

Energy loss and dynamical evolution of quark p_T spectra

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Average energy loss of light quarks has been calculated in a two stage equilibrium scenario where the quarks are executing Brownian motion in a gluonic heat bath. The evolution of the quark p_T spectra is studied by solving Fokker-Planck equation in an expanding plasma. Results are finally compared with experimentally measured pion p_T spectrum at RHIC.

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Production of high transverse momentum (p_T) particles in heavy ion collision, in recent years, has assumed special interest. This is related with the phenomenon of high p_T particle suppression dubbed as jet quenching. Actually energetic partons while passing through plasma lose energy which degrades the population of high p_T hadrons. Experimentally such suppression has been observed at Relativistic Heavy Ion Collider (RHIC) at high p_T domain [1]. This phenomenon of “jet quenching” can be used to extract the properties of early stage of the plasma temporarily produced in the high energy heavy ion collision [2].

An essential component to study the modified p_T spectrum is to estimate parton energy loss in the thermal bath of quarks and gluons. Significant progress has been made in recent years to calculate this energy loss [2–9]. Partons in a plasma can dissipate energy in two ways, either by two body scattering (collisional loss) or via the emission of gluons (radiative loss). In many of these calculations, first, path length dependent energy loss is estimated, which, consequently is used to modify the fragmentation functions that depopulate high p_T hadrons [10]. The present approach is different in the sense that here we dynamically evolve the spectra (quark) from an initial distribution and investigate its shape at different time scales. The importance of this has already been discussed in Refs. [3,11] where the focus has been on the radiative loss. Even though this is the main mechanism of energy loss, under certain kinematic conditions, collisional loss could be comparable, or even be more, to its radiative counterpart [9]. In view of this, to estimate energy loss, we inject quarks with a narrow distribution of energy and study the broadening of the same as the system expands and cools. Likewise, p_T spectra is also evolved dynamically.

In the present model gluons are thermalized at a much smaller time scale than quarks. Such two stage equilibration scenario, albeit in a different context, was considered sometime ago [12,13] comparing gg , $gq(\bar{q})$ and $qq(\bar{q})$ scattering rates. The possibility of earlier thermalization (even faster) of (soft) gluons is further accentuated, if the gluons are assumed to be in color glass condensate (CGC) state initially. In CGC scenario, it has been shown that thermal bath for the gluons could be formed much earlier. The typical time scale for RHIC and LHC energies are estimated to be $t_i = 1.40 \text{ GeV}^{-1}$, $t_i = 0.62 \text{ GeV}^{-1}$, respectively [14].

Under such a simplifying scenario, it thus reduces to the Brownian motion problem where quarks are executing random motion in the gluonic bath. Further evolution of the quarks are, therefore, governed by the Fokker-Planck equation (FPE). So we avoid solving full Boltzmann kinetic equation (BKE) and approximate relevant collision integral in terms of appropriately defined drag and diffusion coefficients [15–17].

The drag and diffusion coefficients are calculated using techniques of finite temperature perturbative quantum chromodynamics. The former is related to the quark energy loss, while the latter is related to the square of the momentum transfer [15,16]. Both of these quantities show infrared divergences as the collisions are dominated by soft (t -channel) gluons. To cure this problem, we screen the interaction via hard thermal loop (HTL) corrected propagator which makes long range Coulomb interaction finite.

To arrive at the relevant FP equation from BKE we assume that there is no external force and therefore,

$$\left(\frac{\partial}{\partial t} + \mathbf{v}_p \cdot \nabla_{\mathbf{r}} \right) f(\mathbf{p}, \mathbf{x}, t) = C[f(\mathbf{p}, \mathbf{x}, t)]. \quad (1)$$

Here, quarks have a phase space distribution which evolves in time and the collision term is evaluated by considering ultrarelativistic scattering of the quarks and gluons which eventually are expressed in terms of transport coefficients. Considering that the system expands in the longitudinal direction and taking z along the collision axis, the above equation takes the form [18]

$$\frac{\partial f(\mathbf{p}, z, t)}{\partial t} + v_{pz} \frac{\partial f(\mathbf{p}, z, t)}{\partial z} = C[f(\mathbf{p}, z, t)]. \quad (2)$$

Here, $v_{pz} = p_z/E_p$ (for light partons $E_p = |p|$). This equation can be simplified further for the central rapidity region which is boost invariant in rapidity. This implies

$$f(\mathbf{p}_t, p_z, z, t) = f(\mathbf{p}_t, p'_z, \tau). \quad (3)$$

Here $p'_z = \gamma(p_z - u_z p)$, the transformation velocity $u_z = z/t$, $\gamma = (1 - u_z^2)^{-1/2} = t/\tau$ and $\tau = \sqrt{t^2 - z^2}$ denotes the proper time. Using the Lorentz transformation relation $\partial\tau/\partial z|_{z=0} = 0$, $\gamma_{z=0} = 1$ and $\partial p'_z/\partial z|_{z=0} = -p/t$,

one finds

$$v_{pz} \frac{\partial f}{\partial z} = -\frac{p_z}{t} \frac{\partial f}{\partial p_z}. \quad (4)$$

Therefore the Boltzmann equation takes the following form:

$$\left. \frac{\partial f(\mathbf{p}_t, p_z, t)}{\partial t} \right|_{p_z, t} = \left(\frac{\partial}{\partial t} - \frac{p_z}{t} \frac{\partial}{\partial p_z} \right) f(\mathbf{p}_t, p_z, t), \quad (5)$$

$$\left(\frac{\partial}{\partial t} - \frac{p_z}{t} \frac{\partial}{\partial p_z} \right) f(\mathbf{p}_t, p_z, t) = C[f(\mathbf{p}_t, p_z, t)]. \quad (6)$$

Evidently in Eq. (6), the term on the left side involving p_z/t arises because of the expansion while the right hand side characterizes the collisions. The latter can be written in terms of the differential collision rate $W_{\mathbf{p}, \mathbf{q}}$

$$C[f(\mathbf{p}_t, p_z, t)] = \int d^3 q [W_{\mathbf{p}+\mathbf{q}, \mathbf{q}} f(\mathbf{p} + \mathbf{q}) - W_{\mathbf{p}, \mathbf{q}} f(\mathbf{p})] \quad (7)$$

which quantifies the rate of change of the quark momentum from \mathbf{p} to $\mathbf{p} - \mathbf{q}$, $W_{\mathbf{p}, \mathbf{q}} = d\Gamma(\mathbf{p}, \mathbf{q})/d^3 q$.

In a partonic plasma, small angle collisions, with parametric dependence of $O(g^2 T)$, are more frequent than the large angle scattering rate. The latter goes as $\sim O(g^4 T)$. Therefore the distribution function does not change much over the mean time between two soft scatterings. This allows us to approximate $f(\mathbf{p} + \mathbf{q}) \simeq f(\mathbf{p})$. In contrast, $W_{\mathbf{p}, \mathbf{q}}$, being sensitive to small momentum transfer, falls off very fast with increasing q . Therefore, we write

$$\begin{aligned} W_{\mathbf{p}+\mathbf{q}, \mathbf{q}} f(\mathbf{p} + \mathbf{q}) &\simeq W_{\mathbf{p}, \mathbf{q}} f(\mathbf{p}) + q_i \frac{\partial}{\partial p_i} (W_{\mathbf{p}, \mathbf{q}} f) \\ &+ \frac{1}{2} q_i q_j \frac{\partial^2}{\partial p_i \partial p_j} (W_{\mathbf{p}, \mathbf{q}} f). \end{aligned} \quad (8)$$

With this approximation, Eq. (6) can be written as

$$\begin{aligned} \left(\frac{\partial}{\partial t} - \frac{p_z}{t} \frac{\partial}{\partial p_z} \right) f(\mathbf{p}_t, p_z, t) \\ = \frac{\partial}{\partial p_i} A_i(\mathbf{p}) f(\mathbf{p}) + \frac{1}{2} \frac{\partial}{\partial p_i \partial p_j} [B_{ij}(\mathbf{p}) f(\mathbf{p})], \end{aligned} \quad (9)$$

where we have defined the following kernels:

$$A_i = \int d^3 q W_{\mathbf{p}, \mathbf{q}} q_i, \quad (10)$$

$$B_{ij} = \int d^3 q W_{\mathbf{p}, \mathbf{q}} q_i q_j, \quad (11)$$

$$\begin{aligned} A_i &= \frac{v}{16p(2\pi)^5} \int \frac{d^3 k'}{k'} \frac{d^3 k}{k} \frac{d^3 q}{p'} d\omega q_i |\mathcal{M}|_{t \rightarrow 0}^2 f(k) \\ &\times (1 + f(k')) \delta^3(\mathbf{q} - \mathbf{k}' + \mathbf{k}) \delta(\omega - \mathbf{v}_{\mathbf{k}'} \cdot \mathbf{q}) \delta(\omega - \mathbf{v}_{\mathbf{k}} \cdot \mathbf{q}), \end{aligned} \quad (12)$$

$$\begin{aligned} B_{ij} &= \frac{v}{16p(2\pi)^5} \int \frac{d^3 k'}{k'} \frac{d^3 k}{k} \frac{d^3 q}{p'} d\omega q_i q_j |\mathcal{M}|_{t \rightarrow 0}^2 f(k) \\ &\times (1 + f(k')) \delta^3(\mathbf{q} - \mathbf{k}' + \mathbf{k}) \\ &\times \delta(\omega - \mathbf{v}_{\mathbf{k}'} \cdot \mathbf{q}) \delta(\omega - \mathbf{v}_{\mathbf{k}} \cdot \mathbf{q}). \end{aligned} \quad (13)$$

First we consider Compton scattering ($gq(\bar{q}) \rightarrow gq(\bar{q})$) for which

$$|\mathcal{M}|^2 = g^4 \left[\frac{s^2 + u^2}{t^2} - \frac{4}{9} \frac{s^2 + u^2}{us} \right]. \quad (14)$$

In terms of explicit momentum variables it reduces to

$$|\mathcal{M}|_{t \rightarrow 0}^2 = 8g^4 \frac{p' k' p k}{(q^2 - \omega^2)^2} (1 - \cos \theta_{pk}) (1 - \cos \theta_{p'k'}). \quad (15)$$

In general we level the incoming four-momentum of the bath particle (gluon) as K and the final momentum of the out going particles as P' and K' . The usual Mandelstums are written as s, u, t with $t = \omega^2 - q^2$, $s = -(u + t)$. The scattering angles, following Ref. [19], can be written as

$$\begin{aligned} \cos \theta_{pq} &= \frac{\omega}{q} + \frac{t}{2pq}, \\ \cos \theta_{kq} &= \frac{\omega}{q} - \frac{t}{2pq}. \end{aligned} \quad (16)$$

The parton parton collision is dominated by the soft momentum transfer. To calculate the leading contribution, therefore, we consider small angle scatterings only. In this limit $t \rightarrow 0$ and $s = -u$. Hence the collision is dominated by the t channel, $|\mathcal{M}|^2 \rightarrow g^4 2s^2/t^2$. We also have $\cos \theta_{kq} \sim \cos \theta_{k'q} \sim \omega/q \sim \cos \theta_{pq} \sim \cos \theta_{p'q}$ and $\cos \theta_{pk} = \omega^2/q^2 + (1 - \omega^2/q^2) \cos \phi_{pk}$. With these azimuthal angle averaged matrix element becomes

$$\begin{aligned} \langle \mathcal{M} \rangle_{t \rightarrow 0}^2 &\simeq 12g^4 p' k' p k (1 - \omega^2/q^2)^2 / (q^2 - \omega^2)^2 \\ &\simeq 12g^4 p^2 k^2 / q^4. \end{aligned} \quad (17)$$

It might be noted that Eq. (12) is symmetric under $k \leftrightarrow k'$, which allows one to write

$$\begin{aligned} A(p^2) &= \frac{v}{16p^2(2\pi)^5} \int \frac{d^3 k'}{k'} \frac{d^3 k}{k} \frac{d^3 q}{p'} d\omega p \cdot q \langle \mathcal{M} \rangle_{t \rightarrow 0}^2 \\ &\times \frac{1}{2} [f(k)(1 + f(k')) - f(k')(1 + f(k))] \\ &\times \delta^3(\mathbf{q} - \mathbf{k}' + \mathbf{k}) \delta(\omega - \mathbf{v}_{\mathbf{k}'} \cdot \mathbf{q}) \delta(\omega - \mathbf{v}_{\mathbf{k}} \cdot \mathbf{q}). \end{aligned} \quad (18)$$

Recognizing the fact that ω is small compared to the momentum, we have

$$f(k) - f(k') = -\omega \frac{\partial f}{\partial k}. \quad (19)$$

Furthermore,

$$-\int k^2 \frac{\partial f(k)}{\partial k} dk = \int 2kf(k) dk = \frac{\pi^2 T^2}{3}. \quad (20)$$

With the help of these identities, the drag coefficient can easily be calculated at the leading log order:

$$A(p^2) = \frac{v\pi\alpha_s^2 T^2}{6p} \mathcal{L}, \quad (21)$$

where $\mathcal{L} = \int \frac{dq}{q}$ [14]. Evidently this is infrared singular. Such divergences do not arise if close and distant collisions are treated separately. For very low momentum transfer the concept of individual collision breaks down and one has to

take collective excitations of the plasma into account. Hence there should be a lower momentum cutoff above which bare interactions are valid description, while for the softer collisions medium modified hard thermal loop corrected propagator should be used [8,9]. It is evident that Eq. (12) actually gives dE/dt or the energy loss rate [9] that can be related to the drag coefficient.

B_{ij} can be decomposed into longitudinal and transverse components:

$$B_{ij} = B_t \left(\delta_{ij} - \frac{p_i p_j}{p^2} \right) + B_l \frac{p_i p_j}{p^2}. \quad (22)$$

Explicit calculation shows that the off diagonal components of B_{ij} vanish,

$$B_{t,l} = \frac{v}{(2\pi)^5} \int \frac{d^3 k d^3 q d\omega}{2k 2k' 2p 2p'} \delta(\omega - \mathbf{v}_p \cdot \mathbf{q}) \delta(\omega - \mathbf{v}_k \cdot \mathbf{q}) \times \langle \mathcal{M} \rangle_{t \rightarrow 0}^2 f(k) \left[1 + f(k) + \omega \frac{\partial f}{\partial k} \right] q_{t,l}^2. \quad (23)$$

Here, in the small angle limit $q_l \simeq \omega$ and $q_t \simeq \sqrt{q^2 - q_l^2}$. With all these, in the leading log approximation

$$B_t = \frac{2v\pi\alpha_s^2}{3} T^3 \mathcal{L}, \quad (24)$$

$$B_l = \frac{v\pi\alpha_s^2}{3} T^3 \mathcal{L}. \quad (25)$$

Here the fluctuation-dissipation theorem, is found to be satisfied automatically. This connects drag and momentum diffusion constant giving rise to Einstein's relation $B(p^2) = 2TEA(p^2)$, where $B_{ij}(p^2) = \delta_{ij}B(p^2)$. To remove arbitrariness related with the momentum cutoff scales hidden in \mathcal{L} , hard thermal loop corrected propagator is used [9].

In the Coulomb gauge, we can define $D_{00} = \Delta_l$ and $D_{ij} = (\delta_{ij} - q^i q^j / q^2) \Delta_t$ where Δ_l and Δ_t denote the longitudinal and transverse gluon propagators given by

$$\Delta_l(q_0, q)^{-1} = q^2 - \frac{3}{2}\omega_p^2 \left[\frac{q_0}{q} \ln \frac{q_0 + q}{q_0 - q} - 2 \right], \quad (26)$$

$$\Delta_t(q_0, q)^{-1} = q_0^2 - q^2 + \frac{3}{2}\omega_p^2 \times \left[\frac{q_0(q_0^2 - q^2)}{2q^3} \ln \frac{q_0 + q}{q_0 - q} - \frac{q_0^2}{q^2} \right]. \quad (27)$$

With this matrix element in the limit of small angle scattering, we get the following expression for the squared matrix element:

$$|\mathcal{M}|^2 = g^4 C_{qq} 16 (EE_1)^2 |\Delta_l(q_0, q) + (v \times \hat{q}) \cdot (v_1 \times \hat{q}) \Delta_t(q_0, q)|^2 \quad (28)$$

with $v = \hat{p}$, $v_1 = \hat{p}_1$ and C_{qq} is the color factor. With this screened interaction, further calculation for the drag and diffusion constant proceeds along the line of Ref. [9].

Now to calculate average energy loss of the light quarks in an expanding partonic plasma we inject test quarks having

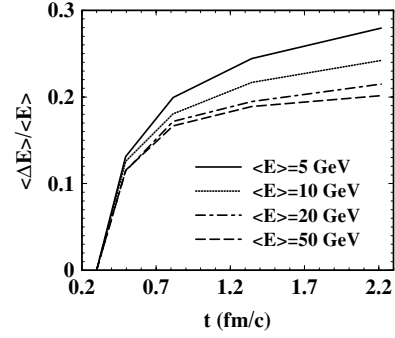


FIG. 1. Average energy loss of quarks as a function of time for various initial average energies.

following distribution at time t_i :

$$f(p_T, p_z, t = t_i) = N \delta^2(p_T - p_{T0}) \delta(p_z - p_{z0}). \quad (29)$$

As the time progresses the system expands and $f(p_T, p_z, t)$ evolves according to Eq. (9). The average energy loss is given by

$$\langle \Delta E \rangle = \langle E \rangle - E_0, \quad (30)$$

where $\langle E(t) \rangle = \int d^3 p E f(p_T, p_z, t) / \int d^3 p f(p_T, p_z, t)$. E_0 corresponds to the peak of the initial spectrum, i.e., $E_0 = \langle E(t_i) \rangle$.

In Fig. 1, we present results for the energy loss as a function of time for various initial energies with means indicated in the legend. To describe the expansion dynamics we implement Bjorken cooling law [20] $\tau_i T_i^{1/c_s^2} = \tau T^{1/c_s^2}$, where c_s is the velocity of sound. The results are compared with the measured pion p_T spectrum at RHIC [21] in Fig. 2. The initial p_T distribution of quarks is assumed to be proportional to the pion p_T spectrum as measured in p - p collision [22]:

$$f(p_T, p_z, t = t_i) = \frac{N_0}{p_T} \frac{dN(y=0)}{d^2 p_T dy} = \frac{\bar{N}_0}{p_T} \frac{1}{(1 + \frac{p_T}{p_0})^v}, \quad (31)$$

where, $v = 9.97$ and $p_0 = 1.212$ and N_0 is the normalization constant.

The expanding system approach the transition temperature, $T_c \sim 190$ MeV at a time $t \sim 2$ fm/c. Assuming that the high p_T partons materialize into hadrons outside the system and the hadronization process does not affect the shape

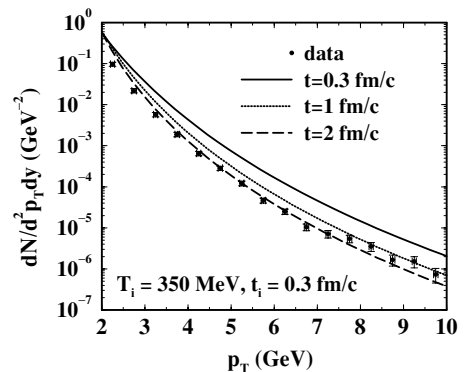


FIG. 2. Time evolution of the transverse momentum distributions.

of the p_T distribution drastically, the quark spectra at $t \sim 2$ fm/c may be compared with the pion spectrum obtained experimentally.

In conclusion, we, in the present work, estimate quark energy loss considering stochastic nature of the interaction for which three dimensional FP equation is solved. Relevant

drag and diffusion coefficient have been calculated in the soft collision limit. To highlight the importance of collisional energy loss radiative processes is excluded. We find that two body scattering gives reasonable amount of quenching so as to accommodate RHIC data and also describes the shape of the p_T spectra reasonably well.

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