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First test of the E(5/4) Bose-Fermi symmetry: The structure of ^{135}Ba

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The first case of a Bose-Fermi critical point symmetry, E(5/4), representing the coupling of a $j = 3/2$ fermion to an E(5) core, was recently proposed. Since ^{134}Ba has been found to be an empirical manifestation of E(5), we carried out a β-decay experiment to study levels in ^{135}Ba, where the last neutron can occupy the $2d_{3/2}$ orbit, as the first test of E(5/4). The comparison shows significant areas of agreement as well as significant discrepancies. Comparison of interacting boson-fermion approximation and shell model calculations with the data are also presented.

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The standard geometric symmetries of the harmonic vibrator, γ-unstable rotor, and deformed symmetric rotor represent stable limits of structure. However, most nuclei occur in regions of structural change. Recently, a new class of geometrical symmetries [1,2], called critical point symmetries, has been proposed for systems undergoing phase transitions between dynamical symmetries. The X(5) description [1], at the phase transition between the vibrator and the rotor, is well explored [3–5]. The E(5) symmetry [2], at the phase transition between the vibrator and γ-soft limits, is less explored [6,7].

Very recently, the first case of a critical point Bose-Fermi symmetry for odd-mass nuclei was developed [8]. Called E(5/4), it describes, analytically, a γ-soft critical point E(5) core coupled to a $j = 3/2$ particle, where E(5) represents a second order phase transition from a vibrator U(5) to a γ-soft rotor O(6). This class of problem is of general interest because it involves Bose-Fermi coupling, a topic of considerable activity in many branches of physics today. It is intriguing that atomic nuclei offer one of the best testing grounds for these ideas. Indeed, the concept of supersymmetry in nuclei has been much discussed recently for fermions coupled to an O(6) γ-soft core [9]. The present case is, of course, even more challenging as it involves coupling a fermion to a bosonic (even-even) core balanced at the critical point where different degrees of freedom compete.

It is the purpose of this Rapid Communication to provide the first experimental test of the E(5/4) Bose-Fermi critical point symmetry. Since ^{134}Ba ($N = 78$) is the first empirical realization [6] of E(5), and since the last neutrons in this mass region can occupy the $d_{3/2}$ orbit, ^{135}Ba is the natural initial test case for E(5/4). To complement the comparison with the present data and provide a perspective, we will also show comparisons with shell model and interacting boson-fermion approximation (IBFA) calculations.

To provide a reliable level scheme—the existing scheme [10–13] is based mainly on early γ-ray singles and low efficiency coincidence data—we carried out a ^{136}Ba$(p, 2n)^{135}$La β-decay experiment. A ^{136}Ba (300μg/cm^2) target was bombarded with a proton beam of 18 MeV and 30 enA from the ESTU tandem accelerator at WNSL. We used nine Compton suppressed clover detectors from YRAST Ball [14]. The experiment was carried out with beam-off–beam-on cycling in 2 h segments for 72 h. With beam-off, we recorded the ^{135}Ba data from ^{135}La β-decay. Sources of ^{152}Eu and ^{133}Ba were used for energy and intensity calibration. Radware software [15] was used to analyze the data.

The data are illustrated in Fig. 1 and the level scheme from our data and the literature [10–13] in Fig. 2. Our singles and coincidence spectra provide evidence for the placement of ten transitions and show no coincidences of three others, suggesting they feed the ground state of ^{135}Ba.

We note that two results from the existing literature must be wrong. Although we confirm the intensity and placement of the $3/2^+_2 \to 5/2^+_1$ transition, the existing $E2/M1$ mixing ratio, δ, leads to an obviously erroneous $1144 < B(E2) < 1664$ W.u. value, and similarly for the $3/2^+_3 \to 5/2^+_1$ transition. These transitions are therefore ignored below. While further experimental work on ^{135}Ba (especially additional δ values) would be valuable, the present level scheme provides the needed observables for a sensitive first test of E(5/4).

The E(5/4) scheme is shown in Fig. 3 (left). States are labeled [8,16] by three quantum numbers according to the notation $\xi\pm, [\tau, \tau_1]$. Major families of levels are grouped according to the quantum number $\xi = 1, 2, \ldots$ while τ is the characteristic O(5) phonon-like quantum number [17]. The coupling of the spin $1/2\hbar$ fermion to the bosonic core is represented by τ_1 which takes on the values $\tau \pm 1/2$ depending on whether the coupling is parallel (labelled by $\xi+$) or antiparallel ($\xi-$).

This level scheme superficially resembles a $3/2^+$ particle weakly coupled to a vibrator core, showing a $3/2^+$ ground state ($\xi+ = 1+, [\tau, \tau_1] = [0, 1/2]$) and a first multiplet

†Deceased.

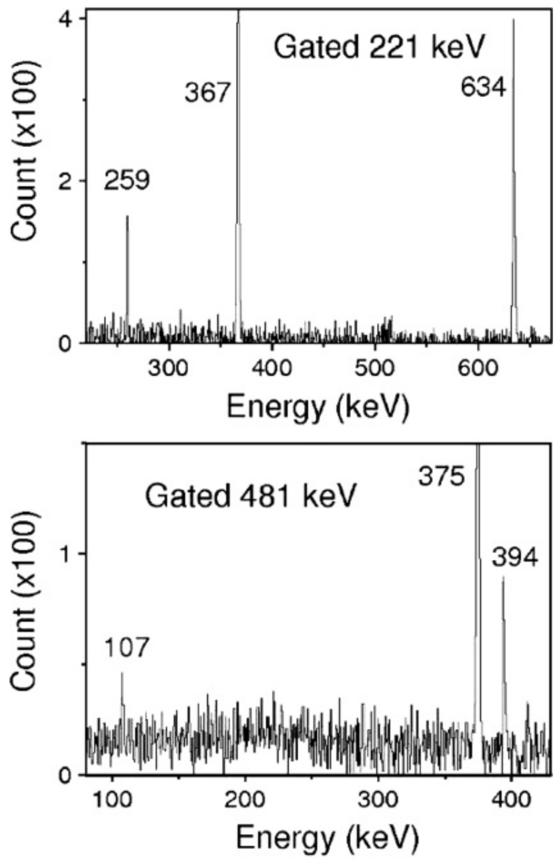

FIG. 1. Upper panel: Spectrum gated on the 221 keV transition. Lower panel: Spectrum gated on the 481 keV transition.

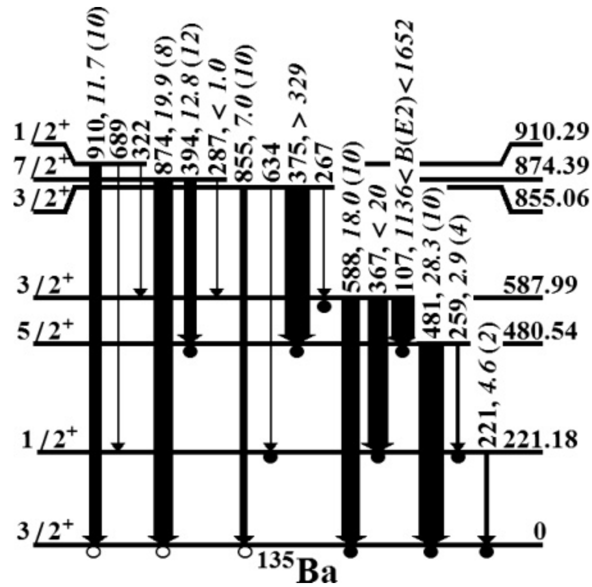

FIG. 2. Level scheme for positive parity states in ^{135}Ba up through 910 keV with $B(E2)$ values (W.u.) (italics) and nominal γ-ray energies shown, obtained with data from the present work and the literature [10–13]. The arrow widths are proportional to the $B(E2)$ values. The solid circles denote transitions confirmed from the coincidence spectra. The open circles are transitions not found in coincidence with any other γ-rays, suggesting ground state transitions. Literature δ values were used to deduce certain $B(E2)$ values. Some $B(E2)$ values (for the transitions $3/2^+_2$ and $3/2^+_3$ to $5/2^+_1$) are very large, suggesting the importance of further measurements of the δ values. Transitions with no $B(E2)$ values are indeterminant because of lack of δ values for multiple branches or because they cannot be an $E2$ transition ($1/2^+_2 \rightarrow 1/2^+_1$).

$3/2^+ \otimes 2^+_1(1+, [1, 3/2])$. However, the multiplet lacks the $3/2^+_2$ level. This level reappears higher as the bandhead state labeled 1−, [1,1/2]. Note that this state was, for simplicity, *not* included in Ref. [8] but was recently discussed [16] in the context of IBFA calculations. It is essential for any adequate test of E(5/4). A second $\xi = 1+$ multiplet ([2,5/2]) appears higher and resembles coupling to two-phonon core states, followed by, still higher, an entire $\xi = 2+$ sequence. Note that the present ξ notation differs from that of Ref. [16]. Our $\xi = 1-$ state is labeled $\xi = 2$ there and our $\xi = 2+$ level is called $\xi = 3$ there.

E(5/4) in Ref. [8] is almost parameter-free except for scale. The Bose-Fermi coupling is specified by a strength k. Energies of the $\xi+$ levels depend on k at the level of a couple percent. However, in the present, extended, version of E(5/4), the energies of the $\xi-$ states depend significantly on k. In strict supersymmetry $k = -1$: the $3/2^+_2(1-, [1, 1/2])$ state is then degenerate with the two-phonon $1+, [2, 5/2]$ multiplet. For $k = -0.5$, it lies lower. We adopt $k = -0.5$ to be consistent with Ref. [16].

E(5/4) has approximate selection rules, which resemble but are not identical to weak coupling in the vibrator. The allowed collective transitions have $\Delta\tau = \pm 1$ and $\Delta\xi = 0$ including transitions from $\xi-$ to $\xi+$ states. Transitions with $\Delta\tau = 0$, $\Delta\tau_1 = 0, \pm 1$, (i.e., within a multiplet) are weak but not forbidden. Similarly, $\Delta\xi = 1$ transitions can be moderately strong if $\Delta\tau = 1$.

We compare E(5/4) with the data in Fig. 3. The energy and $B(E2)$ scales are normalized to the $5/2^+_1$ level which is the lowest lying member of the first multiplet. The comparison shows elements of good agreement and significant discrepancies. We first note that $1/2^+_1$ state (221 keV) has a large spectroscopic factor in (d, p) implying that its structure is largely an $s_{1/2}$ particle coupled to the ^{134}Ba core ground state. Therefore, the lowest $1/2^+$ state of E(5/4) should be compared with the 910 keV $1/2^+_2$ state. This focuses attention on the most obvious discrepancy between E(5/4) and the data—the highly irregular energies of the first multiplet ($7/2^+_1$, $5/2^+_1$, $1/2^+_2$) which are degenerate in E(5/4). While relaxations [8] to E(5/4) can break the degeneracy, the energies will still vary monotonically with spin, inconsistent with the data. Despite strong energy anharmonicity, the ground state $B(E2)$ values from the lowest multiplet are in rather good agreement with E(5/4). This perhaps suggests that the anharmonicities arise largely from diagonal interactions (see IBFA discussion below). The first excited $3/2^+$ state is of particular interest. With a large $B(E2)$ value to the ground state it is clearly associated with the 1−, [1,1/2] state. Its identification (omitted in Ref. [8]) significantly improves the comparison with E(5/4).

The $3/2^+_3$ state at 855 keV has a moderate ground state $B(E2)$ value. $B(E2)$ values for its other decay branches are not known. Therefore it is difficult to assign E(5/4) quantum

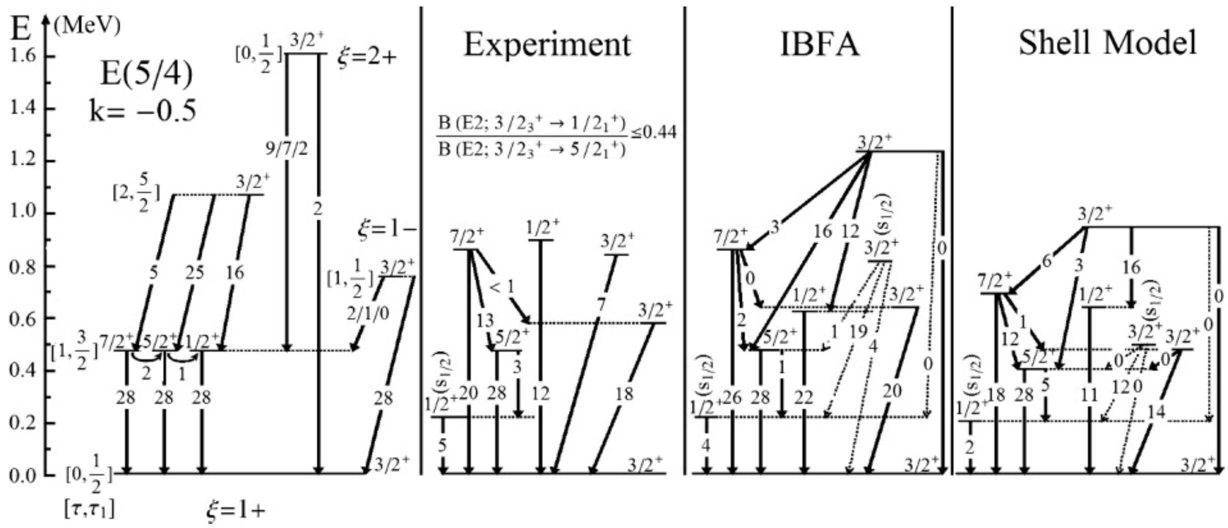

FIG. 3. Comparison of the data for ^{135}Ba with E(5/4), as well as IBFA and shell model calculations (see text for details). Numbers on the transitions are $B(E2)$ values in W.u. The triplets of numbers on two E(5/4) transitions refer to $B(E2)$ values to the $7/2_1^+$, $5/2_1^+$ and $1/2_1^+$ states, respectively.

numbers. The most likely assignment is 1+, [2, 5/2]. Note that E(5/4) predicts a large $B(E2)$ value from this state to the $5/2_1^+$ level.

To provide a more complete discussion, we have also carried out numerical fits with the IBFA [18] that include both $3s_{1/2}$ and $2d_{3/2}$ single particle states. We used the Hamiltonian $H = H_{\rm sd} + H_F + V_{\rm BF}$, where $H_{\rm sd}$ is the boson (core), H_F is the single nucleon degree of freedom, and $V_{\rm BF}$ is a Bose-Fermi interaction. The $d_{3/2}$ and $s_{1/2}$ energy difference was chosen to be 198 keV. Parameters that fit the data for the even-even core, in the notation of the IBFA code ODDA [19], are: pair = 0.148, oct = 0.002, ell = 0.02, eps = 0.92, qq = −0.016 MeV, chq = 0.0, chi = 0.0. The core wave functions are admixtures of U(5) basis states (while preserving O(5) core symmetry). This is particularly important in reproducing the properties of the $3/2_2^+$ state (which is analogous to the 1−, [1,1/2] E(5/4) state). $V_{\rm BF}$ has three terms, corresponding to monopole, quadrupole, and exchange interactions (with strengths chosen as bfm = 0.04, bfq = 0.05, and bfe = 0.8 MeV, respectively). The quadrupole term is the usual boson-fermion quadrupole coupling. For systems with O(5) symmetry ($\chi = 0$), such a term cannot break the multiplet degeneracies. It does, however, raise the energy of the first excited $3/2^+$ state. The monopole term expands or contracts the energy scale. The exchange interaction is extremely important. It reflects the fact that the bosons are comprised of fermions that can be exchanged with the odd fermion. It is predominantly diagonal and breaks multiplet degeneracies. Note that, in Ref. [16], the exchange term was ignored.

The IBFA results are shown in Fig. 3 and information on the dominant components in the wave function is summarized in Table I. The comparison of the IBFA with ^{135}Ba is, overall, quite good. The IBFA accounts for the energy (fitted) and ground state $B(E2)$ value of the $s_{1/2}$ state. More impressively, the IBFA, because of the exchange term, can reproduce the highly non-monotonic splitting of the lowest multiplet while leaving intact the large ground state $B(E2)$ values.

The energy of the lowest excited $3/2^+$ state is also well reproduced, and the $B(E2)$ value to the ground state is collective, and in excellent agreement with the data. This sensitively tests the critical point description as this state originates (in the vibrator limit) in the first multiplet (along with the $7/2_1^+$, $5/2_1^+$ and $1/2_1^+$ states) but decouples from that multiplet as the phase transition commences, as seen in the systematic core-U(5) to core-O(6) IBFA calculations of Ref. [16].

The most significant discrepancy is that while $B(E2; 7/2_1^+ \rightarrow 5/2_1^+)$ is 12.8 W.u., the IBFA value is 1.9 W.u. These two levels both have the same dominant structure ($d_{3/2} \otimes 2_1^+$) (Table I) and thus the intra-multiplet transition [($\Delta(\tau, \tau_1 = (0, 0)$) in E(5/4)] is naturally weak in the IBFA. Note that, if χ is allowed to be finite in the $E2$ operator (as seen in Ref. [13]), the O(5) selection rules are broken and larger $B(E2; 7/2_1^+ \rightarrow 5/2_1^+)$ values result.

TABLE I. The amplitudes $a^2_{d_{3/2}}$ and $a^2_{s_{1/2}}$ of the coupling of the ^{134}Ba core with $d_{3/2}$ and $s_{1/2}$ fermion wave functions respectively, in the IBFA. For the larger of a_d and a_s, the $0_1^+/2_1^+$ core decomposition $\alpha^2(0_1^+)$ and $\beta^2(2_1^+)$ is shown.

J^π	IBFA		
	$a^2_{d_{3/2}}$	$a^2_{s_{1/2}}$	$\alpha^2(0_1^+)/\beta^2(2_1^+)$%
$3/2_1^+$	99.8	0.2	98/1
$3/2_2^+$	79.5	20.5	1/78
$3/2_3^+$	20.9	79.1	0/79
$3/2_4^+$	92.1	7.9	0/0
$1/2_1^+$	1.2	98.8	99/0
$1/2_2^+$	99.4	0.6	0/98
$5/2_1^+$	95.3	4.7	0/95
$7/2_1^+$	99.8	0.2	0/98

The nature of the $3/2_3^+$ state is interesting. It has a moderate (7 W.u.) ground state transition. Also, it turns out that the data allow the extraction of a limit on the $B(E2)$ branching ratio to the $1/2_1^+$ and $5/2_1^+$ states of 0.44 (see Fig. 3). The $3/2_3^+$ state in the IBFA is largely $1/2^+ \otimes 2_1^+$ and has a large ratio of $\sim$16, while the ratio is $\sim$0 for the $3/2_4^+$ state, which is mostly (see Table I) a $3/2^+ \otimes$ two-phonon state, and has relative $B(E2)$ values to the first excited multiplet very similar to those from the 1+, [2, 5/2], $3/2^+$ state in E(5/4). Both $3/2^+$ states have weak ground state transitions. We favor the empirical association of the $3/2_3^+$ state with the $3/2_4^+$ level of the IBFA, but urge a remeasurement of δ ($3/2_3^+ \rightarrow 5/2_1^+$).

Finally, we performed shell model (SM) calculations for ^{135}Ba with the NL3 interaction Hamiltonian [20], using the code OXBASH [21,22]. This interaction gives reasonable absolute $B(E2)$ values in $^{100-132}$Sn, level spacings in ^{132}Sn, and single particle energies in ^{101}Sn. The calculations were performed for a ^{100}Sn core with all configurations in the $1g_{7/2}$ and $2d_{5/2}$ proton and the $2d_{3/2}$, $3s_{1/2}$ and $1h_{11/2}$ neutron orbits. The results are included in Fig. 3. The agreement is overall rather good, and certainly the shell model is the best of the models for both relative energies and $B(E2)$ values.

The $1/2_1^+$ level is largely a $3s_{1/2}$ single particle state, in agreement with the spectroscopic factor data. The observed non-monotonic arrangement of the $7/2_1^+, 5/2_1^+$, and $1/2_1^+$ states is also correctly reproduced. Unique to the models presented, the shell model reproduces the collective $7/2_1^+ \rightarrow 5/2_1^+$ $B(E2)$ value. The collectivity originates in differing structures in the proton sector. Again, as with the IBFA, the data for the empirical $3/2_3^+$ state, in particular, the weak branching to the $1/2_1^+$ state, seem to be better reproduced by the $3/2_4^+$ state.

To summarize, we have performed the first test of the E(5/4) Bose-Fermi symmetry in ^{135}Ba. The results show clearly that E(5/4) can account for some of the observables in ^{135}Ba but that it does not provide a fully satisfactory description. Specifically, many of the collective and forbidden $B(E2)$ values of E(5/4) agree well with the data. The inclusion of the 1−, [1, 1/2], $3/2_2^+$ level, omitted in Ref. [8] and highlighted in Ref. [16], is essential. However, E(5/4) cannot explain the highly nonmonotonic energies of the first (2^+ core-coupled) multiplet, or the strong intramultiplet $7/2_1^+ \rightarrow 5/2_1^+$ transition. That the comparison with E(5/4) is mixed is perhaps not surprising: E(5/4) is an extremely simplified scheme and critical point nuclei show a delicate balance of phases which can be disturbed by the addition of a fermion. Therefore, symmetries such as $E(5/4)$ may be more significantly broken than their counterparts [such as E(5)] in even-even nuclei. IBFA and Shell Model calculations are in better agreement with the data, at the cost of additional parameters. The $7/2_1^+ \rightarrow 5/2_1^+$ transition, which is quite collective experimentally and in the Shell Model, is almost forbidden in the IBFA if O(5) symmetry is preserved. The exchange term in the IBFA is critical to breaking the multiplet degeneracies seen experimentally. The microscopic shell model is complementary to symmetry-based approaches to the structure of many-body systems. Both are essential to understanding the complexity of these systems and the simplicities that they show.

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