\tau_0$, where n_j is an integer. The recurrence time $\tau_{\rm rec}$ is the least common multiple (LCM) for the ensemble $(n_j; j = 1, 2, \ldots, M)$ multiplied by τ_0. The frequency component ω_j is propotional to the inverse integers $\omega_j = (2\pi/\tau_0)/n_j$.

The total excitation energy $E_x = S_n + E_0$ divided into these harmonics is,

$$\begin{aligned} E_x &= \hbar(\omega_1 + \omega_2 + \cdots + \omega_M) \\ &= (k_1 + k_2 + \cdots + k_M)(m/l)E_0, \end{aligned} \tag{5a}$$

which is also written as

$$= \frac{2\pi\hbar}{\tau_0}\sum_{j=1}^{M}\frac{1}{n_j} = G\sum_{j=1}^{M}\frac{1}{n_j}\ (n_j\text{: integer}), \tag{5b}$$

where $G = 2\pi\hbar/\tau_0$. [For multiple excitation, the numerators 1 in Eq. (5b) will be replaced by small integers a_j.]

Then, the ratio E_x/E_0 is an integer ratio,

$$\begin{aligned} \frac{E_x}{E_0} &= (m/l)(k_1 + k_2 + \cdots + k_M) \\ &= \frac{m}{l}\,\mathrm{LCM}(n_1, n_2, \ldots, n_M)\sum_{j=1}^{M}\frac{1}{n_j}. \end{aligned} \tag{6}$$

For a set of different prime numbers $n_1, n_2, \ldots, n_M$, $\mathrm{LCM}(n_1, n_2, \ldots, n_M) = \prod_{j=1}^{M} n_j$. The right-hand side of Eq. (6) is $(m/l)(n_1 + n_2)$ for $M = 2$ and $(m/l)(n_1n_2 + n_2n_3 + n_3n_1)$ for $M = 3$, and so on.

Therefore, integer ratios exist among E_x, E_0, and $S_n (= E_x - E_0)$. This will be the reason for the observation of integer ratios in E_x/S_n for resonances of many light nuclei in a several-hundred keV or MeV region, as described in the previous sections. It is considered that these integer ratios will exist for resonances down to the eV region of medium and heavy nuclei, where the ratios E_x/E_0 increase to $\sim 10^{4\sim 7}$ because of large M and large $\mathrm{LCM}(n_1, n_2, \ldots, n_M)$. However, in the eV region, confirming Eq. (6) by use of observed data will be difficult because of the large values of E_x/E_0.

These normal modes will be fast deformations or particles in orbit with definite frequencies. The average energies of these normal modes are $E_x/M \sim 1$ MeV for medium and heavy nuclei ($M = 6 \sim 10$) [8]. Nevertheless, the response function $F(\tau)$ behaves like a pulse array with a pulse separation $\tau_{\rm rec}$, like intermittent pulses, and like breathing of the CN. During the pulse, nuclear potential deformation becomes larger so that the neutron wave penetrates easily through the nuclear surface, and the interference takes place between the passing component and the trapped component of the incident wave. This behavior is similar to a time window that opens periodically with $\tau_{\rm rec}$. For incident neutron of off-resonance energy, the compound nucleus does not respond and stays quiet and only the potential scattering cross section is observed. By varying the incident neutron energy, sets of normal modes will be excited as resonance reactions if the sets obey Eq. (6).

For proton-induced reactions, similar normal modes will be excited and the integer ratios among E_x/S_p are observed.

The reaction mechanism of breathing model of CN is essentially different from that of the prevailing doorway-state model, where the initially excited one-particle/one-hole state $(1p\text{-}1h)$ progresses to $(2p\text{-}2h), \ldots$, to $(np\text{-}nh)$ final compound states, leading to internal mixing as a one-way street. In the scenario, the recurrence of the CN is not included, which is essentially important in resonance reactions.

V. ORIGIN OF INTEGER RATIOS AMONG DIFFERENT NUCLEI

In addition to the integer ratios described above, it is remarked that unexpected simple integer ratios are found among S_n in Table II; $S_n(^4\mathrm{He})/S_n(^{11}\mathrm{B}) = 9/5$, $S_n(^{10}\mathrm{Be})/S_n(^{14}\mathrm{C}) = 5/6$, $S_n(^{16}\mathrm{N})/S_n(^{17}\mathrm{O}) = 3/5$, $S_n(^9\mathrm{Be})/S_n(^{17}\mathrm{O}) = 2/5$, etc., with sufficient accuracy. Though statistical test is not made, this facts will not be a matter of chance but because of some physical reason.

We have encountered many cases of integer ratios among E_x, E_n, S_n, D_0, including different nuclei as described above and in Refs. [1–7]. Therefore we are inclined to think that there will be some constant value at the origin and some mechanism of integer splitting or branching with hierarchy structures that is exerted depending on the situation of each nucleus. If we accept these facts, the "breathing model" tells us that G in Eq. (5b) is a common value for many nuclei. Through the analyses of experimental data, we obtained a preliminary value $G = 34.5$ MeV [15] with deviations of $\sim$1% depending on the excitation levels. Many of the observed excitation energies E_x and S_n are found to form simple integer ratios multiplied by G. For example, S_n and E_x in Table II are simply written as $S_n(^4\mathrm{He}) = (3/5)G$, $S_n(^{11}\mathrm{B}) = (1/3)G$, $S_n(^{17}\mathrm{O}) = (3/25)G$, etc., and $E_x(^4\mathrm{He} : 27500) = (4/5)G$, $E_x(^{11}\mathrm{B}: 15290) = (4/9)G$, $E_x(^{17}\mathrm{O}: 6860) = (1/5)G$, etc. This will be the true reason of integer ratios 4/3 or 5/3 in E_x/S_n described in this article. The framework here seems to be useful to classify the E_x and S_n, though the theoretical foundation is not concrete yet.

We briefly comment on the physical meaning of $G = 34.5$ MeV and τ_0. The value $G = 34.5$ MeV derived as a parent energy of many excitation energies is almost equal to the observed Fermi energy (the maximum energy of a nucleon trapped in nuclear potential) for light nuclei.

As for $\tau_0 = 2\pi\hbar/G = 1.20 \times 10^{-22}s = 36$ fm$/c$, we think that it is the minimum unit time required to transfer the response in the normal modes excited in CN. It is noted that τ_0 is almost equivalent to the measured evaporation time $30 \sim 50$(fm$/c$), with which nucleons come out from hot compound nuclei in heavy-ion collisions at high energy.

The recurrence times of M normal modes excited in parallel in CN are $n_1\tau_0, n_2\tau_0, \ldots, n_M\tau_0$, and the global recurrence time is $\mathrm{LCM}(n_1, n_2, \ldots, n_M) \times \tau_0$, as described in Sec. IV. If τ_0 keeps a constant value for many resonances of different nuclei, integer ratios will exist among these E_x. However, τ_0 will deviate slightly depending on the situations of the states, which causes the deviation observed in G.

Analyses of observed data and several considerations using the breathing model and G are reported in Refs. [9,10,16,17].

VI. SUMMARY AND DISCUSSIONS

Integer ratios in E_x/S_n and E_n/S_n are found in many resonances of $^{16}\mathrm{O} + n$. Assuming these levels are distributed at random, the probability of occurrence at integer ratios is calculated to be 1×10^{-2}. For light compound nuclei, ^{4}He, ^{7,8}Li, ^{9,10}Be, ^{11,12}B, ^{13,14}C, ^{15,16}N, ^{17,19}O, the observed resonance levels of many nuclei are situated at positions corresponding to integer ratios in E_x/S_n and in particular at $E_x/S_n = 4/3$ or 5/3. The probability of occurrence at these points is calculated to be $\sim 10^{-3}$, assuming a random distribution of levels for each nucleus. Despite the small number statistics, it is inferred that E_x, which are generally determined by nuclear potential interactions preferentially locate at nearby points where E_x/S_n are simple integer ratios.

To understand these integer ratios, we have developed a classical model of resonance reactions called "breathing of the CN coherent with the incident wave," where time-periodic behaviors are explicitly considered. The integer ratios in E_x/S_n and E_x/E_n for a nucleus come out of the model, where normal modes with higher harmonic frequencies are assumed.

To understand widely observed integer ratios among E_x and S_n of different nuclei, it is inferred that G in the breathing model must be a constant value for many nuclei. A preliminary value $G = 34.5$ MeV is obtained through the analyses of dominant spacings of different nuclei. S_n and E_x in Table II are found to form simple integer ratios multiplied by G, and the meaning of the 4/3 or 5/3 ratios in E_x/S_n becomes evident.

The physical meanings of $G = 34.5$ MeV and $\tau_0 = 1.20 \times 10^{-22}\,s$ are discussed in relation with the Fermi energy and the time unit required to transfer the response in the nucleus. Properties of the normal modes excited on each resonance (quasistable states) and the selection rules for transitions, integer splitting mechanisms with hierarchy structures will be a problem to consider in the future.

To understand the precise mechanism of fine structures resonances, including $J\pi$, strengths, and level clusters in a wide energy region, a more sophisticated approach will be needed, including non-linear Schrödinger equation in nuclear potential. There will be an interesting field of nuclear physics in the 21st century where dynamic behaviors of the many degrees of freedom are essentially important. Neutron resonances may be a clue to this problem. On the experimental side, however, further high-resolution measurements on E_x are needed.

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