Consistent interpretation of $B(E2)$ values and g factors in deformed nuclei

Jing-ye Zhang,[1,2] R. F. Casten,[1] A. Wolf,[1,3] Z. Berant,[1,3] R. B. Cakirli,[1,4] N. V. Zamfir,[5] and E. A. McCutchan[1]
[1]*Wright Nuclear Structure Laboratory, Yale University, New Haven, Connecticut 06520, USA*
[2]*Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996, USA*
[3]*Nuclear Research Center Negev, Beer-Sheva, Israel*
[4]*Department of Physics, University of Istanbul, Istanbul, Turkey*
[5]*National Institute of Physics and Nuclear Engineering, Bucharest-Magurele, Romania*
(Received 28 November 2005; published 8 March 2006)

A simple phenomenological model is discussed that simultaneously accounts for the saturation of $B(E2; 0_1^+ \rightarrow 2_1^+)$ values and the newly recognized near constancy of $g(2_1^+)$ factor values in deformed nuclei. The model invokes reduced effective contributions to these observables from the valence neutrons and protons. Empirical evidence supporting this ansatz comes from recently extracted proton-neutron interaction strengths.

DOI: 10.1103/PhysRevC.73.037301 PACS number(s): 21.10.Ky, 21.60.Ev, 23.20.Js, 27.70.+q

The variation of $g(2_1^+)$ factors across broad ranges of nuclei shows striking features that can give clues to shell structure, residual interactions, correlations, and collective effects. For example, in the $N = 90$ region, it has been shown [1] that the growth of $g(2_1^+)$ factors with neutron number can be interpreted in terms of the disappearance of the $Z = 64$ shell gap and the onset of a shape/phase transition. There now exists a wealth of data [2,3] on $g(2_1^+)$ factors throughout the rare-earth region that reveals at least two other, rather general, features. Starting at the onset of deformation around $N = 90$, the $g(2_1^+)$ factors at first decrease with N in a systematic way. Then, from $N \sim$ 94–96 through $N \sim 108$, i.e., in the well-deformed region, they are remarkably constant. Finally, they increase with N beyond 108.

These features reveal interesting aspects of structural evolution. It is the purpose of this brief report to discuss the data beyond $N = 90$ and, in particular, to present a phenomenological model for the constancy of the $g(2_1^+)$ factors in the well-deformed region. It will be seen that, although this model is somewhat *ad hoc* and in need of a microscopic justification, it is based on existing ideas about the reduction of proton-neutron *(p-n)* interaction strengths near midshell, simultaneously accounts for the saturation in $B(E2; 0_1^+ \rightarrow 2_1^+)$ values in the same region, and is empirically supported by a recent study of *p-n* interactions [4].

We first look at the full set of $g(2_1^+)$ factors and $B(E2)$ values in this region, as shown in Fig. 1. Considering first the $g(2_1^+)$ factor data, we note that they decrease in value at the beginning of the deformed region just above $N = 88$ and then are nearly constant between $N =$ 94–108, as mentioned above. Note that we specifically do not show the $g(2_1^+)$ factors or $B(E2)$ values for $N < 88$ because they have been discussed earlier [1] and relate to the $Z = 64$ shell gap: our focus here is on the nuclei beyond the onset of deformation (roughly, beyond $R_{4/2} \sim 3.0$). At the end of the rare-earth region, for $N \geqslant 108$, there is a behavior nearly symmetric to that at the beginning, namely a rise in the $g(2_1^+)$ factors when the deformation begins to decrease in W.

The changes in g factors in the transitional regions ($88 \leqslant N \leqslant 94$, and $N \geqslant 108$) have an interesting behavior, namely their slopes as a function of Z change smoothly. This is shown in Fig. 1(a), where we have drawn straight lines in the regions of changing $g(2_1^+)$ factors for each Z value. The decrease in $g(2_1^+)$ factors with N for $90 \leqslant N \leqslant 94$ is probably related to the development of extensive mixing and correlations in the increasingly deformed wave functions. In particular, as the *p-n* interaction accumulates, the contribution of the protons to the wave functions of the low-lying states is diluted and the $g(2_1^+)$ factors, which for collective states arise dominantly from proton motion, decrease. For $N \geqslant 108$, the decrease of the number of neutron holes as N increases causes an opposite effect. The constancy of empirical $g(2_1^+)$ factors in the well-deformed regions is then interpreted as a relative *rise* in the $g(2_1^+)$ factors compared to their expected continuing decrease to midshell. These ideas certainly need to be worked out more rigorously. Our intention here is to point out the phenomenological behavior itself as an encouragement to further theoretical study.

In Fig. 1(b) we note a similar, but inverted, behavior in the $B(E2)$ values. They increase rapidly as deformation sets in and collectivity increases; then they become asymptotically almost constant before decreasing somewhat after midshell. Note that the effect of growing correlations as deformation and collectivity set in has opposite effects on $g(2_1^+)$ factors and $B(E2)$ values, reducing the former and increasing the latter.

We now turn to a discussion of the asymptotic behavior near midshell. The constancy of $g(2_1^+)$ factors and $B(E2)$ values for $94 \leqslant N \leqslant 108$ means that adding neutrons to the nucleus does not affect these observables significantly; that is, the data exhibit a saturation effect. The asymptotic behavior of the $B(E2)$ values has been described [5] in terms of reduced effective *p-n* interactions near midshell because of reduced average overlaps of protons and neutrons in orbital planes at varying angles (K values) to one another, as briefly explained below. We exploit the same idea to account for the behavior of the $g(2_1^+)$ factors.

First we note that standard models do not predict such constancy in $g(2_1^+)$ factors. In the stably deformed region, geometric models predict a monotonically decreasing Z/A dependence, and bosonic models predict a parabolic dependence

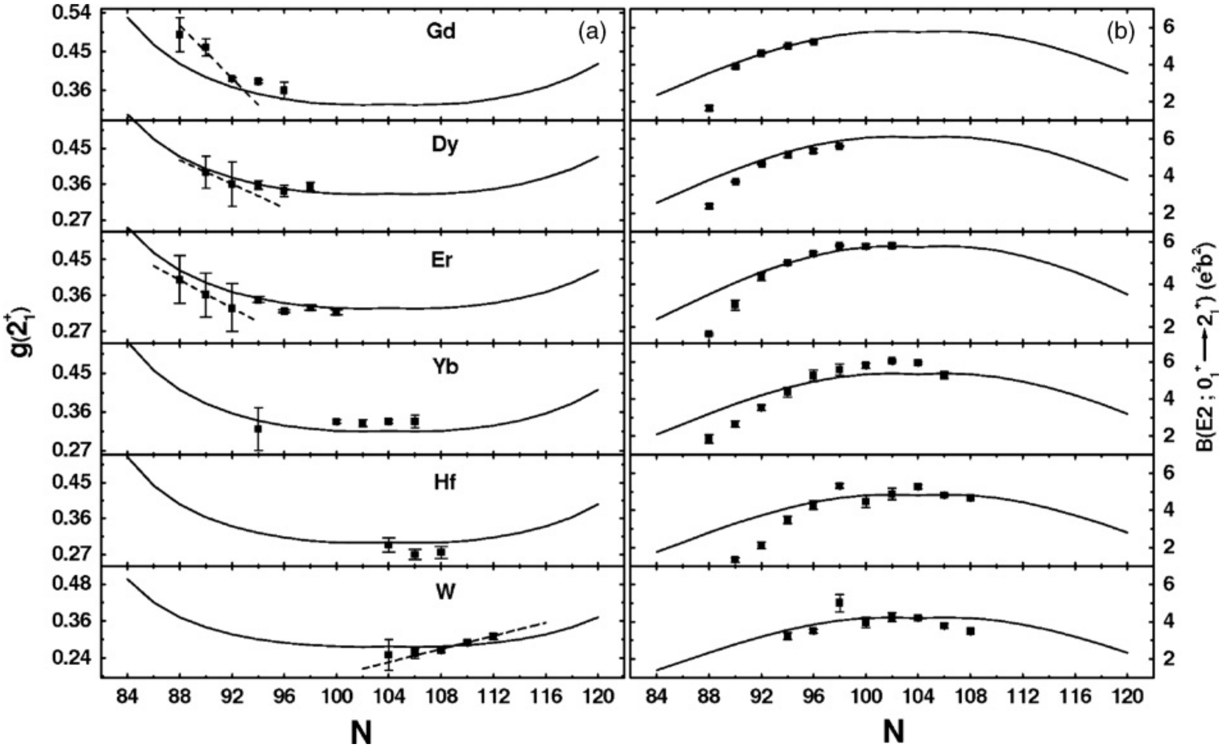

FIG. 1. Comparison between data and calculated (solid lines) of $g(2_1^+)$ factors and $B(E2;\, 0_1^+ \to 2_1^+)$ values in e^{2}b^2. (a) $g(2_1^+)$ factors and (b) $B(E2;\, 0_1^+ \to 2_1^+)$ values for nuclei from Gd through W with $N = 88$ through 112. The solid curves are calculated values with Eqs. (1) to (3) with the parameters $f = 0.05$, $g_\pi = 0.63$, $g_\nu = 0.05$, $e_\pi = 0.180$ eb, and $e_\nu = 0.153$ eb. The dashed lines in (a) are linear fits in the transitional region to guide the eye and show the change in the slope for the various isotopic chains at the beginning of the deformed region.

on $N_p\ /(N_p + N_n)$, centered on midshell, where N_p and N_n are the numbers of valence protons and neutrons. Both are in disagreement with the data. This is illustrated in Fig. 2(a) for the Yb isotopes where data span a rather large range of N. Neither predicted dependence matches the empirical results. Numerical IBA calculations of $B(E2)$ values, with parameters obtained from the detailed fits in Ref. [6], are also shown in Fig. 2 and confirm that this strong dependence is a general feature of the boson model. There has not been, to our knowledge, a satisfactory consistent explanation of the constancy of both the $g(2_1^+)$ factors and $B(E2)$ values.

We offer here an interpretation in terms of effective valence nucleon numbers (thus modifying the IBA predictions). Our ansatz is essentially the same as that which accounts for the saturation in $B(E2)$ values [as seen in Fig. 2(b)], namely a reduction in the effective strength of the valence *p*-*n* interaction across the midshell region. Here we stress that this strength reduction, which leads to a reduction in the growth of correlations, has the effect of lowering the $B(E2)$ values (relative to, say, the expected dependence on the square of the number of valence nucleons, or N_pN_n, or that given by the IBA), but it raises the $g(2_1^+)$ factors relative to the same expectations (that is, relative to the downward parabolic dependence predicted by the IBA assuming normal valence nucleon numbers as illustrated in Fig. 2). Therefore, a key element of this work is to show that the *same* effective valence nucleon numbers used for the $g(2_1^+)$ factors also account for the $B(E2;\, 0_1^+ \to 2_1^+)$ data.

The present interpretation is phenomenological. That is, it presents a rationale for the observed behavior. It is not microscopic but, hopefully, will encourage detailed microscopic calculations. Exposure of a microscopic linkage between the

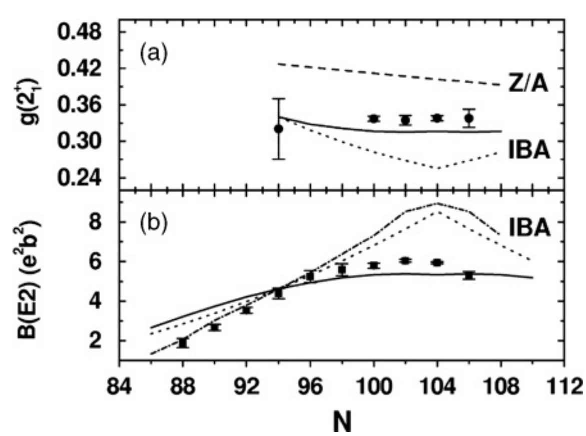

FIG. 2. (a) The $g(2_1^+)$ factors and (b) $B(E2)$ values for Yb isotopes. In addition to the data points, the dotted line is from the IBA model with normal boson numbers and the dashed line from the geometrical model, *Z*/*A*. The solid curve is from the present model with effective boson numbers. In both (a) and (b), the IBA calculated values have been normalized to the present estimated value at $N =$ 94 for comparison. The dot-dash line is from IBA calculations using the parameters of Ref. [6] with normal boson numbers

$g(2_1^+)$ factor and $B(E2)$ observables throughout the deformed nuclei region would be of considerable interest. It would complement other associations of $M1$ and $E2$ collectivity such as found in Ref. [7] and in mixed symmetry states [8].

It seems likely that residual interactions have a tendency to saturate near midshell when the number of valence nucleons maximizes. There could be several reasons for this effect. In the deformed field, orbits are labeled by a K quantum number defining the angle of the orbital plane relative to the symmetry axis. Nucleons in orbits with similar K values will overlap highly but those in rather different orbits will not. If there are only a few valence protons and neutrons, they will occupy similar orbits, with high overlap. However, near midshell, the valence nucleons occupy a variety of different orbits. Therefore, even if the last nucleons near midshell are in orbits of similar K (and even this is not guaranteed as inspection of any Nilsson diagram will show), their overlaps with *other* valence nucleons will not be uniformly large. Hence the *average* overlap should be less than near the beginning of a shell. This was the rationale behind the explanation in Ref. [5]. Another explanation is similar to that which leads to the well-known saturation in nuclear binding energies, namely any given nucleon tends to interact (bind) only with a few others.

Regardless of the microscopic rationale, an empirical reduction in average *p-n* interaction strengths near the middle of the rare earth region has, in fact, recently been discovered [4] through a systematic study of *p-n* interactions extracted using an interaction filter (a double difference of masses) that isolates the average *p-n* interaction of the last proton with the last neutron. In Fig. 3, we give a condensed overview of this effect. The figure shows that the average *p-n* interactions, called δV_{pn}, decrease from the ^{132}Sn region into the midshell nuclei (e.g., the Er-Hf nuclei with $N \sim 100$) where g factors are nearly constant and the $B(E2)$ values saturate. These empirical *p-n* interaction strengths have been connected [9] with growth rates of collectivity—faster growth rates of collectivity occur in regions of larger δV_{pn}. Hence, the midshell reduction in δV_{pn} is plausibly associated with the saturation phenomena that we discuss. Thus, the explanation we offer below has a sound empirical basis.

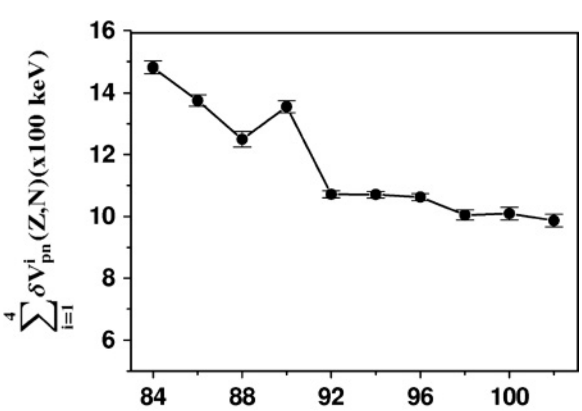

FIG. 3. Illustration of the reduction of δV_{pn} values near the midshell nuclei involved in this study. The ordinate gives the sum of empirical δV_{pn} values (taken from Refs. [4,9]) over four proton numbers for each neutron number. The sums are taken for the isotones $N = 84$ with $Z = 54, 56, 58, 60$, for the isotones $N = 86$ with $Z = 56$–62, and so on, incrementing the starting Z value for each successive N. This provides a guide to the average proton-neutron interaction values for the relevant nuclei. Note the small increase at $N = 90$ and the gradual fall-off across the figure.

To develop a specific model, we therefore assume that a saturation effect exists and we parametrize the effective valence nucleon number by the following expression:

$$N_\tau^{\text{eff}} = N_\tau(1 - N_\tau f) \qquad (\tau = \pi, \nu), \qquad (1)$$

where N_π and N_ν are half the numbers of valence protons and neutrons and N_π^{eff} and N_ν^{eff} are their effective values, respectively. f is a parameter to be fit to the data. Equation (1) applies to both protons and neutrons. Here we focus on the neutron number dependence where the data are plentiful. We expect a similar saturation effect against proton number. The data in Fig. 2 clearly suggests that the saturation effect increases with N, and therefore in Eq. (1) we assume a linear dependency of the correction on the neutron number.

We use Eq. (1) with the following IBA expressions [10,11] for the $g(2_1^+)$ factor and the $B(E2)$ values:

$$g(2_1^+) = \left(g_\pi N_\pi^{\text{eff}} + g_\nu N_\nu^{\text{eff}}\right)/\left(N_\pi^{\text{eff}} + N_\nu^{\text{eff}}\right) \qquad (2)$$

and

$$B(E2; 0_1^+ \to 2_1^+) = \left(\frac{2N^{\text{eff}} + 3}{N^{\text{eff}}}\right)\left(e_\pi N_\pi^{\text{eff}} + e_\nu N_\nu^{\text{eff}}\right)^2, \quad (3)$$

where $g_\pi = 0.63$ and $g_\nu = 0.05$ are proton and neutron g factors, taken from ref. [1], e_π and e_ν are boson effective charges, and $N^{\text{eff}} = N_\pi^{\text{eff}} + N_\nu^{\text{eff}}$. Note that the specific expression used for the $B(E2)$ value corresponds to the SU(3) limit. Most deformed nuclei deviate somewhat from this limit, with the low-lying states being admixtures of SU(3) basis states from several representations (see Ref. [12]). Nevertheless, the quadratic dependence on proton and neutron boson numbers is quite generally applicable for well-deformed nuclei, as seen in the dot-dash line in Fig. 2(b), which uses existing IBA parameters for the Yb nuclei and normal boson numbers.

To make the analysis as simple as possible we use the *same* f value for both protons and neutrons and for *all* nuclei studied. From a fit of all the $g(2_1^+)$ factor data from $N = 94$–108 with Eq. (2), using Eq. (1), we obtain $f = 0.05$. To calculate g factors and $B(E2;\, 0_1^+ \to 2_1^+)$ values, we use the values of g_π and g_ν mentioned above, choose constant values of the boson effective charges e_π and e_ν, namely $e_\pi = 0.180$ eb, $e_\nu = 0.153$ eb for all nuclei studied. Thus, the calculated results for both observables in about 30–40 nuclei are obtained by fitting only a single parameter in the context of an interpretation that has both microscopic and empirical support.

The results of the fits are shown as solid curves in Fig. 1 and Fig. 2 for $g(2_1^+)$ factors (left) and for $B(E2)$ values (right). We note that the calculated values for the g factors reflect the quality of fit to the data with the single parameter f. Clearly the agreement with the data is quite good, both as concerning the neutron number dependence and the Z dependence. There are no significant discrepancies except perhaps for the $B(E2)$

values in the light Hf isotopes. The fact that both $B(E2)$ values and $g(2_1^+)$ factors can be reproduced in such a simple scheme, in terms of an effective number of valence nucleons, suggests a common origin for the saturation in both observables.

To summarize, we have presented a simple one parameter model for the effective size of the proton and neutron valence spaces that allows one to account for the saturation phenomena observed in both $g(2_1^+)$ factor and $B(E2;\,0_1^+\,2\rightarrow 2_1^+)$ values in the rare-earth region. The ansatz in this phenomenological model is supported by empirical valence p-n interaction strengths. Finally, further tests of this model for nuclei in this mass region that have not yet been studied would be useful. Further $g(2_1^+)$ factor experiments are currently being planned to fill in the systematics.

We are grateful to Ho-chiang Ai and Volker Werner for useful discussions. Work supported by the US DOE under grant DE-FG02-91ER-40609 and DE-FG02-96ER-40983.

[1] A. Wolf and R. F. Casten, Phys. Rev. C **36**, 851 (1987).
[2] A. E. Stuchbery, S. S. Anderssen, A. P. Byrne, P. M. Davidson, G. D. Dracoulis, and G. J. Lane, Phys. Rev. Lett. **76**, 2246 (1996).
[3] Z. Berant *et al.*, Phys. Rev. C **69**, 034320 (2004); A. Wolf *et al.*, *ibid.* **72**, 027301 (2005).
[4] R. B. Cakirli, D. S. Brenner, R. F. Casten, and E. A. Millman, Phys. Rev. Lett. **94**, 092501 (2005).
[5] R. F. Casten, K. Heyde, and A. Wolf, Phys. Lett. **B208**, 33 (1988).
[6] E. A. McCutchan, N. V. Zamfir, and R. F. Casten, Phys. Rev. C **69**, 064306 (2004).
[7] C. Rangacharyulu, A. Richter, H. J. Wörtche, W. Ziegler, and R. F. Casten, Phys. Rev. C **43**, R949 (1991).
[8] N. Pietralla *et al.*, Phys. Rev. Lett. **83**, 1303 (1999).
[9] R. B. Cakirli and R. F. Casten, submitted to publication.
[10] M. Sambataro and A. E. L. Dieperink, Phys. Lett. **107B**, 249 (1981).
[11] F. Iachello and A. Arima, *The Interacting Boson Model* (Cambridge University Press, Cambridge, 1987).
[12] *Collective Bands in Nuclei: Progress in Particle and Nuclear Physics*, edited by D. Wilkinson (Pergamon, New York, 1983), Vol. 9.