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Normal and superdeformed high-spin structures in ^{161}Lu

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High-spin states in ^{161}Lu were populated in the ^{139}La(^{28}Si, 6n) reaction and γ-ray coincidences were measured with the EUROBALL spectrometer. On the basis of these data, the previously known level scheme is extended with new band structures and is partly revised. Configuration assignments are made to all bands based on comparison of experimental properties with cranked shell model calculations. A strongly populated band with parity and signature $(\pi, \alpha) = (+, -1/2)$ is found to be yrast above spin $I \simeq 33$. This band shows characteristics resembling those of two triaxial superdeformed bands in this nucleus based on the occupation of the shape-driving $i_{13/2}$ proton orbital. This structure, unique to ^{161}Lu within the chain of even-N Lu isotopes, is discussed in terms of a quasiparticle configuration in a local triaxial minimum with a larger triaxiality and a smaller quadrupole deformation than calculated for the $i_{13/2}$ proton excitation.

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I. INTRODUCTION

The nucleus ^{161}Lu is the lightest of the even-N Lu isotopes for which triaxiality has been demonstrated by the existence of a wobbling excitation built on an $i_{13/2}$ proton configuration [1]. We refer to Ref. [2] and references therein for an overview of searches and results on triaxial superdeformed (TSD) bands in the region. Theoretical calculations predict local TSD minima with $(\varepsilon, \gamma) \sim (0.4, 20^{\circ})$ in the potential energy surfaces for nuclei around $Z = 72$ and $N = 94$. As examples potential energy surfaces for ^{161}Lu, calculated using the ultimate cranker (UC) code [3–5], are shown in Fig. 1 for two different configurations.

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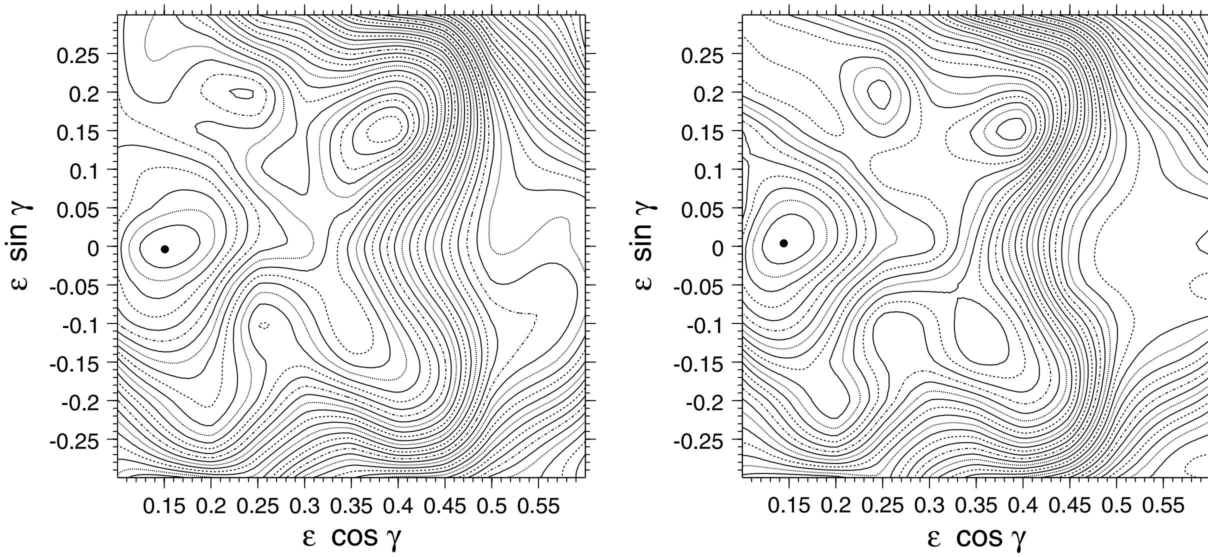

FIG. 1. Total energy surfaces for ^{161}Lu calculated for parity and signature $(\pi, \alpha) = (+, +1/2)$ at spin $I = 61/2$ (left panel) and $(\pi, \alpha) = (+, -1/2)$ at spin $I = 63/2$ (right panel). The energy difference between the contour lines is 0.2 MeV.

In the nuclei with firm evidence for triaxiality it is also possible to probe 'regular' quasi-particle excitations in a triaxial minimum, coexisting with the wobbling excitations [6]. However, only scarce information of this kind exists to date. Although many bands suggested to be of TSD nature have been found, only a few of them have been connected to the known normal deformed (ND) structures, thereby yielding information on excitation energy and possibly also spin and parity [7,8], which is compulsory for an analysis of their structure.

In the even-N neighbor, ^{163}Lu, the triaxial $i_{13/2}$ proton band, TDS1, is populated with a strength of $\simeq\sim$10%. In the data from the present experiment two TSD rotational bands, TSD1 and TSD2, were discovered in ^{161}Lu [1] which show all the characteristics of $n_w = 0$ and 1 wobbling excitations. However, their population is much weaker and the excitation energy, which could not be firmly determined, is probably considerably higher. Therefore, the TSD minimum in ^{161}Lu is less favored than the one in ^{163}Lu.

A peculiar observation in ^{161}Lu that will be addressed in the present paper is a strongly populated band which is yrast above spin $I \simeq 33$, showing aligned angular momenta and dynamic moments of inertia different from the ND bands, but similar to the $i_{13/2}$ proton band TSD1 and its wobbling excitation TSD2. To comply with the experimental parity and signature, $(+, -1/2)$, of this band a configuration is suggested involving in addition to the favored $i_{13/2}$ proton also an $h_{9/2}$ and an $h_{11/2}$ proton.

The work presented here also covers several additions to the former level scheme of ^{161}Lu from Ref. [9], both at high and low spin, together with some revisions. New ND bands and many cross-band links throughout the low-spin part of the level scheme have been established, which ensures relative excitation energies for all bands. The $I = 1/2^+$ ground state could not be identified in this experiment. Therefore, all energies are given relative to the lowest state in a band with the Nilsson configuration $[411]1/2^+$ with $I = 3/2^+$, which most likely is very close in energy to the $1/2^+$ ground state as observed in ^{163}Lu [6].

In the following section, details of the experiment and the analysis methods are given and in Sec. III the results are summarized. The configuration assignments to the observed structures are discussed in Sec. IV.

II. EXPERIMENTAL DETAILS AND ANALYSIS METHODS

The present experiment was performed at the Vivitron accelerator at IReS, Strasbourg. The nucleus ^{161}Lu was populated up to high spins in the reaction ^{139}La(^{28}Si, 6n) at a beam energy of 175 MeV. The target consisted of a stack of two self-supporting La foils of 520 μg/cm^2 thickness each. To prevent oxidation, the targets were handled in an argon atmosphere. The EUROBALL spectrometer [10,11], equipped with 30 conventional Compton-suppressed large-volume Ge detectors and 41 Compton-suppressed composite Ge detectors, was used to detect the emitted γ radiation. Of the composite detectors 26 were Clovers, each consisting of four Ge crystals, and 15 were Clusters, each composed of seven Ge crystals. In order to select high-fold coincidence events, an 'inner ball' of 210 BGO crystals was used as multiplicity filter. With a hardware trigger condition requiring at least four γ rays in the Ge detectors before Compton suppression and 11 γ rays detected in the BGO filter, a total of 4.7×10^9 coincidence events were recorded. After presorting with rejecting bad events and performing add-back of scattered γ rays in the composite detectors, 2.8×10^9 events with a Ge coincidence fold $f \geqslant 3$ remained for the off-line analysis.

These events were first sorted into three- and four-dimensional coincidence arrays (cubes and hypercubes, respectively) using a narrow energy-dependent time gate and then analyzed with programs from the RADWARE package [12].

The time gate excluded events caused by neutrons, but did not lead to a loss in efficiency for low-energy γ rays. Frequently used techniques to determine multipolarities using multi-detector arrays are measuring either DCO ratios [13] or angular distribution ratios [14], the latter providing better statistics for weak transitions. In order to derive these ratios, three asymmetric matrices, $(E_{\rm fb}, E_{90})$, $(E_{\rm all}, E_{90})$ and $(E_{\rm all}, E_{\rm fb})$ were constructed using different detector combinations. The events measured at forward (f) and backward (b) angles were taken from detectors mounted in rings between $\theta = 15.5^\circ$ and 34.6° and between $\theta = 149.3^\circ$ and 163.46°, respectively. The 90° events were taken from detectors in rings between $\theta = 80.8^\circ$ and 107.9°. The information on multipolarities was extracted from both DCO ratios, $R_{\rm DCO} = I_{\rm fb}^{\gamma_2}({\rm Gate}_{90}^{\gamma_1})/I_{90}^{\gamma_2}({\rm Gate}_{\rm fb}^{\gamma_1})$, and angular distribution ratios, $R_{\rm ang} = I_{\rm fb}^{\gamma_2}({\rm Gate}_{\rm all}^{\gamma_1})/I_{90}^{\gamma_2}({\rm Gate}_{\rm all}^{\gamma_1})$. Information about the de-alignment parameter, σ/I, was obtained from the angular distributions of clean transitions of known stretched $E2$ multipolarity. A value of $\sigma/I = 0.25 \pm 0.02$ was derived in the spin range around $I = 27/2$. The expected angular distribution ratios for stretched quadrupole and for stretched dipole transitions lie around 0.86 and 0.58, respectively. For transitions in the low-spin region, no angular correlation measurement was possible due to the loss in alignment. While the alignment stays approximately constant at high spins, it is reduced at low spins by the interaction between the nuclear moments and the fields produced by the atomic electrons.

III. EXPERIMENTAL RESULTS

Prior to this work, four rotational band structures belonging to the ND minimum in the potential-energy surface of ^{161}Lu were observed [9]. They are the two signature-partner bands with negative parity assigned as $\pi h_{11/2}$ $[514]9/2^-$ ($\alpha = \pm 1/2$), and two positive-parity signature-partner bands observed only for $I^\pi \geqslant 21/2^+$. In the data from the present experiment, two rotational bands, named TSD1 and TSD2, were identified up to high spins. Results on these bands were already reported in our recent publication [1]. They have been interpreted as wobbling bands belonging to the TSD minimum with the wobbling-phonon quantum numbers $n_w = 0$ and 1 [1,15]. All of these structures could not be connected to the $[411]1/2^+$ ground-state level extracted from laser spectroscopy [16]. Other unassigned states have been observed in radioactive β decay studies of ^{161}Hf [17] but were not observed in the nuclear-reaction-spectroscopy measurements.

The level scheme of ^{161}Lu obtained from the present work is given in Fig. 2. The level scheme has been constructed on the basis of the γ-ray coincidence data and intensity balances. The widths of the arrows are proportional to the observed γ-ray intensities. Large parts of the ND level structure are in agreement with the results of Yu *et al.* [9], but were extended to higher and lower spins, and some parts, both for negative- and positive-parity bands, had to be revised. Several new rotational structures were discovered and connected to known levels in the low-spin region and partly also in the high-spin region. The spins were determined from DCO and angular distribution ratios, and parities were deduced assuming $E2$ multipolarity for stretched quadrupole transitions and using general systematics. The many inter-band transitions give mutual constraints on spins and parities. Level and transition energies are listed in Table I for the important inter-band transitions, together with the angular distribution ratios, $R_{\rm ang}$, and DCO ratios, $R_{\rm DCO}$. Some of the new bands established in ^{161}Lu have been labeled according to the Nilsson orbital occupied by the odd-proton, see the discussion below. The new excited bands have a multi-quasi-particle structure. Those with positive-parity have been labeled X1 and X2 and those with negative parity are labeled X3 and X4. In the following, the new results are summarized in some detail.

A. The positive-parity bands

1. The [404]7/2$^+$ coupled bands

This pair of coupled bands was previously reported from spin/parity $I^\pi = 23/2^+$ to $83/2^+$ and from $21/2^+$ to $49/2^+$ for the positive and negative signature branches, respectively [9]. Both bands were extended to higher spins and the positive-signature partner also to lower spin. The new evidence for the high-spin levels, see the γ-ray spectrum of Fig. 3(b), for both signature partners and the connecting $\Delta I = 1$ interband transitions made a revision of the positive-signature band above $I^\pi = 47/2^+$ necessary. High-spin transitions, previously assigned to the positive-signature branch, now form part of a new band labeled X2, which will be discussed in a following subsection. The two signature partners now extend from $I^\pi = 21/2^+$ to $73/2^+$ and from $7/2^+$ to $71/2^+$ for the negative and positive signatures, respectively. Additional transitions of 474 and 559 keV were observed in coincidence with the low-spin portion of the $[404]7/2^+$ sequences, however it was not possible to definitively establish their position in the level scheme. These transitions are probably associated with the $\alpha = -1/2$ signature band below spin 21/2. Two yrare levels with $I^\pi = 21/2^+$ and $23/2^+$ were observed to be connected thru several feeding and decay-out transitions with the $[404]7/2^+$ sequences.

The population of the $[404]7/2^+$ coupled bands relative to the yrast $[514]9/2^-$ structure is about 31% at spin 31/2. The previously reported linking transitions to the $[514]9/2^-$ band of 651.9 and 943.3 keV were confirmed. Angular distribution and DCO ratios confirm the stretched dipole character of these two transitions (see Table I). Three, possibly four, additional connecting transitions to the $[514]9/2^-$ structure between spin $15/2^+$ and $25/2^+$ were observed. The angular distribution and DCO ratios of the 704.1 keV transition are compatible with stretched dipole character, see Table I. Additional decay to the $[514]9/2^-$ band occurs thru the yrare $21/2^+$ and $23/2^+$ levels, but the transitions of 736.6 and 1137.2 keV are too weak for extracting angular correlation ratios.

Two direct connections to the $[411]1/2^+$ structure were observed, decaying from the $7/2^+$ band head into the $3/2^+$ and $(5/2^+)$ states. In addition, the $[404]7/2^+$ structure is connected to the $[411]1/2^+$ ground state band via the newly established $[402]5/2^+$ coupled bands thru two $\Delta I = 1$ and two $\Delta I = 2$ decays. Angular correlation and DCO ratios for

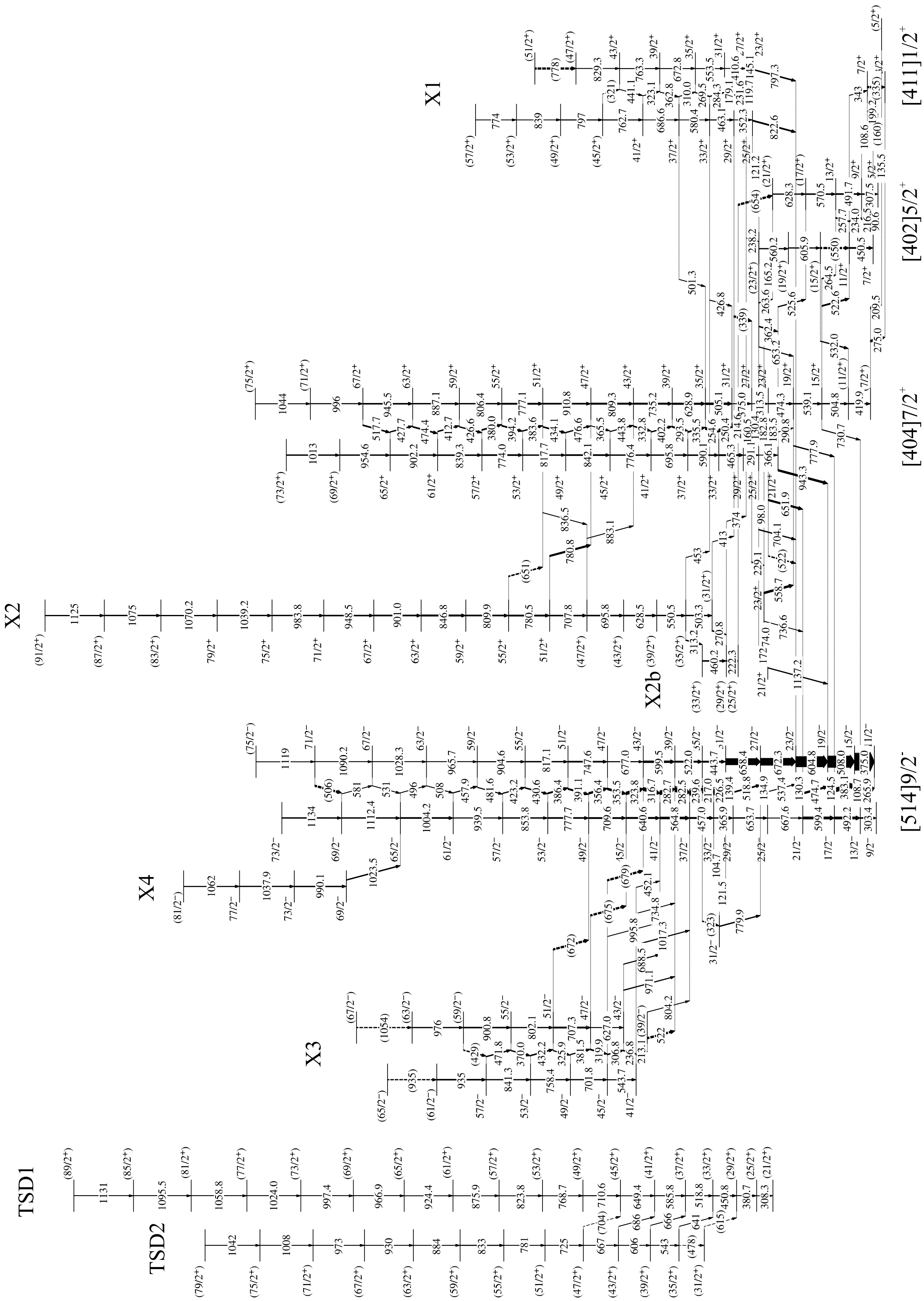

FIG. 2. Level scheme of ^{161}Lu.

TABLE I. Angular distribution and DCO ratios for different connecting transitions in ^{161}Lu.

Band	E_γ keV	$I_i^\pi \rightarrow I_f^\pi$	R_{ang}	R_{DCO}
[404]7/2$^+$	522.6	$15/2^+ \rightarrow 11/2^+$	0.77 ± 0.07	1.05 ± 0.15
[404]7/2$^+$	943.3	$21/2^+ \rightarrow 19/2^-$	0.54 ± 0.05	0.49 ± 0.12
[404]7/2$^+$	651.9	$23/2^+ \rightarrow 21/2^-$	0.51 ± 0.04	0.53 ± 0.09
[404]7/2$^+$	704.1	$25/2^+ \rightarrow 23/2^-$	0.51 ± 0.12	0.76 ± 0.56
X1	822.6	$25/2^+ \rightarrow 23/2^-$	0.56 ± 0.16	–
X2	(780.8–780.5)	$51/2^+ \rightarrow 47/2^+$	0.93 ± 0.06	1.02 ± 0.16
X3	971.1	$43/2^- \rightarrow 37/2^-$	0.97 ± 0.09	1.17 ± 0.77
X3	688.5	$43/2^- \rightarrow 39/2^-$	0.73 ± 0.12	0.50 ± 0.29
X4	1023.5	$69/2^- \rightarrow 65/2^-$	0.98 ± 0.17	–

the most intense $15/2^+ \rightarrow 11/2^+$ transition of 522.6 keV are compatible with a stretched $E2$ transition (see Table I).

2. The [411]1/2$^+$ and [402]5/2$^+$ coupled bands

Spin and parity assignments for the two new bands, [411]1/2$^+$ and [402]5/2$^+$, are based on several coincidence connections established with the [404]7/2$^+$ band, which is itself fixed thru the stretched dipole, presumably $E1$, transitions of 651.9, 704.1 and 943.3 keV to the [514]9/2$^-$ structure, as mentioned above.

Only three levels are associated with the [411]1/2$^+$ configuration. The coupled partner bands labeled [402]5/2$^+$ are observed up to $I^\pi = 21/2^+$ and tentatively up to $23/2^+$ for the positive and negative signatures, respectively. In-band and inter-band transitions for both structures are seen in connection with the [404]7/2$^+$ band in the spectrum of Fig. 3(a). The part above the $11/2^+$ level in the $\alpha = -1/2$ signature branch of the [402]5/2$^+$ band is uncertain, since the expected $15/2^+ \rightarrow 11/2^+$ $E2$ transition of 550 keV could not be firmly identified. Similarly, in the [411]1/2$^+$ band the expected $7/2^+ \rightarrow 3/2^+$ $E2$ transition of 335 keV has not been

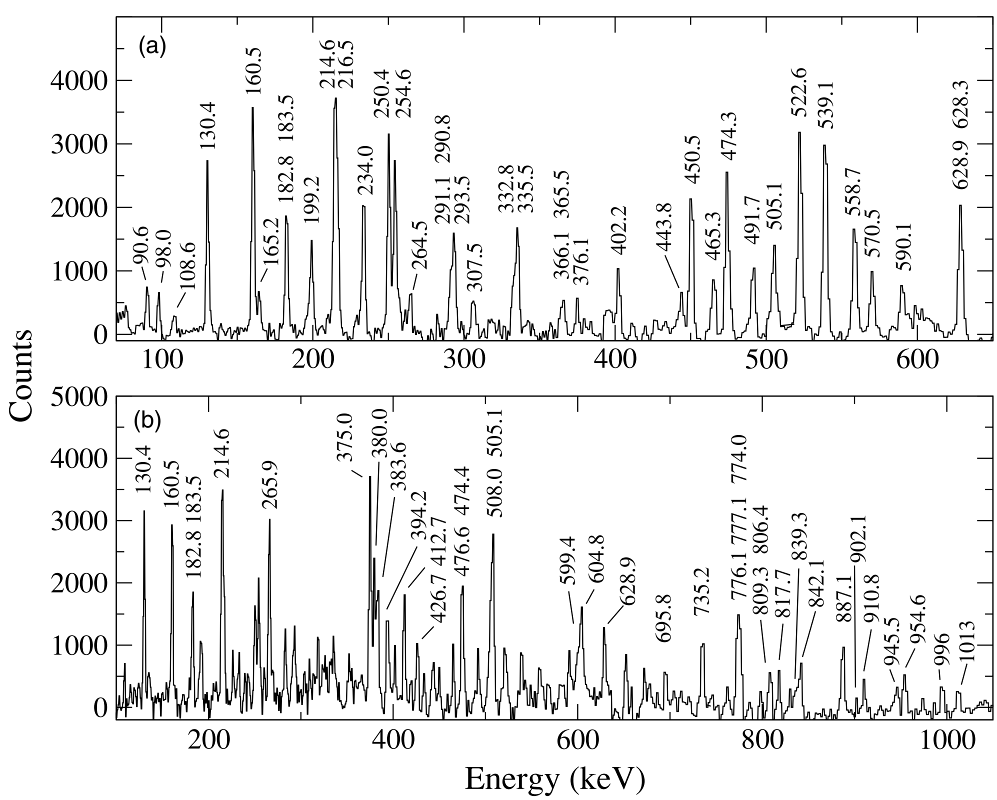

FIG. 3. (a) Gamma-ray coincidence spectrum documenting the [404]7/2$^+$ and [402]5/2$^+$ bands in ^{161}Lu. The spectrum was obtained using double-gate lists. The first list contains the $5/2^+ \rightarrow 3/2^+$ transition of 135.5 keV and the second list contains the $\Delta I = 1$ transitions from spin $21/2^+$ up to spin $35/2^+$ from the [404]7/2$^+$ band. (b) Gamma-ray coincidence spectrum documenting the high spin states of [404]7/2$^+$ in ^{161}Lu. The spectrum was obtained using double-gate lists. The first list contains the $65/2^+ \rightarrow 63/2^+$ transition at 427.7 keV and the second list contains the $\Delta I = 1$ transitions from spin $35/2^+$ up to spin $63/2^+$ from [404]7/2$^+$.

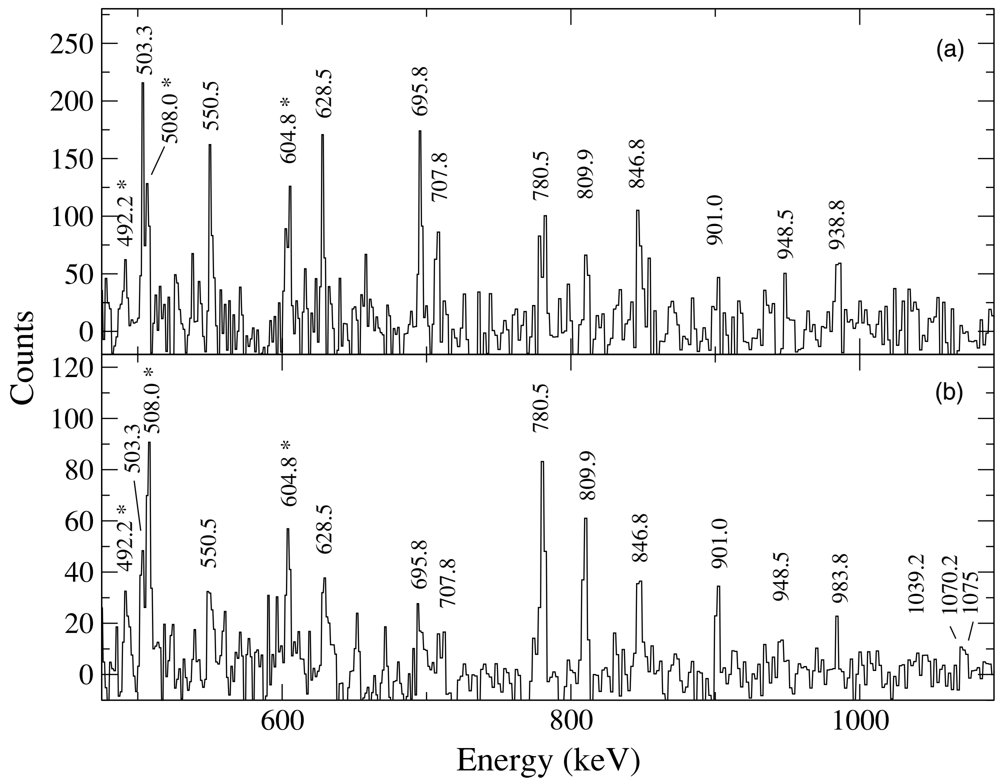

FIG. 4. (a) Gamma-ray coincidence spectrum of the X2 band in ^{161}Lu. The spectrum was obtained using triple-gate lists. The first list contains the $(31/2^+) \rightarrow (29/2^+)$ transition of 270.8 keV and the second and third lists contain all the transitions of X2 above spin $(35/2^+)$. Transitions belonging to other ND band of ^{161}Lu are indicted by an asterisks. (b) Gamma-ray coincidence spectrum documenting the connection between the newly established levels at lower spin and the high spin transitions of X2. The spectrum was obtained using triple gates on all transitions between spin $(35/2^+)$ and $51/2^+$.

observed. Connections between the two structures occur thru three, possibly four, $\Delta I = 1$ transitions. The main decay flow goes via the 135.5 keV transition from the band head of the $[402]5/2^+$ sequence into the $3/2^+$ state of the $[411]1/2^+$ band. No further transitions below the 135.5 keV line have been observed, thereby establishing the $3/2^+$ state as the lowest observed energy level in ^{161}Lu in the present experiment. The missing $1/2^+$ band head is probably close to the $3/2^+$ state, preventing the observation of transitions since they will have very low energy. Both structures are only seen in the low-spin region and no angular correlation measurements are available due to the loss in alignment.

3. The band X1

A new coupled band, labeled X1, which decays into the $[514]9/2^-$ and $[402]5/2^+$ bands from the $23/2^+$ and $25/2^+$ states via transitions of 797.3, 822.6 and 121.2 keV, has been found. The intensity of this band, measured at spin 33/2, is about 8.2% relative to the yrast $[514]9/2^-$ structure. The angular correlation ratio for the 822.6 keV transition is compatible with pure stretched dipole character, see Table I. Therefore, it is presumably a stretched $E1$ transition, since an $M1$ transition of such a high energy would most likely have an $E2$ admixture and, in addition, a stretched $E2$ transition of higher energy to the $21/2^-$ state of the $[514]9/2^-$ band would be expected to compete with an $M1$ transition.

4. The bands X2 and X2b

The band labeled X2 was previously assigned to the positive-signature branch of the $[404]7/2^+$ sequences above the $47/2^+$ state [9]. As explained above, it is now established as a separate structure which could be extended to higher as well as lower spins. Figure 4(a) shows an example of the coincidence spectra of X2. Figure 4(b) shows the direct connection between the known levels at high spin and the newly established transitions at lower spins. Angular correlation ratios suggest for all transitions above the $51/2^+$ level stretched quadrupole, presumably $E2$ multipolarity. Due to the weakness of the lower-spin transitions and the difficulty to find clean gates, it was not possible to extract DCO ratios below the $51/2^+$ state.

Strong interaction with the $[404]7/2^+$ ($\alpha = +1/2$) band occurs between the states with $I^\pi = 47/2^+$ and $51/2^+$. Transitions of 780.8 and 883.1 keV are observed from the X2 to the $[404]7/2^+$ band and one transition of 836.5 keV is observed from the $[404]7/2^+$ to the X2 band at this crossing. The fact that transitions are observed both ways completely fixes the spin and parity of band X2 relative to those of the

$[404]7/2^+$ band with signature $\alpha = -1/2$ at the crossing. Even if the 780.8 keV transition is a doublet, angular distribution and DCO ratios are compatible with stretched $E2$'s for both of them (see Table I), supporting the spin and parity assignment deduced from the interaction scenario.

In the low-spin region, at $I^\pi = 35/2^+$ and $31/2^+$, band X2 starts to decay to other positive-parity bands, namely to X1, via two transitions, and to X2b, also via two low-energy transitions. The multipolarity could not be determined for any of these, but the $\Delta I = 1$ character is compatible with the spins of X1 together with the stretched $E2$ character of the transitions below the $I^\pi = 51/2^+$ state in the X2 band.

The levels of the weak structure X2b are established through the two in-band transitions together with the feeding transitions from band X2, and a 374 keV decay to the $27/2^+$ level of X1. However, no decay-out has been firmly observed from the lowest level with $I^\pi = (25/2^+)$.

B. The negative-parity bands

1. The [514]9/2$^-$ coupled bands

The coupled bands built on the $[514]9/2^-$ proton configuration, previously established up to $I^\pi = 61/2^-$ [9], are partly confirmed. On the basis of the observation of the coincident $25/2^- \rightarrow 23/2^-$ and $27/2^- \rightarrow 25/2^-$ inter-band $M1$ transitions with energies of 537.4 and 134.9 keV, respectively, the $E2$ transitions with energies of 667.6 and 653.7 keV have been interchanged. A weak transition, assumed to be the missing $13/2^- \rightarrow 9/2^-$ E2 decay, is observed with an energy of 303.4 keV. New levels have also been added at the top of the band, up to the $73/2^-$ and $75/2^-$ states for the positive- and negative-signature branches, respectively. An yrare level at $I^\pi = 31/2^-$ is observed through two, possibly three connecting transitions.

2. The band X3

A new weakly populated coupled band, labeled X3 in Fig. 2, has been established. This band is connected to the yrast $[514]9/2^-$ band via several transitions between the $39/2^-$ and $45/2^-$ states, and tentatively up to the $51/2^-$ level.

The intensity relative to the yrast $[514]9/2^-$ structure has been measured to about 4.6% at spin 45/2. Angular correlation ratios for the strongest inter-band transitions suggest quadrupole character for the 971.1 keV and dipole character for the 688.5 keV transition (see Table I).

3. The band X4

The band X4 was already observed in a previous in-beam experiment [9]. It decays into the $65/2^-$ level of the $[514]9/2^-$ sequence thru a transition with an energy of 1023.5 keV. The angular distribution and DCO ratios (see Table I) agree with a $\Delta I = 2$, presumably $E2$, transition. This band could be extended up to $I^\pi = (81/2^-)$.

IV. CONFIGURATION ASSIGNMENTS AND DISCUSSION

Nilsson and quasiparticle configurations are assigned to the ND bands of ^{161}Lu by examining properties such as aligned angular momenta, band crossing frequencies, excitation energies, and signature splittings compared with expectations from cranked shell model (CSM) [18,19] calculations, performed with the UC code [3–5]. $B(M1)/B(E2)$ ratios are extracted from γ-ray branching ratios, $T_\gamma(I \rightarrow I-1)/T_\gamma(I \rightarrow I-2)$, for the coupled bands and compared with predictions from the geometrical-coupling model of Ref. [20]. In the calculations of $B(M1)/B(E2)$ ratios experimental alignments are used, and the g factors and quadrupole moments are given in the text below for the individual cases.

The neutron and proton Nilsson orbitals close to the Fermi level, together with the quasiparticle labels commonly used within the CSM, are given in Table II. In addition to the known $[514]9/2^-$ signature-partner bands, the Nilsson configurations $\pi g_{7/2}[404]7/2^+$ for the already known positive-parity band and $\pi d_{5/2}[402]5/2^+$ for a new structure could be assigned. From the present data, it was also possible to bind the established rotational structures to the $\pi d_{3/2}[411]1/2^+$ ground band for which some levels have been added. An interpretation of the excited bands, X1 and X3, in terms of quasiparticle excitations in the ND well of the potential energy surface of ^{161}Lu is also given. For the new bands X2 and X4, which both have large alignments resembling the behavior observed for the two TSD bands [1], shapes different from ND have to be considered.

Figures 5 and 6 show the experimental aligned angular momenta, i_x, as a function of the rotational frequency, $\hbar\omega$, for the positive- and negative-parity ND structures, respectively. Figures 7 and 8 show the excitation energies versus spin for these bands. For the positive-parity structures, the assumed excitation energies of the two TSD bands, TSD1 and TSD2 [1], are added for comparison. In Fig. 9 the alignments of the bands X2 and X4 are compared with those of the two TSD bands. Backbends or up-bends in the alignment plots are understood as being due to crossings with structures in which different pairs of quasi-particles are aligned. In the rare-earth region, the first backbend is attributed to the alignment of $i_{13/2}$ neutrons, at a crossing frequency of $\hbar\omega_c \simeq 0.27$ MeV, which in the CSM terminology is denoted as the AB crossing [18,19]. The first proton alignment is from a pair of $h_{11/2}$ quasiparticles, denoted *ef* and expected at $\hbar\omega_c \simeq 0.4$ MeV.

TABLE II. Labeling of the lowest Nilsson orbitals for neutrons and protons.

Nilsson orbital	$\alpha = +1/2$	$\alpha = -1/2$
$\nu i_{13/2}[660]1/2^+$	A	B
$\nu i_{13/2}[660]1/2^+$	C	D
$\nu h_{9/2}[521]3/2^-$	E	F
$\pi d_{3/2}[411]1/2^+$	a	b
$\pi g_{7/2}[404]7/2^+$	c	d
$\pi d_{5/2}[402]5/2^+$	i	j
$\pi h_{11/2}[514]9/2^-$	e	f

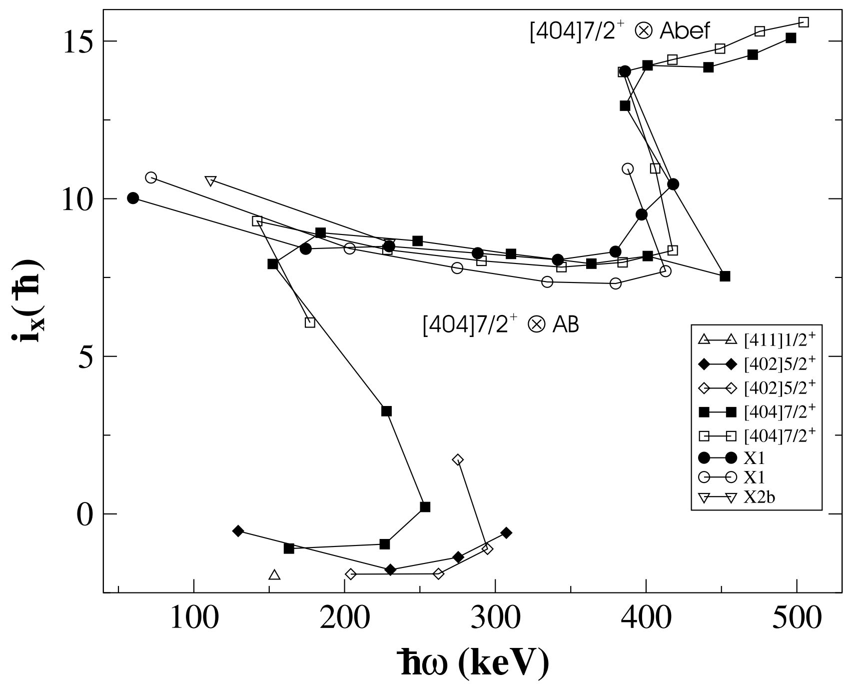

FIG. 5. Aligned angular momenta, i_x, relative to a reference $i_{\rm ref} = \Im_0\omega + \Im_1\omega^3$ with $\Im_0 = 30\hbar^2\,{\rm MeV}^{-1}$ and $\Im_1 = 40\hbar^4\,{\rm MeV}^{-3}$ for the positive-parity bands in ^{161}Lu.

A. The positive-parity bands

1. The bands [411]1/2+, [402]5/2+ and [404]7/2+

In the vicinity of the Fermi surface, the three positive-parity Nilsson orbitals $[411]1/2^+$, $[402]5/2^+$ and $[404]7/2^+$ lie close in energy at low spin. This leads to the possibility of strong mixing between the levels of the same spin for the three bands built on these configurations. Such interactions are indeed observed in the level scheme in Fig. 2. Below the first neutron $i_{13/2}$ AB crossing, the level scheme is quite complicated. Several interband transitions between the three structures are observed, competing with the in-band transitions.

The lowest observed state in the $[411]1/2^+$ sequence is established as $3/2^+$. The expected ground state with $I^\pi = 1/2^+$ [16] is not observed. Conversion-electron measurements suggest mixed $M1/E2$ multipolarity for the 135.5 keV transition [21]. Therefore, this transition is assigned as $\Delta I = 1, 5/2^+ \rightarrow 3/2^+$. Moreover, a $\Delta I = 3$ transition of 275.0 keV from the $7/2^+$ state is unlikely. Systematics in neighboring Lu isotopes show the same feature. No $1/2^+$ state has been observed in ^{165}Lu [22] and ^{167}Lu [23], while in the case of

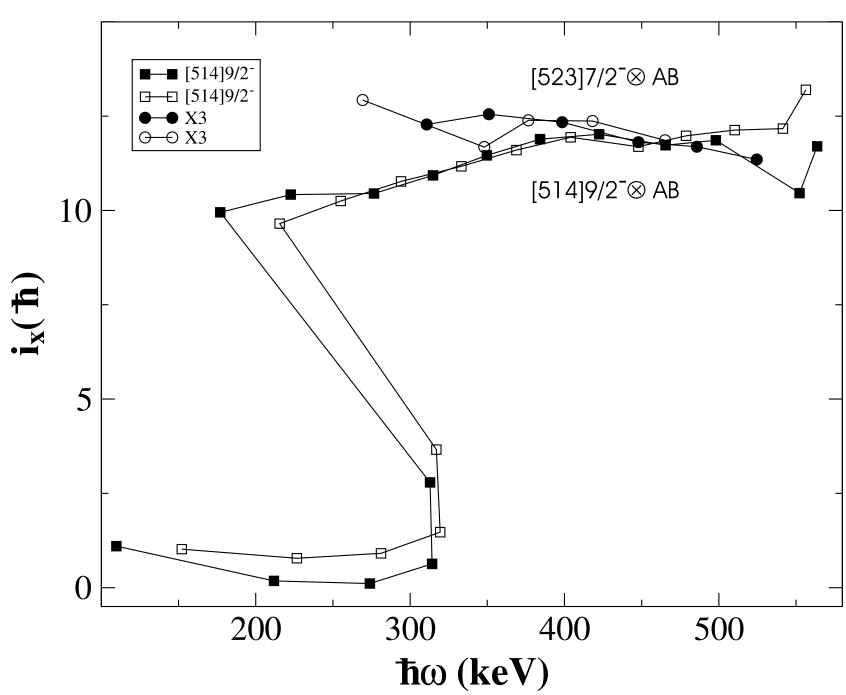

FIG. 6. Aligned angular momenta, i_x, relative to a reference $i_{\rm ref} = \Im_0\omega + \Im_1\omega^3$ with $\Im_0 = 30\hbar^2\,{\rm MeV}^{-1}$ and $\Im_1 = 40\hbar^4\,{\rm MeV}^{-3}$ for the negative-parity bands in ^{161}Lu.

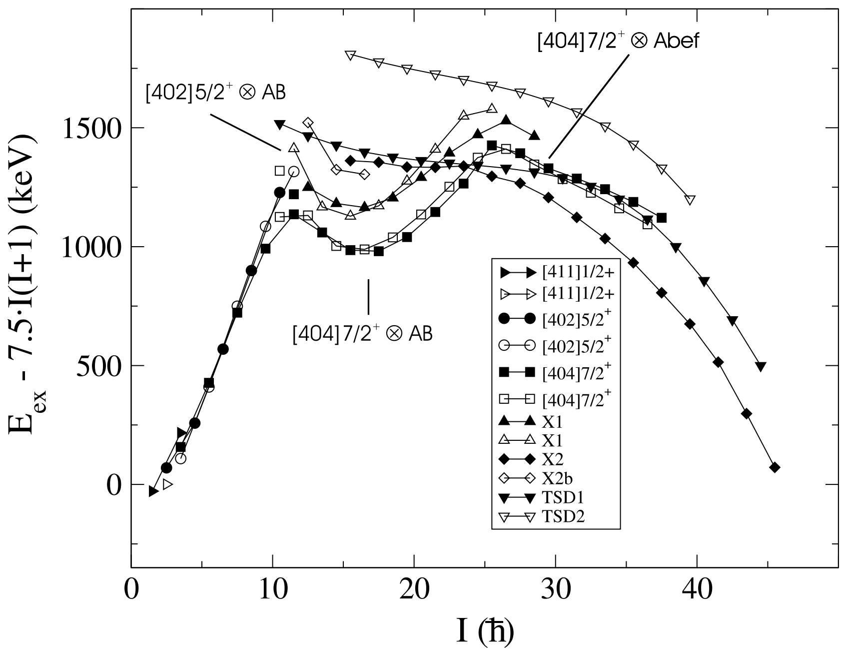

FIG. 7. Excitation energies as a function of spin for the positive-parity bands in ^{161}Lu. The excitation energy of the two TSD bans, TSD1 and TSD2, is estimated [1].

^{163}Lu [6] the $1/2^+$ level lies just 17 keV below the $3/2^+$ state.

Due to the strong mixing between the three structures and the resulting discontinuities in the level scheme, it was not possible to follow the [411]$1/2^+$ sequence and the $\alpha = +1/2$ signature partner of the [404]$7/2^+$ bands up to the frequency of the first band crossing. In the [404]$7/2^+$ structure both signatures become well defined above the frequency of the *AB* neutron crossing. In the spin range 37/2 to 43/2 the $B(M1)/B(E2)$ ratios are $\simeq\sim 1.04(8)\mu_N^2/e^2\mathrm{b}^2$ which is in agreement with Ref. [9] and compatible with calculations using $g_R = Z/A = 0.4$, $g_\Omega = 0.61$ and $Q_0 = 4.7$ b. The two close-lying and connected additional levels at $I^\pi = 21/2^+$ and $23/2^+$ are identified as the continuation of the yrare three-quasi-particle band [404]$7/2^+\otimes$ *AB* below the crossing.

A second crossing in the [404]$7/2^+$ bands at a rotational frequency of $\hbar\omega_c \simeq 0.42$ MeV can be seen in Fig. 5 with an increase in alignment of about $6\hbar$. Systematics of neighboring even-even ^{160}Yb [24] and ^{162}Hf [25] show similar backbends at similar rotational frequencies. Both have been interpreted as the first $h_{11/2}$ proton alignment, *ef*. The similarity in the crossing frequency and the increase in alignment suggest the crossing in ^{161}Lu to be interpreted also as the first $h_{11/2}$ proton, *ef*, rather than the second $i_{13/2}$ neutron alignment, *CD*, which are both predicted to occur in the same frequency range by the CSM calculations. Thus, the resulting configuration is a five quasi-particle band, [404]$7/2^+\otimes$ *ABef*.

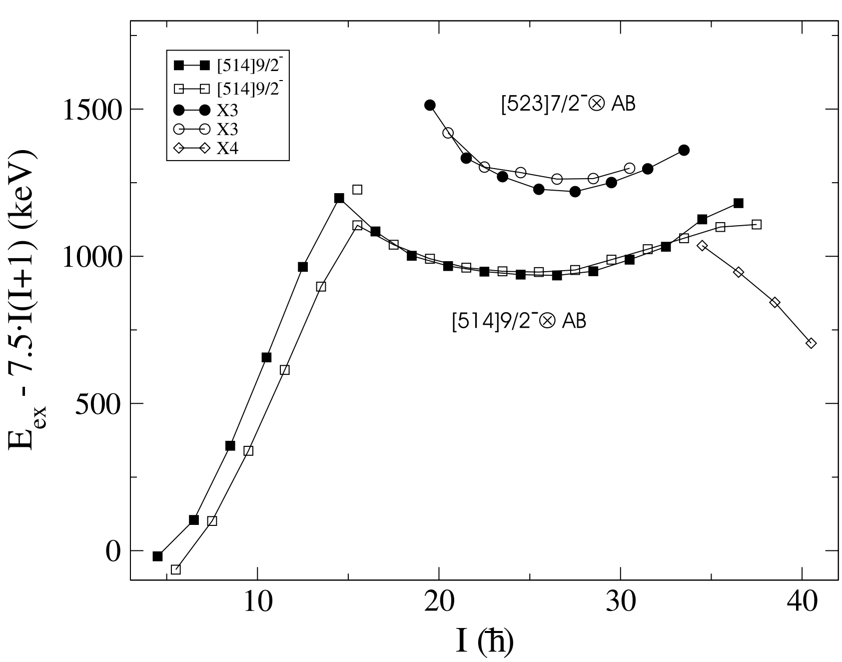

FIG. 8. Excitation energies as a function of spin for the negative-parity bands in ^{161}Lu.

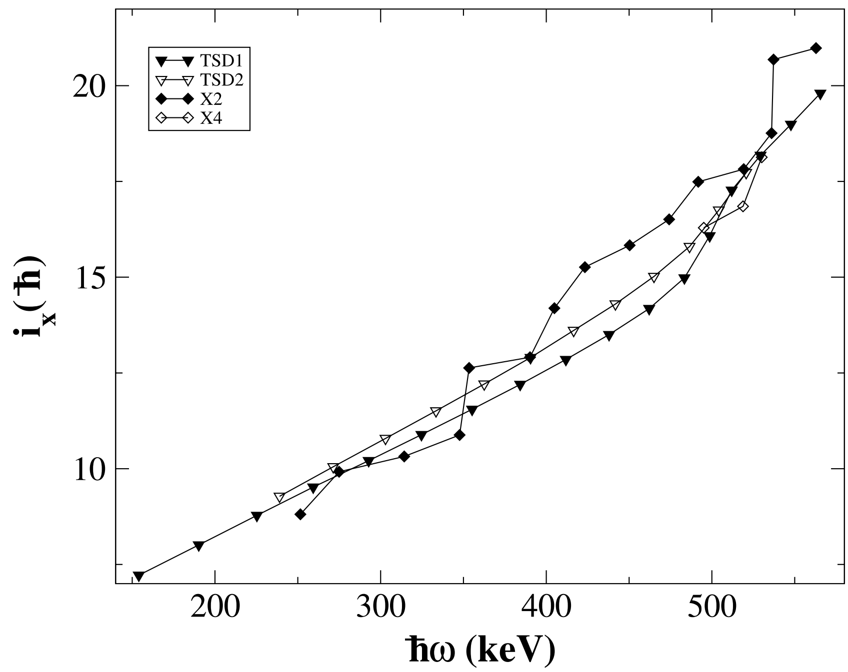

FIG. 9. Aligned angular momenta, i_x, relative to a reference $i_{ref} = \Im_0\omega + \Im_1\omega^3$ with $\Im_0 = 30\hbar^2\,\mathrm{MeV}^{-1}$ and $\Im_1 = 40\hbar^4\,\mathrm{MeV}^{-3}$ for the bands X2 and X4 compared with the TSD bands 1 and 2 in ^{161}Lu.

2. The band X1

Over the range of observed frequencies, the aligned angular momenta of band X1 are similar to those of the $[404]7/2^+$ bands, see Fig 5. The band lies about 180 keV higher in excitation energy than the $[404]7/2^+$ bands. The alignment and the parity of the band suggest a three-quasi-particle configuration of a positive-parity proton coupled to the *AB* neutrons. The best candidate is the $[402]5/2^+ \otimes AB$ configuration on the basis of the observed signature splitting which is expected to be much larger for the alternative $[411]1/2^+ \otimes AB$ configuration. Moreover, the experimental B(M1)/B(E2) ratios are $\simeq\sim 1.72(36)\mu_N^2/e^2\mathrm{b}^2$ at $I^\pi = 35/2^+$ to $37/2^+$ which are considerably larger than those of the $[404]7/2^+ \otimes AB$ bands. The calculated value is $\simeq 2.3\mu_N^2/e^2\mathrm{b}^2$ for a $[402]5/2^+ \otimes AB$ configuration, using $g_R = Z/A = 0.4$, $g_\Omega = 1.57$ and $Q_0 = 4.7\,\mathrm{b}$. A different three-quasi-particle structure with positive parity observed in the neighbors, ^{163}Lu and ^{165}Lu, competing with the $[404]7/2^+ \otimes AB$ and $[402]5/2^+ \otimes AB$ structures, is $[514]9/2^- \otimes AE$. This possibility is excluded for X1 since the expected *BC* neutron crossing at $\hbar\omega \simeq 0.32$ MeV, corresponding to $[514]9/2^- \otimes AE \to [514]9/2^- \otimes AEBC$, is not observed in band X1, probably due to blocking.

3. The bands X2 and X2b

The band X2 has an alignment which, over the covered frequency range, resembles that of the two TSD bands [1], as illustrated in Fig. 9. The irregularity at $\hbar\omega \simeq 0.37$ MeV is caused by the interaction with the $[404]7/2^+$ band with $\alpha = -1/2$. By removing the induced perturbations of -8 and $+17$ keV for the $I^\pi = 51/2^+$ and $47/2^+$ states, respectively, this irregularity can be eliminated. The observed level distances are 130 and 62 keV at $I^\pi = 51/2^+$ and $47/2^+$, respectively. Assuming two-state mixing, these perturbations derive consistently from an interaction strength of $\simeq$31 keV at both states. Unfortunately, the branching ratios of out-of-band to in-band transition strength could not be determined with sufficient accuracy for a further analysis of the relative transition quadrupole moments for the two interacting bands. Similarly, a shift of $\simeq -11$ keV for the $I^\pi = (35/2^+)$ level would make the alignment curve smooth at the bottom. This perturbation could be explained by an interaction with the non-observed $35/2^+$ state in the $\alpha = -1/2$ signature partner of band X2b, which would naturally be close in energy. The decay-out to band X1, assigned as the $[402]5/2^+ \otimes AB$ configuration, could be mediated by this interaction.

Since the alignment behavior of band X2 is so similar to that of the superdeformed bands TSD1 and TSD2, it is reasonable to assume a large deformation also for X2. We have investigated the potential energy surfaces for the parity and signature of band X2, $(+, -1/2)$, in comparison with the surface relevant for the TSD1 and TSD2 bands. The structure of TSD1 is mainly an $i_{13/2}$ proton coupled to a pair of aligned $i_{13/2}$ neutrons in the triaxial minimum at $(\varepsilon\cos\gamma, \varepsilon\sin\gamma) \simeq (0.39, 0.15)$ as illustrated in the left-hand part of Fig. 1, and TSD2 is interpreted as a wobbling excitation built on TSD1 [1]. With positive parity and $\alpha = -1/2$, X2 may belong to the triaxial minimum with smaller quadrupole deformation and larger triaxiality, $(\varepsilon\cos\gamma, \varepsilon\sin\gamma) \sim (0.25, 0.20)$, seen in the right-hand panel of Fig. 1. The lowest state in the triaxial minimum with the larger γ is also based on the $i_{13/2}$ proton with $\alpha = +1/2$ as for TSD1, but here two additional proton orbitals, $h_{9/2}$ and $h_{11/2}$ with $\alpha = +1/2$ are also occupied. Again, these proton excitations are coupled to a pair of aligned $i_{13/2}$ neutrons. The total signature of this configuration is 3/2 which is equivalent to $\alpha = -1/2$. The suggested configuration may seem somewhat speculative, but it is the most likely one based on the UC calculations. However, it should be noted that the UC predicts a signature partner to the suggested configuration fairly close in energy which has not been observed experimentally. Furthermore, the fact that X2 is very low in energy, even yrast above spin $I \geqslant 33$, whereas the excitation energy of TSD1 is considerably higher, does not agree with the results of the UC calculations. The relatively large interaction of X2 with the ND band $[404]7/2^+ \otimes AB$ of $\simeq$31 keV at $I \simeq 25$ indicates that the ND potential-energy minimum and the TSD minimum of band X2 are not well separated.

For X2b, which is fed in the decay of X2 and comprises only three states, the excitation energy and alignment resemble those observed for band X1 with the $[402]5/2^+ \otimes AB$ configuration. Most likely this short sequence can be assigned as the favored partner of the $[411]1/2^+ \otimes AB$ configuration which should be expected around the observed energy of X2b.

B. The negative-parity bands

1. The [514]9/2⁻ coupled bands

With the observation of the new $9/2^-$ band-head state, the pair of $[514]9/2^-$ signature-partner bands becomes similar to the low-lying negative-parity sequences in ^{165}Lu [22,26] and ^{167}Lu [23]. In both these nuclei the band head with $I^\pi = 9/2^-$ has been observed with connections to their respective ground state bands. However, no further transitions below the $9/2^-$ state were observed in ^{161}Lu. This is different to ^{163}Lu [6], where a $7/2^-$ state is seen and, therefore, the $[523]7/2^-$ Nilsson configuration was assigned to that band. In the present case of ^{161}Lu no decay-out from the $[514]9/2^-$ bands to the ground state was observed in prompt coincidence. However, an unobserved 7.3 ms isomeric transition has been suggested [21] in coincidence with a γ line of 135.5 keV, identified as the mixed $M1/E2$ $5/2^+ \rightarrow 3/2^+$ decay between the $[402]5/2^+$ ($\alpha = +1/2$) and $[411]1/2^+$ ($\alpha = -1/2$) bands. It has been suggested [21] that the unobserved isomeric transition could be a highly converted $M2$ transition of less than the K binding energy. With an energy separation of 33.4 keV between the $9/2^-$ and $5/2^+$ states, the assumption of a $9/2^-$ isomeric state is well supported by our results.

Similar to the positive-parity $[404]7/2^+$ structure, the additional level with $I^\pi = 31/2^-$ can be interpreted as the extension of the yrare three-quasi-particle band $[514]9/2^- \otimes AB$ to lower frequencies.

In contrast to the $[404]7/2^+$ sequence there is no alignment gain observed at $\hbar\omega \sim 0.42$ MeV. This is in agreement with the expected blocking of the first proton crossing, *ef*, in the $[514]9/2^- \otimes AB$ bands, *eAB* and *fAB*. The irregularity at the top of the $\alpha = +1/2$ signature partner is most likely caused by an interaction with band X4 at $I^\pi = 69/2^-$. A shift of $\simeq$−30 keV makes the alignment behavior smooth.

2. The band X3

The observed aligned angular momenta of the two signatures of band X3 are similar to those of the $[514]9/2^-$ structure. It lies about 300 keV higher in excitation energy than the band $[514]9/2^-$. Based on the same arguments as for band X1, band X3 is interpreted as a three-quasi-particle band coupling a negative-parity proton to the AB neutrons. The only available negative-parity proton orbital close to the Fermi level for which no signature splitting is expected, as observed experimentally, has the $[523]7/2^-$ configuration. Therefore, the three-quasiparticle configuration $[523]7/2^- \otimes AB$ is assigned to band X3. Unfortunately, no $B(M1)/B(E2)$ values could be extracted due to the lack of clean coincidence gates in this weakly populated band.

The decay pattern with both $\Delta I = 2$, $E2$, and $\Delta I = 1$, $M1$ transitions to the $[514]9/2^-$ bands supports this assignment. The stretched $E2$ transitions are possible through first-order Coriolis interaction. Large $M1$ matrix elements may also be expected between $h_{11/2}$ orbitals with $\Delta K = 1$.

3. The band X4

A band similar to band X4 with large alignment has been observed in neighboring ^{163}Lu [6]. However, an important difference is that in ^{161}Lu this band feeds into the $\alpha = +1/2$ signature partner, e, and not into the $\alpha = -1/2$ branch, f, of the $[514]9/2^-$ band through mixing as discussed above. Therefore, a different assignment must be made. The most likely candidate here is the $\pi h_{9/2}[541]1/2^-$ orbital, which is not observed in ^{163}Lu. At the high frequencies where it is observed in ^{161}Lu it has most likely the structure $[541]1/2^- \otimes ABef$, in agreement with the large alignment. A comparison to the $[404]7/2^+ \otimes ABef$ bands shows a Routhian energy difference of $\simeq$0.25 MeV at $\hbar\omega \simeq 0.5$ MeV, X4 being the lowest. In the UC calculations with the ND minimum at $(\varepsilon, \gamma) \simeq (0.2, 0^\circ)$ the two orbitals should coincide in Routhian energy at this frequency. The $h_{9/2}$ orbital depends strongly on ε and is known to be shape-driving. Therefore, a larger deformation of this band compared with bands e and f as well as the other ND bands is quite likely.

V. SUMMARY

The level scheme of ^{161}Lu is considerably expanded by the present experiment, and configurations are suggested for the new bands. Of the Nilsson orbitals with positive parity, $[404]7/2^+$, $[402]5/2^+$ and $[411]1/2^+$, expected close to the Fermi level at normal deformation, $(\varepsilon, \gamma) \simeq (0.2, 0^\circ)$, only the band built on the $[411]1/2^+$ orbital, which provides the ground state, is not well developed. The bands with the two other configurations are established up to high spins, above the first neutron $i_{13/2}$ crossing. From the negative-parity bands, the one based on the second $h_{11/2}$ orbital, $[523]7/2^-$, has been identified as a three quasi-particle configuration, i.e. with a pair of aligned $i_{13/2}$ neutrons. The band based on the strongly shape-driving $h_{9/2}$ orbital, $[541]1/2^-$, is established at very high spins, where it is a five-quasi-particle configuration with the $h_{9/2}$ proton coupled to a pair of aligned $i_{13/2}$ neutrons as well as a pair of aligned $h_{11/2}$ protons. This band most likely has a larger deformation than the other ND bands which are observed at an excitation energy similar to that of the $[514]9/2^- \otimes AB$ band at $I \geqslant 32$.

The new level scheme also comprises a band with characteristics similar to the triaxial bands in this nucleus, TSD1 and its wobbling excitation TSD2. A plausible explanation for this positive parity, $\alpha = -1/2$ band which becomes yrast at $I \simeq 33\hbar$, is a triaxial shape with a larger γ and smaller ε than the one corresponding to TSD1 and TSD2. The relatively strong interaction at $I \simeq 24$ suggests that this minimum may be less well separated from the ND minimum than the minimum for TSD1 and TSD2. Lifetime measurements, from which transition quadrupole moments can be derived, could shed more light on the question of deformation of this interesting structure.

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