$p_{1/2}$ hole states in odd-A Na nuclei

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Simple weak-coupling calculations for ^{23,25,27}Na demonstrate that the lowest $J = 1/2$ proton-hole state should lie near 5 MeV in ^{27}Na.

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Properties of nuclei near neutron and proton drip lines depend sensitively on subtle details of nuclear structure. Effective single-particle energies vary with distance from the valley of stability, resulting in sub-shell gaps that are different from those near the valley of stability. For proton- and neutron-rich nuclei in the region just above ^{16}O, these effects primarily involve the $1d_{5/2}$ and $2s_{1/2}$ orbitals—which are close together and reasonably well separated from the $1d_{3/2}$ orbital. Small changes in $d_{5/2}$-$s_{1/2}$ splitting can seriously alter the properties of even the lowest-lying states in these nuclei and can bring about new magic numbers—nuclei that are more stable than expected. It is not yet known if these changes can be described as smooth variations in relevant quantities or if sudden sharp changes occur for small changes in neutron or proton number. For these reasons, it is important to study carefully the low-lying levels of nuclei at the limits of stability. One recent experiment [1] involves ^{27}Na and the possibility of two low-lying $J = 1/2$ states.

In an experimental *tour-de-force*, Cooper *et al*. [1] used a radioactive beam on a radioactive target and detected protons and gammas in coincidence, in the reaction ^{14}C(^{14}C, $p\gamma$)^{27}Na. A few $p\gamma\gamma$ coincidences were also detected. They measured energies of 27 gamma transitions, which they were able to explain with a set of 22 excited states (including four for which the gammas did not provide the energy of a state because it was unclear whether they fed the g.s. or first-excited state). Energies of the gamma-decaying states could be determined to within a few hundred keV by the energies of the detected protons. In some cases, construction of the decay scheme was aided by the fact that some γ energies summed to the same total energy, indicating different intermediate paths from initial to final state.

Earlier, Fifield *et al*. [2], with a resolution of 330 keV, used the ^{26}Mg(^{18}O,^{17}F) reaction at 104 MeV to measure the ^{27}Na g.s. mass excess and the excitation energies of four states, up to an energy of 5.6 MeV. Their mass excess of -5514 ± 60 keV was 106–139 keV less negative than four other values, but they all agreed within the combined uncertainties. Their value agrees well with the newest value [3] of -5516 ± 4 keV. Fifield *et al*. were aware that the g.s. should be part of a closely-spaced doublet, but they argued that the excited state would have little effect on their mass excess measurement because of the small expected separation, and probable smaller cross section than the g.s. They suggested, on the basis of an *sd*-shell model calculation, that their state at 1.72 ± 0.04 MeV was probably a $1/2^+$ state.

Cooper *et al*. confirmed the first excited state at 62 keV. From systematics, it should have $J^\pi = 3/2^+$ (the g.s. is $5/2^+$ [4]). They saw two states at reasonably low excitation that they identified as $J = 1/2$ states. A state labeled $(1/2)^+1815$ in their Fig. 3 is fed by a 2901-keV γ and decays via a 1753-keV gamma, presumably to the 62-keV first-excited state. A state labeled $(1/2)^-1725$ in their Fig. 3 is associated with a 1663-keV γ, again assumed to be to the first-excited state. The authors considered the parity of both supposed $J = 1/2$ states. They realized their finding could be important because only one $1/2^+$ is predicted by standard shell-model calculations, but they appear to suggest that one of the two states is probably a $1/2^-$ intruder state.

Cooper *et al*. thus had two apparent candidates for the state seen at 1.72 ± 0.04 MeV by Ref. [2]—viz at 1725 and 1815 keV. The lower is closer in energy, but the upper is taken by them to be the *sd*-shell state, probably because it is connected to another state by a γ transition. In their Table I, which compares their results with an *sd*-shell model calculation (and which, of course, includes only positive-parity states), it is the 1815-keV state they list. They suggest in the text that the other supposed $J = 1/2$ state might be the $p_{1/2}$ intruder state. They state that "the opposite parity assignment to the two spin 1/2 states is also possible."

Thus, there are three possibilities:

(i) Two $J = 1/2$ states, both with positive parity,
(ii) Two $J = 1/2$ states, one of each parity,
(iii) Only one $J = 1/2$ state.

The first would require some major modification to the shell model, but it is by far the most exciting of the three possibilities. The third would imply that one of these supposed $J = 1/2$ states has been mis-identified. It is the second possibility that we examine further here.

Weak coupling can provide the expected energies of the mp-1h states in ^{A}Na, using the Bansal-French-Zamick [5] prescription. This formulism has allowed an understanding of a large number of particle-hole states in nuclei near closed shells. The essence of weak coupling is the assumption that in an mp-nh state (with particles and holes in different major shells), the interactions among the mp are the same as in the mp-0h state, and similarly for the n holes. The interaction between the mp and nh is taken [5] to be $mna + b\mathbf{T}_{mp} \cdot \mathbf{T}_{nh} + m_\pi n_\pi c$, where a and c are independent of m and n, T is isospin and the subscript π refers to protons. Because the Na nuclei have an odd proton, a proton hole state (hole coupled to an

TABLE I. Energies of $1/2^-$ single-hole states in odd-A Na nuclei.

Nucleus	Configuration	Excitation energy (weak coupling)	E_x(exp)
^{23}Na	8p-1h	0.437 MeV $+ 8a + 4c$	2.64 MeV[a]
^{25}Na	10p-1h	-2.016 MeV $+ 10a + b/2 + 4c$	3.995 MeV[a]
^{27}Na	12p-1h	-4.53 MeV $+ 12a + b + 4c$	4.9?[b]

[a]Reference [9].
[b]Expected.

even-even core) will be considerably lower in energy than a neutron hole state (hole coupled to an odd-odd core).

Thus, in weak coupling [5], the excitation energy of the $p_{1/2}$ proton hole state in ^{27}Na is given by

$$E_x(^{27}\text{Na(12p-1h)}) + \text{M}(^{27}\text{Na(g.s.)}) = \text{M}(^{28}\text{Mg(g.s.)}) + \text{M}(^{15}\text{N(g.s.)}) - \text{M}(^{16}\text{O(g.s)}) + 12a + b + 4c.$$

In ^{23}Na, the $p_{1/2}$ hole excitation energy is

$$E_x(^{23}\text{Na (8p-1h)}) + \text{M}(^{23}\text{Na(g.s.)}) = \text{M}(^{24}\text{Mg(g.s.)}) + \text{M}(^{15}\text{N(g.s.)}) - \text{M}(^{16}\text{O(g.s.)}) + 8a + 0b + 4c,$$

where the M's are mass excesses. Using values from Ref. [6] provides the numerical results listed in Table I, where we also include ^{25}Na. We note that if we take *differences* of energies in different Na isotopes, the Coulomb term does not enter, and the parameters a and b occur only in the combination $a + b/4$:

$$E_x(^{27}\text{Na(12p-1h)}) - E_x(^{23}\text{Na(8p-1h)}) = -4.967\ \text{MeV} + 4(a + b/4),$$

and

$$E_x(^{25}\text{Na(10p-1h)}) - E_x(^{23}\text{Na(8p-1h)}) = -2.453\ \text{MeV} + 2(a + b/4),$$

i.e., the energy increases linearly from ^{23}Na to ^{27}Na. The $p_{1/2}$ hole states are also known in ^{21}Na, ^{22}Na, and ^{24}Na. We do not include ^{21}Na because we prefer to stay on one side of the valley of stability. Also, the 6p-1h case in ^{21}Na involves two values of the T_z parentage (0 and 1) for the particles and the associated isospin Clebsch-Gordan coefficients, whereas that parentage is unique in ^{23,25,27}Na. Weak coupling does reproduce the energy in ^{21}Na. In odd-odd nuclei, the strength is split between two J^π values. Also, there the high density of states at quite low excitation causes some mixing with other states.

The quantity $a + b/4$ is well established from the heavy N isotopes—from ^{16}N [7] we get $a + b/4 = 1.826$ MeV, whereas ^{19}N [8] provides 1.806 MeV. For our present purposes, the difference is insignificant and we use the average value of 1.816 MeV. Similar values arise from ^{17,18}N.

Thus, the excitation energy of the 12p-1h state in ^{27}Na should be 2.30 MeV higher than that for the 8p-1h state in ^{23}Na. Experimentally, the latter is at 2.64 MeV, [9] implying $E_x = 4.94$ MeV for the 12p-1h state in ^{27}Na. The same type of computation gives 3.82 MeV for the excitation energy of the 10p-1h state in ^{25}Na—reasonably close to a known state at 3.995(4) MeV [9] that is populated strongly via $l = 1$ in ^{26}Mg(d,^{3}He), [10] as would be expected for such a state.

Therefore, we conclude that the 1.7-(or 1.8-) MeV state [1] of ^{27}Na cannot be the $1/2^-$ $p_{1/2}$ hole state, unless a very drastic shift in structure has taken place. Rather, that state should be near 5 MeV in excitation. Whatever the nature of the extra state near 1.8 MeV (if it is really there), it would appear that conventional methods of calculation would have difficulty reproducing it. It is too low to correspond to an *fp*-shell particle excitation, and other hole states, e.g., 14p-3h or 13p-2h are expected to be significantly higher.

The $p_{1/2}$ hole state should be strongly populated in pickup from ^{28}Mg. Using a beam of ^{28}Mg on a deuterium target detecting ^{3}He, or on a ^{12}C target detecting ^{13}N should find it. The latter has the advantages of a simple target and an outgoing nucleus with only one bound state. Other heavy-ion reactions might suffer from too many states excited in the outgoing nucleus.

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