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Possible existence of multiple chiral doublets in ^{106}Rh

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Adiabatic and configuration-fixed constrained triaxial relativistic mean field (RMF) approaches are developed for the first time. A new phenomenon, the existence of multiple chiral doublets (MχD), i.e., more than one pair of chiral doublet bands in one single nucleus, is suggested for ^{106}Rh based on the triaxial deformations and their corresponding proton and neutron configurations.

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Handedness or chirality is a subject of general interest in molecular physics, elementary particles, and optical physics. The occurrence of chirality in nuclear physics was suggested in 1997 [1], and the predicted patterns of spectra exhibiting chirality, i.e., the chiral doublet bands, were experimentally observed in 2001 [2].

Since the pioneer work on chirality in nuclear physics, much effort has been made to understand the new phenomena and explore their possible existence in the nuclear chart, e.g., [3–10]. Experimentally, chiral doublet bands have been identified in many nuclei in the $A \sim 130$ mass region with the earlier suggested configuration $\pi h_{11/2} \otimes \nu h_{11/2}^{-1}$, $A \sim 100$ with $\pi g_{9/2}^{-1} \otimes \nu h_{11/2}$, and $A \sim 190$ with $\pi h_{9/2} \otimes \nu i_{13/2}^{-1}$. From a theoretical aspect, the chiral symmetry breaking was first predicted in the particle-rotor model (PRM) and tilted axis cranking (TAC) approach for triaxially deformed nuclei [1]. It was investigated later in a hybrid Woods-Saxon and Nilsson model combined with the shell correction method [7] as well as the Skyrme-Hartree-Fock cranking approach [8], and its selection rules for electromagnetic transitions have been discussed in a simple PRM [9].

For a triaxially deformed rotational nucleus, the collective angular momentum favors alignment with the intermediate axis, which in this case has the largest moment of inertia. However, the valence particle and hole angular momentum vectors align along the nuclear short and long axis, respectively. These orientations maximize the overlap of the particle densities with the triaxial core and minimize the interaction energy. The three mutually perpendicular angular momenta can be arranged to form two systems with opposite chirality, a left- and a right-handedness. They are transformed into each other by the chiral operator which combines time reversal and spacial rotation of 180°, $\chi = \mathcal{T}\mathcal{R}(\pi)$. Chiral symmetry in an atomic nucleus is observed because quantum tunneling occurs between systems with opposite chirality.

The description for quantum tunneling of chiral partners is beyond the mean field, as the usual cranking approach is a semiclassical model and the total angular momentum is not a good quantum number [11]. In contrast, the PRM is better suited for this purpose; however, it is so far confined to only one-particle and one-hole configurations [1,9,10]. Its generalization for more particles and/or holes is still under development. Furthermore, the deformation γ and configurations in PRM are not self-consistent as inputs of the model.

The relativistic mean field (RMF) theory has received wide attention because of its success in describing the properties of nuclei and many nuclear phenomena in the past few years [12–14]. It is interesting to search for nuclei with triaxial deformation and configurations having not only one particle and one hole but also several particles and several holes suitable for chirality in RMF theory. A multidimensional microscopic cranking RMF calculation is very time consuming and has only been applied in magnetic rotation so far [15].

In this Brief Report, we will develop the adiabatic and configuration-fixed constrained RMF approaches to investigate the triaxial shape coexistence and possible chiral doublet bands in the $A \sim 100$ mass region. The existence of multiple chiral doublets (MχD), i.e., more than one pair of chiral doublet bands in one single nucleus, will be suggested after examining the deformation and corresponding configurations.

In RMF theory, the nuclei are characterized by an attactive scalar field $S(\mathbf{r})$ and a repulsive vector field $V(\mathbf{r})$ in the Dirac equation

$$\{-i\boldsymbol{\alpha} \cdot \boldsymbol{\nabla} + V(\mathbf{r}) + \beta[M + S(\mathbf{r})]\}\psi_i = \varepsilon_i \psi_i, \qquad (1)$$

which can be solved by expanding separately the upper and lower components of the spinor ψ_i in terms of eigenfunctions of the three-dimensional deformed oscillator in Cartesian coordinates $\phi_\alpha(\mathbf{r})$ and its time reversal state $\phi_{\tilde{\alpha}}(\mathbf{r}) = \hat{T}\phi_\alpha(\mathbf{r})$ with $\hat{T} = i\hat{\sigma}_y K$,

$$\psi_i(\mathbf{r}) = \begin{pmatrix} \sum_\alpha f_\alpha^{(i)} \phi_\alpha(\mathbf{r}) \\ \sum_{\tilde{\alpha}} i g_{\tilde{\alpha}}^{(i)} \phi_{\tilde{\alpha}}(\mathbf{r}) \end{pmatrix}. \qquad (2)$$

In order to avoid the complex matrix diagonalization problem in Ref. [16], $\phi_\alpha(\mathbf{r})$ is written as a product of three Hermite polynomials [17],

$$\phi_\alpha(\mathbf{r}) = \frac{i^{n_y}}{\sqrt{2}} \phi_{n_x}(x)\phi_{n_y}(y)\phi_{n_z}(z) \begin{pmatrix} 1 \\ (-1)^{n_x+1} \end{pmatrix}. \qquad (3)$$

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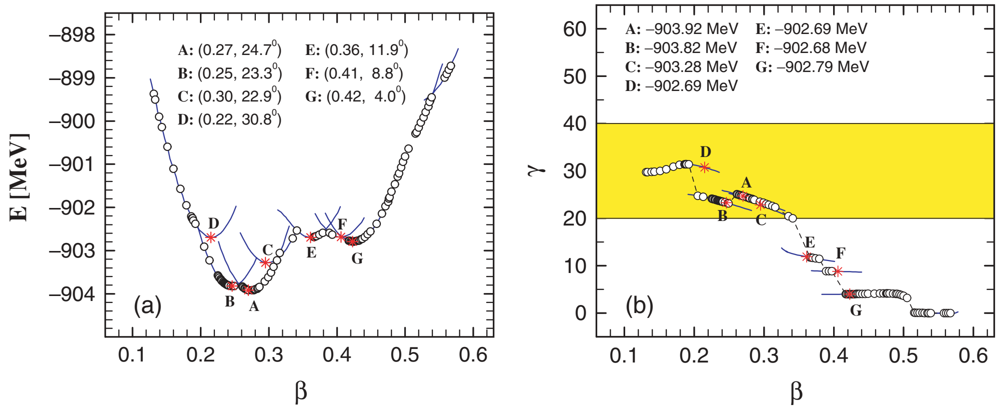

FIG. 1. (Color online) Energy surfaces (a) and deformations γ (b) as functions of deformation β in adiabatic (open circles) and configuration-fixed (solid lines) constrained triaxial RMF calculation with PK1 for ^{106}Rh. The minima in the energy surfaces are represented as stars and labeled A–G with their corresponding deformations β and γ (a) and energies (b).

The meson fields are expanded similarly with the same deformation β_0 and γ_0 for the basis, but an oscillator length smaller by a factor of $\sqrt{2}$ than that for the nucleons ($b_B = b_0/\sqrt{2}$) has been used in order to simplify the calculations and to avoid the necessity of additional parameters. The oscillator frequency is $\hbar\omega_0 = 41A^{-1/3}$ MeV.

The convergence of the results, i.e., the binding energies and the deformations, β and γ, with the number of expanded oscillator shells for fermions n_{0f} has been checked for ^{106}Rh as an example. It is found that as long as $n_{0f} \geqslant 10$, the binding energies as well as the deformations β and γ obtained are almost the same. Furthermore, they are independent on the deformation β_0 of the basis. A similar convergence check for the bosons has also been done. In the following, a spherical basis with 12 major oscillator shells for fermions and 10 shells for bosons will be used, which gives an error less than 0.1% for the binding energy. The present investigation of odd-odd nuclei neglected the pairing correlation.

In general, the triaxial RMF calculation leads to only some local minima. In order to get the ground state for the triaxial deformed nucleus, constrained calculations are necessary; and in principal, such calculations should be carried out in the two-dimensional β and γ plane. However, two-dimensional constrained calculations are expensive even for modern computer facilities; therefore $\langle \hat{Q}_{20}^2 + 2\hat{Q}_{22}^2 \rangle$, i.e., β^2 constrained calculations were carried out to search for the ground state for triaxially deformed nucleus.

The energy surface and deformation γ as functions of deformation β in adiabatic constrained triaxial RMF calculation with PK1 [18] for ^{106}Rh are presented as open circles in Figs. 1(a) and 1(b), respectively. There are some irregularities in the energy surface. Furthermore, some local minima are too obscure to be recognized, and it is technically difficult to understand their corresponding single-particle configurations. Therefore, we performed the configuration-fixed constrained calculation similar to what had been done in the nonrelativistic case [19]. Starting from any point in the energy surface in the adiabatic constrained calculations, the configuration-fixed constrained calculation requires that the occupied single-particle orbits are fixed during the constrained calculation, i.e.,

$$|\langle \psi_j(\beta + \delta\beta)|\psi_i(\beta)\rangle| \approx 1. \quad (4)$$

The energy surfaces and the deformations γ in configuration-fixed calculations with PK1 for ^{106}Rh are given as solid lines in Figs. 1(a) and 1(b), respectively. For each fixed configuration, the constrained calculation gives a continuous, smooth curve for the energy surface and deformation γ as a function of deformation β. The irregularities in the adiabatic energy surface disappear. In comparison, the minima in the energy surfaces of the configuration-fixed constrained calculations become obvious, which are represented by stars and labeled A–G. Their corresponding deformations β and γ and their binding energies are given in Figs. 1(a) and 1(b), respectively. It is interesting to note that for each fixed configuration, the deformation γ is approximately a constant [as in Fig 1(b)], which means that the deformation γ is mainly determined by its corresponding configuration.

The energies for these minima, including the ground state, are within 1.3 MeV to each other but correspond to different deformations β and γ, which is a good example of shape coexistence. The shape coexistence here is different from the spherical, oblate, and prolate shape coexistence, e.g., in neutron-deficient Pt, Hg, and Pb isotopes. It is the triaxial shape coexistence for the ground state A, the excited minima B, C, and D. The states A, B, C, and D have deformation β and γ suitable for chirality [1,10]. As these states are all in one single nucleus, the building of chiral doublet bands on them may lead to a new phenomenon, the existence of MχD in one single nucleus. Therefore, in the following, we will investigate their proton and neutron configurations in detail to see whether the particle and hole configurations required by chirality are available.

Performing the configuration-fixed constrained calculations for the ground state, the single-particle levels can be obtained as functions of deformation β. The difference in single-particle levels obtained by choosing other minima is negligible. The neutron and proton single-particle levels obtained in such a way are presented in Fig. 2. The positive (negative) parity states are marked by solid (dashed) lines,

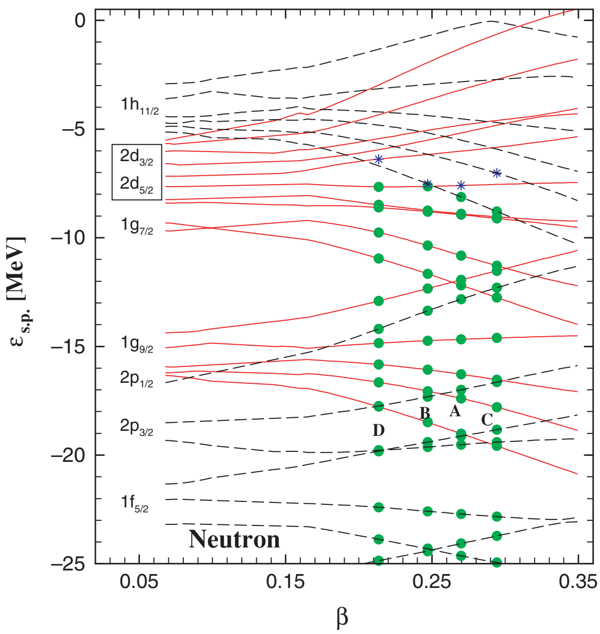

FIG. 2. (Color online) Neutron and proton single-particle levels obtained in configuration-fixed constrained triaxial RMF calculations with PK1 for ^{106}Rh as functions of deformation β. Positive (negative) parity states are marked by solid (dashed) lines. Occupations corresponding to the minima in Fig. 1 are represented by filled circles (two particles) and stars (one particle).

and the occupations corresponding to the minima in Fig. 1 are represented by filled circles (two particles) and stars (one particle). The corresponding quantum numbers for the spherical case are labeled at the left side of the levels. As the energy curves become very stiff for small deformation in Fig. 1, the present configuration-fixed constrained calculations cannot be performed for deformation $\beta < 0.06$. Therefore, we cannot distinguish the occupation of some low-j orbits, e.g., the last occupied neutron orbit for state A may come from $2d_{5/2}$ or $2d_{3/2}$, as marked in the figures. However, as the nuclear chirality is essentially determined by the high-j orbits, particularly the orbit $1h_{11/2}$, such illegibility does not influence the conclusions here. One may note that a similar situation happens also for some levels from $1g_{7/2}$.

The binding energies, deformations β and γ as well as the corresponding configurations extracted from Fig. 2 for the minima A, B, C, and D in ^{106}Rh obtained in the configuration-fixed constrained triaxial RMF calculations with PK1 are listed in Table I. Except the state D, in which there is no high-j neutron valence particle, we found the high-j proton and neutron configuration for the ground state (state A) as $\pi(1g_{9/2})^{-3} \otimes \nu(1h_{11/2})^2$, state B as $\pi(1g_{9/2})^{-3} \otimes \nu(1h_{11/2})^1$, and state C as $\pi(1g_{9/2})^{-3} \otimes \nu(1h_{11/2})^3$. All of them have high-$j$ proton holes and high-j neutron particle configurations, which together with their triaxial deformations favor the construction of the chiral doublet bands. It is interesting to note that states A and B compete strongly with each other in energy. However, due to different parities, states A and B do not mix up and could produce the new phenomenon MχD. So far, a pair of chiral doublet bands with negative parity have been observed in ^{106}Rh [5]. It will be interesting to search for other chiral doublet bands in this nucleus.

Similar to ^{106}Rh, detailed constrained calculations have also been performed for other isotopes in the $A \sim 100$ mass region, and the possibilities of MχD exist in other nuclei as well. The results will be published elsewhere in detail. Here, in Fig. 3, the deformations β and γ for ground states in $^{98-114}$Rh, $^{102-116}$Ag, and $^{100-118}$In isotopes are presented. The shaded area represents the favorable deformation γ for nuclear chirality. The nuclei, ^{104}Rh and ^{106}Rh, in which the chiral doublet bands have been observed [4,5], are marked as filled circles. Apart from ^{104}Rh and ^{106}Rh, the favorable deformation γ for chirality has been found in ^{102,108,110}Rh, $^{108-112}$Ag, and ^{112}In, which implies that more chiral doublet bands can be

TABLE I. Binding energies, deformations β and γ, and the corresponding configurations for the minima A, B, C, and D in ^{106}Rh obtained in the configuration-fixed constrained triaxial RMF calculations with PK1.

State	E (MeV)	β	γ	Configurations
A	903.9150	0.270	24.7°	$\pi(1g_{9/2})^{-3} \otimes \nu\{(1h_{11/2})^2[(2d_{5/2})^1$ or $(2d_{3/2})^1]\}$
B	903.8196	0.246	23.3°	$\pi(1g_{9/2})^{-3} \otimes \nu\{(1h_{11/2})^1[(2d_{5/2})^2$ or $(2d_{3/2})^2]\}$
C	903.2790	0.295	22.9°	$\pi(1g_{9/2})^{-3} \otimes \nu(1h_{11/2})^3$
D	902.6960	0.215	30.8°	$\pi(1g_{9/2})^{-3} \otimes \nu[(2d_{5/2})^3$ or $(2d_{3/2})^3]$

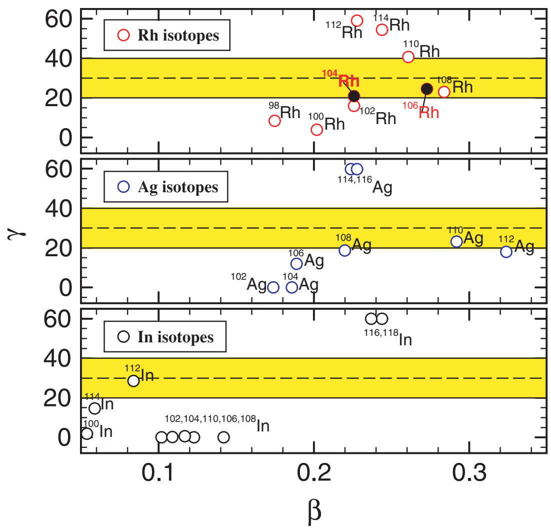

FIG. 3. (Color online) Deformations β and γ for ground states in Rh, Ag, and In isotopes in constrained triaxial RMF calculations with PK1. Shaded areas represent the favorable deformation γ for nuclear chirality. Nuclei ^{104}Rh and ^{106}Rh, in which chiral doublet bands have been observed, are marked as filled circles.

expected in the $A \sim 100$ mass region. For each isotope chain, the deformation γ varies with the neutron number due to different occupation in $\nu h_{11/2}$.

In summary, adiabatic and configuration-fixed constrained triaxial RMF approaches are developed for the first time to investigate the triaxial shape coexistence and possible chiral doublet bands. A new phenomenon, the existence of multiple chiral doublets (MχD), i.e., more than one pair of chiral doublet bands in one single nucleus, is suggested in ^{106}Rh from the results of our examination of the deformation and the corresponding configurations. Similar investigations have also been done for Rh, Ag, and In isotopes and with other effective interactions. The observation of MχD is very promising in this mass region.

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