General classification and analysis of neutron β-decay experiments

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A general analysis of the sensitivities of neutron β-decay experiments to manifestations of possible interaction beyond the standard model is carried out. In a consistent fashion, we take into account all known radiative and recoil corrections arising in the standard model. This provides a description of angular correlations in neutron decay in terms of one parameter, which is accurate to the level of $\sim 10^{-5}$. Based on this general expression, we present an analysis of the sensitivities to new physics for selected neutron decay experiments. We emphasize that the usual parametrization of experiments in terms of the tree-level coefficients a, A, and B is inadequate when the experimental sensitivities are at the same or higher level relative to the size of the corrections to the tree-level description.

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I. INTRODUCTION

The relative simplicity of the decay of the free neutron makes it an attractive laboratory for the study of possible extensions to the standard model. As is well known, measurements of the neutron lifetime and neutron decay correlations can be used to determine the weak vector coupling constant, which, in turn, can be combined with information on strange particle decay to test such notions as the universality of the weak interaction or to search for (or put a limit on) nonstandard couplings (see, for example, Refs. [1–8] and references therein). It is less widely appreciated that precision measurements of the correlations in neutron decay can, in principle, be used as a test of the standard model without appeal to measurements in other systems. In particular, the detailed shape of the decay spectra and the energy dependence of the decay correlation are sensitive to nonstandard couplings. The extraction of such information in a consistent fashion requires a rather delicate analysis, as the lowest order description of the correlation coefficients (and their energy dependencies) must be modified by a number of higher order corrections that are incorporated within the standard model. These include such effects as weak magnetism and radiative corrections. Recently [9] effective field theory has been used to incorporate all standard model effects in a consistent fashion in terms of one parameter with an estimated theoretical accuracy on the order of 10^{-5}. Because this accuracy is well below that anticipated in the next generation of neutron decay experiments (see, for example, articles in Ref. [10]), this analysis provides a useful framework for the exploration of the sensitivity of various experiments to new physics.

In this article, we extend the description of neutron β-decay of Ref. [9] by including the most general nonstandard β-decay interactions. Our framework provides a consistent description of the modifications of the β-decay observables at a level well below that anticipated in the next generation of experiments. Not surprisingly, we find that the different experimental observables have quite different sensitivities to the form of hypothetical nonstandard couplings (i.e., vector, scalar, etc.).

II. NEUTRON β DECAY BEYOND THE STANDARD MODEL

The most general description of neutron β decay can be done in terms of low-energy constants C_i by the Hamiltonian [11,12]

$$\begin{aligned} H_{\text{int}} = {} & (\hat{\psi}_p \psi_n)(C_S \hat{\psi}_e \psi_\nu + C'_S \hat{\psi}_e \gamma_5 \psi_\nu) \\ & + (\hat{\psi}_p \gamma_\mu \psi_n)(C_V \hat{\psi}_e \gamma_\mu \psi_\nu + C'_V \hat{\psi}_e \gamma_\mu \gamma_5 \psi_\nu) \\ & + \tfrac{1}{2}(\hat{\psi}_p \sigma_{\lambda\mu} \psi_n)(C_T \hat{\psi}_e \sigma_{\lambda\mu} \psi_\nu + C'_T \hat{\psi}_e \sigma_{\lambda\mu} \gamma_5 \psi_\nu) \\ & - (\hat{\psi}_p \gamma_\mu \gamma_5 \psi_n)(C_A \hat{\psi}_e \gamma_\mu \gamma_5 \psi_\nu + C'_A \hat{\psi}_e \gamma_\mu \psi_\nu) \\ & + (\hat{\psi}_p \gamma_5 \psi_n)(C_P \hat{\psi}_e \gamma_5 \psi_\nu + C'_P \hat{\psi}_e \psi_\nu) \\ & + \text{Hermitian conjugate}, \end{aligned} \quad (1)$$

where the index $i = V, A, S, T$, and P corresponds to vector, axial-vector, scalar, tensor, and pseudoscalar nucleon interactions. In this presentation, the constants C_i can be considered as effective constants of nucleon interactions with defined Lorentz structure, assuming that all high-energy degrees of freedom (for the standard model and any given extension of the standard model) are integrated out. In this article we consider only time-reversal conserving interactions, therefore

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the constants C_i can be chosen to be real. (The violation of time-reversal invariance in neutron decay at the level of considered accuracy would be a clear manifestation of new physics and thus does not require an analysis of the form contained here.) Ignoring electron and proton polarizations, the given effective Hamiltonian will result in the neutron β-decay rate [12] in the tree approximation (neglecting recoil corrections and radiative corrections) in terms of the angular correlations coefficients a, A, and B:

$$\frac{d\Gamma^3}{dE_e d\Omega_e d\Omega_\nu} = \Phi(E_e) G_F^2 |V_{ud}|^2 (1+3\lambda^2)\left(1 + b\frac{m_e}{E_e} + a\frac{\vec{p}_e\cdot\vec{p}_\nu}{E_e E_\nu} + A\frac{\vec{\sigma}\cdot\vec{p}_e}{E_e} + B\frac{\vec{\sigma}\cdot\vec{p}_\nu}{E_\nu}\right). \quad (2)$$

Here, $\vec{\sigma}$ is the neutron spin, m_e is the electron mass, E_e, E_ν, $\vec{p}_e$, and $\vec{p}_\nu$ are the energies and momenta of the electron and antineutrino, respectively, and G_F is the Fermi constant of the weak interaction (obtained from the μ-decay rate). The function $\Phi(E_e)$ includes normalization constants, phase-space factors, and standard Coulomb corrections. For the standard model the angular coefficients depend only on one parameter, $\lambda = -C_A/C_V > 0$, the ratio of axial-vector to vector nucleon coupling constant ($C_V = C'_V$ and $C_A = C'_A$):

$$a = \frac{1-\lambda^2}{1+3\lambda^2}, \quad A = -2\frac{\lambda^2-\lambda}{1+3\lambda^2}, \quad B = 2\frac{\lambda^2+\lambda}{1+3\lambda^2}. \quad (3)$$

(The parameter b is equal to zero for vectoraxial-vector weak interactions.)

As was shown in Ref. [1] the existence of additional interactions modifies the above expressions and can lead to a nonzero value for the coefficient b. To explicitly see the influence of a nonstandard interaction on the angular coefficients and on the decay rate of neutron one can rewrite the coupling constants C_i as a sum of a contribution from the standard model $C_i^{\rm SM}$ and a possible contribution from new physics δC_i:

$$\begin{aligned} C_V &= C_V^{\rm SM} + \delta C_V \\ C'_V &= C_V^{\rm SM} + \delta C'_V \\ C_A &= C_A^{\rm SM} + \delta C_A \\ C'_A &= C_A^{\rm SM} + \delta C'_A \\ C_S &= \delta C_S \\ C'_S &= \delta C'_S \\ C_T &= \delta C_T \\ C'_T &= \delta C'_T. \end{aligned} \quad (4)$$

We neglect the pseudoscalar coupling constants because we treat Ref. [12] nucleons nonrelativistically. Defining the term proportional to the total decay rate in Eq. (2) as $\xi = (1+3\lambda^2)$ one can obtain corrections to parameters ξ, a, b, A, and B because of new physics as $\delta\xi$, δa, δb, δA, and δB, correspondingly. Then, using results of Ref. [1],

$$\begin{aligned} \delta\xi &= C_V^{\rm SM}(\delta C_V + \delta C'_V) + \big(\delta C_V^2 + \delta C_V'^2 \\ &\quad + \delta C_S{}^2 + \delta C_S'^2\big)/2 + 3\big[C_A^{\rm SM}(\delta C_A + \delta C'_A) \\ &\quad + \big(\delta C_A^2 + \delta C_A'^2 + \delta C_T^2 + \delta C_T'^2\big)/2\big], \\ \xi\delta b &= \sqrt{1-\alpha^2}\big[C_V^{\rm SM}(\delta C_S + \delta C'_S) + \delta C_S \delta C_V + \delta C'_S \delta C'_V \\ &\quad + 3\big(C_A^{\rm SM}(\delta C_T + \delta C'_T) + \delta C_T \delta C_A + \delta C'_T \delta C'_A\big)\big], \\ \xi\delta a &= C_V^{\rm SM}(\delta C_V + \delta C'_V) + \big(\delta C_V^2 + \delta C_V'^2 \\ &\quad - \delta C_S^2 - \delta C_S'^2\big)/2 - C_A^{\rm SM}(\delta C_A + \delta C'_A) \\ &\quad - \big(\delta C_A^2 + \delta C_A'^2 - \delta C_T^2 - \delta C_T'^2\big)/2, \\ \xi\delta A &= -2C_A^{\rm SM}(\delta C_A + \delta C'_A) + \delta C'_A \delta C'_A - \delta C'_T \delta C'_T \\ &\quad - \big[C_V^{\rm SM}(\delta C_A + \delta C'_A) + C_A^{\rm SM}(\delta C_V + \delta C'_V) \\ &\quad + \delta C_V \delta C'_A + \delta C'_V \delta C_A - \delta C_S \delta C'_T - \delta C'_S \delta C_T\big], \\ \xi\delta B &= \frac{m\sqrt{1-\alpha^2}}{E_e}\big[2C_A^{\rm SM}(\delta C_T + \delta C'_T) + C_A^{\rm SM}(\delta C_S + \delta C'_S) \\ &\quad + C_V^{\rm SM}(\delta C_T + C'_T) + 2\delta C_T \delta C'_A + 2\delta C_A \delta C'_T \\ &\quad + \delta C_S \delta C'_A + \delta C_A \delta C'_S + \delta C_V \delta C'_T + \delta C_T \delta C'_V\big] \\ &\quad + 2C_A^{\rm SM}(\delta C_A + \delta C'_A) - C_V^{\rm SM}(\delta C_A + \delta C'_A) \\ &\quad - C_A^{\rm SM}(\delta C_V + \delta C'_V) - \delta C_S \delta C'_T - \delta C_T \delta C'_S \\ &\quad - \delta C_V \delta C'_A - \delta C_A \delta C'_V. \end{aligned} \quad (5)$$

It should be noted that we have neglected radiative corrections and recoil effects for the new physics contributions, because these are expected to be very small. However, Coulomb corrections for the new physics contributions are taken into account because they are important for a low-energy part of the electron spectrum.

From the above equations one can see that contributions from possible new physics to the neutron decay distribution function is rather complicated. To be able to separate new physics from different corrections Eq. (2), obtained in the tree level of approximation, one must describe the neutron decay process with accuracy that is better than the expected experimental accuracy. Assuming that the accuracy in future neutron decay experiments can reach a level of about 10^{-3}–10^{-4}, we wish to describe the neutron decay with theoretical accuracy by about 10^{-5} and our description must include all recoil and radiative corrections [13–21]. To do this we will use recent results of calculations [9] of radiative corrections for neutron decay in the effective field theory (EFT) with some necessary modifications. The results of Ref. [9] can be used because they take into account both recoil and radiative corrections in the same framework of the EFT with estimated theoretical accuracy that is better than 10^{-5}. However, the EFT approach does not provide all parameters but rather gives a parametrization in terms of a few (two, in the case of neutron decay) low-energy constants that must be extracted from independent experiments. Therefore, the neutron β-decay distribution function is parameterized in terms of one unknown parameter (the second parameter is effectively absorbed in the axial vector coupling constant). If this parameter would be extracted from an independent experiment, it gives a model-independent description of neutron β decay in the standard model with accuracy better than 10^{-5}. A rough estimate of this parameter based on a "natural" size of strong interaction contribution to radiative corrections gives an accuracy for the expressions for the rate and the angular correlation coefficients that is better than 10^{-3} (see Ref. [9]).

We vary the magnitude of this parameter in a wide range for the given numerical analysis and show that variations of the parameters in the allowed range do not significantly change our results at a level well below 10^{-3}. Also, unlike Ref. [9], we use the exact Fermi function for numerical calculations to take into account all corrections because of interactions with the classical electromagnetic field. This gives us the expression for neutron decay distribution function as

$$\frac{d\Gamma^3}{dE_e d\Omega_{\hat{p}_e} d\Omega_{\hat{p}_\nu}} = \frac{(G_F V_{ud})^2}{(2\pi)^5} |\vec{p}_e| E_e \left(E_e^{\max} - E_e\right)^2 F(Z, E_e) \times \left\{ f_0(E_e) + \frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} f_1(E_e) + \left[\left(\frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} \right)^2 - \frac{\beta^2}{3} \right] f_2(E_e) + \frac{\vec{\sigma} \cdot \vec{p}_e}{E_e} f_3(E_e) + \frac{\vec{\sigma} \cdot \vec{p}_e}{E_e} \frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} f_4(E_e) + \frac{\vec{\sigma} \cdot \vec{p}_\nu}{E_\nu} f_5(E_e) + \frac{\vec{\sigma} \cdot \vec{p}_\nu}{E_\nu} \frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} f_6(E_e) \right\}, \tag{6}$$

where the energy-dependent angular correlation coefficients are

$$f_0(E_e) = (1 + 3\lambda^2) \left(1 + \frac{\alpha}{2\pi} \delta_\alpha^{(1)} + \frac{\alpha}{2\pi}\, e_V^R \right) - \frac{2}{m_N} \left[\lambda(\mu_V + \lambda) \frac{m_e^2}{E_e} + \lambda(\mu_V + \lambda) E_e^{\max} - (1 + 2\lambda\mu_V + 5\lambda^2) E_e \right], \tag{7}$$

$$f_1(E_e) = (1 - \lambda^2) \left[1 + \frac{\alpha}{2\pi} \left(\delta_\alpha^{(1)} + \delta_\alpha^{(2)} \right) + \frac{\alpha}{2\pi}\, e_V^R \right] + \frac{1}{m_N} \left[2\lambda(\mu_V + \lambda) E_e^{\max} - 4\lambda(\mu_V + 3\lambda) E_e \right], \tag{8}$$

$$f_2(E_e) = -\frac{3}{m_N} (1 - \lambda^2) E_e, \tag{9}$$

$$f_3(E_e) = (-2\lambda^2 + 2\lambda) \left[1 + \frac{\alpha}{2\pi} \left(\delta_\alpha^{(1)} + \delta_\alpha^{(2)} \right) + \frac{\alpha}{2\pi}\, e_V^R \right] + \frac{1}{m_N} \left[(\mu_V + \lambda)(\lambda - 1) E_e^{\max} + (-3\lambda\mu_V + \mu_V - 5\lambda^2 + 7\lambda) E_e \right], \tag{10}$$

$$f_4(E_e) = \frac{1}{m_N} (\mu_V + 5\lambda)(\lambda - 1) E_e, \tag{11}$$

$$f_5(E_e) = (2\lambda^2 + 2\lambda) \left(1 + \frac{\alpha}{2\pi} \delta_\alpha^{(1)} + \frac{\alpha}{2\pi}\, e_V^R \right) + \frac{1}{m_N} \left[-(\mu_V + \lambda)(\lambda + 1) \frac{m_e^2}{E_e} - 2\lambda(\mu_V + \lambda) E_e^{\max} + (3\mu_V\lambda + \mu_V + 7\lambda^2 + 5\lambda) E_e \right], \tag{12}$$

$$f_6(E_e) = \frac{1}{m_N} \left[(\mu_V + \lambda)(\lambda + 1) E_e^{\max} - (\mu_V + 7\lambda)(\lambda + 1) E_e \right]. \tag{13}$$

Here e_V^R is the finite renormalized low-energy constant (LEC) corresponding to the "inner" radiative corrections because of the strong interactions in the standard QCD approach; $F(Z, E_e)$ is the standard Fermi function; and the functions $\delta_\alpha^{(1)}$ and $\delta_\alpha^{(2)}$ are

$$\delta_\alpha^{(1)} = \frac{1}{2} + \frac{1+\beta^2}{\beta} \ln\left(\frac{1+\beta}{1-\beta}\right) - \frac{1}{\beta} \ln^2\left(\frac{1+\beta}{1-\beta}\right) + \frac{4}{\beta} L\left(\frac{2\beta}{1+\beta}\right) + 4\left[\frac{1}{2\beta} \ln\left(\frac{1+\beta}{1-\beta}\right) - 1\right] \times \left\{ \ln\left[\frac{2\left(E_e^{\max} - E_e\right)}{m_e}\right] + \frac{1}{3}\left(\frac{E_e^{\max} - E_e}{E_e}\right) - \frac{3}{2} \right\} + \left(\frac{E_e^{\max} - E_e}{E_e}\right)^2 \frac{1}{12\beta} \ln\left(\frac{1+\beta}{1-\beta}\right). \tag{14}$$

$$\delta_\alpha^{(2)} = \frac{1-\beta^2}{\beta} \ln\left(\frac{1+\beta}{1-\beta}\right) + \left(\frac{E_e^{\max} - E_e}{E_e}\right) \frac{4(1-\beta^2)}{3\beta^2} \times \left[\frac{1}{2\beta} \ln\left(\frac{1+\beta}{1-\beta}\right) - 1\right] + \left(\frac{E_e^{\max} - E_e}{E_e}\right)^2 \frac{1}{6\beta^2} \times \left[\frac{1-\beta^2}{2\beta} \ln\left(\frac{1+\beta}{1-\beta}\right) - 1\right], \tag{15}$$

where $\beta = p_e/E_e$. The only unknown parameter e_V^R is chosen to satisfy the estimate [20] for an "inner" part of the radiative corrections: $\alpha/2\pi\, e_V^R = 0.02$. In Eq. (6) the custom of expanding the nucleon recoil correction of the three-body phase space has been used. These recoil corrections are included in the coefficients $f_i, i = 0, 1, \ldots, 6$ defined in the partial decay-rate expression, Eq. (6). It should be noted that the expression for f_2 is an exclusive three-body phase-space recoil correction, whereas all other $f_i, i = 0, 1, 3, \ldots, 6$ contain a mixture of regular recoil and phase space $(1/m_N)$ corrections.

The above expression presents all contributions from the standard model. Therefore, the difference between this theoretical description and an experimental result can only be because of effects not accounted for by the standard model. From Eqs. (5) we can see that the only contributions from new physics in neutron decay are

$$\begin{aligned} f_0(E_e) &\longrightarrow f_0(E_e) + \delta\xi + \frac{m}{E_e}\delta b, \\ f_1(E_e) &\longrightarrow f_1(E_e) + \delta a, \\ f_3(E_e) &\longrightarrow f_3(E_e) + \delta A, \\ f_5(E_e) &\longrightarrow f_5(E_e) + \delta B, \end{aligned} \tag{16}$$

Because possible contributions from models beyond the standard one are rather complicated, we have to use numerical analysis for calculations of experimental sensitivities to new physics.

III. THE ANALYSIS OF THE EXPERIMENTAL SENSITIVITY TO NEW PHYSICS

To calculate the sensitivity of an experiment with a total number of events N to the parameter q we use the standard technique of the minimum variance bound estimator (see, for example, Refs. [22,23]). The estimated uncertainties provided by this method correspond to one-sigma limits for a normal distribution. The statistical error (variance) σ_q of parameter q

TABLE I. Relative statistical error (K) of the standard experiments to different types of interactions from new physics (C_i constants) provided that these constants have the same values of 1×10^{-3}.

Interactions	a	A	B
V	5.26	3.60	6.95
A	1.73	1.90	1.91
T	2.59	7.25	1.50
S	8.70	26.70	1.46
$V+A$	2.01	1.58	3.86

in the given experiment can be written as

$$\sigma_q = \frac{K}{\sqrt{N}}, \tag{17}$$

where

$$K^{-2} = \frac{\int w(\vec{x})\left[\frac{1}{w(\vec{x})}\frac{\partial w(\vec{x})}{\partial q}\right]^2 d\vec{x}}{\int w(\vec{x}) d\vec{x}}. \tag{18}$$

Here $w(\vec{x})$ is a distribution function of measurable parameters $\vec{x}$. We can calculate the sensitivity of the experiment to a particular coefficient C_i or to a function of these coefficients. The results for these integrated sensitivities for each type of interaction (C_i) and for the left-right model are given in Table I for the standard experiments measuring a, A, and B coefficients in neutron decay, assuming that all coefficients C_i have the same value of 1×10^{-3}. The numerical test shows that results for the coefficients K can be linearly rescaled for the parameters C_i in the range from 10^{-2} to 10^{-4} with an accuracy of better than 10%. We can see that different experiments have different sensitivities (discovery potentials) for the possible manifestations of new physics.

The given description of neutron β-decay experiments in terms of low-energy constants related to the Lorentz structure of weak interactions is general and complete. All models beyond the standard one (new physics) contribute to the C_i values in different ways. Therefore, each model can be described by a function of the C_i parameters. To relate these C coefficients explicitly to the possible models beyond the standard one we can use the parametrization of Ref. [7]. It should be noted that the definitions of Ref. [12] used for the C_i coefficients are the same as in Ref. [7], except for the opposite sign of C'_V, C'_S, C'_T, and C_A. Therefore, we can rewrite the relations of the δC_i, which contain contributions to the C_i from new physics, in terms of the parameters $\bar{a}_{jl}$ and $\bar{A}_{jl}$ defined in article [7] as

$$\begin{aligned}
\delta C_V &= C_V^{\rm SM}(\bar{a}_{\rm LL} + \bar{a}_{\rm LR} + \bar{a}_{\rm RL} + \bar{a}_{\rm RR}),\\
\delta C'_V &= -C_V^{\rm SM}(-\bar{a}_{\rm LL} - \bar{a}_{\rm LR} + \bar{a}_{\rm RL} + \bar{a}_{\rm RR}),\\
\delta C_A &= -C_A^{\rm SM}(\bar{a}_{\rm LL} - \bar{a}_{\rm LR} - \bar{a}_{\rm RL} + \bar{a}_{\rm RR}),\\
\delta C'_A &= C_A^{\rm SM}(-\bar{a}_{\rm LL} + \bar{a}_{\rm LR} - \bar{a}_{\rm RL} + \bar{a}_{\rm RR})\\
\delta C_S &= g_S(\bar{A}_{\rm LL} + \bar{A}_{\rm LR} + \bar{A}_{\rm RL} + \bar{A}_{\rm RR}),\\
\delta C'_S &= -g_S(-\bar{A}_{\rm LL} - \bar{A}_{\rm LR} + \bar{A}_{\rm RL} + \bar{A}_{\rm RR}),\\
\delta C_T &= 2g_T(\bar{\alpha}_{\rm LL} + \bar{\alpha}_{\rm RR}),\\
\delta C'_T &= -2g_T(-\bar{\alpha}_{\rm LL} + \bar{\alpha}_{\rm RR}).
\end{aligned} \tag{19}$$

TABLE II. Relative statistical error (K) of the standard experiments to different types of interactions from new physics ($\bar{a}_{ij}$ constants) provided that these constants have the same values of 1×10^{-3}.

	$\bar{a}_{\rm LL}$	$\bar{a}_{\rm LR}$	$\bar{a}_{\rm RL}$	$\bar{a}_{\rm RR}$	$\bar{A}_{\rm LL}$	$\bar{A}_{\rm LR}$	$\bar{A}_{\rm RL}$	$\bar{A}_{\rm RR}$	$\bar{\alpha}_{\rm LL}$	$\bar{\alpha}_{\rm RR}$
a	0.17	0.25	135	487	1.43	1.43	283	283	0.19	79
A	1.53	0.63	423	1026	13.1	13.1	860	860	1.82	223
B	0.58	1.21	89	347	0.72	0.72	958	958	0.37	59

The parameters $\bar{a}_{jl}$, $\bar{\alpha}_{jl}$, and $\bar{A}_{jl}$ describe contributions to the low-energy Hamiltonian from current-current interactions in terms of j-type of leptonic current and i-type of quark current. For example, $\bar{a}_{\rm LR}$ is the contribution to the Hamiltonian from left-handed leptonic current and right-handed quark current normalized by the size of the standard model (left–left current) interactions. g_S and g_T are formfactors at zero-momentum transfer in the nucleon matrix element of scalar and tensor currents. For more details, see the article [7]. It should be noted, that $\delta C_i + \delta C'_i$ involve left-handed neutrinos and $\delta C_i - \delta C'_i$ is related to right-handed neutrino contributions in corresponding lepton currents. The analysis of the three experiments under consideration (a, A, and B coefficient measurements) in terms of sensitivities (K^{-1}) to $\bar{a}_{jl}$, $\bar{\alpha}_{jl}$, and $\bar{A}_{jl}$ parameters is presented in Table II. For the sake of easy comparison the sensitivities in this table are calculated under assumptions that all parameters ($\bar{a}_{jl}$, $\bar{\alpha}_{jl}$, and $\bar{A}_{jl}$) have exactly the same value, 1×10^{-3}. The expected values of these parameters vary over a wide range from 0.07 to 10^{-6} (see Table III and Ref. [7] for the comprehensive analysis). The numerical results for the coefficients K in the table can be linearly rescaled for the parameters $\bar{a}_{ij}$, $\bar{\alpha}_{jl}$, and $\bar{A}_{ij}$ in the range from 10^{-2} to 10^{-4} with an accuracy better than 10%. The relative statistical errors presented in the table demonstrate discovery potentials of different experiments to new physics in terms of parameters $\bar{a}_{ij}$, $\bar{\alpha}_{jl}$, and $\bar{A}_{ij}$. It should be noted that the parameter $\bar{a}_{\rm LR}$ cannot provide sensitive information on new physics at the quark level, unless we obtain the axial-vector coupling constant g_A from another experiment, because in correlations $\bar{a}_{\rm LR}$ appears in a product with g_A (see Ref. [7]). For discussion of significance of each of these parameters to models beyond the standard one see Ref. [7].

It should be noted the results in the Tables I and II are calculated with the estimated value of the parameter

TABLE III. Possible manifestations of new physics.

Model	L-R	Exotic Fermion	Lepto-quark	Contact inter-actions	SUSY	Higgs
$\bar{a}_{\rm RL}$	0.067	0.042				
$\bar{a}_{\rm RR}$	0.075		0.01			
$\bar{A}_{\rm LL} + \bar{A}_{\rm LR}$				0.01	7.5×10^{-4}	3×10^{-6}
$\bar{A}_{\rm RR} + \bar{A}_{\rm RL}$				0.1		
$-\bar{A}_{\rm LL} + \bar{A}_{\rm LR}$			3×10^{-6}			
$\bar{A}_{\rm RR} - \bar{A}_{\rm RL}$			4×10^{-4}			

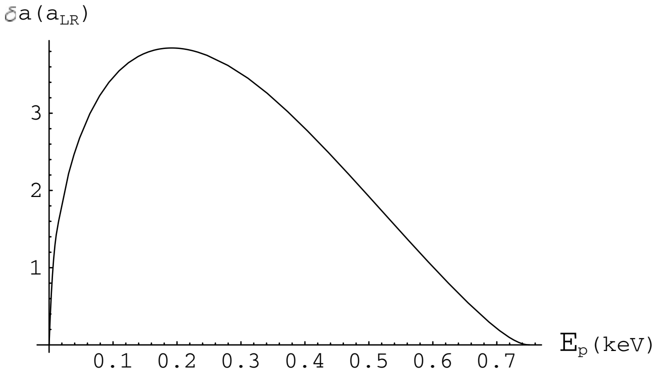

FIG. 1. Manifestation of a_{LR}-type interactions on the a coefficient.

$\alpha/(2\pi)\, e_V^R = 0.02$. Numerical tests show that a change of this parameter by a factor of 2 leads to changes of results in the tables by about 1%.

The calculated integral sensitivities of different experiments to a particular parameter related to new physics can be used for the estimation of the experimental sensitivity when the experimental statistics is not good enough. For the optimization of experiments it is useful to know how manifestations of new physics contribute to the energy spectrum of the measurable parameter. As an example, the contributions from $\bar{a}_{\mathrm{LR}}$, $\bar{a}_{\mathrm{RL}}$, and $\bar{a}_{\mathrm{RR}}$ to the spectra for the a, A, and B correlations are shown in Figs. 1–8. For uniform presentation all graphs in the figures are normalized by $N_f = G_F^2 |V_{ud}|^2 \int f(E) dE$, where $f(E)$ is $a(E_p)$, $A(E_e)$, and $A(E_e)$, correspondingly. One can see that these contributions have different shapes and positions of maxima both for different model parameters and for different angular correlations. This gives the opportunity for fine tuning in the search for particular models beyond the standard one in neutron decays.

Using the approach developed here one can calculate the exact spectrum for a given model. For example, manifestations of the left-right model ($\bar{a}_{\mathrm{RL}} = 0.067$ and $\bar{a}_{\mathrm{RR}} = 0.075$) in the measurements of the A and B coefficients are shown in solid lines in Figs. 9 and 10. The dashed lines show contributions from recoil effects and radiative corrections (without Coulomb corrections) assuming that $\alpha/(2\pi)\, e_V^R = 0.02$. From these plots one can see the importance of the corrections at the level of the possible manifestations of new physics. Figure 11

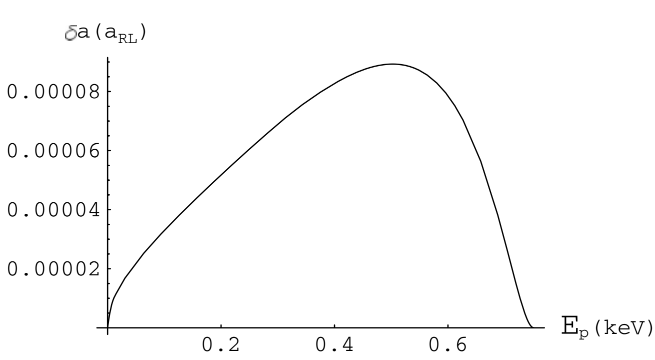

FIG. 2. Manifestation of a_{RL}-type interactions on the a coefficient.

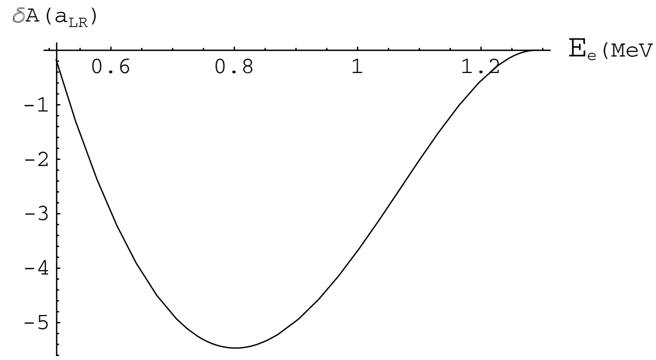

FIG. 3. Manifestation of a_{LR}-type interactions on the A coefficient.

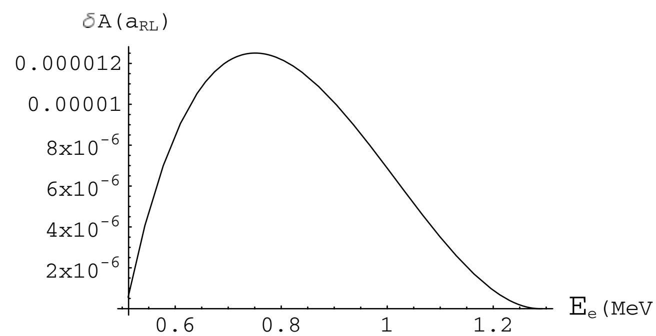

FIG. 4. Manifestation of a_{RL}-type interactions on the A coefficient.

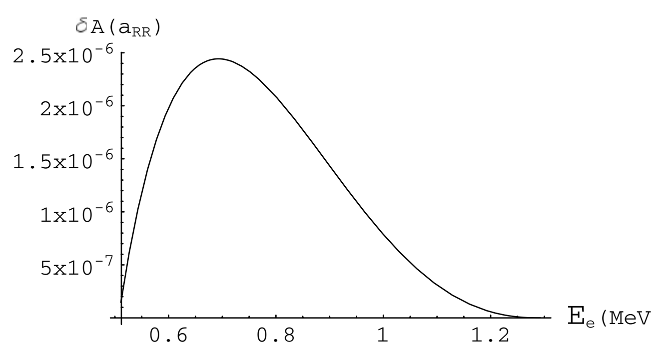

FIG. 5. Manifestation of a_{RR}-type interactions on the A coefficient.

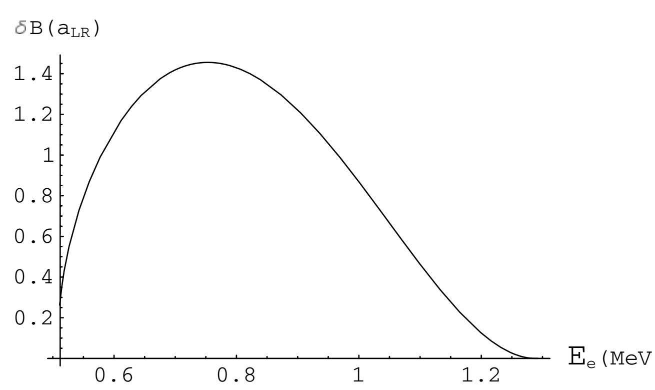

FIG. 6. Manifestation of a_{LR}-type interactions on the B coefficient.

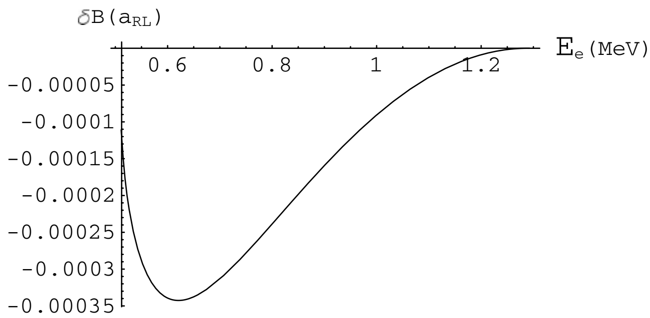

FIG. 7. Manifestation of a_{RL}-type interactions on the B coefficient.

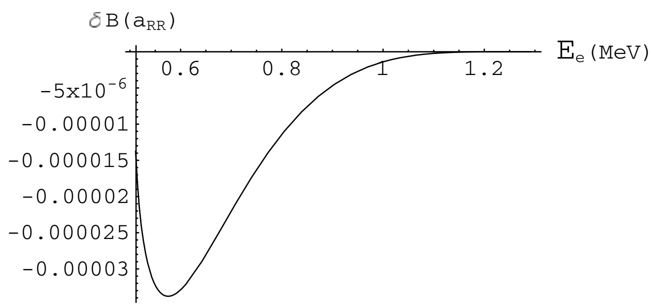

FIG. 8. Manifestation of a_{RR}-type interactions on the B coefficient.

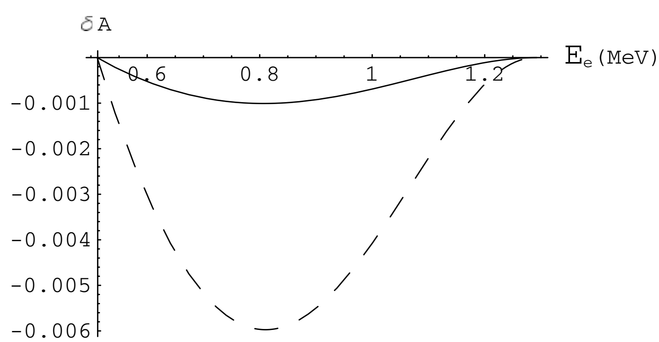

FIG. 9. Contributions from radiative and recoil corrections (dashed line) and from the left-right model (solid line) to the A coefficient. The curves are explained in the text.

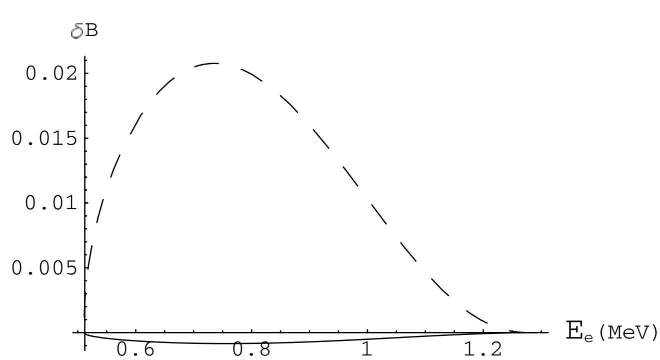

FIG. 10. Contributions from radiative and recoil corrections (dashed line) and from the left-right model (solid line) to the B coefficient. The curves are explained in the text.

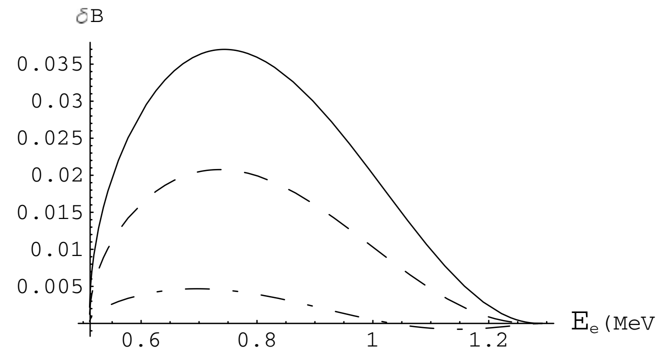

FIG. 11. Contributions from radiative and recoil corrections to the B coefficient for $\alpha/(2\pi)\, e_V^R = 0.01$ (dashed-dotted line), $\alpha/(2\pi)\, e_V^R = 0.02$ (dashed line), and $\alpha/(2\pi)\, e_V^R = 0.03$ (solid line).

shows how these corrections for the coefficient B affected by the value of the parameter $\alpha/(2\pi)\, e_V^R$ related to nuclear structure: dashed-doted, dashed, and solid lines correspond to 0.01, 0.02 and 0.03 values for the parameter.

We presented here results of analysis for only a number of parameters $\bar{a}_{ij}$ to illustrate a different level of sensitivities of experiments to the parameters. For the complete analysis of future experiments all $\bar{a}_{ij}$, $\bar{\alpha}_{ij}$, and $\bar{A}_{ij}$ parameters should be analyzed with specific experimental conditions taken into account.

IV. CONCLUSIONS

The analysis presented here provides a general basis for comparison of different experiments of neutron β decay from the point of view of the discovery potential for new physics. It is also demonstrates that various parameters measured in experiments have quite different sensitivities to the detailed nature of the (supposed) new physics and can, in principle, be used to differentiate between different extensions to the standard model. Thus neutron decay can be considered as a promising tool to search for new physics, which may not only detect the manifestations of new physics but also define the source of the possible deviations from predictions of the standard model. Our results can be used for optimization of new high-precision experiments to define important directions and to complement high-energy experiments. Finally we emphasize that the usual parametrization of experiments in terms of the tree-level coefficients a, A, and B is inadequate when experimental sensitivities are comparable or better to the size of the corrections to the tree-level description. This is expected in the next generation of neutron decay experiments. Therefore, such analysis is needed for these experiments. One has to use the full expression for neutron β-decay in terms of the coupling constants. In other words, the high-precision experiments should focus on the parameters important for physics rather than on the coefficients a, A, and B that are sufficient only for low-accuracy measurements.

ACKNOWLEDGMENTS

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