PHYSICAL REVIEW C 73, 042201(R) (2006)

Hints on the quadrupole deformation of the $\Delta(1232)$

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(Received 29 December 2005; published 18 April 2006)

The E2/M1 ratio (EMR) of the $\Delta(1232)$ is extracted from the world data in pion photoproduction by means of an effective Lagrangian approach (ELA). This quantity has been derived within a crossing symmetric, gauge invariant, and chiral symmetric Lagrangian model which also contains a consistent modern treatment of the $\Delta(1232)$ resonance. The *bare s*-channel $\Delta(1232)$ contribution is well isolated and final state interactions (FSI) are effectively taken into account fulfilling Watson's theorem. The obtained EMR value, EMR = $(-1.30 \pm 0.52)\%$, is in good agreement with the latest lattice QCD calculations [Phys. Rev. Lett. **94**, 021601 (2005)] and disagrees with results of current quark model calculations.

DOI: 10.1103/PhysRevC.73.042201 PACS number(s): 14.20.Gk, 14.20.Dh, 25.20.Lj, 13.60.Le

From general symmetry principles, the emission of a photon by a spin-3/2 system that becomes spin-1/2, involves transverse electric quadrupole (E2) and magnetic dipole (M1) multipolarities. Likewise, this is the case of the absorption of a real photon by a spin-1/2 to reach spin-3/2. In the absence of knowledge of the internal structure of the system, an estimate of the ratio between the two multipolarities can be made by resorting to Weisskopf [1] units for multipole strengths in nuclear systems. For the excitation of a nucleon into a $\Delta(1232)$ ($\gamma + N \rightarrow \Delta$) this estimate gives

$$R_W = \sqrt{\left(\frac{S_{\rm E2}}{S_{\rm M1}}\right)} = 1.07 \cdot 10^{-3} R_0^2 (M_\Delta - M_N) \tag{1}$$

with the nucleon radius R_0 in fm and the mass difference in MeV. In what follows we refer to this value as the Weisskopf ratio (R_W) . Taking a radius $R_0 = 0.875$ [2] and a mass difference $(M_\Delta - M_N) \simeq 270$ MeV one gets $R_W \simeq 0.22$.

Within the quark model, a single quark spin flip is the standard picture for the photoexcitation of the nucleon into a Δ , assuming spherically symmetric (L = 0) radial wave functions of both parent and daughter. Under these premises, an E2 transition cannot take place, as it was first noticed by Becchi and Morpurgo in their 1965 paper [3], where they concluded that a value of the E2/M1 ratio (EMR) much smaller than R_W should be considered as a test of the model. As early as 1963 values of EMR small but different from zero were reported in the literature [4] which was supported by further experiments later on [5–7]. A nonvanishing E2 multipolarity evokes a deformed nucleon picture [8]. In an extreme rotational model approximation the nucleon could be considered as the head of a $K^{\pi}=\frac{1}{2}^+$ rotational band $(\frac{1}{2}^+,\frac{3}{2}^+,\frac{5}{2}^+,\ldots)$, in analogy to rotational nuclear bands. In this picture the electromagnetic current and multipoles for the transition between the members of the band can be parametrized in terms of intrinsic single particle and collective multipoles [9]. In particular, the E2

multipole for the transition $(\gamma + N \rightarrow \Delta)$ would be given in terms of the intrinsic quadrupole moment (Q_0) by the relation [9,10]

$$\mathcal{M}(E2) = \left\langle \frac{1}{2} \frac{1}{2} 20 \middle| \frac{3}{2} \frac{1}{2} \right\rangle \sqrt{\frac{5}{8\pi}} Q_0 = 0.282 Q_0.$$
 (2)

In turn, Q_0 would be related to the spectroscopic quadrupole moment of the Δ by

$$Q_0 = -5Q_{\Lambda}. (3)$$

Hence, the relationship between the static $\Delta(1232)$ quadrupole moment and the E2 multipole for the $N \to \Delta$ transition is

$$\mathcal{M}(E2, N \to \Delta) = -\frac{5}{\sqrt{4\pi}}Q_{\Delta}.$$
 (4)

Within this picture, a negative (positive) static quadrupole moment implies a prolate (oblate) intrinsic deformation, which is not always well stated in the literature.

Over the last few years much effort has been invested in the determination of quadrupole deformation in the nucleon [11,12]. Because the spin of the nucleon is 1/2, a possible intrinsic quadrupole deformation is not directly observable and its study requires research on its lowest-lying excitation— $\Delta(1232)$ —and its decay through pion emission. Hints on the possible deformation will be deduced via the EMR. In the context of the quark model, De Rújula, Georgi, and Glashow [13] were the first to suggest a tensor force arising from one-gluon exchange and leading to d-state admixtures. On the other hand Buchmann and collaborators [14] pointed out that a nonzero E2 transition could be due to one-gluon or meson-exchange currents. While debate on the physical interpretation of the EMR may still be far from closed, a more precise determination of the EMR value is both possible and mandatory.

Extensive experimental programs have been developed at Brookhaven [6] and Mainz [7], that have resulted in an improvement in the quantity and quality of the pion photoproduction data [15]. However, in order to extract the

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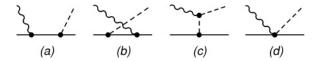


FIG. 1. Feynman diagrams for Born terms: (a) s-channel, (b) u-channel, (c) t-channel, and (d) Kroll-Rudermann.

EMR from experiment, a realistic model of the reaction must be employed that takes into account the final state interaction (FSI) of the outgoing pion as well as the relevant symmetries. Only then can the ratio deduced from the experimental data be compared to the predictions of nucleonic models—namely, quark models [3,14], Skyrme models [16], and lattice QCD [17,18]. Theoretical interest in this topic has been strongly renewed and either new or well-known approaches have been (re)investigated with the latest theoretical advances such as new dynamical models [19–21] and nonpathological spin-3/2 treatments [21,22]. A complete account of the experimental and theoretical work done on this topic goes well beyond the scope of this paper. For a review of the subject we refer the reader to Ref. [12].

A key point in the extraction of the EMR is the reaction model used for the analysis of data. Reaction models have to be developed carefully in order to consider the underlying physics and to minimize model dependencies as well as theoretical uncertainties. Ambiguities in the contribution of the background terms, unitarization, or even formal elements (such as the recently improved spin-3/2 description or the crossing symmetry) can spoil the determination of the parameters of the resonances. This is so even for a well isolated resonance as is the $\Delta(1232)$. A determination of the $\Delta(1232)$ parameters requires one to study the photoproduction reaction not only in the first resonance region, as commonly has been done, but in further kinematical regions in order to keep under control the high energy behavior of the resonance contribution. For example, in a Breit-Wigner model, the inclusion of Regge poles, which take into account heavy meson exchanges, does affect the determination of the $\Delta(1232)$ coupling constants because of the modification of the tail of the resonance [23].

From the theoretical point of view, the effective Lagrangian approach (ELA) is a very suitable and appealing method to study pion photoproduction and nucleon excitations. It is also a reliable, accurate, and formally well established approach in the nucleon mass region.

In this Rapid Communication we employ a realistic model for pion photoproduction on free nucleons from threshold up to 1 GeV based on the ELA that we have recently elaborated. Details on the model will be published somewhere else and can be found in Ref. [22]. In what follows we provide a brief description of the model. In addition to Born (Fig. 1 and vector meson exchange terms [ρ and ω , diagram (e) in Fig. 2], the model includes all the four star resonances in

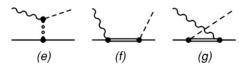


FIG. 2. Feynman diagrams for vector meson exchange (e) and resonance excitations: (f) s-channel and (g) u-channel.

Particle Data Group (PDG) [2] up to 1.7 GeV mass and up to spin-3/2: $\Delta(1232)$, N(1440), N(1520), $\Delta(1620)$, N(1650), and $\Delta(1700)$ —diagrams (f) and (g) in Fig. 2. The main advantages of our model compared to previous ones [24] resides on the treatment of resonances. In particular, we avoid some pathologies in the Lagrangians of the spin-3/2 resonances [such as $\Delta(1232)$], present in previous models, implementing a modern approach due to Pascalutsa [25]. Under this approach the (spin-3/2 resonance)-nucleon-pion and the (spin 3/2 resonance)-nucleon-photon vertices have to fulfill the condition $q_{\alpha}\mathcal{O}^{\alpha...}=0$ where q is the four-momentum of the spin-3/2 particle, α the vertex index which couples to the spin-3/2 field, and the dots stand for other possible indices. In particular, we write the simplest interacting (spin-3/2 resonance)-nucleon-pion Lagrangian as [25]

$$\mathcal{L}_{\text{int}} = -\frac{h}{f_{\pi} M^*} \bar{N} \epsilon_{\mu\nu\lambda\beta} \gamma^{\beta} \gamma^{5} (\partial^{\mu} N_{j}^{*\nu}) (\partial^{\lambda} \pi_{j}) + \text{H.c.} \quad (5)$$

where H.c. stands for Hermitian conjugate, h is the strong coupling constant, $f_{\pi} = 92.3$ MeV is the leptonic decay constant of the pion, M^* the mass of the resonance, and π_j , N, and $N_j^{*\nu}$, the pion, nucleon, and spin-3/2 fields, respectively.

The model also displays chiral symmetry, gauge invariance, and crossing symmetry. The dressing of the resonances [26] is considered by means of a phenomenological width which takes into account decays into one π , one η , and two π . The width is built in order to fulfill crossing symmetry and contributes to both s- and u-channels of the resonances. In order to regularize the high energy behavior of the model we include a crossing symmetric and gauge invariant form factor for Born and vector meson exchange terms [27], as well as form factors in the resonance contributions consistent with the phenomenological widths. We assume that the FSI factorizes and can be included through the distortion of the πN final state wave function. Factorization of FSI has been successfully applied to electron scattering knock-out reactions [28].

A detailed calculation of the distortion would require one to calculate higher order pion loops or to develop a phenomenological potential FSI model. The first approach is overwhelmingly complex and the second would introduce additional model-dependencies, which are to be avoided in the present analysis, in as much as we are concerned here with the bare properties of the $\Delta(1232)$. We rather include FSI in a phenomenological way by adding a phase δ_{FSI} to the electromagnetic multipoles. We determine this phase so that the total phase of the electromagnetic multipole is identical to the one of the energy dependent solution of SAID [29]. In this way Watson's theorem [30] is fulfilled below the two pion threshold and we are able to disentangle the electromagnetic vertex from FSI effects.

In order to obtain a reliable set of electromagnetic coupling constants of the nucleon resonances we have fitted the experimental electromagnetic multipoles using modern minimization techniques based upon genetic algorithms. We have obtained different fits which are compared in Ref. [22]. In this Rapid Communication we focus on two fits obtained including FSI and using two different prescriptions for the determination of the masses of the resonances. The first one uses the set of masses and widths provided by Vrana, Dytman,

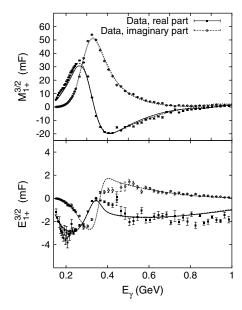


FIG. 3. Comparison among the fits from set nos. 1 (solid: real part, short-dashed: imaginary part) and 2 (dashed: real part, dotted: imaginary part) and the SAID energy independent solution (data) for $M_{1+}^{3/2}$ and $E_{1+}^{3/2}$ electromagnetic multipoles [29]. A detailed discussion on electromagnetic multipoles can be found in Ref. [22].

and Lee [31], and the second one uses a set established by means of a speed plot calculation from the current solution of the SAID πN partial wave analysis [29].

the SAID πN partial wave analysis [29]. In Fig. 3 we show our fits to $M_{1+}^{3/2}$ and $E_{1+}^{3/2}$ multipoles for both sets of parameters.

Caution must be taken with the various definitions of EMR employed in the literature. We should distinguish between the intrinsic [or *bare* EMR of the $\Delta(1232)$] and the directly measured value which is often called *physical* or *dressed* EMR value [19,21] and which is obtained as the ratio between the imaginary parts of $E_{1+}^{3/2}$ and $M_{1+}^{3/2}$ at the E_{γ} value at which Re $M_{1+}^{3/2}=0=\text{Re}\,E_{1+}^{3/2}$. Since all the reaction models are fitted to the experimental electromagnetic multipoles, they generally reproduce the physical EMR value. As seen in Fig. 3 this is also the case in our model, where we get

$$\frac{\text{Im } E_{1+}^{3/2}}{\text{Im } M_{1+}^{3/2}} = (-3.9 \pm 1.1)\% \tag{6}$$

for 328 MeV \leq $E_{\gamma} \leq$ 343 MeV. This value compares well with the value obtained by LEGS Collaboration in Ref. [6], [-3.07 \pm 0.26 (stat. + syst.) \pm 0.24 (model)]%, and is somewhat higher than the PDG value (-2.5 \pm 0.5)%.

However, this measured EMR value is not easily computed with the theoretical models of the nucleon and its resonances. Instead, in order to compare to models of nucleonic structure, it is better to extract the *bare* EMR value of $\Delta(1232)$ which is defined as

$$EMR = \frac{G_E^{\Delta}}{G_M^{\Delta}} \times 100\%. \tag{7}$$

This depends only on the intrinsic characteristics of the $\Delta(1232)$ and can thus be compared directly to predictions

from nucleonic models. It is not, however, directly measurable but must be inferred (in a model dependent way) from reaction models, precisely what we aim in this Rapid Communication.

The connection between both definitions of EMR values is straightforward when FSI are neglected as can be found in the paper by Jones and Scadron [33]. In our formalism, both values can be connected from the definitions of the electromagnetic multipoles [22] and their connection to the $\gamma + N \rightarrow \Delta$ transition Lagrangian

$$\mathcal{L}_{\text{em}} = \frac{3e}{2MM^{+}} \bar{N} [ig_{1}\tilde{F}_{\mu\nu} + g_{2}\gamma^{5}F_{\mu\nu}] \partial^{\mu}N_{3}^{*\nu} + \text{H.c.}, \quad (8)$$

where $M^+=M+M_\Delta$, $F_{\mu\nu}$ is the electromagnetic field, $\tilde{F}_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta}$, and g_1 and g_2 are the coupling constants that can be related to he electric and magnetic form factors through $G_E^\Delta=-\frac{1}{2}\frac{M_\Delta-M}{M^+}g_2$ and $G_M^\Delta=g_1+\frac{1}{2}\frac{M_\Delta-M}{M^+}g_2$ [32]. In our calculation, the numerical differencies between the dressed and the bare EMR values are attributed to FSI.

In Table I we quote our extracted bare EMR values obtained from Eq. (7) together with the mass, helicity amplitudes, and electromagnetic form factors at the photon point of the $\Delta(1232)$.

In our calculations we have considered the pole mass of the resonance instead of the Breit-Wigner mass [19–21]. One must be aware of the fact that electromagnetic coupling constants are very sensitive to the mass and that the width of the $\Delta(1232)$ and the multipoles vary rapidly in the region around the peak of the $\Delta(1232)$. Thus, a variation in the mass of the resonance affects the determination of the EMR value. This is also seen in Table I. Out of the two results given in Table I we adopt as our final result the average value for the bare EMR= $(-1.30\pm0.52)\%$.

In Table II we compare our average EMR values (bare and dressed) to the ones extracted by other authors using other models for pion photoproduction, as well as to predictions of nucleonic models. Our bare result is similar to that from Ref. [19]. However, it disagrees with the bare value derived with the dynamical model of Pascalutsa and Tjon [21], where a positive deformation of the $\Delta(1232)$ [EMR = $(3.8 \pm 1.6)\%$] is inferred. We compare to their model because, together with the one we employ in this work, they were the only available models that include nonpathological $\Delta(1232)$ Lagrangians.

TABLE I. Intrinsic (or bare) EMR [from Eq. (7)] and parameters of $\Delta(1232)$ for the two fits considered. M_{Δ} is the mass, $A_{1/2}^{\Delta}$ and $A_{3/2}^{\Delta}$ the helicity amplitudes, G_E^{Δ} the electric form factor, and G_M^{Δ} the magnetic form factor. Masses and widths for set no. 1 have been taken from Ref. [31] and for set no. 2 they have been calculated using the speed plot technique [22].

	Set no. 1	Set no. 2
M_{Δ} (MeV)	1215 ± 2	1209 ± 2
$A_{1/2}^{\Delta} (\text{GeV}^{-1/2})$	-0.123 ± 0.003	-0.123 ± 0.003
$A_{3/2}^{\Delta}$ (GeV ^{-1/2})	-0.225 ± 0.005	-0.224 ± 0.004
G_E^{Δ}	-0.076 ± 0.042	-0.071 ± 0.042
G_M^Δ	5.650 ± 0.070	5.701 ± 0.071
EMR	$(-1.35 \pm 0.74)\%$	$(-1.24 \pm 0.74)\%$

TABLE II. Comparison of EMR values from nucleonic models and EMR values extracted from data predicted through several reaction models (see text).

Physical EMR, experiments	EMR	Ref.
Particle Data Group	$(-2.5 \pm 0.5)\%$	[2]
LEGS Collaboration	$(-3.07 \pm$	[6]
	0.26 (stat. + syst.) \pm	
	0.24 (model))%	
Physical EMR, reaction models		
Sato and Lee	-2.7%	[19]
Fuda and Alharbi	-2.09%	[20]
Pascalutsa and Tjon	$(-2.4 \pm 0.1)\%$	[21]
Present work (average)	$(-3.9 \pm 1.1)\%$	
Extractions of bare EMR, reaction models		
Sato and Lee	-1.3%	[19]
Pascalutsa and Tjon	$(3.8 \pm 1.6)\%$	[21]
Present work (average)	$(-1.30 \pm 0.52)\%$	
Bare EMR, predictions from nucleonic models		
Nonrelativistic quark model	0%	[3]
Constituent quark model	-3.5%	[14]
Skyrme model	$(-3.5 \pm 1.5)\%$	[16]
Lattice QCD (Leinweber et al.)	$(3 \pm 8)\%$	[17]
Lattice QCD (Alexandrou et al.)		[18]
$(Q^2 = 0.1 \text{ GeV}^2, m_\pi = 0)$	$(-1.93 \pm 0.94)\%$	
$(Q^2 = 0.1 \text{ GeV}^2, m_\pi = 370 \text{ MeV})$	$(-1.40 \pm 0.60)\%$	

The discrepancy is not so worrisome if we recall that dynamical models have ambiguities in the determination of the bare value of EMR [34] that is highly model dependent as it stems from the comparison among different dynamical models, namely Refs. [19–21]. More recently [35] the dependence of the effective chiral perturbation theory on the small expansion parameters was fully exploited to reconcile the (bare) lattice QCD calculations with the physical EMR values

In conclusion, the bare EMR value derived from the multipole experimental data with our realistic ELA model is compatible with some of the predictions of the nucleonic models. In particular it agrees very well with the latest lattice QCD calculations [18] and suggests the need for further improvements in quark models. The comparison of our extracted EMR value to R_W is indicative of a small oblate deformation of the $\Delta(1232)$. In our work we show that an ELA which takes into account FSI is also able to reconcile the physical EMR value with the lattice QCD calculations prediction for EMR. We consider that our picture and that of Ref. [35] are complementary. Thus, both pictures will help to understand the issue of the $\Delta(1232)$ deformation as well as the properties of other resonances.

C.F.-R. work is being developed under Spanish Government grant UAC2002-0009. This work has been supported in part under contracts of Ministerio de Educación y Ciencia (Spain) BFM2002-03562, BFM2003-04147-C02-01, and FIS2005-00640.

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