Nuclear structure of ^{102}Mo

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(Received 13 December 2005; published 19 May 2006)

Nuclear structure of the ^{102}Mo nucleus has been studied using the ^{100}Mo$(t, p)^{102}$Mo reaction with the triton beam energy of 12 MeV obtained from the tandem Van de Graaff accelerator and a multichannel magnetic spectrograph. Proton spectra are obtained at 12 different angles from 5° to 87.5°, at an interval of 7.5° and are detected in nuclear emulsion plates. Thirty-five levels in the energy range from 0.000 to 3.248 MeV have been observed. The results yield a number of new levels with spin assignments. Absolute differential cross sections for the levels have been measured. The experimental angular distributions are compared with the theoretical distorted-wave Born approximation calculations to determine L and J^π values. The present results are compared with the previous results.

DOI: 10.1103/PhysRevC.73.054311 PACS number(s): 21.10.Hw, 24.10.Eq, 25.55.Hp, 27.60.+j

I. INTRODUCTION

The study of the nuclear structure of neutron-rich nuclei near the $A = 100$ region has been an interesting subject from both the theoretical and the experimental viewpoints because of the coexistence of spherical and deformed shapes in this region [1–5]. These shape transitional nuclei demonstrate a characteristic feature different from the adjacent nuclei and are sensitive to two-nucleon transfer reactions since these reactions populate states most strongly in overlapping configuration. The two-nucleon transfer reactions exhibit a strong selectivity based on the degree to which the transferred nucleons are correlated in the final state [6]. The (t, p) reaction is more attractive than the other two-nucleon transfer reactions since two neutrons must be captured with their spin antiparallel ($S = 0$). This permits the unambiguous determination of spins and parities of the final states, where the target nucleus such as ^{100}Mo has zero spin. This reaction is a useful probe for examining the details of the nuclear structure from shell-model and pairing-multipole viewpoints. Neutron-rich nuclei not easily accessible through other reactions can be formed with this reaction.

The energies of the first 2^+ levels of the even-even Mo nuclei with $N \leqslant 58$ exhibit a striking feature of high values, particularly, 0.880 MeV in ^{94}Mo, 0.783 MeV in ^{98}Mo, and 0.539 MeV in ^{100}Mo, systematically changing to low values ~0.2 MeV for isotopes with $N \geqslant 60$. This indicates some characteristic changes in structure of the Mo nuclei. The lightest, ^{92}Mo, is spherical whereas the heavier ^{104}Mo and ^{106}Mo have permanent shape of deformation [2,7]. Therefore, the isotope ^{102}Mo is believed to exist near the well-deformed region [8]. The deformation in this region of nuclei can be due to the strong n-p correlation when the protons occupy the $1g_{9/2}$ orbital and the neutrons occupy its spin orbit partner the $1g_{7/2}$ orbital [9,10].

The neutron-rich ^{102}Mo nucleus was studied with the (t, p), (^{18}O, ^{16}O), and $(t, p\gamma\gamma)$ reactions [3,11,12] and from γ decay [13]; however, the information on the properties of the levels in ^{102}Mo from experimental and theoretical investigations has, so far, been very meager. The angular distributions of the differential cross sections for the levels in ^{102}Mo in the energy range 0.000–3.248 MeV have been measured for a wide range of angle up to 87.5° and are analyzed by using the distorted-wave Born approximation (DWBA) calculations in the present experiment.

II. EXPERIMENTAL PROCEDURE

A beam of 12-MeV tritons from the tandem Van de Graaff accelerator at the Atomic Weapons Research Establishment (AWRE), Aldermaston, was used to bombard the thin target, ^{100}Mo. The reaction products were analyzed with a multichannel magnetic spectrograph, from 5° to 87.5°, in steps of 7.5° in a 10.804-kG magnetic field [14]. The integrated beam current was 6001 μC in this experiment. The triton beam was focused as a rectangular spot 1.5 mm wide and 1.0 mm high on the target. Outgoing protons were detected by 50-μm-thick Ilford K2 emulsion plates mounted in the focal plane of each channel. The energy resolution of the reaction was ~12 keV (full width at half maximum), and the measurement was made at 12 angles between 5° and 87.5°.

The present (t, p) experiment has been measured using a target approximately 100 μg cm^{-2} thick. The target obtained from Oak Ridge National Laboratory was self-supporting and isotopically enriched. The isotopic composition (in percentage) of the target is as follows:

Target	^{92}Mo	^{94}Mo	^{95}Mo	^{96}Mo	^{98}Mo	^{99}Mo	^{100}Mo
^{100}Mo	1.30	0.32	0.20	0.09	0.20	0.24	97.65

To improve the quality of the proton tracks 0.25-mm-thick polythene absorbers were placed in contact with the nuclear emulsion so that the protons penetrated the polythene while all other particles were absorbed. The emulsion plates were taken out from the spectrograph after completing the exposure and then were developed by using the standard method [15]. The Q values were obtained from the input plate position of proton groups using a computer program at five different angles (20°, 35°, 42.5°, 50°, and 57.5^0). The excitation energy of a particular group was measured by subtracting the Q value of that group from the Q value of the ground state.

TABLE I. Triton and proton optical-model parameters used in the DWBA calculations.

Particle	V (MeV)	r_0 (fm)	a_0 (fm)	W (MeV)	r_1 (fm)	a_1 (fm)	V_s (MeV)	W_s (MeV)	r_s (fm)	a_s (fm)
t	156.50	1.240	0.700	12.500	1.530	0.799	7.340	0.066	1.305	0.320
p	57.619	1.170	0.750	0.000	1.320	0.620	6.200	10.720	1.010	0.750

The excitation energy quoted for each level is the mean of the excitation energies at those angles.

The (t, t) angular distributions were normalized to the cross sections predicted by the optical-model calculation in determining the absolute triton elastic cross sections, which produced the best fit to the experimental data. The cross section for the ground-state transition of the ^{100}Mo$(t, p)^{102}$Mo reaction was measured in a separate short exposure by recording the proton group in the bottom half of the plate as the elastically scattered triton group was being recorded on the top half of the plate of the ^{100}Mo$(t, t)^{100}$Mo exposure. The result was finally normalized to the ground-state count of the ^{100}Mo$(t, p)^{102}$Mo long exposure to obtain the absolute differential cross section for different levels in the ^{102}Mo nucleus.

III. DWBA ANALYSIS

Distorted-wave Born approximation (DWBA) calculations were carried out using the computer code DWUCK4 [16]. The optical-model potential was of the form

$$U(r) = V_c(r) - V(1+e^x)^{-1} + 4iW\,d/dx'(1+e^{x'})^{-1} + (\hbar/m_\pi c)^2 r^{-1}(V_{so} + iW_{so})\,d/dr(1+e^{x_s})^{-1}\,\sigma\cdot 1,$$

where $x = (r - r_0A^{1/3})/a_0$, $x' = (r - r_1A^{1/3})/a_1$, $x_s = (r - r_sA^{1/3})/a_s$, and σ is the spin of the incident particle. The Coulomb potential $V_c(r)$ is due to a uniformly charged spherical nucleus of radius $R_c = r_cA^{1/3}$ and charge Z. The Coulomb radius was $1.3A^{1/3}$ fm. V and W are the depths of the real and imaginary potential wells, $r_0A^{1/3}$ is the mean radius of the well, and a is a measure of surface diffuseness. Optical-model parameters obtained from the analysis of elastic scattering of tritons [17] and protons [18] are summarized in Table I.

The DWBA program was run for different L transitions with $0 \leqslant L \leqslant 5$ at a number of excitation energies in the present experiment. The L value assignments were based on the closeness of the DWBA fit to the experimental points. The angular distributions corresponding to an L value transfer of 0 are forward peaked with a secondary maximum around 32.5°. The angular distribution for $L = 1$, 2, 3, 4, and 5 transfers are peaked at 12.5°, 20°, 30°, 37.5°, and 47.5°, respectively.

IV. RESULTS AND DISCUSSION

The results obtained from the present experiment are summarized in Table II, where the energy of 35 levels with uncertainties, L transfers, spins, and parities are presented. This table also contains previous results. A typical proton energy spectrum measured at an angle of 20° to the incident beam direction is shown in Fig 1. The mean ground-state Q value was found to be 7.795 ± 0.009 MeV. The angular distributions for the observed levels are shown in Fig. 2, where smooth curves represent the prediction of DWBA calculations. The calculations were performed for different L transfer ranging from $L = 0$ to 5 for each level in the energy range from 0.000 to 3.248 MeV using optical-model parameters with a spin-orbit term. The transferred angular momentum L has been assigned by comparing the shape of the experimental angular distribution with the calculated one. The peak value of the experimental cross section for each level has been normalized with the DWBA differential cross section of that level at the same angle.

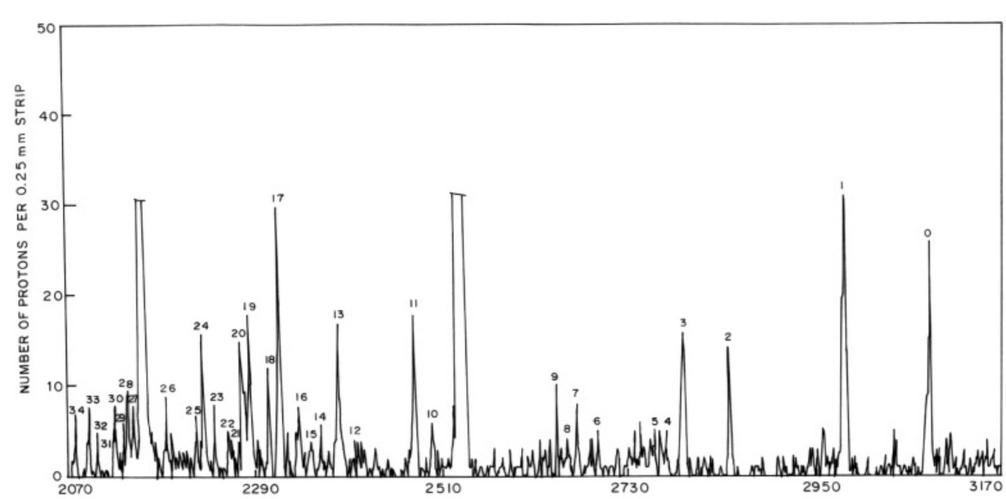

FIG. 1. Energy spectrum of protons from the ^{100}Mo$(t, p)^{102}$Mo reaction at a lab angle of 20°.

TABLE II. Results of the ^{100}Mo$(t, p)^{102}$Mo reaction and previous results.

^{100}Mo$(t, p)^{102}$Mo				^{100}Mo$(t, p)^{102}$Mo[a]		^{100}Mo$(t, p\gamma\gamma)^{102}$Mo[b]	
Group	Energy (MeV)	L	J^π	Energy (MeV)	J^π	Energy (MeV)	J^π
0	0.000	0	0^+	0.000	0^+	0.000	0^+
1	0.298 ± 0.002	2	2^+	0.296	2^+	0.296	2^+
2	0.697 ± 0.004	0	0^+	0.699	0^+	0.696	0^+
						0.743	4^+
3	0.850 ± 0.002	2	2^+	0.850	2^+	0.847	2^+
4	1.248 ± 0.004	(0+2)	$(0^+,2^+)$	1.251	2^+	1.244	2^+
5	1.330 ± 0.002	0	0^+	1.334	0^+	1.327	6^+
						1.397	(4^+)
6	1.608 ± 0.002	2	2^+				
7	1.878 ± 0.002	3	3^-	1.881	3^-		
						2.018	8^+
8	2.108 ± 0.003	1	1^-				
9	2.122 ± 0.001	0	0^+	2.12			
10	2.234 ± 0.003	4	4^+	2.239	(4^+)		
11	2.248 ± 0.007	2	2^+				
12	2.305 ± 0.003	2	2^+				
				2.321			
13	2.366 ± 0.001	2	2^+				
				2.389			
14	2.412 ± 0.004			2.416		2.417	(10^+)
15	2.485 ± 0.004	2	2^+			2.480	
16	2.502 ± 0.001	4	4^+	2.504			
17	2.522 ± 0.002	3	3^-	2.523			
				2.545			
18	2.608 ± 0.001			2.617			
19	2.659 ± 0.004	4	4^+	2.662			
20	2.684 ± 0.007	3	3^-				
21	2.704 ± 0.004	0	0^+	2.705	2^+		
22	2.742 ± 0.002						
23	2.797 ± 0.004						
24	2.851 ± 0.001			2.855			
25	2.872 ± 0.003	2	2^+	2.875			
26	2.943 ± 0.004	0	0^+				
				2.972			
27	2.988 ± 0.011	4	4^+				
28	3.010 ± 0.007	2	2^+	3.011			
29	3.063 ± 0.002	4	4^+				
30	3.091 ± 0.003	3	3^-				
31	3.125 ± 0.003	2	2^+				
32	3.162 ± 0.005	4	4^+				
33	3.193 ± 0.007	2	2^+				
34	3.248 ± 0.001						

[a]Reference [3].
[b]Reference [12].

The agreements between the measured angular distributions and the theoretical predictions are satisfactory and the L values and parities of 29 levels have been assigned unambigously.

The initial spin and parity of the target nucleus ^{100}Mo are known to be 0^+. In the (t, p) reaction, the L value of the transferred particles plays an important role in determining the spin and parity of the state of the residual nucleus as two identical particles must be in a spin of antisymmetric state ($S = 0$). Therefore, the L value of the transferred particles uniquely determines the spin and parity of the final state of the ^{102}Mo nucleus. The agreement between our results and those reported previously is generally good, but there are discrepancies in the assignment of J^π values of a few levels.

1. 1.248-MeV level

The level at 1.248 MeV is an unresolved doublet. The possible assignments to the two components of this unresolved

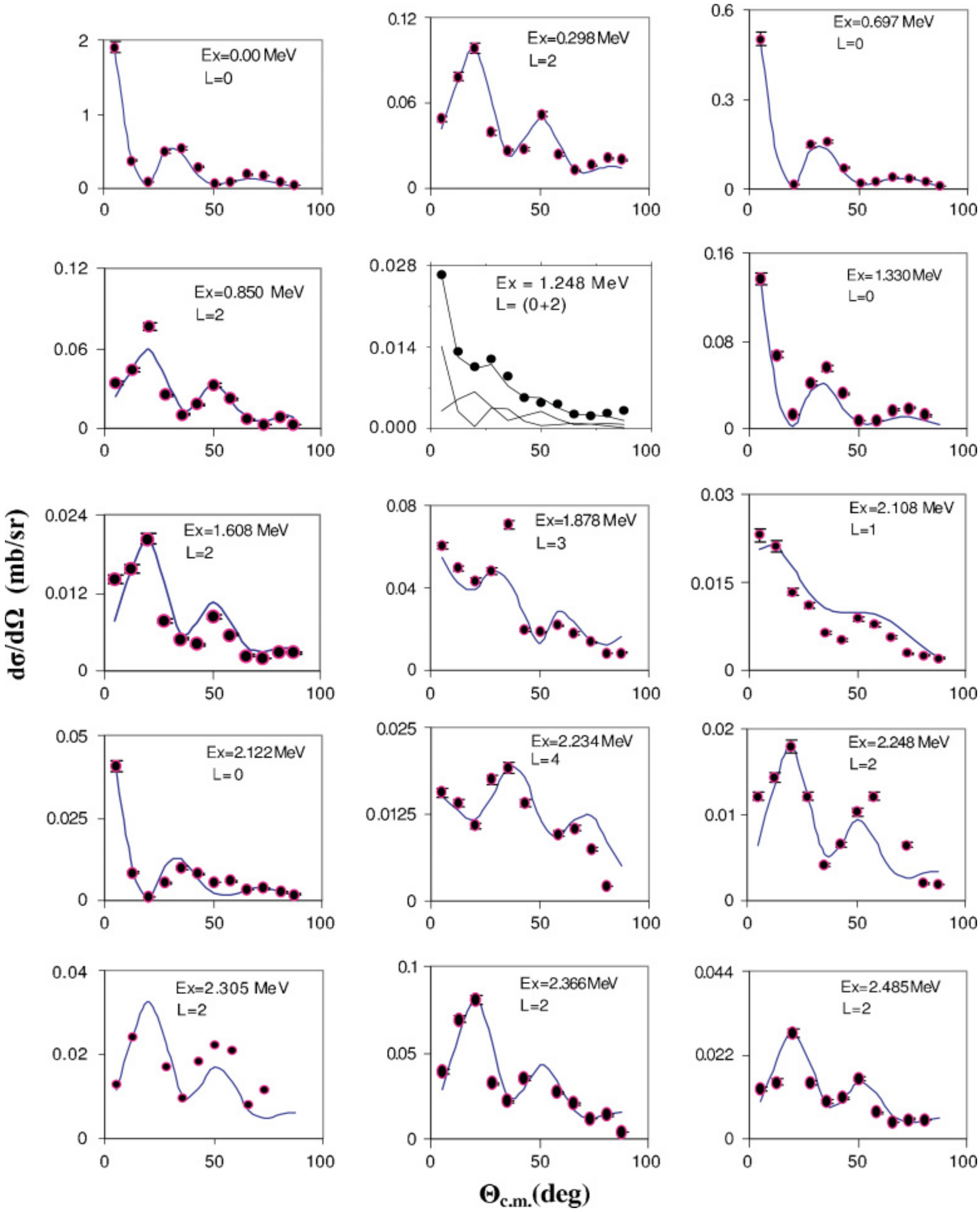

FIG. 2. (Color online) Angular distributions for the transition to the ^{102}Mo levels whose excitation energy (MeV) and L transfer value are indicated. Experimental cross sections are shown as points with error bars. Solid lines are the results of DWBA calculations.

level are based on a combination of $L = 0$ transfer (45%), whose J^{π} value is 0^{+}, and $L = 2$ transfer (55%), whose $J^{\pi} = 2^{+}$. This level was reported as $J^{\pi} = 2^{+}$ in previous works [3,12].

2. 1.330-MeV level

The 6^{+} spin-parity value assigned in the $(t, p\gamma\gamma)$ study [12] for the 1.327-MeV level does not agree with our assignment. The present angular distribution strongly favors a 0^{+} spin-parity assignment for the 1.330-MeV level because of an $L = 0$ transfer. The present J^{π} value is in agreement with the J^{π} value at 1.334 MeV in the (t, p) study [3].

3. 2.234-MeV level

Flynn *et al.* [3] assigned a doubtful $J^{\pi} = 4^{+}$ to the level at 2.239 MeV. We have assigned a definite $J^{\pi} = 4^{+}$ since $L = 4$ fits the experimental data well.

4. 2.485-MeV level

The level at 2.485 MeV is assigned a definite $J^{\pi} = 2^{+}$ in this work. Estep *et al.* [12] reported this level but could not predict the value of J^{π}. This level was not observed in the previous (t, p) work [3].

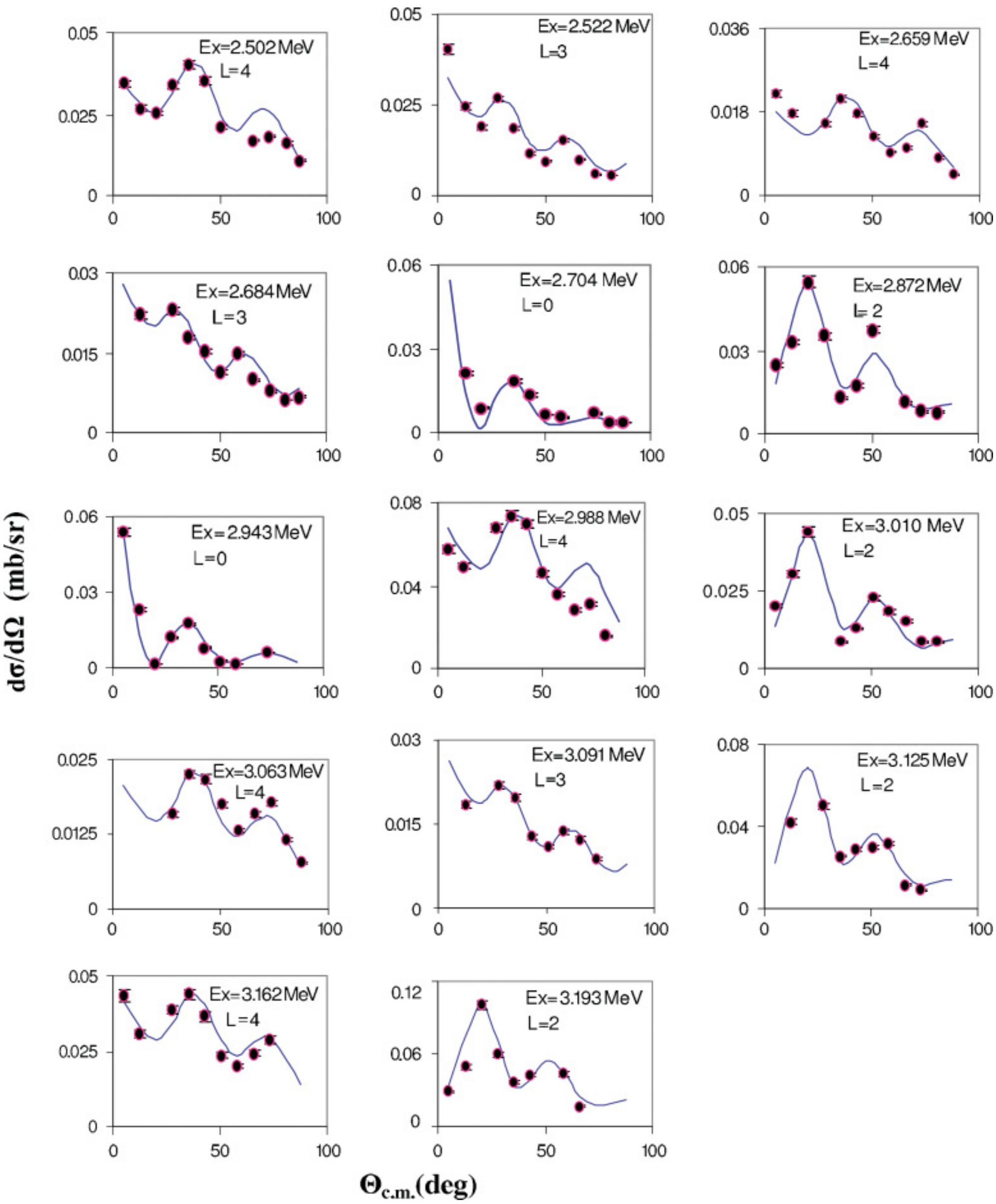

FIG. 2. *(Continued.)*

5. 2.704-MeV level

The level at 2.704 MeV is assigned a definite $J^\pi = 0^+$, which is not in agreement with the assignment $J^\pi = 2^+$ for this level in the previous (t, p) work [3].

The angular distributions of the levels at 2.412, 2.608, 2.742, 2.797, 2.851, and 3.248 MeV are not shown because sufficient data could not be obtained at different angles owing to emulsion disturbances. The present results in ^{102}Mo are compared with those observed in ^{96,98,100,104,106}Mo [2,7] to establish the systematic behavior of the first 2^+, 0^+, and 3^- levels among the even-even Mo isotopes in Fig. 3. It is seen that there is a systematic trend of lowering energy of these levels with increasing neutron number. As neutrons are added, the appearance of downward trends of the excitation energies of these states suggests the existence of a transition in the molybdenum isotopes. The transition in shapes from spherical to deformed occurs suddenly in Zr and Nb [5,19–21] but gradually in Mo with increasing neutron number [3,7]. The coexistence of spherical and deformed properties plays a vital role in these isotopes. These coexisting shapes are particularly sensitive to two-nucleon transfer reactions and are excited with unusually large strength in such reactions. The shape transitional nuclei exhibit substantial strength in the first excited 0^+ state with the reduced ground-state two-nucleon transfer strength in these reactions resulting from mixing of the states of different shapes and from states that have maximum overlap with the target ground-state configuration [8]. It is also observed in Fig. 3 that there is a gradual increase in $L = 0$ strength with increasing neutron number. The 0^+ state at 0.697 MeV in ^{102}Mo is excited more strongly-carrying almost

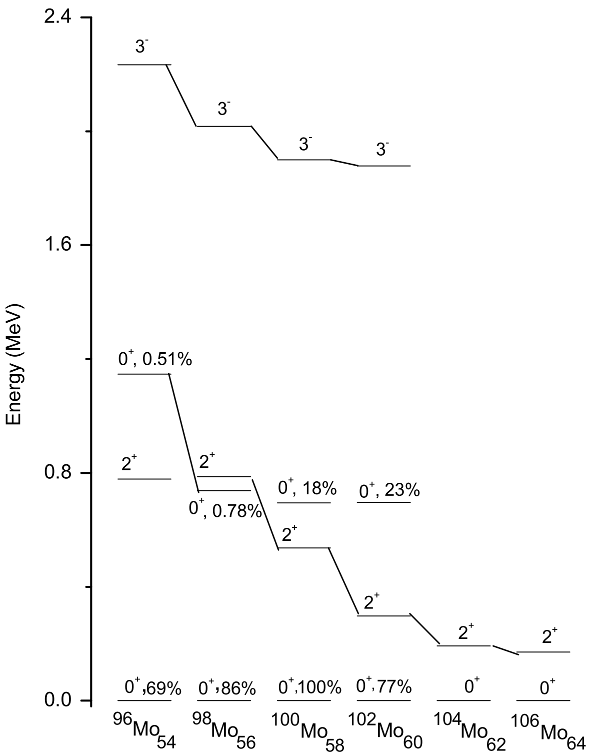

FIG. 3. Systematics of the low-lying first 2^+, 0^+, and 3^- states. The $L = 0$ transition strength has been normalized to 100 units for the transitions of ^{100}Mo.

30% of the ground-state strength in the present (t, p) reaction. The behavior of the first excited 0^+state might indicate that this 0^+ state becomes the bandhead of a deformed band, which in the heavier Mo nuclei becomes the ground state [8,22]. The deformation can be attributed to the isoscalar $n - p$ interaction when protons occupy the $1g_{9/2}$ orbital and neutrons occupy its spin orbit partner $1g_{7/2}$ orbital [9,10]. The deformation parameter β_2 increases with neutron number. It is 0.175 for ^{98}Mo, 0.217 for ^{100}Mo, 0.311 for ^{102}Mo, and 0.33 for ^{104}Mo [2,23]. It may therefore be noted that the ^{102}Mo nucleus exists at the edge of a region of well-deformed shape.

V. CONCLUSIONS

The nuclear properties of the levels in ^{102}Mo have been studied with the ^{100}Mo$(t, p)^{102}$Mo reaction. A number of new energy levels have been found, and spin assignments were made for many of them. The present results are in good agreement with the previous results. The systematics of the first 2^+, 0^+, and 3^- states in Fig. 3 and the appearance of the low-lying 0^+ state as the bandhead of a deformed band that becomes the ground state in the heavier ^{104,106}Mo nuclei suggest that the ^{102}Mo nucleus exists at the end of the transitional region. Theoretical calculation of the level scheme of ^{102}Mo$_{60}$ is not yet available. It will be useful to have further experimental and theoretical investigations of ^{102}Mo.

ACKNOWLEDGMENTS

The authors would like to thank the operating staff of the tandem accelerator at AWRE, Aldermaston, for their cooperation and Professor P. D. Kunz, University of Colorado, for sending us the DWUCK4 program. One of the authors (MSC) is grateful to the University of Bradford for financial support and to Professor G. Brown and Dr. W. Booth, University of Bradford, England, for their help with the exposed plates. Mr. Rahman acknowledges gratefully a research fellowship from the Bose Centre for Advanced Study and Research in Natural Sciences, Dhaka University. The authors would also like to thank the staff of IIT, University of Dhaka, for the computer facilities.

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