Relation between nuclear and nucleon structure functions and their moments

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Calculations of nuclear structure functions (SFs) $F^A_{k=1,2}(x, Q^2)$ routinely exploit a generalized convolution, involving the SFs for nucleons F^N_k and the linking SF $f^{PN,A}$ of a fictitious nucleus, composed of point particles, with the latter usually expressed in terms of hadronic degrees of freedom. For finite Q^2 the approach seemed to be lacking a solid justification and the same is the case for recently proposed, effective nuclear parton distribution functions, which exactly reproduce the above-mentioned hadronically computed F^A_k. Many years ago Jaffe and West proved the above convolution in the plane-wave impulse approximation for the nuclear components in the convolution. We extend the above proof to include classes of nuclear final-state interactions. One and the same function appears to relate parton distribution functions in nuclei and nucleons and SFs for nuclear targets and for nucleons. That relation is the previously conjectured one, with an entirely different interpretation of $f^{PN,A}$. We conclude with an extensive analysis of moments of nuclear SFs based on the generalized convolution. Characteristics of those moments are shown to be quite similar to those for a nucleon. We conclude that the above is evidence of asymptotic freedom of a nucleon in a medium and not the same for a composite nucleus.

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I. INTRODUCTION

This paper concerns two related topics. The first is a generalized convolution, involving structure functions (SFs) F^A_k and F^N_k, which compose cross sections for inclusive scattering of unpolarized leptons from composite targets A and for a nucleon. The second one deals with implications of the above for moments of F^A_k.

Standard approaches employing hadronic degrees of freedom have used generalized convolutions of the form

$$F^A = f^A * F^N, \tag{1.1}$$

$$F^A_k(x, Q^2) = \sum_\alpha \int_x^A \frac{dz}{z^{2-k}} f^{\alpha,A}(z, Q^2) F^a_k\left(\frac{x}{z}, Q^2\right) \tag{1.2}$$

$$\approx \int_x^A \frac{dz}{z^{2-k}} f^{PN,A}(z, Q^2) F^{\langle N\rangle}_k\left(\frac{x}{z}, Q^2\right), \tag{1.3}$$

$$F^{\langle N\rangle}_k = \frac{ZF^p_k + NF^n_k}{A} = \frac{1}{2}\left[1 - \frac{\delta N}{A}\right]F^p_k + \frac{1}{2}\left[1 + \frac{\delta N}{A}\right]F^n_k. \tag{1.4}$$

The involved SFs depend on the squared four-momentum transfer $q^2 = -Q^2 = -(|\boldsymbol{q}|^2 - \nu^2)$ and on the Bjorken variable x in terms of the nucleon mass M with support $0 \leqslant x = Q^2/2M\nu \leqslant M_A/M \approx A$.

Equation (1.2) decomposes F^A_k into contributions from various constituents $'a'$, such as nucleon, virtual bosons, etc. For the kinematic region of our main interest, $x \gtrsim 0.2$, it suffices to retain only nucleons, or more precisely, the averaged nucleon with SF $F^{\langle N\rangle}_k$ [Eq. (1.4)], which we obtained by weighting $F^{p,n}_k$ with Z and N; $\delta N/A$ is the relative neutron excess.

Within the framework of hadron dynamics, the convolution (1.3) can be proven in the plane-wave impulse approximation (PWIA). In that approximation the linking function f in convolution (1.3), which in general is the SF of a fictitious nucleus composed of point nucleons, is approximated by $f^{PN,A} \to f^{\rm PWIA}$, with the latter related to the spectral function of the knocked-out nucleon in the target [1]. For finite Q^2, convolution (1.3) stood as a conjecture.

The same is the case for an alternative, nonperturbative Gersch-Rodriguez-Smith (GRS) approach [2], which was originally formulated for a nonrelativistic system of point particles [3]. It was subsequently extended to systems of composite constituents, such as quantum gases and liquids, H_2, D_2, He, etc. Since the energy scales for electronic, rotation-vibration modes, etc., differ appreciably, the Born-Oppenheimer approximation applies. As a consequence, the SF (or "linear response") of the composite system is accurately given as a repeated regular convolution, Eq. (1.1) involving the SF of the translation of the centers of mass of inert molecules and of internal modes of each molecule [4]:

$$F^{\rm qugas}(|\mathbf{q}|, \nu) = \int d\nu_1 F^{\rm trsl}(|\mathbf{q}|, \nu - \nu_1) * \int d\nu_2 F^{\rm rot}(|\mathbf{q}|, \nu_1 - \nu_2) * \int d\nu_3 F^{\rm vibr}(|\mathbf{q}|, \nu_2 - \nu_3) * \ldots \tag{1.5}$$

The next step in the development has been a covariant generalization of the above GRS theory, first for the SF of a system of point particles, i.e., for f in convolution (1.3) [5]. For increasing Q^2, internal degrees of freedom need ultimately to be included through $F^{\langle N\rangle}_k$, as described by the generalized convolution (1.3).

It stands to reason that, in general, convolution (1.3) for a composite nucleus rests on different energy scales for the participating modes. In fact, convolution (1.3) was proven for a model with quarks clustered in nucleons, in which the energy scale for internal excitations is much in excess of that for NN forces [2]. For higher but not asymptotic Q^2, it seemed difficult to derive a covariant version, and convolution (1.3) has been considered a conjecture.

Calculations were based on data for F_2^p and on some adopted F_k^n [6], such that a calculation of F_k^A amounts to that of $f^{\rm GRS}$. The latter can be evaluated by use of purely hadronic notions, such as single-nucleon spectral functions, nuclear density matrices of various orders, NN forward-scattering amplitudes (fsa's), etc. Support for the validity of convolution (1.3) came mainly from the reasonable description of a large body of inclusive scattering cross-section data for $Q^2 \gtrsim Q_0^2 \approx 2.5\,{\rm GeV}^2$ [7–9].

For later reference, we mention that convolution (1.3) has its deficiencies. For example, $F_k^{\langle N\rangle}$ is taken to be the SF of a free averaged nucleon, which generally is off its mass shell. In addition, convolution (1.3) lacks explicit spin-isospin structure and, in particular f is usually computed from spin-isospin-averaged input.

Next we recall an alternative representation of a nuclear SF, which uses nuclear parton distribution functions (pdf's) $q_i^A(x, Q^2)$ for finite Q^2, which have to be computed from their nucleonic analogs $q_i(x, Q^2)$. Those nuclear pdf's are effective ones: We do not aim for an underlying theory, and in particular not for accounting for Q^2 dependence, compatible with evolution from a scale Q_0^2. The only requirement is the exact reproduction of $F_2^A(x, Q^2)$, as computed in hadronic representation (1.3).

The above requirement is not anywhere sufficient to determine those pdf's, and the apparent freedom is exploited by two deliberate choices [10]. An inessential choice is one for which we assume F_2^A to be the same combination of nuclear pdf's, as $F_2^{\langle N\rangle}$ is of nucleon ones; thus (for clarity we drop the x, Q^2 dependence in arguments)

$$F_2^{\langle N\rangle} = \sum_i a_i x q_i = \frac{5x}{18}\left[u_v + d_v + 2\bar{u} + 2\bar{d} + \frac{4}{5}s - \frac{3\delta N}{5A}(u_v - d_v + 2\bar{u} - 2\bar{d})\right], \quad (1.6)$$

$$F_2^A \equiv \sum_i a_i x q_i^A = \frac{5x}{18}\left[u_v^A + d_v^A + 2\bar{u}^A + 2\bar{d}^A + \frac{4}{5}s^A - \frac{3\delta N}{5A}\left(u_v^A - d_v^A + 2\bar{u}^A - 2\bar{d}^A\right)\right]. \quad (1.7)$$

Next we choose to relate nuclear pdf's of given species i to its analog for the averaged nucleon $\langle N\rangle$, in precisely the same way as hadronic representation (1.3) links nuclear and nucleon SF; thus

$$q_{i/A}(x, Q^2) = \sum_a \int_0^A dz f_{a/A}(z, Q^2) q_{i/a}\left(\frac{x}{z}, Q^2\right) \quad (1.8)$$

$$\approx \int_0^A dz f_{N/A}(x, Q^2) q_{i/N}\left(\frac{x}{z}, Q^2\right). \quad (1.9)$$

Approximation (1.9) does not mix flavors and uses a single linking function $f_{N/A} = f^{PN,A}$, independent of the species, whether valence, sea quarks, or gluons. By construction the computed nuclear SF F_2^A (1.7) in the parton representation (1.9) are identical to their hadronic analog (1.3), provided the same input is used. In practice the input F_2^n for the two differs (see Ref. [10] for a discussion). Between parentheses we add that, being the same SF as in that of convolution (1.3), f carries along the above-mentioned deficiencies.

For both the hadron and pdf representations of F_2^A, there seems to be missing proof of convolution (1.3), as well as of an estimate of the lower-limit Q_0^2 beyond which Eq. (1.7) is approximately valid. However, we recently stumbled on 20-year-old papers by Jaffe and West, which contain the basics of the desired proof [11,12]. Judging from the lack of citations, even cognoscenti apparently overlooked or forgot those papers, possibly because those were published in the proceedings of a summer school and of an American Institute of Physics meeting. In the above publications the generalized convolution is derived by use of a parton model as well as pQCD, both in the special case of the PWIA. In the following discussion we generalize their results to include the nuclear final-state interaction (FSI).

Since the article of Jaffe [11] is fairly self-contained, it will suffice to cite only some essentials, in particular the central relation between forward γ-target scattering amplitudes and pdf's. We then show that, although the inclusion of general FSIs usually spoils their accommodation in a convolution for F_k^A [11], this is not the case for some nuclear FSIs not involving partons. The above holds, for instance, for the distorted-wave impulse approximation (DWIA) in the form given in Ref. [13]. This is also the case for the GRS version for finite, relatively large Q^2 and for those $f^{PN,A}$ are just the SFs in the representations (1.9) and (1.3); the above completes the proof for what previously was called a conjecture. We conclude this paper with an analysis of data on moments of high-Q^2 nuclear SFs and present pQCD results for the above as was done in the past for a proton.

II. DERIVATION OF NUCLEAR PARTON DISTRIBUTION FUNCTIONS

We start with a proton and consider the forward scattering amplitude (fsa) $a(\gamma p)$ as a two-step process in which the proton emits a quark, which in turn absorbs the virtual photon (Fig. 1). That amplitude can be evaluated, given an expression for the current in terms of parton fields. For instance, in a model with

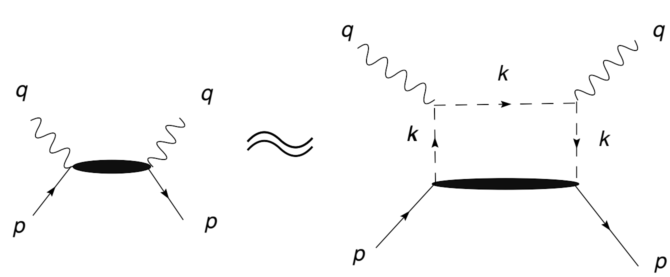

FIG. 1. The decomposition of the forward γp amplitude in the PWIA and its link to the quark-p scattering amplitude.

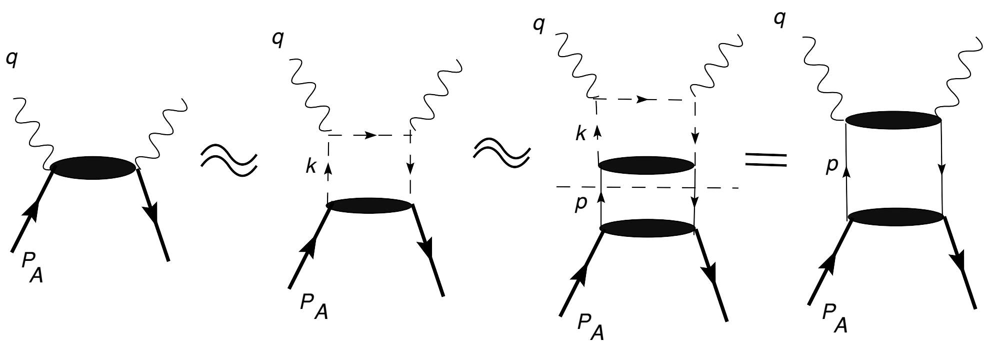

FIG. 2. Same as Fig. 1 for a composite target. The inclusion of an intermediate set of free nucleon and spectator states and a recombination of terms (marked by dashed horizontal lines), lead to a generalized convolution of forward amplitudes $a(\gamma^N)$ and $a(N\text{-Sp})$.

free parton fields, the result is [11]

$$F_2^p(x,Q^2) = x\sum_i e_i^2 q_i(x,Q^2), \tag{2.1}$$

$$q_i(x,Q^2) = \int \frac{d^4k}{(2\pi)^4}\delta\left(x-\frac{kq}{pq}\right)\chi_i(k,p), \tag{2.2}$$

$$q_i^{\rm scal}(x) = \lim_{Q^2\to\infty} q_i(x,Q^2) = \int \frac{d^4k}{(2\pi)^4}\delta\left(x-\frac{k^+}{p^+}\right)\chi_i(k,p), \tag{2.3}$$

where the sum in Eq. (2.1) is over quarks with charge $e_i \cdot \chi_i = \chi_{i/p}$ in Eq. (2.3) is the fsa $a(q_i p)$ in Fig. 1. Above one neglects spin and color: Their inclusion is straightforward and is immaterial for the reasoning. To lowest order, i.e., in the PWIA, Eq. (2.2) is proportional to what in nuclear physics parlance is called the spectral function of a parton i in the p. The δ function in the integrand of Eq. (2.2) selects the momentum fraction x of the quark in the proton as determined by the four-momenta k, p, q of the quark, proton, and virtual photon, and the integrand in Eq. (2.2) holds for finite Q^2. Equation (2.3) is the Bjorken limit of (2.2), in which case the argument of the δ function can be expressed in terms of the dominant light-cone components k^+, p^+: The resulting q_i, F_2^p are pdf's and SFs in the scaling limit and depend only on x.

Of an entirely different nature is the Q^2 dependence generated by the FSI beyond the PWIA, coming from quarks that emit gluons, from gluon pair production, triple gluon coupling, etc. Those add $\ln(Q^2)$ and $[1/Q^2]^n$ corrections to the above scaling limits for pdf's and SFs. For the present purpose it is irrelevant whether those ultimately derive from the operator product expansion (OPE) or are calculated in pQCD by evolution.

Much of the above for a p target, a neutron, or averaged N holds also for a general target A: One can copy Eqs. (2.1)–(2.3), replacing $p(N)$ with a composite target. However, it is awkward to deal with the spectral function of a parton in a nucleus, as is the fsa $\chi_{i/A}$ in the PWIA.

A more natural way is the evaluation of that amplitude on insertion of an intermediate set of states for free nucleons and a fully interacting daughter nucleus. The product of the fsa's $a(\gamma q_i)$ and $a(q_i N)$ is subsequently integrated over the intermediate momentum to form $a(\gamma N) \propto F^N$. The result, illustrated in Fig. 2, amounts to the following relation between the three involved fsa's:

$$\chi_{i/A}(k,P) = \sum_a \int \frac{d^4p}{(2\pi)^4}\chi_{i/a}(k,p)\chi_{a/A}(p,P), \tag{2.4}$$

where the two subamplitudes for γq and N-Sp are in the PWIA. The fsa $a(N\text{-Sp})$ in the PWIA is now related to the familiar spectral function of a nucleon in the target.

As in Eq. (2.2) for a p, one now projects out of each fsa the appropriate pdf; Eq. (2.4) is converted to

$$q_{i/A}(x) = \sum_a \int_x^A dz \int dp_0^2 f_{a/A}(z;p_0^2)q_{i/a}\left(\frac{x}{z};p_0^2\right) \tag{2.5}$$

$$\approx \int_x^A dz f_{N/A}(z)q_{i/N}\left(\frac{x}{z}\right). \tag{2.6}$$

Again Eq. (2.5) relates to several constituents/clusters, all of which may be off their mass shell ($p_0^2 \neq M_a^2$), while in approximation (2.6) one retains only nucleons and in addition disregards those off-shell effects. Approximation (2.6) is clearly approximation (1.9) in the Bjorken limit.

Next, on inclusion of gluon emissions from quarks, nuclear pdf's acquire Q^2-dependence, changing Eq. (2.5) and approximation (2.6) into

$$q_{i/A}(x,Q^2) = \sum_a \int_x^A dz \int dp_0^2 f_{a/A}(z,Q^2;p_0^2)q_{i/a} \times\left(\frac{x}{z};Q^2;p_0^2\right) \tag{2.7}$$

$$\approx \int_x^A dz f_{N/A}(z,Q^2)q_{i/N}\left(\frac{x}{z},Q^2\right). \tag{2.8}$$

Above $f_{N/A}$ is the distribution function (df) of nucleons in the nucleus in the PWIA, while $q_{i/N}$ are pdf's beyond their scaling limit. Now just as gluon effects may be viewed as FSIs on the fsa $a(\gamma q_i)$ in the scaling limit, one should consider FSIs pertinent to the nuclear part.

As emphasized by Jaffe, most classes of those FSIs cannot be accommodated in a generalized convolution. However, the above does not hold for selected, nuclear FSIs generated by the interaction between the above-assumed free N and the spectator nucleus. An illustrative example is a ladder of N-spectator collisions, which turn the PWIA into the DWIA (Fig. 3). The same holds for a description in the alternative, nonperturbative

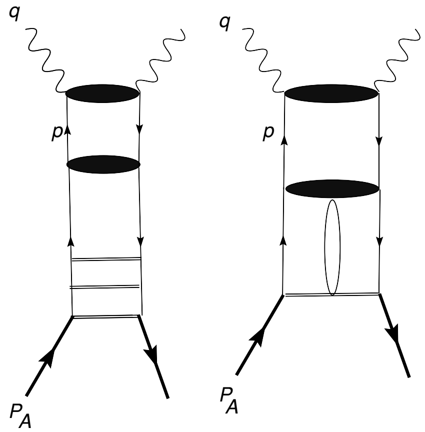

FIG. 3. Ladder of N-Sp nucleus collisions, which are accommodated in a convolution, and an example of nuclear FSIs that cannot be accommodated.

GRS theory for FSIs: $f_{N/A} \to f^{PN,A} \to f^{\rm GRS}$, which leads to the GRS version of convolution (1.3) [2,5].

In the last step we take the proper combinations (1.6) and (1.7) of respectively, nucleon and nuclear pdf, and obtain

$$F_2^A(s, Q^2) = \int_x^A dz f^{PN,A}(z, Q^2) F_2^{\langle N\rangle}\left(\frac{x}{z}, Q^2\right). \qquad (2.9)$$

Approximation (2.8) and Eq. (2.9) are manifestly the same as Eqs. (1.9) and (1.3), but Eq. (2.3) is a choice, whereas Eq. (2.8) is the result of a *derivation*. Just as for the descriptions outlined in Secs. I and II, here we deal with one species-independent $f_{N/A}$, which relates the df of partons in nuclei and nucleons without flavor mixing. We recall that the above correspondence holds for the two discussed approaches, in which quite similar approximations have been applied, e.g., the use of averaged spin-isospin observables and the neglect of off-shell effects. Finally, not all, even purely nuclear FSI components can be accommodated in a generalized convolution of the form of Eq. (2.9) [14].

In spite of the established formal correspondence, the interpretation and calculation of the components are entirely different. For instance, the nuclear point-nucleon SFs $f^{PN,A}$ in relation (1.9) are calculated with characteristic nuclear tools and input, such as the single-N spectral function, A particle density matrices of various orders, the effective NN scattering amplitude, etc., whereas in Eq. (2.8) those relate to the fsa $a(N\text{-Sp})$. Likewise, F_2^N in convolution (1.3) is plainly taken from data, whereas in Eq. (2.9) it is the result of an elaborate pQCD calculation.

We conclude this section by emphasizing the different scales involved in the two factors of the integrand in Eq. (2.9), as has been illustrated above in the example of quantum gases. In Eq. (2.9) by far the strongest Q^2 dependence resides in $F_2^{\langle N\rangle}(x, Q^2)$, while that in the nuclear component f is soft. For $Q^2 \gtrsim Q_0^2 \approx 3\,\text{GeV}^2$ a parton description of the nucleon SF is largely sufficient, whereas the nuclear part including the FSI is most conveniently evaluated in a plain hadronic description. The above value of Q_0^2 is approximately the one above which convolution (1.3) has empirically been found to hold.

This concludes our generalization of the proofs of Jaffe and West on the "factorization" of nuclear pdf's and SFs. The next section deals with moments or Mellin transforms of nuclear SFs in their obvious relation to $F_2^{\langle N\rangle}(x, Q^2)$.

III. MOMENTS OF NUCLEAR STRUCTURE FUNCTIONS

We recall the role played by moments M of F_2^p for a p, for instance the Cornwall-Norton moments [15]:

$$M^p(n, Q^2) = \left[M_2^p(n, Q^2\right] = \int_0^A dx x^{n-2} F_2^p(x, Q^2) \qquad (3.1)$$

For lowest twist (LO), nonsinglets (NSs) and large enough Q^2, asymptotic freedom of QCD predicts that moments of various rank raised to known powers are linear in $\ln(Q^2)$. In terms of the strong coupling constant α_c,

$$\frac{M^p(n, Q^2)}{M^p(n, Q_0^2)} \approx \left[\frac{\alpha_c(Q_0^2)}{\alpha_c(Q^2)}\right]^{-d(n)}, \qquad (3.2)$$

$$\frac{\alpha_c(Q_0^2)}{\alpha_c(Q^2)} \approx 1 + \frac{\beta_0}{4\pi}\alpha_c(Q_0^2)\ln\left(\frac{Q^2}{Q_0^2}\right) + \mathcal{O}\left\{\left[\alpha_c(Q_0^2)\right]^2\right\}, \qquad (3.3)$$

where Q_0 is some scale and $\beta_0(N_f) = 11 - 2N_f/3$ in terms of the number of flavors N_f. The exponents $d^{\rm NS}(n, N_f)$ in approximation (3.2) are expressed in terms of the NS anomalous dimension $\gamma_0^{\rm NS}(n)$:

$$d^{\rm NS}(n, N_f) = \frac{\gamma_0^{\rm NS}(n)}{2\beta_0(N_f)}, \qquad (3.4)$$

$$\gamma_0^{\rm NS}(n) = \frac{8}{3}\left[1 - \frac{2}{n(n+1)} + 4\sum_{2\leqslant j\leqslant n}\frac{1}{j}\right]. \qquad (3.5)$$

For conciseness we define

$$\begin{aligned}\mathcal{S}^A &= [M^A]^{-1/d(n)},\\ \mathcal{L}^A &= \ln(\mathcal{S}^A),\end{aligned} \qquad (3.6)$$

and find, in view of $|\frac{\beta_0}{4\pi}\alpha_c(Q_0^2)| \ll 1$,

$$\begin{aligned}\mathcal{S}^p(n, Q^2) &\approx \mathcal{S}^p(n, Q_0^2)\left[1 + \frac{\beta_0}{4\pi}\alpha(Q_0^2)\ln\left(\frac{Q^2}{Q_0^2}\right)\right]\\ &= c^p(n)\ln(Q^2) + b^p(n)\\ &\approx c^p(n)\ln(Q^2) + b^p,\end{aligned} \qquad (3.7)$$

$$\begin{aligned}\mathcal{L}^p(n, Q^2) &\approx \mathcal{L}^p(n, Q_0^2) + \frac{\beta_0}{4\pi}\alpha(Q_0^2)\ln(Q^2/Q_0^2)\\ &\approx \zeta^p(n)\ln(Q^2) + \eta^p(n).\end{aligned} \qquad (3.8)$$

Slopes $c^P(n)$ for order n and the common intercept b^p are in principle, determined by the scale or coupling constant, (3.3). Decades ago, predictions (3.7) and (3.8) were checked against available proton data [16]. A recent JLab experiment, covering $Q^2 \lesssim 4.5\,\text{GeV}^2$ and $x \lesssim x_M(Q^2)(\approx 0.8$ for $Q^2 = 4.5\,\text{GeV}^2)$, led to a detailed analysis of the effects of higher-twist components in the moments $M^p(n, Q^2)$ [17].

There has been hardly any interest in moments of nuclear SFs for moderate [18] and large Q^2 (see for instance Ref. [8]). In a straightforward way one can generalize the above for any target A, including the averaged N. The latter requires in addition to F_2^p knowledge of F_2^n, for which there is no direct experimental information. We refer to Ref. [6] for the description of an indirect extraction of F_2^n or $C(x, Q^2) = F_2^n(x, Q^2)/F_2^p(x, Q^2)$ from inclusive scattering data on various targets. Once obtained,

$$F^{\langle N\rangle}(x, Q^2) = \frac{Z + NC(x, Q^2)}{Z + N} F_2^p(x, Q^2). \quad (3.9)$$

Since $F_2^n \neq F_2^p$, the parameter functions c and b in Eq. (3.7) for a neutron will differ from those for a proton and the same is the case for the averaged N, or for any target A. To relate the latter two, one naturally exploits convolution (1.3) and its Mellin transform ($m^{\langle N\rangle} \equiv 1$):

$$M^A(n, Q^2) = m^A(n + 1, Q^2) M^{\langle N\rangle}(n, Q^2), \quad (3.10)$$

with

$$\begin{aligned} M^A(n, Q^2) &= \int_0^A dx x^{n-2} F_2^A(x, Q^2), \\ m^A(n, Q^2) &= \int_0^A dx x^{n-2} f^{PN,A}(x, Q^2), \quad (3.11) \\ \sigma^A(n, Q^2) &= [m^A(n, Q^2)]^{-1/d(n)}. \end{aligned}$$

A remark on m^A is in order here. First, for $Q^2 \gtrsim 20\,\mathrm{GeV}^2$, one may neglect FSI parts in the calculated SF $f^{PN,A}$ from which m^A is computed. Next, as moments of a peaked, normalized $f^{PN,A}$, $m^A(n = 2, Q^2)$ has a minimum value of 1, independent of A and Q^2. For increasing n, $m^A(n)$ slowly increases, the least for D and He and about to the same measure for all $A \gtrsim 12$. Those moments moreover carry the weak Q^2 dependence of f [19] and reach for $n = 7$ the asymptotic limits ≈1.027 for D and ≈1.082 for medium and heavy A.

For use below we also briefly discuss the behavior of σ^A, Eqs. (3.11). For n between 2 and 7, the exponent $d(n, N_f = 6)$ increases from 0.507 to 1.397 and causes σ^A for D to barely decrease from 1.000 to 0.977, and for $A \gtrsim 12$, from 1.000 to ≈0.931. It suffices to illustrate (Fig. 4) the n dependence of $m^A(n, Q^2 = 20\,\mathrm{GeV}^2)$ for D and Fe, representative for a target with $A \gtrsim 12$: The choice made for Q^2 is irrelevant, since the Q^2 dependence of m^A is negligible for all practical purposes.

For target-independent anomalous dimensions, Eqs. (3.9)–(3.11) enable the generalizations of Eqs. (3.7) and (3.8),

$$\mathcal{S}^{\langle N\rangle}(n, Q^2) \approx c^{\langle N\rangle}(n) \ln(Q^2) + b^{\langle N\rangle}(n), \quad (3.12)$$

$$\mathcal{L}^{\langle N\rangle}(n, Q^2) \approx \zeta^{\langle N\rangle}(n) \ln(Q^2) + \eta^{\langle N\rangle}(n), \quad (3.13)$$

as well as

$$\mathcal{S}^A(n, Q^2) \approx c^A(n) \ln(Q^2) + b^A(n), \quad (3.14)$$

$$\mathcal{L}^A(n, Q^2) \approx \zeta^A(n) \ln(Q^2) + \eta^A(n). \quad (3.15)$$

For given Q^2 we compared separate expansions (3.14) and (3.15) and found that the logarithm of the first is close to that of the second.

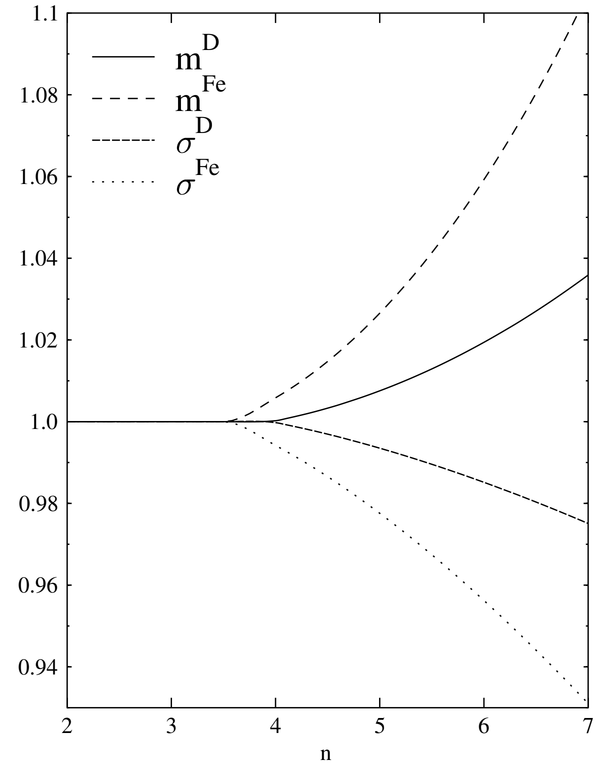

FIG. 4. Moments $m^A(n, Q^2)$ and their characteristic power $\sigma^A(n, Q^2)$, Eq. (3.7), for $A = D$, Fe; $n = 2$–7, $Q^2 = 20\,\mathrm{GeV}^2$.

From the above one infers that slopes and intercepts c, b will differ for p, n and thus for $\langle N\rangle$, while for general A one checks from Eq. (3.10) the following approximations:

$$\begin{aligned} c^A(2, Q^2) &\approx \sigma^A(3, Q^2) c^{\langle N\rangle}(n) \approx c^{\langle N\rangle}(n), \\ b^A(n, Q^2) &\approx \sigma^A(n + 1, Q^2) b^{\langle N\rangle}(n) \approx b^{\langle N\rangle}(n), \end{aligned} \quad (3.16)$$

$$\begin{aligned} \zeta^A(n) &\approx \zeta^{\langle N\rangle}(n), \\ \eta^A(n, Q^2) &\approx \eta^{\langle N\rangle}(n) + \ln[\sigma^A(n + 1, Q^2)] \approx \eta^{\langle N\rangle}(n). \end{aligned} \quad (3.17)$$

Medium changes are governed by σ^A, Eqs. (3.6): Target-to-target differences between slopes and intercepts for general targets and $\langle N\rangle$ never exceed a few percent [see Fig. 4, the text after Eqs. (3.11), and also point 5 below].

In what follows we distinguish between computed and experimenal SFs F_2^A and their moments, as well as ratios ρ^A, which derive from Mellin transform (3.10) of convolution (1.3). Using Eq. (3.10) one checks

$$\rho^{A,\mathrm{th}}(n, Q^2) \equiv \frac{M^{A,\mathrm{th}}(n, Q^2)}{m^A(n + 1, Q^2)} = M^{\langle N\rangle}(n, Q^2), \quad (3.18)$$

$$\begin{aligned} [\rho^{A,\mathrm{th}}(n, Q^2)]^{-1/d(n)} &\approx c^A(n, Q^2) \ln(Q^2) + b^A(n, Q^2) \\ &\approx c^{\langle N\rangle}(n, Q^2) \ln(Q^2) + b^{\langle N\rangle}(n, Q^2). \end{aligned} \quad (3.19)$$

For isosinglet targets $M^{\langle N\rangle}(n, Q^2)$ does not depend on A, whereas for $I \neq 0$, there is a weak A dependence, that is due to the small neutron excess $\delta N/A$, Eq. (1.4).

Using the measured $F^{A,\mathrm{dat}}$, we consider the corresponding moments $M^{A,\mathrm{dat}}$, Eqs. (3.11), and the ratios $\rho^{A,\mathrm{dat}}$, Eq. (3.18). In contrast to $\rho^{A,\mathrm{th}}$, the ratios $\rho^{A,\mathrm{dat}}$ do depend on $f^{PN,A}$. A reliable computation of the latter and thus indirectly of m^A is currently possible only for $A \leqslant 4$.

Understanding the n dependence of $M^A(n, Q^2)$ relies on the knowledge that all SFs $F_2^A(x, Q^2)$ reach maxima for the smallest x, then decrease with increasing x, and become negligibly small beyond $x \approx 0.8$. The derived moments $M^A(n, Q^2)$ of lowest order thus critically depend on the values of $F_2^A(x, Q^2)$ for very small x. There is only meager experimental information available on F_2^A for large Q^2. In spite of the fact that second-generation EMC ratios $\mu^A = F_2^A/F_2^D$ have been measured for large Q^2, the individual F_2^A are only rarely available. We know of CERN NA-4 data on F_2^A, $A = D$, C [20,21], and NA-2 data for Fe [20,22], which are not dense and do not extend over the entire required critical x range. To those we added a few data points from a JLab experiment [23], although the relevant Q^2 is low for a LO analysis.

For growing n, $M^A(n)$ draws more and more on increasing x. Since for medium x, F_2^A have fallen by at least an order of magnitude from their maxima, it becomes increasingly difficult to reliably compute $M^{A,\mathrm{dat}}(n, Q^2)$ for large n. We now mention results for $N_f = 6$.

(i) $M^{A,\mathrm{th}}(n, Q^2)$ is barely A-dependent, and for various n slowly approaches its asymptotic Q^2 limit. In particular [10,12]

$$M^{A,\mathrm{th}}(n = 2, Q^2 \to \infty) \to \frac{5}{6}\frac{N_f}{(3N_f + 16)} = 0.1471.$$

(ii) In detail, the NA-4 F_2^D data show substantial scatter [20], which reflects in their moments and in $\mathcal{S}^D = [M^D]^{-1/d(n)}$. In spite of the above remarks, $\mathcal{S}^D$ for low n accurately follows the theoretical curves, but for increasing n data overshoots predictions up to ≈15% (Fig. 5).

It is instructive to make a similar comparison for a large body of D data, which have been parametrized by Arneodo *et al.* [24]. Very good agreement now obtains for $n \leqslant 4$. Discrepancies grow again with n, but are definitely smaller than for the above-mentioned data (Fig. 6). The cause is clearly few percent differences between the two data sets. The comparison also illustrates the effect of experimental scatter.

(iii) The above data for F_2^C lack values for small x [20,21] without which one cannot compute low-order moments. We therefore took recourse to a previously proposed method, which is based on the observation that *all* $F_2^A(x \approx 0.18, Q^2)$ have a common value ≈0.30, approximately independent of A and Q^2 (see for instance Ref. [6]). If nuclear SFs are well known for $X_m > X_0$, one may extrapolate F^A down to X_0 (Fig. 7).

(iv) Before discussing Fe, we mention the result of a comparison of the high-Q^2 data of Refs. [20–22] for $F_2^A(x, Q^2)$: (a) $F_2^D \approx F_2^C$, (b) for small x both D and C are a few percent lower than F_2^{Fe}, but for $x \gtrsim 0.18$ the situation appears reversed and $F_2^{D,C}$ are ≈15%−20% larger than for Fe. No similar behavior has been observed for lower Q^2. It is conceivable that the above Fe data [20] have a normalization error of the order of 15%–20% for $x \gtrsim 0.22$. In Fig. 8 we entered adjusted $\mathcal{S}^{\mathrm{Fe,dat}}$.

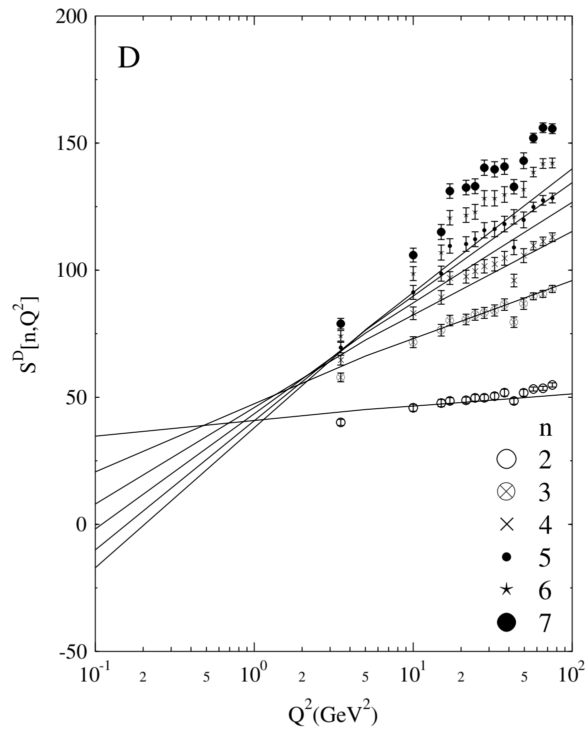

FIG. 5. Characteristic powers of moments $\mathcal{S}^D(n, Q^2)$, Eqs. (3.6), for D as functions of In Q^2. Data points for underlying SFs are from Ref. [20]; n increases for lines with increasing slopes.

(v) All $\mathcal{S}^{A,\mathrm{th}}(n, Q^2)$ intersect around $Q^2 \approx (0.6{-}1.0)\,\mathrm{GeV}^2$, which is reflected in the approximate equality of all $b^A(n)$. The exception is $n = 2$ for which $\mathcal{S}^A$ has a very small slope, which may reflect the sensitivity of $M^A(n = 2, 3)$ to the small-x behavior of the nuclear SF. The fact that there is little A dependence seems to exclude screening effects in F_2^A for $x \lesssim 0.15$ as a cause, but quarks emitted by virtual bosons in the same small-x range may contribute [25].

(vi) Results for $\rho^{A,\mathrm{dat}}$ and for $\rho^{A,\langle N\rangle;\mathrm{th}}$ are assembled in Table I. There is overall agreement for D and C and a deficiency for the nonadjusted Fe data. The former imply that the extracted $M^n(n, Q^2)$ are practically independent of A, as they ought to be.

(vii) We tested whether expansions (3.16) and (3.17) for $\mathcal{S}^{A,\mathrm{th}}$ and $\mathcal{L}^{A,\mathrm{th}}$ and varying n are approximately linear functions of $\ln(Q^2)$ with only weakly A-dependent coefficients. Table II confirms the above for our targets.

The small but marked influence of m^A is manifest in a comparison between $\mathcal{S}^{\langle N\rangle}(n, Q^2)$ (for which $m^{\langle N\rangle} \equiv 1$) and $\mathcal{S}^A$. As expression (3.16) predicts, intercepts $b^{\langle N\rangle}(n) \approx b^A(n)$ are quite similar, while for the slopes one has approximately $c^{\langle N\rangle}(n) \approx c^A(n = 2)$.

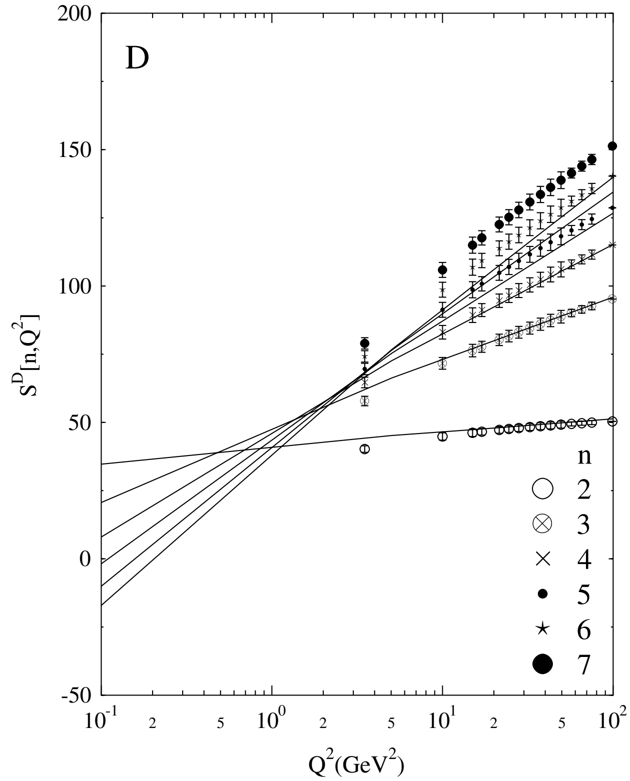

FIG. 6. Same as Fig. 5 for parametrizations for the average of a vast body of *D* data [24].

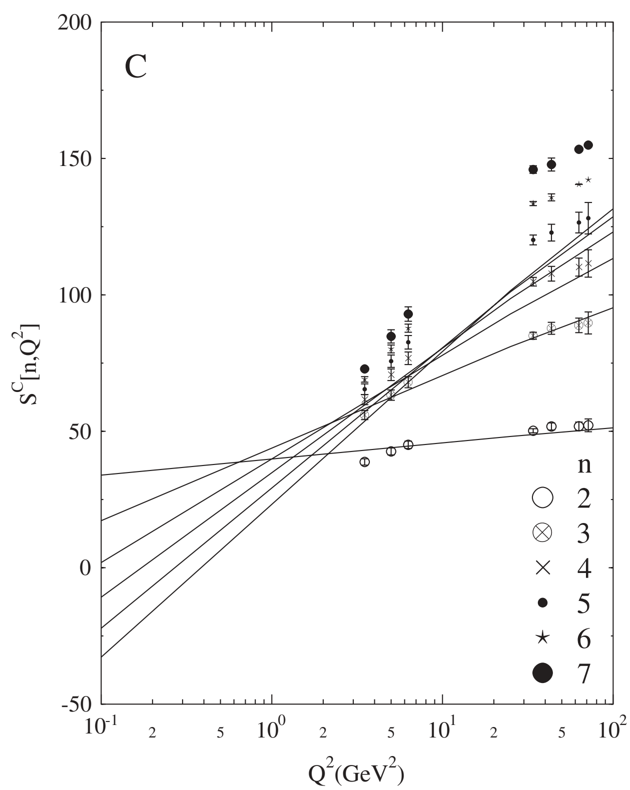

FIG. 7. Same as Fig. 5 for *C*. Data are from Refs. [20,23].

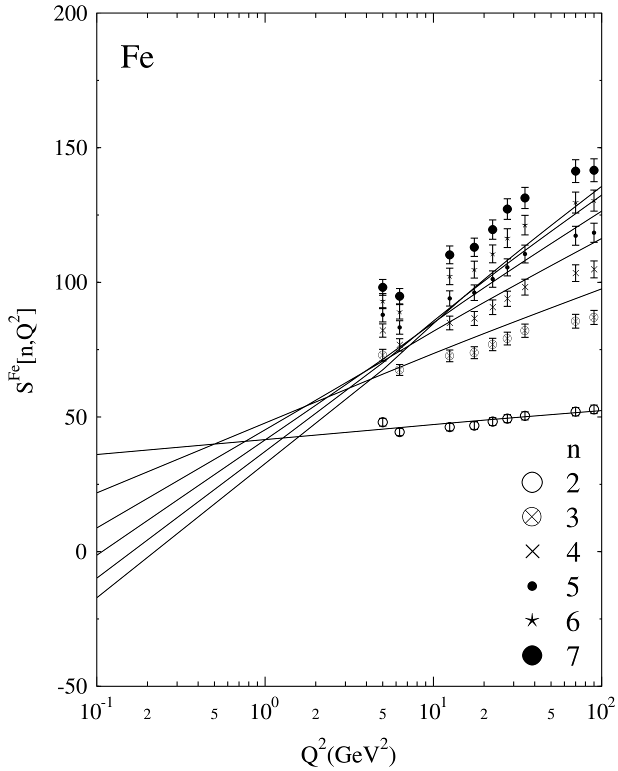

FIG. 8. Same as Fig. 5 for Fe for partly renormalized F_2^{Fe} data.

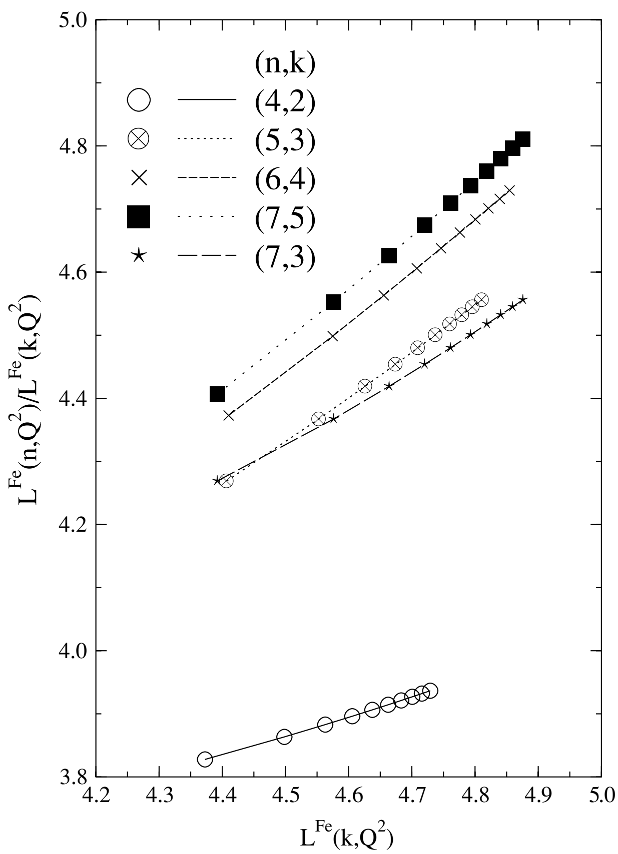

FIG. 9. $\mathcal{L}^{\mathrm{fe,th}}(n, Q^2)$ versus $\mathcal{L}^{\mathrm{fe,th}}(k, Q^2)$, approximation (3.20), for $(n, k) = (4, 2), (5, 3), (6, 4), (7, 5), (7, 3)$. Data points as in Fig. 7.

TABLE I. Ratios $p^{A,\mathrm{dat}}(n,Q^2)$, Eq. (3.18), $A=D,C$ for $n=2-7$ and a number of roughly common Q^2 values m (Fe data are not adjusted). Also shown are $p^{A,\mathrm{th}}(n,Q^2)$ for the averaged N, pertinent to an isoscalar nucleus and for Fe.

Target		$p^{A,\mathrm{dat}}(n,Q^2)$				
Q^2	n	3.5	17	35	50	72
D	2	0.1548	0.1400	0.1372	0.1354	0.1315
	3	0.0401	0.0309	0.0298	0.0290	0.0276
	4	0.0156	0.0104	0.0099	0.0096	0.0090
	5	0.0073	0.0043	0.0040	0.0039	0.0036
	6	0.0038	0.0020	0.0019	0.0018	0.0017
	7	0.0021	0.0010	0.0010	0.0009	0.0008
C	2	0.1555	0.140	0.1373	0.1351	0.1347
	3	0.0398	0.031	0.0291	0.0284	0.0281
	4	0.0152	0.011	0.0094	0.0091	0.0089
	5	0.0069	0.004	0.0037	0.0036	0.0035
	6	0.0034	0.003	0.0017	0.0016	0.0016
	7	0.0017	0.001	0.0008	0.0008	0.0008
Fe	2	0.1499	0.1325	0.1290	0.1270	0.1267
	3	0.0354	0.0276	0.0260	0.0253	0.0251
	4	0.0121	0.0087	0.0080	0.0077	0.0076
	5	0.0049	0.0034	0.0030	0.0029	0.0029
	6	0.0021	0.0014	0.0013	0.0012	0.0012
	7	0.0009	0.0007	0.0006	0.0006	0.0006
$\langle N\rangle_{I=0}$	2	0.1469	0.1414	0.1393	0.1380	0.1369
	3	0.0376	0.0315	0.0296	0.0285	0.0275
	4	0.0149	0.0111	0.0100	0.0096	0.0092
	5	0.0073	0.0050	0.0044	0.0041	0.0038
	6	0.0041	0.0026	0.0022	0.0020	0.0019
	7	0.0025	0.0015	0.0012	0.0011	0.0010
$\langle N\rangle_{\mathrm{Fe}}$	2	0.1448	0.1396	0.1380	0.1374	0.1353
	3	0.0368	0.0308	0.0295	0.0288	0.0272
	4	0.0145	0.0109	0.0098	0.0094	0.0090
	5	0.0071	0.0048	0.0044	0.0041	0.0037
	6	0.0039	0.0025	0.0022	0.0020	0.0018
	7	0.0024	0.0014	0.0012	0.0011	0.0010

(viii) Finally we exploited the fact that $\eta^A(n)\gg\zeta(n)\ln(Q^2)$ in approximation (3.15). Consequently

$$\frac{\mathcal{L}^A(n,Q^2)}{\mathcal{L}^A(k,Q^2)}\approx\frac{\eta^A(n)}{\eta^A(k)}\left\{1+\left[\frac{\zeta^A(n)}{\eta^A(n)}-\frac{\zeta^A(k)}{\eta^A(k)}\right]\ln(Q^2)\right\}$$
$$\approx\frac{\eta^A(n)}{\eta^A(k)}. \tag{3.20}$$

The form in curly braces exceeds 1 by less than 10% and predicts only a weak A and Q^2 dependence of the above ratios for pairs n,k, which gently grows with n-k. It thus suffices to illustrate the above for one species. We chose Fe and the pairs $n,k=(4,2)$, (5,3), (6,4), (7,5), (7,3) (Fig. 9).

The data are seen to follow prediction (3.20) remarkably well, including the weak $\ln(Q^2)$ dependence in prediction (3.20).

For a proton the linear dependence of $\mathcal{S}^p$ on $\ln(Q^2)$ has been regarded as experimental evidence for asymptotic freedom. With quite similar results for nuclei, we do not tend to conclude the same for composite systems. It is more likely that the de facto separation of nuclear and nucleon components reflects the manifestation of asymptotic freedom of (on-shell!) nucleons and allocates to the medium controlled modifications of slopes and intercepts (see Ref. [26] for a differently argued separation).

TABLE II. Expansion coefficients of $\mathcal{S}^{\langle N\rangle,A;\mathrm{th}}(n,Q^2)$, approximations (3.12), (3.13), (3.14), (3.15), for $n=2-7$, for $A=D,C$, Fe compared with the same for the average nucleon $\langle N\rangle_{I=0}$.

Target	n	$c^A(n)$	$b^A(n)$	$\zeta^A(n)$	$\eta^A(n)$
D	2	2.085	46.503	0.0426	3.840
	3	9.937	73.035	0.1175	4.297
	4	14.273	82.415	0.1444	4.421
	5	17.717	87.177	0.1607	4.480
	6	19.379	89.868	0.1731	4.512
	7	21.111	91.332	0.1831	4.530
C	2	2.284	46.017	0.0469	3.830
	3	10.286	71.608	0.1232	4.278
	4	14.690	79.540	0.1526	4.386
	5	17.641	82.447	0.1723	4.425
	6	19.875	82.875	0.1891	4.433
	7	21.655	81.655	0.2049	4.420
Fe	2	2.241	47.245	0.0449	3.856
	3	10.404	73.705	0.1214	4.307
	4	14.917	82.088	0.1505	4.418
	5	17.937	85.366	0.1697	4.460
	6	20.210	86.173	0.1856	4.471
	7	22.002	85.414	0.2002	4.465
$\langle N\rangle_{I=0}$	2	2.137	40.989	0.0443	3.725
	3	9.918	49.888	0.1182	4.020
	4	14.246	50.006	0.1443	4.093
	5	17.205	48.929	0.1598	4.126
	6	19.504	47.498	0.1711	4.144
	7	21.379	46.007	0.1800	4.154

It is instructive to compare the above with an extension of the bag model of nucleons with nuclei with comparable average internucleon spacings and sizes of bags, which may overlap and cause conceptual complications. No such problems occur in the above interpretation of convolution (1.3).

Finally we remark that the above analysis is complicated by the presence of color singlet contributions, which are coupled to those for gluons. Only for sufficiently high $n\gtrsim 4-5$ are those approximately decoupled [16], allowing an analysis of actual moments and not of the assumed NSs. This is also the reason why we do not study medium effects on slopes and intercepts in greater detail.

IV. CONCLUSION

This note generalizes old work by Jaffe and West, who by means of a parton model and pQCD in the PWIA for large Q^2 proved that pdf's and SFs of composite targets and of nucleons are related by a generalized convolution. Their publications did not appear in the standard literature and have apparently been forgotten or disregarded. This paper is therefore in part an *amende honorable* to their work.

We first reviewed facets of the conjectured convolution for finite Q^2, working in both a hadronic and an effective nuclear pdf representation. In those we did not aim to check whether the Q^2 dependence is actually reproduced by, or in agreement with, evolution from a scale Q_0^2.

Next, we mentioned crucial points in the publications of Jaffe and West. Those are foremost the general relation between fsa's and pdf's. Next we cited the decomposition of fsa $a(\gamma A)$ into the fsa's $a(\gamma N)$ and $a(N$, spectator-nucleus). Jaffe and West studied those first in the PWIA and in the Bjorken limit, leading to the scaling results. Those have subsequently been supplemented by contributions that are due to gluon emission by quarks, etc., which, as regards photon-parton scattering, extend results beyond the above limit. We generalized the above, and included classes of FSIs between a nucleon, intermediately emitted by a target and the remaining spectator nucleus. The LO expressions, relating nuclear and nucleon pdf's, and consequently the same for SFs, continue to be of the convolution type. Moreover, those are identical to the same, previously conjectured ones in the above hadron and effective pdf representations. That correspondence is a formal one: The interpretation of the two results is entirely different.

The existence of an "ultimate" description does not imply a preference over an "effective" one under all circumstances. It is easier to compute the SF $f^{PN,A}$ from nuclear physics concepts than from pQCD, or to use data on nucleon SFs as opposed to a calculation of F^N: Results from effective theories are frequently quite accurate.

The above is not at all specific for descriptions of nuclear SFs, but holds for many effective theories. A classical example is the interatomic interaction of the centers of the atoms in diatomic molecules. The "true" potential ought in principle to be computed quantum-mechanically, which is extremely laborious, but in practice one uses Lennard-Jones or Morse potentials. Those do contain the essentials of the physics, including a short-range repulsion, which mimics the effect of the Pauli principle for overlapping electron configurations. The spectroscopy of diatomic molecules and the physics of gases and liquids of diatomic molecules are accurately accounted for by effective dynamics.

The last part of this paper concerns moments of nuclear SFs. The behavior of moments M^p of the SF F_2^p, specifically the linear dependence of $\mathcal{S}^P$ on ln (Q^2), has in the past been shown to be a consequence of asymptotic freedom of QCD. Quite similar properties are shared by nuclear moments. However, rather than concluding that inclusive scattering data on nuclei support asymptotic freedom for composite systems, we prefer a sober point of view. The formal factorization of F_2^A [or the actual one of moments $M^A(n, Q^2)$] separates nucleonic and nuclear dependencies without changing the required separation of parts with hard and soft Q^2 dependence as is the case of a proton. The observed $\ln(Q^2)$ behavior simply reflects asymptotic freedom of isolated nucleons, with characteristic medium modifications of nucleon parameters.

After completion of this paper Simonetta Liutti alerted us to previous work on moments of nuclear SFs. [27].

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