Structure of and $E2$ transition in ^{16}C in a ^{14}C + n + n model

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A three-body model of $^{14}\mathrm{C} + n + n$ is applied to study the energy spectrum and the hindered $E2$ transition in ^{16}C. A realistic two-nucleon potential is used for the valence neutrons. Both spin-singlet and spin-triplet components for the neutrons are taken into account. The three-body problem with a Pauli constraint is solved in a stochastic variational method. For the n-^{14}C potential chosen to reproduce the properties of ^{15}C, the low-lying energy spectrum agrees reasonably well with experiment, but the ground state is predicted to be about 1 MeV high. The calculated $B(E2; 2_1^+ \to 0_1^+)$ value is about twice the measured value if the polarization charge of the valence neutrons is taken to be the same as that required to fit the ^{15}C data. The correlated motion of the valence neutrons is displayed through the two-neutron density distribution.

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Recently the $E2$ transition from the first 2^+ state to the ground 0^+ state in ^{16}C has been studied through a lifetime measurement by use of a recoil shadow method [1] and $^{16}\mathrm{C} + ^{208}$Pb inelastic scattering [2]. The $B(E2)$ value is found to be $0.63 \pm 0.12\ e^2\mathrm{fm}^4$, which corresponds to an anomalously small strength of about 0.26 W.u. The anomaly is apparent by comparison with other C isotopes. The $B(E2; 2_1^+ \to 0_1^+)$ values of ^{10}C, ^{12}C, and ^{14}C decrease gradually as expected from the increasing excitation energies. However, the case of ^{16}C strongly deviates from this expectation: The excitation energy of the 2_1^+ state of ^{16}C is only 1.77 MeV, much smaller than the case of ^{14}C (7.01 MeV), but the $B(E2)$ value of ^{16}C is nevertheless much smaller than that of ^{14}C, which is 3.74 e^2 fm^4 (1.87 W.u.). The $B(E2)$ value for the $\frac{5}{2}^+_1 \to \frac{1}{2}^+_1$ transition in ^{15}C is also small, $0.97 \pm 0.02\ e^2\,\mathrm{fm}^4$ (0.44 W.u.) [3].

In a previous paper [4], a three-body model of $^{14}\mathrm{C} + n + n$ was applied to study the anomalous $E2$ transition in ^{16}C under the assumptions that the ^{14}C core has a neutron shell closure of the p shell and that the two valence neutrons are in a spin-singlet state. The $^{14}\mathrm{C} + (sd)^2$ model for the low-lying levels of ^{16}C is supported by experiment as well [5,6]. The calculated $B(E2)$ value was 1.38 e^2 fm^4, 2.2 times larger than that of experiment if one uses the same neutron polarization charge as that required for ^{15}C. The deformations of both proton and neutron densities of the C isotopes are studied in antisymmetrized molecular dynamics calculation [7], in which the neutron shape is found to change significantly with the neutron number and the $B(E2)$ value of ^{16}C in a variation after projection calculation is 4–6 times too large compared with that of experiment. The $B(E2)$ value is overestimated also in the deformed Hartree-Fock model and the shell-model calculations [8]. The purpose of the present investigation is to study the structure of ^{16}C thoroughly and to reexamine a mechanism that leads to the hindered transition. Our basic assumption is the same as before, so the relevant levels of ^{16}C are assumed to be generated from the $^{14}\mathrm{C} + n + n$ model. There are, however, some noticeable differences. Here we use more general basis functions, allowing for both spin-singlet and spin-triplet neutrons. This makes it possible to obtain unnatural parity states such as 3^+. In addition, we use a realistic interaction for the two neutrons in order to avoid uncertainties of the model, while in the previous paper an effective interaction was used and its strength was increased so as to fit the ground-state energy.

The wave function for ^{16}C is determined from the following Hamiltonian:

$$H = T_{\boldsymbol{R}} + T_{\boldsymbol{r}} + U_1 + U_2 + v_{12}, \tag{1}$$

where the subscripts of the kinetic energies stand for the relative distance vector, $\boldsymbol{R}$, from the center of mass of ^{14}C to that of the two neutrons and the relative distance vector of the neutrons, $\boldsymbol{r}$. The two-neutron potential v_{12} is taken from the G3RS (case 1) potential [9], which contains central, tensor, and spin-orbit forces and reproduces the nucleon-nucleon scattering data as well as the deuteron properties. The n-^{14}C potential U takes the form

$$U = -V_0 f(r) + V_1 \boldsymbol{\ell} \cdot \boldsymbol{s} \frac{1}{r}\frac{d}{dr} f(r), \tag{2}$$

where $f(r) = [1 + \exp(\frac{r-R_c}{a})]^{-1}$ with $R_c = r_0 A_c^{\frac{1}{3}}$ ($A_c = 14$). The parameters of U are determined to reproduce the energies of the $\frac{1}{2}^+_1$ and $\frac{5}{2}^+_1$ states of ^{15}C. To make the $1s_{1/2}$ state lower than the $0d_{5/2}$ state, the $\boldsymbol{\ell} \cdot \boldsymbol{s}$ strength V_1 was chosen in the previous study to be about half of the standard value [10]. This potential is called set A hereafter. We also test other possibilities to fit these single-particle (s.p.) energies. In set B we use the standard value of V_1 but weaken V_0 for all the partial waves other than the s wave, while in set C a potential with a larger diffuseness parameter is chosen. The parameters of each set are listed in Table I. These potentials generate almost the same s.p. wave function for both $1s_{1/2}$ and $0d_{5/2}$.

TABLE I. Parameters of the n-^{14}C potential.

Set	V_0 (MeV)	V_1 (MeV fm^2)	r_0 (fm)	a (fm)
A	−50.31	16.64	1.25	0.65
B	−50.31 ($l = 0$), −47.18 ($l \neq 0$)	31.25	1.25	0.65
C	−51.71	26.24	1.20	0.73

Trial wave functions are assumed as a combination of correlated Gaussian bases:

$$\Psi_{JM}(1,2) = \sum_{i=1}^{K} C_i \Psi_{JM}(\lambda_i, A_i), \tag{3}$$

$$\Psi_{JM}(\lambda, A) = (1 - P_{12}) \times \{e^{-\frac{1}{2}\tilde{\boldsymbol{x}}A\boldsymbol{x}}[[\mathcal{Y}_{\ell_1}(\boldsymbol{x}_1)\mathcal{Y}_{\ell_2}(\boldsymbol{x}_2)]_L \chi_S(1,2)]_{JM}\}, \tag{4}$$

where P_{12} is a permutation of the two neutrons. The basis function is specified by a set of angular momenta $\lambda = (\ell_1, \ell_2, L, S)$ and a 2×2 positive-definite, symmetric matrix A. Here $\tilde{\boldsymbol{x}}A\boldsymbol{x}$ stands for $A_{11}\boldsymbol{x}_1^2 + 2A_{12}\boldsymbol{x}_1 \cdot \boldsymbol{x}_2 + A_{22}\boldsymbol{x}_2^2$, and $\boldsymbol{x}_1 = \boldsymbol{R} + \frac{1}{2}\boldsymbol{r}$ and $\boldsymbol{x}_2 = \boldsymbol{R} - \frac{1}{2}\boldsymbol{r}$ are the distance vectors of the neutrons from the center of mass of the ^{14}C core. The two neutrons are explicitly correlated because of the presence of the cross term $A_{12}\boldsymbol{x}_1 \cdot \boldsymbol{x}_2$, the inclusion of which is very important to obtain a precise solution [11,12]. The angular parts of the two-neutron orbital motion are described with $\mathcal{Y}_{\ell m}(\boldsymbol{r}) = r^\ell Y_{\ell m}(\hat{\boldsymbol{r}})$, and they are coupled with the spin part χ_S to the total angular momentum J. In the previous study, the angular motion was described with much simpler functions specified by a global vector [13], but here a general form is used to take into account important L, S contents of the wave functions.

The energy and the corresponding wave function are determined from a solution of the generalized eigenvalue problem:

$$\sum_{j=1}^{K}[H_{ij} - EB_{ij}]C_j = 0 \,(i = 1, 2, \ldots, K), \tag{5}$$

$$\begin{pmatrix} H_{ij} \\ B_{ij} \end{pmatrix} = \langle \Psi_{JM}(\lambda_i, A_i) \mid \begin{pmatrix} H \\ 1 \end{pmatrix} \mid \Psi_{JM}(\lambda_j, A_j) \rangle. \tag{6}$$

It is vital to take into account the Pauli principle for the motion of the valence neutrons. Under the assumption that ^{14}C is neutron closed, the Pauli constraint is fulfilled by the condition that the trial wave function has no overlap with any orbit $u_{n\ell jm}$ occupied in the ^{14}C core:

$$\langle u_{n\ell jm}(i) | \Psi_{JM}(1,2) \rangle = 0 \quad (i = 1, 2), \tag{7}$$

where the s.p. orbit $u_{n\ell jm}$ is generated from U and $n\ell j$ runs over $0s_{1/2}$, $0p_{3/2}$, and $0p_{1/2}$. Condition (7) is practically achieved by the orthogonal projection method [14]. The accuracy of the present calculation is such that the probability of mixing in of the occupied orbits is of the order of 10^{-4}.

Before superposing basis functions with different λ, we did a pilot calculation in a single λ channel. We use the algorithm called the stochastic variational method (SVM) [11,12] to optimize the parameter matrices A. The SVM increases the basis dimension one by one by testing a number of candidates that are chosen randomly and in addition fine tunes the already chosen parameters by a refinement process. The basis selection with the SVM is effectively performed not only to take care of the short-range repulsion of v_{12} but also to satisfy condition (7). For $J^\pi = 0^+$, those channels that give an energy lower than the $^{14}\mathrm{C} + n + n$ threshold are only $\lambda = (0, 0, 0, 0)$ and $(2, 2, 0, 0)$, while for $J^\pi = 2^+$ such a bound solution is obtained only for $\lambda = (0, 2, 2, 0)$. Channels with $\ell_i \geqslant 3$ play a minor role. We also studied levels with $J^\pi = 3^+$ and 4^+. The most important channel is $\lambda = (0, 2, 2, 1)$ for 3^+ and $\lambda = (2, 2, 4, 0)$ and $(2, 2, 3, 1)$ for 4^+. Thus the $S = 1$ channel is important for 4^+.

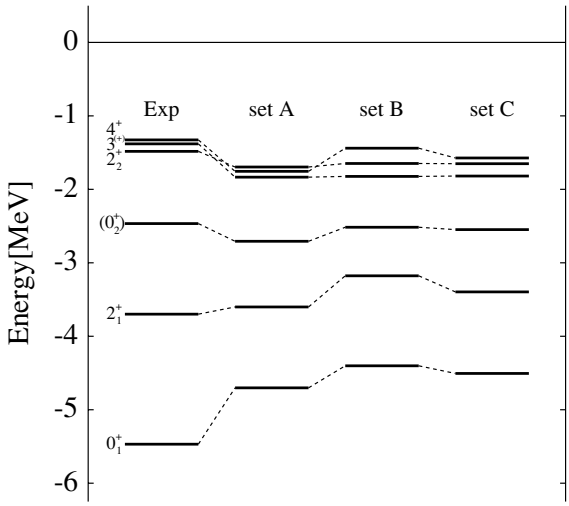

FIG. 1. Low-lying energy levels of ^{16}C for different sets of the n-^{14}C potential. Energy is given from the $^{14}\mathrm{C} + n + n$ threshold.

The basis sets chosen in the single-channel calculation serve as the basis functions for a coupled-channels calculation. Channels are truncated to those with $\ell_1 + \ell_2 \leqslant 4$. A basis dimension K used in the calculation is about 450 for 0^+ and 500–650 for the other cases. Figure 1 compares the calculated energy levels of ^{16}C with experiment. The U dependence of the spectrum is moderate. The ground-state energy is about 1 MeV too high compared with that of experiment, but the energies of the other J^π states are in fair agreement with experiment. No state with $J^\pi = 1^+$ was obtained below the $^{15}\mathrm{C} + n$ threshold.

Table II summarizes some properties of the states calculated with the set B potential. The other potentials give similar results. The root-mean-square radius of ^{16}C, $r(^{16}\mathrm{C})$, for the point nucleon distribution is calculated from the equation $r^2(^{16}\mathrm{C}) = \frac{14}{16}r^2(^{14}\mathrm{C}) + \frac{7}{64}\langle \boldsymbol{R}^2 \rangle + \frac{8}{256}\langle \boldsymbol{r}^2 \rangle$, where we substitute the point proton radius, 2.35 fm [3], for $r(^{14}\mathrm{C})$. The $r(^{16}\mathrm{C})$ value for the ground state agrees well with that extracted from the reaction cross-section analysis [15]. The probability of spin-singlet components, $P_{S=0}$, shows that our previous calculation [4] with no $S = 1$ components is not very good for the 2^+_1 state but quite acceptable for the ground state. P_{ss}, P_{sd}, and P_{dd} stand for the probabilities for the two neutrons to occupy the $(1s_{1/2})^2$, $(1s_{1/2}0d_{5/2})$, and $(0d_{5/2})^2$ configurations, respectively. The sum of the probabilities, $P_{ss} + P_{sd} + P_{dd}$, is less than unity, which signals the importance of unbound s.p. orbits or $n + {}^{14}$C continuum states. Its effect appears more important in the 0^+_1 and 2^+_1 states. The previous calculation

TABLE II. Properties of the states in ^{16}C. Set B parameters are used. Energy and length are in mega-electron-volts and femtometers, respectively.

J^π	$E_{\rm cal}$	$E_{\rm exp}$	$\langle \boldsymbol{x}_1^2 \rangle$	$\langle \boldsymbol{R}^2 \rangle$	$\langle \boldsymbol{r}^2 \rangle$	$\langle \boldsymbol{x}_1 \cdot \boldsymbol{x}_2 \rangle$	$r(^{16}\mathrm{C})$	$P_{S=0}$	P_{ss}	P_{sd}	P_{dd}
0_1^+	−4.40	−5.469	17.8	10.4	29.5	3.04	2.63	0.92	0.52	–	0.37
0_2^+	−2.52	−2.466	19.7	10.4	37.3	1.10	2.67	0.86	0.43	–	0.53
2_1^+	−3.18	−3.699	16.8	9.39	29.7	1.95	2.61	0.75	–	0.67	0.24
2_2^+	−1.65	−1.483	17.1	8.95	32.5	0.84	2.62	0.55	–	0.26	0.72
3_1^+	−1.82	−1.381	22.3	11.5	43.1	0.73	2.73	0.00	–	0.99	–
4_1^+	−1.44	−1.327	15.9	8.30	30.4	0.70	2.59	0.35	–	–	0.96

gave $P_{ss} = 0.49$ and $P_{dd} = 0.39$ for the ground state, which is consistent with the present result. We thus expect that the momentum distribution data of ^{15}C fragments from ^{16}C breakup [16] are well reproduced in the present model as in the previous paper [4].

The two-neutron correlation is examined with the density distribution function

$$\rho(x_1, x_2, \theta) = \frac{1}{2J+1} \sum_M \langle \Psi_{JM}(1,2) \mid \Psi_{JM}(1,2) \rangle_{\rm spin}, \qquad (8)$$

where θ is the angle between $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ and $\langle \cdots \rangle_{\rm spin}$ indicates that the integration is to be done over the spin coordinates only. Figure 2 displays the contour map of $8\pi^2 x^4 \sin\theta\, \rho(x, x, \theta)$ for the 0^+ states. For the 0_1^+ state, we have two distinct peaks: One peak with smaller angles is the highest (about 0.02 fm^{-2} in height), suggesting the correlation of "dineutron" type, and the other with larger angles reaches half the highest peak, corresponding to a "cigarlike" configuration in which the two neutrons sit on the opposite sides of the core. For the 0_2^+ state, however, only one distinct peak with a height of about 0.014 fm^{-2} appears around $\theta = 90^\circ$ (a "boomerang" shape). These different characteristics are expected from the expectation value of $\cos\theta$, which is estimated from $\langle \boldsymbol{x}_1 \cdot \boldsymbol{x}_2 \rangle / \langle \boldsymbol{x}_1^2 \rangle$. This value is 0.17 for 0_1^+ and 0.06 for 0_2^+, as seen from Table II. We also confirmed that the 2_1^+ state has two peaks, similar to the ground state, but the other states all have one peak at the boomerang configuration.

The $E2$ operator, $\mathcal{M}_\mu$, for ^{16}C in the $^{14}\mathrm{C} + n + n$ model is expressed as [4]

$$\mathcal{M}_\mu = \tfrac{1}{2}\,\delta\, e\, \mathcal{Y}_{2\mu}(\boldsymbol{r}) + q_{\rm eff}\, e\, \mathcal{Y}_{2\mu}(\boldsymbol{R}), \qquad (9)$$

with $q_{\rm eff} = \frac{3}{32} + \frac{49}{32}\delta$, where δ is a polarization charge of the valence neutron. The value of δ can be estimated so as to fit the $B(E2)$ value of ^{15}C within the $^{14}\mathrm{C} + n$ model, which leads to $\delta = 0.160$ (set A), 0.159 (set B), and 0.151 (set C). The first term on the right-hand side of Eq. (9) is the $E2$ operator corresponding to the relative motion of the two neutrons, while the second term corresponds to the relative motion between the ^{14}C core and the center of mass of the two neutrons. If the valence neutrons have no charge ($\delta = 0$), the $E2$ operator reduces to the second term of Eq. (9) with a small $q_{\rm eff}$. In this case, the $B(E2)$ value is contributed by the difference between the center of mass of ^{14}C and that of ^{16}C. For nonzero δ, both terms of Eq. (9) contribute to the $E2$ transition matrix element. The $B(E2; 2_1^+ \to 0_1^+)$ values calculated with the different potentials are shown in Fig. 3. All the cases give similar results. The $B(E2)$ value is about

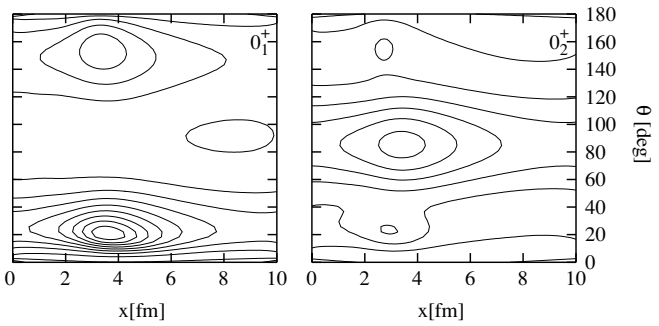

FIG. 2. Contour maps of the two-neutron densities $\rho(x, x, \theta)$, weighted by $8\pi^2 x^4 \sin\theta$, for the 0_1^+ and 0_2^+ states of ^{16}C. The contour shows peaks, and the difference between any two neighboring contour levels is 0.002 fm^{-2}. See text.

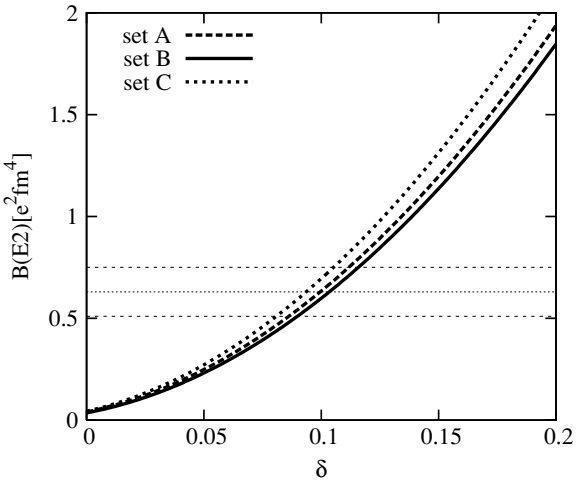

FIG. 3. The $B(E2)$ value for the transition from the 2_1^+ state to the ground state in ^{16}C as a function of the polarization charge δ. The measured value [1,2] $0.63 \pm 0.12\ e^2$ fm^4 is indicated by horizontal lines.

twice the measurement for such δ that is determined from the ^{15}C data. It is interesting to see how the $B(E2)$ value changes with the admixture of the $S = 1$ components. $E2$ operator (9) is independent of spin, so its reduced matrix element consists of two parts:

$$\langle 0_1^+||\mathcal{M}||2_1^+\rangle = \sqrt{(1-a^2)(1-b^2)}\langle\mathcal{M}(S=0)\rangle + ab\langle\mathcal{M}(S=1)\rangle, \quad (10)$$

where a and b are the probability amplitudes of the $S = 1$ components in the 0_1^+ and 2_1^+ states, respectively, and $\langle\mathcal{M}(S)\rangle$ stands for the reduced matrix element between their (normalized) wave-function components with spin S. For set B and for $\delta = 0.159$, we have $\langle\mathcal{M}(S=0)\rangle = 2.75\, e\,\mathrm{fm}^2$ and $\langle\mathcal{M}(S=1)\rangle = 1.56\ e\,\mathrm{fm}^2$. If both of the 0_1^+ and 2_1^+ states are purely spin singlet ($a = b = 0$), the $B(E2)$ value would be $2.75^2/5 = 1.51\ e^2\ \mathrm{fm}^4$. In fact, they have $S = 1$ admixtures whose magnitudes are $a^2 = 0.08$ and $b^2 = 0.25$ (see Table II), and therefore the $B(E2)$ value reduces to $1.25\, e^2\,\mathrm{fm}^4$, as shown in Fig. 3. In the previous calculation with $S = 0$ only [4], $\langle\mathcal{M}(S=0)\rangle$ was $2.63\ e\ \mathrm{fm}^2$, leading to the $B(E2)$ value of $1.38\ e^2\mathrm{fm}^4$. Thus the $S = 1$ admixture has led to reducing the $B(E2)$ value.

We have studied the structure and anomalous $E2$ transition of ^{16}C in the $^{14}\mathrm{C} + n + n$ model, in which the two valence neutrons interact by means of the realistic potential. We have tested three different n-^{14}C potentials that all reproduce the energies of the $\frac{1}{2}_1^+$ and $\frac{5}{2}_1^+$ states of ^{15}C in the $^{14}\mathrm{C} + n$ model. Both spin-singlet and spin-triplet components are included in the calculation. The SVM has been used to solve the three-body problem with the Pauli constraint on the valence neutrons. The energy spectrum of ^{16}C below the $^{14}\mathrm{C} + n + n$ threshold is in fair agreement with experiment except that the ground-state energy is about 1 MeV too high. The dependence of the $B(E2;\ 2_1^+ \to 0_1^+)$ value on the polarization charge of the valence neutrons was studied and found not to be very sensitive to the choice of the n-^{14}C potential. We have seen that admixing the $S = 1$ components in the wave functions reduces the $B(E2)$ value considerably. The calculated $B(E2)$ value is, however, about twice the observed value if the polarization charge is set to reproduce the $B(E2;\ \frac{5}{2}_1^+ \to \frac{1}{2}_1^+)$ of ^{15}C.

Within the $^{14}\mathrm{C} + n + n$ model, the present calculation is probably the most unambiguous because it is free from any adjustable parameters and the result is rather insensitive to the choice of the n-^{14}C potential. The fact that the ground-state energy is predicted to be 1 MeV high and that the theoretical $B(E2)$ value is still twice as large may point to the necessity of other mechanisms such as core distortion or excitation.

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