

# Hadron production in the transfragmentation region in heavy-ion collisions

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We study the production of hadrons in Au+Au collisions in the region  $0.6 < x_F < 1.2$ , which we refer to as the transfragmentation region (TFR), since it corresponds roughly to  $\eta' > 0$ , where  $\eta' = \eta - y_{\text{beam}}$ , depending on the transverse momentum  $p_T$ . We show how hadrons can be produced in that region when the hadronization process is parton recombination. The inclusive  $x$  distributions for proton and pion production are calculated with momentum degradation taken into account. The results show that the proton yield is significantly higher than that of the pions in the TFR. Without particle identification, the existing data cannot be used for comparison with our result on the  $p/\pi$  ratio. Without  $p_T$  determination, it is not feasible to relate the  $x$  distribution to the experimental  $\eta'$  distribution. Nevertheless, on theoretical grounds we have shown why the production of hadrons in the TFR is not forbidden by momentum conservation.

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## I. INTRODUCTION

One of the interesting features of hadron production in heavy-ion collisions (HIC) is the discovery of the scaling property of the pseudorapidity distribution in the fragmentation region over an order of magnitude of variation in the collision energy [1,2]. Such a property was hypothesized in [3] for  $pp$  collision, and was referred to as limiting fragmentation. The independence on energy, however, does not imply independence on the collision system. The pseudorapidity distributions in the fragmentation region show definitive dependence on the centrality of Au+Au collisions for any fixed energy [2,4]. Indeed, one cannot expect on theoretical grounds that the properties of hadron production in that region should be the same in  $pp$  and  $AA$  collisions. Whereas in  $pp$  collisions no particle can be produced with a rapidity greater than the beam rapidity, that is not the case in  $AA$  collisions. In fact, in terms of the variable  $\eta' = \eta - y_{\text{beam}}$ , where  $\eta$  is the pseudorapidity and  $y_{\text{beam}}$  the beam rapidity, PHOBOS data indicate that the charged-particle distribution  $dN_{\text{ch}}/d\eta'$  does not vanish in the  $\eta' > 0$  region [2,5]. We shall refer to that region as the transfragmentation region (TFR). In this paper we describe the physics of hadron production in the TFR in the framework of the recombination model.

Limiting fragmentation is a natural consequence of any formalism that uses momentum fractions of partons as the essential variables to describe hadronization since c.m. energy does not appear explicitly. However, particle production in the TFR is intriguing because it seems to violate momentum conservation, as it certainly would in  $pp$  collisions. In  $AA$  collisions, complications arise because fragments of the noninteracting spectators can get into the detectors at small angles. For peripheral collisions at relatively low energy ( $\sqrt{s} = 19.6$  GeV),  $dN_{\text{ch}}/d\eta'$  seems to approach a constant value as  $\eta' \rightarrow 2$  [2,5]. Such effects seem to diminish at higher  $\sqrt{s}$ . But let us put aside such issues, since our interest here is in the hadronization of the interacting part of nuclear collisions. For that, there is at present no data in the TFR for, say  $\sqrt{s} = 200$  GeV. Nevertheless, it is an interesting and important

question to ask whether there exist theoretical reasons for hadrons to be produced in the TFR.

Since the problem deals with low-transverse-momentum (low- $p_T$ ) physics, one cannot make use of perturbative QCD (pQCD) with any degree of confidence. Nevertheless, if one takes the point of view that hadrons are produced in the fragmentation region (FR) by parton fragmentation, such as in the dual parton model [6], then no hadrons can appear in the TFR, since all partons have momentum fractions less than 1. On the other hand, in parton recombination, the momentum fractions are additive and can result in a hadron momentum fraction greater than 1, provided that the constituents come from different nucleons in the colliding system, a condition that is readily satisfied in  $AA$  collisions. It is with that possibility in mind that we study in detail the problem of hadron production in the TFR in the recombination model (RM) [7,8,9].

Nuclear collisions have the complication of momentum degradation of constituents traversing nuclear matter. We have investigated the degradation effect in  $pA$  collisions, treating the constituents in terms of valons [8,10], and found good agreement with the data on “baryon stopping” in the FR [11,12]. Here, in  $AA$  collisions the medium is dense, and our focus is on the TFR. Nevertheless, similar formalism will be used to take the degradation effect into account.

Since hadronization is a problem that involves the momenta of the constituents in an essential way (as does momentum degradation), we shall be working with the momentum fraction variable  $x$ , instead of the pseudorapidity variable  $\eta$ . In terms of  $x$ , the TFR is more precisely  $x > 1$ , and the FR is for  $x < 1$ , but above, say, 0.2. These regions do not map isomorphically to regions in  $\eta$ , since  $p_T$  is involved in the definition of angle  $\theta$ . Although  $\eta$  is a more convenient variable for experimental detection, we shall work entirely with the  $x$  variables. The mismatch between theory and experiment in that respect can be overcome only when the  $p_T$  values of the detected particles are determined. Until then, we cannot compare our predictions with any data. In our treatment, the physics of hadronization on both sides of  $x = 1$  is continuous, so we shall calculate the spectra in the region  $0.6 < x < 1.2$ , which we broadly

refer to as TFR, as it roughly corresponds to the  $\eta' > 0$  region (although in principle, there is no upper limit on  $x$ ).

We note that in the region  $x \sim 1$  the possibility of formation of very high parton density at very high energy has been pointed out in [13]. Such a possibility may affect the  $p/\pi$  ratio, but is not taken into account in our treatment here.

Apart from details, our main qualitative prediction is that protons dominate the charge particles detected in the TFR. Such a prediction should be easier to confirm or falsify than the verification of our results on the inclusive distributions in  $x$ . It is hoped that this paper will stimulate an effort to improve particle identification in the TFR.

## II. PRELIMINARY CONSIDERATIONS

Let us begin with the kinematics relevant for the TFR in HIC at high energy. For hadrons detected at small angle  $\theta$  relative to the beam axis, we may approximate  $\tan \theta/2$  by  $p_T/2p_L$ , where  $p_T$  and  $p_L$  are the transverse and longitudinal momenta, respectively, of the produced hadron. Thus the pseudorapidity is  $\eta = \ln(2p_L/p_T)$ , while the beam rapidity is  $y_{\text{beam}} = \ln(\sqrt{s}/m_p)$ , where  $m_p$  is the proton mass. With  $\eta'$  being the shifted pseudorapidity,  $\eta' = \eta - y_{\text{beam}}$ , we have for Feynman  $x$ , defined by  $x = 2p_L/\sqrt{s}$ ,

$$x = \frac{p_T}{m_p} e^{\eta'}. \quad (1)$$

The mapping between  $\eta'$  and  $x$  therefore depends on  $p_T$ . If in the forward region  $\langle p_T \rangle < m_p$ , then the  $\eta' > 0$  region corresponds to a range in  $x$  that straddles  $x = 1$ . For that reason, we study the hadron spectra in the range  $0.6 < x < 1.2$  as a representative of the TFR.

Next we consider the geometrical aspect of nuclear collisions. For convenience in identifying the forward direction, let us consider  $AB$  collisions, and we are interested in the production of hadrons in the TFR of  $A$ . In the Glauber model, we write the cross section of  $AB$  collisions in the form

$$\sigma^{AB} = \int d^2b g^{AB}(b), \quad (2)$$

where

$$g^{AB}(b) = \int d^2s T_A(s) [1 - e^{-\sigma T_B(|\vec{s} - \vec{b}|)}] + \int d^2s T_B(s) [1 - e^{-\sigma T_A(|\vec{s} - \vec{b}|)}]. \quad (3)$$

$T_A(s)$  is the thickness function normalized to  $A$ , i.e.,

$$T_A(s) = A \int dz \rho(s, z), \quad \int d^2s T_A(s) = A, \quad (4)$$

$\rho$  being the nuclear density normalized to 1;  $\sigma$  in Eq. (3) is the inelastic nucleon-nucleon cross section.

To calculate the average number of wounded nucleons in  $B$ , we consider only the first term in Eq. (3) and call it  $g^{(A)B}(b)$ . Define  $\Pi_v^{(A)B}(b)$  to be the probability of  $A$  having  $v$  collisions in  $B$  at impact parameter  $b$ , so that

$$g^{(A)B}(b) = \sum_{v=1}^B \Pi_v^{(A)B}(b). \quad (5)$$

Then we have

$$\Pi_v^{(A)B}(b) = \int d^2s T_A(s) \pi_v^{pB}(|\vec{s} - \vec{b}|), \quad (6)$$

where  $\pi_v^{pB}$  is the corresponding probability in  $pB$  collisions [14]

$$\pi_v^{pB}(b) = \frac{1}{v!} [\sigma T_B(b)]^v \exp[-\sigma T_B(b)], \quad (7)$$

from which one can recover the necessary condition

$$\sum_{v=1}^B \pi_v^{pB}(b) = 1 - \exp[-\sigma T_B(b)], \quad (8)$$

as required by Eqs. (5) and (6). Moreover, we obtain

$$\sum_{v=1}^B v \pi_v^{pB}(b) = \sigma T_B(b). \quad (9)$$

Thus for  $pB$  collisions the average number of wounded nucleons in  $B$  at  $b$  is

$$\bar{v}^{pB}(b) = \frac{\sum_{v=1}^B v \pi_v^{pB}(b)}{\sum_{v=1}^B \pi_v^{pB}(b)} = \frac{\sigma T_B(b)}{1 - \exp[-\sigma T_B(b)]}. \quad (10)$$

Now, returning to  $AB$  collisions, the average number of wounded nucleons in  $B$  is

$$\bar{v}^{(A)B}(b) \equiv \frac{\sum_{v=1}^B v \Pi_v^{(A)B}(b)}{\sum_{v=1}^B \Pi_v^{(A)B}(b)} = \frac{\sigma \int d^2s T_A(s) T_B(|\vec{s} - \vec{b}|)}{g^{(A)B}(b)}. \quad (11)$$

Since the number of binary collisions  $N_{\text{coll}}^{AB}(b)$  is the numerator of the last expression in Eq. (11), and the number of participants  $N_{\text{part}}^{AB}$  is  $g^{AB}(b)$ , we have for  $AA$  collisions

$$\bar{v}^{(A)A}(b) = \frac{N_{\text{coll}}^{AA}(b)}{N_{\text{part}}^{AA}(b)/2}. \quad (12)$$

For Au+Au collisions at 200 GeV, the tabulated values of  $N_{\text{coll}}$  and  $N_{\text{part}}$  are 1065 and 351 (at 0–5% centrality) and 220 and 114 (at 30–40%) [15]. Hence we get

$$\begin{aligned} \bar{v}^{(A)A} &= 6.1 & (0\text{--}5\%), \\ &= 3.9 & (30\text{--}40\%). \end{aligned} \quad (13)$$

The impact parameters that correspond to the two centrality bins can be calculated from the overlap function

$$T_{AA}(b) = \int d^2s T_A(s) T_A(|\vec{s} - \vec{b}|). \quad (14)$$

Using the simplified form for  $\rho(s, z)$  with uniform density in Eq. (4) results in a distribution for  $T_{AA}(b)$  that is slightly higher than the tabulated values for various centrality bins given in [15]. Nevertheless, from the shape of the distribution, the corresponding values of  $b$  can reasonably be set at

$$\begin{aligned} b &= 1\text{ fm} & (0\text{--}5\%), \\ &= 8\text{ fm} & (30\text{--}40\%). \end{aligned} \quad (15)$$

The consideration of nuclear geometry and the associated wounded nucleons will become important in the following when the momentum degradation effect is taken into account.

### III. HADRON PRODUCTION AT LARGE $p_L$ IN THE RECOMBINATION MODEL

In  $pp$  collisions, no particle can be produced with  $p_L > \sqrt{s}/2$ . In  $AB$  nuclear collisions, one would initially expect the same to be true. However,  $n$  nucleons in  $A$  have a combined momentum of  $n\sqrt{s}/2$ , which can make possible a particle produced with  $p_L > \sqrt{s}/2$  without violating momentum conservation, if a coherence effect is at work. That would be the case if the valence quarks in three nucleons in  $A$ , each with momentum fraction  $x_i > 1/3$ , say, recombine to form a nucleon, whose Feynman  $x$  can then exceed 1. It is also possible for a pion to be produced with  $x > 1$ , but since an antiquark in the sea (or a gluon converted to  $q\bar{q}$ ) is needed with large enough  $x_i$  to recombine with a valence quark of another nucleon, the probability is much lower. This type of consideration need not be restricted to the  $x > 1$  region. Even for  $x > 0.6$ , which we broadly refer to as TFR, the recombination of partons from different nucleons in  $A$  will dominate over those processes where the partons are from the same nucleon, as in  $pB$  collisions. That dominance is over and above the shifted peak in rapidity due to ‘‘baryon stopping’’ simply on the basis of extra momentum availability in  $AB$  collisions. Thus, even without detail calculations we can predict that the  $p/\pi$  ratio is large in the TFR in  $AB$  collisions. Similarly, the  $\Lambda/K$  ratio is also large.

Hadron production at low  $p_T$  and large  $p_L$  in hadronic collisions has been treated in the RM in good agreement with data [7,8,9]. The extension now to nuclear ( $AB$ ) collisions with emphasis on the TFR region has the same basic recombination formula,

$$H_p^{AB}(x) \equiv x \frac{dN_p^{AB}}{dx} = \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} \frac{dx_3}{x_3} F_{uud}^{AB}(x_1, x_2, x_3) \times R_p(x_1, x_2, x_3, x), \quad (16)$$

for the production of proton.  $R_p$  is the recombination function (RF) that has been studied in the framework of the valon model for the nucleon structure [8,10] by relating  $R_p$  to the valon distribution  $G_p$  in the proton [16],

$$R_p(x_1, x_2, x_3, x) = g_{st} \frac{x_1 x_2 x_3}{x^3} G_p\left(\frac{x_1}{x}, \frac{x_2}{x}, \frac{x_3}{x}\right), \quad (17)$$

where

$$G_p(y_1, y_2, y_3) = g_p (y_1 y_2)^\alpha y_3^\beta \delta(y_1 + y_2 + y_3 - 1), \quad (18)$$

$$g_p = [B(\alpha + 1, \alpha + \beta + 2)B(\alpha + 1, \beta + 1)]^{-1}, \quad (19)$$

and  $g_{st}$  is the statistical factor  $1/6$  [16]. It is by successfully fitting the CTEQ parton distribution functions at low  $Q^2$  that the parameters  $\alpha$  and  $\beta$  are determined to be [10]

$$\alpha = 1.75, \quad \beta = 1.05. \quad (20)$$

The  $\delta$  function in Eq. (18) enforces the momentum sum for recombination:  $\sum_i x_i = x$ .

The key quantity in Eq. (16) is the three-quark distribution  $F_{uud}^{AB}(x_1, x_2, x_3)$ . If it were like in  $pB$  collisions, then the quarks would all originate from the projectile, and the  $x_i$  in  $F_{uud}^{pB}(x_1, x_2, x_3)$  would satisfy  $\sum_i x_i < 1$ . However, in the

TFR of  $AB$  collisions, we consider the dominant component in which each quark is from a separator nucleon in the same longitudinal tube at distance  $s$  from the center of  $A$ , so that we can write in the factorizable form

$$F_{uud}^{(3)B}(x_1, x_2, x_3) = F_v^u(x_1)F_v^u(x_2)F_v^d(x_3), \quad (21)$$

with the labels  $\vec{s}$  and  $\vec{b}$  suppressed. On the left-hand side of Eq. (21) we use (3) instead of  $A$  in the superscript to emphasize that only three nucleons in  $A$  are considered. How to generalize from 3 to  $A$  will be discussed in Sec. IV. The effect of momentum degradation due to the passage through  $B$  will be considered in the next section. The crucial point here is that the variables  $x_i$  in Eq. (21) are independent of one another, so the integrals in Eq. (16) are from 0 to 1 for each  $x_i$ . Thus the maximum possible  $x$  is 3, well beyond the conventional FR. It is this unconventional possibility of producing a proton in the TFR that motivates our investigation here.

In Eq. (21), we used the superscripts  $u$  and  $d$  to denote the flavors of quarks that are to recombine in Eq. (16) where the RF is given by Eqs. (17) and (18) with  $y_1$  and  $y_2$  referring to the  $u$  quark and  $y_3$  to the  $d$  quark. However, if the projectile  $A$  is an isoscalar, which we shall assume, then at every impact parameter  $s$  there are equal numbers of protons and neutrons, so  $F_v^u = F_v^d$ . In the valon model for  $pB$  collisions, the quark distributions are [11]

$$F_v^{u,d}(x_i) = \int_{x_i}^1 dy' \bar{G}'_v(y') K\left(\frac{x_i}{y'}\right), \quad (22)$$

where the valon distribution  $\bar{G}'_v(y')$  differs from  $G(y)$  because of momentum degradation, as will be discussed in detail in the following section.  $K(z)$  is the quark distribution in a valon. Since our valon distribution is flavor independent,  $K(z)$  consists of both valence and sea quarks [10]

$$K(z) = K_{NS}(z) + L(z), \quad (23)$$

where

$$K_{NS}(z) = z^a(1-z)^b/B(a, b+1), \quad (24)$$

$$a = 0.35, \quad b = -0.61.$$

These parameters  $a$  and  $b$  are determined from the moments of  $K_{NS}$  given in [10]. For the sea quark distribution  $L(z)$  we have to do two things: first, we have to average the favored ( $L_f$ ) and unfavored ( $L_u$ ) distributions given [10]; second, we readjust the normalization to saturate the sea.  $L_f(z)$  and  $L_u(z)$  have been determined for the proton sea to distinguish, for example, a  $u$  quark in a  $U$  valon (favored) from a  $d$  quark in a  $U$  valon (or a  $u$  quark in a  $D$  valon, unfavored). They are, for  $Q = 1 \text{ GeV}/c$  [10],

$$\ln L_f(z) = -2.66 + 0.08t - 10.4t^2 - 6t^3, \quad (25)$$

$$\ln L_u(z) = -2.92 + 4.0t - 5.95t^2 - 1.4t^3,$$

where  $t = -\ln(1-z)$ . We define

$$L_q(z) = \frac{1}{2}[L_f(z) + L_u(z)]. \quad (26)$$

This distribution does not include the conversion of gluons to the sea quarks, a process that we must consider in order to

account for the hadronization of all partons, including gluons [8,9]. Thus to saturate the sea, we renormalize as

$$L'_q(z) = ZL_q(z), \quad (27)$$

where  $Z$  is determined by solving the two algebraic equations that express the momentum conservation in the  $u$ ,  $d$ ,  $s$  sections in terms of the second moments [9]

$$\tilde{K}_{\text{NS}}(2) + 2[2\tilde{L}_q(2) + \tilde{L}_s(2)] + \tilde{L}_g(2) = 1 \quad (28)$$

$$\tilde{K}_{\text{NS}}(2) + 2[2\tilde{L}'_q(2) + \tilde{L}'_s(2)] = 1 \quad (29)$$

$$Z = 1 + \frac{\tilde{L}_g(2)}{2[2\tilde{L}_q(2) + \tilde{L}_s(2)]}. \quad (30)$$

In Eq. (29) we have assumed that the  $s$ -quark sea is enhanced also in nuclear collisions, unlike the case of hadronic collision where the gluons are convected to the light-quark sector only through  $g \rightarrow q\bar{q}$  [9]. In forming Eq. (30), we have set  $L'_s = ZL_s$  also. Since the second moments in Eq. (30) are tabulated in [9], we obtain

$$Z = 3.42. \quad (31)$$

In the following we shall consider  $L'_q(z)$  only when the light quarks in the sea are needed, including the  $L(z)$  term in Eq. (23).

For pion production, the physical content of the calculational procedure is the same as for proton production, except that it is a  $q\bar{q}$  recombination. Thus, as in Eqs. (16), (21), and (22), we have

$$H_{\pi}^{AB}(x) \equiv x \frac{dN_{\pi}^{AB}}{dx} = \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} F_{q\bar{q}}^{AB}(x_1, x_2) R_{\pi}(x_1, x_2, x), \quad (32)$$

$$F_{q\bar{q}}^{(2)B}(x_1, x_2) = F_{\bar{v}}^q(x_1) F_{\bar{v}}^{\bar{q}}(x_2), \quad (33)$$

$$F_{\bar{v}}^{\bar{q}}(x_2) = \int_{x_2}^1 dy' \bar{G}_{\bar{v}}(y') L'_q\left(\frac{x_2}{y'}\right), \quad (34)$$

where  $F_{\bar{v}}^{\bar{q}}$  is obtained from the saturated sea. The recombination function for pion is [8,9]

$$R_{\pi}(x_1, x_2, x) = \frac{x_1 x_2}{x^2} \delta\left(\frac{x_1}{x} + \frac{x_2}{x} - 1\right). \quad (35)$$

In an isosymmetric collision system, we need not distinguish the charge states.  $\Lambda$  and  $K$  production can similarly be considered.

#### IV. MOMENTUM DEGRADATION

In the preceding section, we described the quark distribution in Eq. (22) as a convolution of the valon distribution and the quark distribution in a valon, but we did not specify the former. In a  $pp$  collision, the valon distribution  $G(y)$ ,  $y$  beginning the momentum fraction, has been identified as that of the free proton on the basis that at low  $p_T$  and large  $p_L$  the fast partons in the forward direction are unaffected by the opposite-going partons because of the lack of long-range correlation in rapidity. The valon model connects the bound-state problem of a static proton (in terms of constituent quarks) with the

structure problem of a proton in collision (in terms of partons) [8]. In a  $pA$  collision, the effect of momentum degradation in the passage of the projectile through the nuclear target is applied to the valons in Ref. [11], where it is shown not only how baryon stopping can be obtained in agreement with data, but also how pion production in the FR can be determined. We now extend the treatment to the TFR in the  $AB$  collisions.

In a free proton, the single-valon  $U$  and  $D$  distributions are obtained from the exclusive distribution given in Eq. (18) by integration,

$$G^U(y_1) = \int_0^{1-y_1} dy_2 \int_0^{1-y_1-y_2} dy_3 G_p(y_1, y_2, y_3) = g_p B(\alpha + 1, \beta + 1) y_1^{\alpha} (1 - y_1)^{\alpha + \beta + 1}, \quad (36)$$

$$G^D(y_3) = \int_0^{1-y_3} dy_1 \int_0^{1-y_1-y_3} dy_2 G_p(y_1, y_2, y_3) = g_p B(\alpha + 1, \alpha + 1) y_3^{\beta} (1 - y_3)^{2\alpha + 1}. \quad (37)$$

For isoscalar nuclei, we take the average

$$G(y) = \frac{1}{2}[G^U(y) + G^D(y)] \quad (38)$$

before the nucleon traverses the nuclear medium. We note that  $G(y)$  is not an invariant distribution, but  $yG(y)$  is. It is normalized by

$$\int_0^1 dy G(y) = \int dy_1 dy_2 dy_3 G(y_1, y_2, y_3) = 1, \quad (39)$$

i.e., the probability for the proton to consist of three and only three valons is 1.

Suppose now that a nucleon in  $A$  at a fixed impact parameter  $s$  collides with  $\bar{\nu}$  wounded nucleons in  $B$  on the average. Since  $|\vec{s} - \vec{b}|$  can be almost as large as the radius  $R$ , it is possible for  $\bar{\nu}(b, s)$  to be very small. That is the geometrical situation where several nucleons in  $A$  can contribute to the TFR without much momentum degradation. When we consider fluctuations of  $\nu$  from  $\bar{\nu}$ , we must include the possibility of  $\nu$  being 0. Thus we use the Poisson distribution

$$P_{\bar{\nu}}(\nu) = \frac{\bar{\nu}^{\nu}}{\nu!} e^{-\bar{\nu}} \quad (40)$$

with normalization defined by summation from  $\nu = 0$

$$\sum_{\nu=0}^{\infty} P_{\bar{\nu}}(\nu) = 1. \quad (41)$$

If a valon loses a momentum fraction  $1 - \kappa$  at each collision, then after  $\nu$  collisions the modified valon distribution is

$$y' G'_{\nu}(y') = \int_{y'}^1 dy G(y) \delta\left(\frac{y'}{y} - \kappa^{\nu}\right), \quad (42)$$

from which follows

$$G'_{\nu}(y') = \kappa^{-2\nu} G(\kappa^{-\nu} y'). \quad (43)$$

The parameter  $\kappa$  for the reduced momentum fraction is unknown, since it should not be inferred from the  $pA$  collision, which is for a cold nuclear target. In  $AB$  collisions a tube in  $A$  contains many nucleons which cannot all be treated as if each of them collides with  $\nu$  nucleons in a cold nucleus  $B$ .

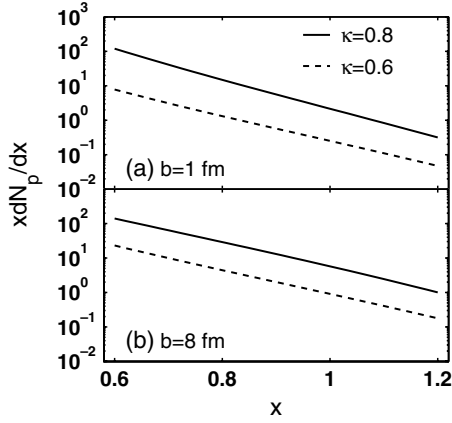


FIG. 1. Proton distributions in the TFR for (a)  $b = 1$  fm and (b)  $b = 8$  fm.

Indeed, it is hard to assess the state of  $B$  when the back part of the tube traverses the medium. We can at best use an adjustable parameter  $\kappa$  to describe in some average sense what contributes to the TFR. From Eq. (43), we obtain after  $\bar{\nu}$  collisions on the average

$$\bar{G}'_{\bar{\nu}}(y') = \sum_{\nu=0}^{\infty} G'_{\nu}(y') P_{\bar{\nu}}(\nu). \quad (44)$$

Identifying  $\bar{\nu}$  with the quantity expressed by Eq. (10) is based on the assumption that each valon experiences the same average number of collisions as the parent nucleon does.

$\bar{G}'_{\bar{\nu}}(y')$ , as given in Eq. (44), is the modified valon distribution that should be used in Eqs. (22) and (34). That takes care of the nuclear effect. What remains is the calculation of the  $p$  and  $\pi$  distributions in the TFR.

## V. PROTON AND PION DISTRIBUTIONS

To calculate the hadron distributions in  $x$ , we return to Eq. (16) as a general formula, which needs, however, some

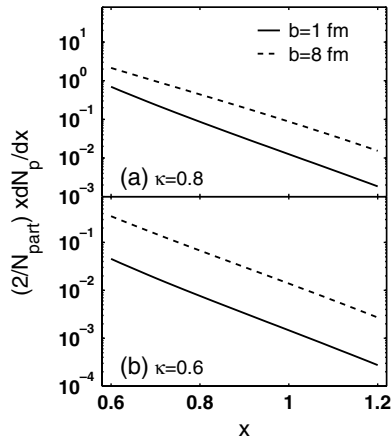


FIG. 2. Proton distributions in the TFR normalized by  $N_{\text{part}}/2$  for (a)  $\kappa = 0.8$  and (b)  $\kappa = 0.6$ , where  $\kappa$  is the survival factor in momentum degradation.

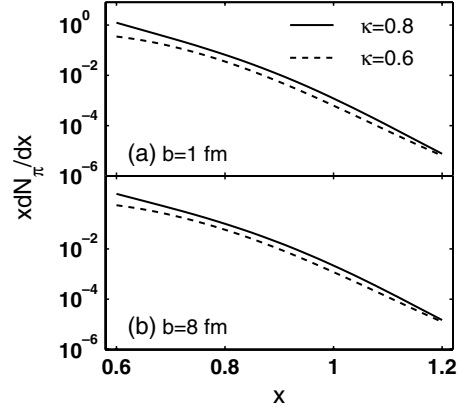


FIG. 3. Same as Fig. 1, but for pion distributions.

more elaboration to account for the size of  $A$ . In Eq. (21) we show the factorizable form of  $F_{uud}^{(3)B}$  when there are only three nucleons in the projectile, each contributing a quark. Now, we specify the details of how to calculate  $F_{uud}^{AB}$ , which is called for in Eq. (16).

In Sec. II, we have in Eq. (7) the probability  $\pi_v^{pB}(b)$  for a nucleon making  $\nu$  collisions in  $B$  at impact parameter  $b$ . Now, consider the same quantity in  $A$  and write

$$\pi_{\mu}^{Ap}(s) = \frac{1}{\mu!} [\sigma T_A(s)]^{\mu} \exp[-\sigma T_A(s)] \quad (45)$$

for the probability that  $\mu$  nucleons in  $A$  colliding with a nucleon in  $B$  at impact parameter  $s$  in  $A$ . In place of Eq. (6), we now have for  $\mu$  nucleons in  $A$  colliding with  $\nu$  nucleons in  $B$  the probability

$$\Pi_{\mu\nu}^{AB}(b) = \int \frac{d^2s}{\sigma} \pi_{\mu}^{Ap}(s) \pi_{\nu}^{pB}(|\vec{s} - \vec{b}|). \quad (46)$$

Since at least three nucleons in  $A$  are needed for our calculation of the proton distribution in the TFR of  $A$ , as done in Sec. III,

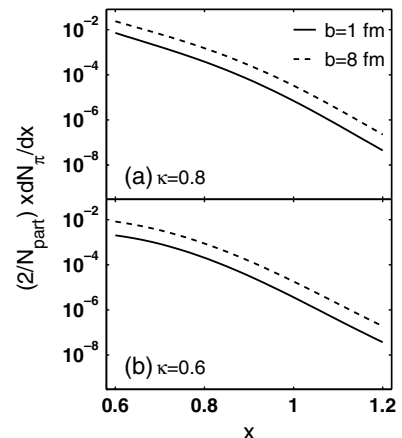
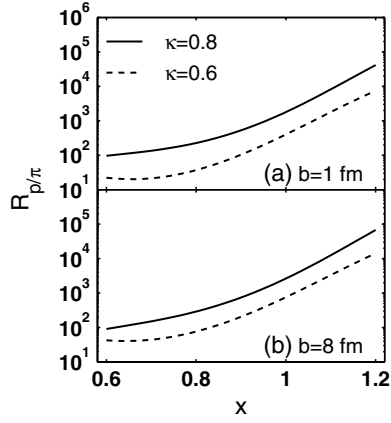


FIG. 4. Same as Fig. 2, but for pion distributions.


 FIG. 5.  $p/\pi$  ratio for (a)  $b = 1$  fm and (b)  $b = 8$  fm.

we must sum over  $\mu$  starting with  $\mu = 3$  and get

$$H_p^{AB}(x, b) = \int \frac{d^2s}{\sigma} \sum_{\mu=3}^{\infty} \pi_{\mu}^{Ap}(s) \binom{\mu}{3} H_p^{(3)B}(x, b, s), \quad (47)$$

where  $H_p^{(3)B}(x, b, s)$  is what we described in Eqs. (16), (21), and (22), i.e.,

$$H_p^{(3)B}(x, b, s) = \int \left[ \prod_{i=1}^3 \frac{dx_i}{x_i} F_{\bar{v}}^q(x_i) \right] R_p(x_1, x_2, x_3, x), \quad (48)$$

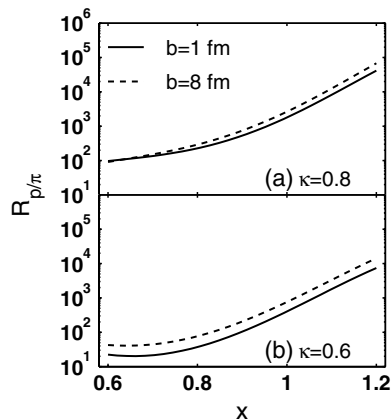
where  $\bar{v}$  is given by Eq. (10) but at  $|\vec{s} - \vec{b}|$ , that is,

$$\bar{v} = \bar{v}^{pB}(|\vec{s} - \vec{b}|) = \frac{\sigma T_B(|\vec{s} - \vec{b}|)}{1 - \exp[-\sigma T_B(|\vec{s} - \vec{b}|)]}. \quad (49)$$

The sum over  $\mu$  in Eq. (47) can be performed, yielding

$$H_p^{AB}(x, b) = \int \frac{d^2s}{\sigma} \frac{[\sigma T_A(s)]^3}{3!} H_p^{(3)B}(x, b, s). \quad (50)$$

Note that with this formula we do not need the results on  $\bar{v}^{(A)A}(b)$  given in Eqs. (12) and (13).


 FIG. 6. Same as Fig. 5, but for (a)  $\kappa = 0.8$  and (b)  $\kappa = 0.6$ .

For pion production, it is straightforward to modify Eq. (50) and get

$$H_{\pi}^{AB}(x, b) = \int \frac{d^2s}{\sigma} \frac{[\sigma T_A(s)]^2}{2!} H_{\pi}^{(2)B}(x, b, s), \quad (51)$$

where

$$H_{\pi}^{(2)B}(x, b, s) = \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} F_{\bar{v}}^q(x_1) F_{\bar{v}}^{\bar{q}}(x_2) R_{\pi}(x_1, x_2, x), \quad (52)$$

since  $F_{\bar{v}}^{\bar{q}}(x_2)$  is severely damped at large  $x_2$ ,  $H_{\pi}^{AB}(x, b)$  is expected to be much more suppressed compared to  $H_p^{AB}(x, b)$  in the TFR. Nevertheless,  $H_{\pi}^{AB}(x, b)$  need not vanish for  $x > 1$ , unlike  $H_{\pi}^{pB}(x, b)$ .

On the basis of Eqs. (50) and (51) we have calculated the proton and pion distributions for Au+Au collisions at  $b = 1$  and 8 fm, corresponding to 0–5% and 30–40% centralities, according to Eq. (15). We used the approximation of uniform nuclear density with  $R = 1.2A^{1/3}$  fm, and  $\sigma = 41$  mb. For the parameter  $\kappa$ , which represents the surviving valon momentum fraction after each collision, we chose two representative values,  $\kappa = 0.8$  and 0.6, where  $\kappa = 1$  implies no momentum degradation. The results for the invariant distribution  $x dN_p/dx$  for the proton, which is just  $H_p^{AuAu}(x, b)$ , are shown in Fig. 1 for  $b = 1$  and  $b = 8$  fm, with  $\kappa = 0.8$  and 0.6. All four lines are nearly straight, i.e., exponential in  $x$ , smoothly throughout the TFR. There can be other contributions to proton production in that region due to the recombination of quarks originating from one or two nucleons, but they are so small that we ignore them.

Figure 1 shows that the  $x$  distributions are suppressed when there is more momentum degradation (smaller  $\kappa$ ), as expected. In the case of  $pA$  collisions, such a suppression would correspond to the qualitative notion of baryon stopping. But in  $AA$  collisions, instead of stopping, we have protons produced at  $x > 1$ . Nevertheless, the overall normalization is lowered when there is more momentum degradation. Thus, there are two features about the inclusive distribution of the proton: it extends smoothly beyond  $x = 1$ , and it is more suppressed at lower  $\kappa$ . The physical value of  $\kappa$  that corresponds to reality can be determined only after data become available and are plotted as in Fig. 1. To summarize the behavior of the  $x$  distributions, we parametrize them in the form

$$x \frac{dN_p}{dx}(b) = \exp[h_0^p(b) - h_1^p(b)x], \quad 0.6 < x < 1.2, \quad (53)$$

which fits the lines in Fig. 1 extremely well with the parameters  $h_0^p(b)$  and  $h_1^p(b)$  given in Table I.

It is of interest to know how the distribution depends on centrality at a given  $\kappa$ . We plot the distributions normalized by

 TABLE I. Parameters  $h_0^p$  and  $h_1^p$  for proton.

	$\kappa = 0.8$		$\kappa = 0.6$	
	$b = 1$	$b = 8$	$b = 1$	$b = 8$
$h_0^p(b)$	10.58	9.90	7.03	7.93
$h_1^p(b)$	9.80	8.18	8.41	8.04

TABLE II. Parameters  $h_0^\pi$ ,  $h_1^\pi$ , and  $h_2^\pi$  for pion.

	$\kappa = 0.8$		$\kappa = 0.6$	
	$b = 1$	$b = 8$	$b = 1$	$b = 8$
$h_0^\pi$	1.96	1.25	0.0063	-0.083
$h_1^\pi$	5.46	7.39	7.40	8.32
$h_2^\pi$	14.15	14.79	14.66	14.87

$N_{\text{part}}/2$  in Fig. 2, which shows how they are suppressed as the collision changes from peripheral to central. At larger  $b$ , the collisions on average have higher  $\mu$  and lower  $\nu$  in Eq. (46), leading to more partons hadronizing in the TFR.

In [2], data from Au+Au collisions at 200 GeV show that the  $\eta'$  distributions for 0–6% and 35–40% crossover at  $\eta' \approx -2$ . According to Eq. (1), that would correspond to  $x \approx 0.135 p_T/m_p \lesssim 0.1$ , which is significantly outside the TFR. For  $-2 < \eta' < 0$ , the data show the peripheral case higher than the central case, not unlike what we have in Fig. 2(a).

For pion production, Fig. 3 shows the rapid decline of the  $x$  distributions, as  $x$  is increased toward 1 and beyond. Since  $\bar{q}$  distribution is suppressed at large  $x_2$ , it is difficult for a pion to acquire enough momentum to go deep into the TFR. Nevertheless, the  $x > 1$  region is not forbidden. It is evident from Fig. 3 that there is no sensitive dependence on  $\kappa$ . The reason is partly because there are only two  $\bar{G}_v'$  functions in Eqs. (33) and (34) for  $\pi$ , instead of three for  $p$ , but mostly because the RF for  $\pi$  is broader than that for  $p$ . The valon distribution in  $\pi$  is flat [9] (corresponding to pion being a tightly bound state), so the wider momentum spread allows the more degraded parton momenta to contribute to the formation of  $\pi$ . In Fig. 4, we show the dependence on  $b$ . Compared to Fig. 2, the pions do not show as much dependence as do the protons, although the vertical scales of the two figures are different and can lead to visual misreading. Quantitatively, we can fit the distributions by

$$x \frac{dN_\pi}{dx}(b) = \exp[h_0^\pi(b) + h_1^\pi(b)x - h_2^\pi(b)x^2], \quad (54)$$

$$0.6 < x < 1.2,$$

with the parameters given in Table II. Qualitatively, the pion distributions are orders of magnitude lower than the proton distributions.

Apart from the details of the  $x$  distributions, our main prediction is that proton production dominates over pion production in the TFR. To give a visual impact of that dominance, we show in Figs. 5 and 6 the  $p/\pi$  ratio. At  $x \sim 1$ , the ratio is roughly  $10^3$  for any combination of  $b$  and  $\kappa$ . It is such a large ratio that particle identification in the TFR would be the most direct way to settle the question of whether our hadronization scheme is in any way close to reality.

## VI. CONCLUSION

We have investigated hadron production in the transfragmentation region. The overwhelming feature of our result is that the  $p/\pi$  ratio is extremely large, roughly  $10^3$  at  $x \sim 1$ . That feature is a direct consequence of parton recombination. For a proton to be produced at  $x \sim 1$ , it is rather easy to find three nucleons in  $A$ , each contributing a quark at  $x_i \sim 1/3$  to form the proton. However, for a pion at  $x \sim 1$ , it is hard to find any antiquark at, for example,  $x_i \sim 1/3$  to help a quark at  $\sim 2/3$  to make up the pion momentum. The quantitative value of the  $p/\pi$  ratio can be obtained only after some rather involved calculations. But to qualitatively have a distribution in  $x$  that crosses the boundary at  $x = 1$  smoothly is possible only by parton recombination. Fragmentation would require all hadrons to be produced at  $x < 1$ , if no parton had momentum  $> 1$ . In  $AA$  collisions, it is, in principle, possible for a parton to acquire momentum  $> 1$ ; however, the probability for that is very low, and the effect of its fragmentation is insignificant compared to the recombination process of producing hadrons at  $x \sim 1$ .

Experimental data do show that particles can be produced at  $\eta' > 0$ , at least at lower energies. At 200 GeV, the data stop at  $\eta' \sim 0$ , but show no evidence of vanishing there [2]. It is unfortunate that we cannot compare our result in  $x$  to the data in  $\eta'$ , since  $p_T$  of the produced particles are unknown. Either particle identification or  $p_T$  determination, preferably both, would greatly help to relate theory and experiment.

The importance of clarifying what happens in the TFR is in the determination of whether there exists new physics in the FR and beyond. In our treatment of the problem, we have considered only low- $p_T$  physics, but we extended it to include recombination of quarks at medium  $x_i$  from different nucleons. If proven correct by data, our results form the basis from which to extend further to higher  $p_T$  in the TFR. Then there should arise a competition between the enhancement effect studied here and the suppression effect found earlier in forward production at intermediate  $p_T$  in  $d + Au$  collisions [17]. The suppression can be due to either initial-state or final-state physics. Any future study of hadron production in a larger domain in  $p_T$  in the TFR will have to be consistent with the physics explored here at low  $p_T$ .

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