Determination of $\pi^{\pm}$ meson polarizabilities from the $\gamma\gamma \to \pi^+\pi^-$ process

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A fit of the experimental data to the total cross section of the process $\gamma\gamma \to \pi^+\pi^-$ in the energy region from threshold to 2500 MeV has been carried out using dispersion relations with subtractions for the invariant amplitudes, where the dipole and the quadrupole polarizabilities of the charged pion are free parameters. As a result, the sum and the difference of the electric and magnetic dipole and quadrupole polarizabilities of the charged pion have been found: $(\alpha_1 + \beta_1)_{\pi^{\pm}} = (0.18^{+0.11}_{-0.02}) \times 10^{-4}\,\mathrm{fm}^3$, $(\alpha_1 - \beta_1)_{\pi^{\pm}} = (13.0^{+2.6}_{-1.9}) \times 10^{-4}\,\mathrm{fm}^3$, $(\alpha_2 + \beta_2)_{\pi^{\pm}} = (0.133 \pm 0.015) \times 10^{-4}\,\mathrm{fm}^5$, $(\alpha_2 - \beta_2)_{\pi^{\pm}} = (25.0^{+0.8}_{-0.3}) \times 10^{-4}\,\mathrm{fm}^5$. These values agree with the dispersion sum rule predictions. The value found for the difference of the dipole polarizabilities is consistent with the results obtained from scattering of high energy π^- mesons off the Coulomb field of heavy nuclei [Yu. M. Antipov *et al.*, Phys. Lett. **B121**, 445 (1983)] and from radiative π^+ photoproduction from the proton at MAMI [J. Ahrens *et al.*, Eur. Phys. J. A **23**, 113 (2005)], whereas it is at variance with the recent calculations in the framework of chiral perturbation theory.

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I. INTRODUCTION

Pion polarizabilities are fundamental structure parameters characterizing the behavior of the pion in an external electromagnetic field. The dipole and quadrupole polarizabilities arise as $\mathcal{O}(\omega^2)$ and $\mathcal{O}(\omega^4)$ terms, respectively, in the expansion of the non-Born amplitude of Compton scattering in powers of the initial photon energy ω. In terms of the electric α_l $(l = 1, 2)$ and magnetic β_l dipole and quadrupole polarizabilities, the corresponding effective interactions of $\mathcal{O}(\omega^2)$ and $\mathcal{O}(\omega^4)$ have the forms [1,2]

$$H^{(2)}_{\mathrm{eff}} = -\tfrac{1}{2}\, 4\pi \,(\alpha_1 \vec{E}^2 + \beta_1 \vec{H}^2), \tag{1}$$

$$H^{(4)}_{\mathrm{eff}} = -\tfrac{1}{12} 4\pi \left(\alpha_2 E^2_{ij} + \beta_2 H^2_{ij}\right), \tag{2}$$

where

$$E_{ij} = \tfrac{1}{2}(\nabla_i E_j + \nabla_j E_i), \quad H_{ij} = \tfrac{1}{2}(\nabla_i H_j + \nabla_j H_i) \tag{3}$$

are the quadrupole strengths of the electric and magnetic fields.

The dipole polarizabilities (α_1 and β_1) of the pion measure the response of the pion to quasistatic electric and magnetic fields. In contrast, the parameters α_2 and β_2 measure the electric and magnetic quadrupole moments induced in the pion in the presence of an applied field gradient. In what follows, the dipole and quadrupole polarizabilities are given in units $10^{-4}\,\mathrm{fm}^3$ and $10^{-4}\,\mathrm{fm}^5$, respectively.

The values of the pion polarizabilities are very sensitive to predictions of different theoretical models. Therefore, an accurate experimental determination of these parameters is very important for testing the validity of such models. In particular, the determination of the difference of the electric and magnetic dipole polarizabilities is of major importance. The value of this difference found from the radiative π^+ meson photoproduction from the proton at MAMI [3] is equal to $11.6 \pm 1.5_{\mathrm{stat}} \pm 3.0_{\mathrm{syst}} \pm 0.5_{\mathrm{mod}}$ and is close to the value obtained from scattering of high-energy π^- mesons off the Coulomb field of heavy nuclei in Serpukhov [4] assuming $\alpha_{1\pi^-} = -\beta_{1\pi^-}$ and equal to $13.6 \pm 2.8 \pm 2.4$. However, these values differ from the prediction of chiral perturbation theory (ChPT) (4.4 ± 1.0 [5]). The experiment of the Lebedev Physical Institute on radiative pion photoproduction from the proton [6] has given $\alpha_{1\pi^+} = 20 \pm 12$. This value has large error bars but nevertheless shows a large discrepancy with regard to the ChPT predictions as well.

Attempts to determine the charged pion dipole polarizabilities from the reaction $\gamma\gamma \to \pi^+\pi^-$ suffered greatly from theoretical and experimental uncertainties. The analyses [7–9] have been performed in the region of low energy ($\sqrt{t} <$ 500 MeV, where t is the square of the total energy in the $\gamma\gamma$ c.m. system). In this region the values of the experimental cross sections of the process under consideration [10–13] are very ambiguous. As a result, the values of $\alpha_{1\pi^{\pm}}$ found lie in the interval 2.2–26.3. The analyses of the data of Mark II [13] have given $\alpha_{1\pi^{\pm}}$ close to the ChPT result. However, even changes of the dipole polarizabilities by more than 100% are compatible with the present error bars in the energy region considered [8].

The experimental information available so far for the dipole polarizabilities of the charged pions is summarized in Table I.

We have determined the charged pion polarizabilities from the analyses of $\gamma\gamma \to \pi^+\pi^-$ reaction data in the energy region from threshold to 2500 MeV.

An investigation of this process at middle energies was also carried out in the framework of different theoretical models [14–17]. However, these authors did not try to extract information about pion polarizabilities.

In Ref. [18] we fitted the data [19] to the total cross section of the process $\gamma\gamma \to \pi^0\pi^0$, using dispersion relations (DRs) at fixed t with one subtraction for the invariant helicity amplitudes in the energy region from 270 to 2000 MeV. This analysis has allowed for the determination of the π^0 dipole polarizabilities. In this work the σ meson was considered as an effective description of the strong S-wave $\pi\pi$ interaction using the broad

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TABLE I. The experimental data presently available for the charged pion dipole polarizabilities.

Experiment	$\alpha_{1\pi^\pm}/10^{-4}$ fm^3	$(\alpha_1-\beta_1)_{\pi^\pm}/10^{-4}$ fm^3
$\pi^- Z \to \gamma\pi^- Z$, Serpukhov (1983) [4]	$6.8 \pm 1.4 \pm 1.2$	
$\gamma p \to \gamma\pi^+ n$, Lebedev Phys. Inst. (1984) [6]	20 ± 12	
$\gamma p \to \gamma\pi^+ n$, MAMI (2005) [3]		$11.6 \pm 1.5_{\rm stat} \pm 3.0_{\rm syst} \pm 0.5_{\rm mod}$
D. Babusci *et al.* (1992) [7]		
$\gamma\gamma \to \pi^+\pi^-$: PLUTO (1984) [10]	$19.1 \pm 4.8 \pm 5.7$	
DM 1 (1986) [11]	17.2 ± 4.6	
DM 2 (1986) [12]	26.3 ± 7.4	
Mark II (1990) [13]	2.2 ± 1.6	
F. Donoghue, B. Holstein (1993) [8]		
$\gamma\gamma \to \pi^+\pi^-$: Mark II	2.7	
A. Kaloshin, V. Serebryakov (1994) [9]		
$\gamma\gamma \to \pi^+\pi^-$: Mark II		5.25 ± 0.95

Breit-Wigner resonance expression. The parameters of such a σ meson have been found from the fit to the experimental data [19] in the energy region 270–825 MeV. As a result, a good description of the experimental data was obtained for $\sqrt{t} = 270$–1700 MeV. However, this work predicted a strong rise in the total cross section at higher energies, in contradiction with the experimental data.

In Ref. [20] we showed that this discrepancy could be eliminated in the energy region at least up to 2.25 GeV by regarding the quadrupole π^0 meson polarizabilities as free parameters. With this aim, we have constructed DRs at fixed t with one subtraction, where the subtraction functions were determined with the help of the DRs with two subtractions. The subtraction constants were connected with the dipole and quadrupole polarizabilities. The analysis of the experimental data [19,21] for the total cross section of the process $\gamma\gamma \to \pi^0\pi^0$ was performed in the energy region 270–2250 MeV. This analysis has resulted in the determination of the quadrupole π^0 meson polarizabilities for the first time. In addition, the values of the effective interaction radius of the $f_2(1270)$ meson and its decay width into two photons have been determined: $r_f = 0.96 \pm 0.01$ fm and $\Gamma_{f_2\to\gamma\gamma} = 3.05 \pm 0.11$ keV.

In the present work we construct the DRs similar to those of Ref. [20] for the amplitudes of the process $\gamma\gamma \to \pi^+\pi^-$. Using the DRs allows one to avoid the problem of double counting and the subtractions in the DRs provide a good convergence of the integrand expressions in these DRs and therefore increases the reliability of the calculations. These DRs are used to fit the experimental data [13,22–26] in the energy region from threshold to 2500 MeV. As a result, we have found the dipole polarizabilities of the charged pions and determined their quadrupole polarizabilities for the first time. The value of the difference $(\alpha_1 - \beta_1)_{\pi^\pm} = 13.0^{+2.6}_{-1.9}$ found is in good agreement with the result of Refs. [3,4] and with the predictions of the dispersion sum rules (DSRs) [20]. However, this result is at variance with the recent calculations in the framework of ChPT [5].

The paper is organized as follows. In Sec. II the DRs for the invariant helicity amplitudes of the process $\gamma\gamma \to \pi^+\pi^-$ are constructed. The determination of the charge pion polarizabilities from the experimental data on the reaction $\gamma\gamma \to \pi^+\pi^-$ is given in Sec. III. The discussion of the results obtained is presented in Sec. IV. Conclusions are presented in Sec. V.

II. DISPERSION RELATIONS FOR THE AMPLITUDES OF THE PROCESS $\gamma\gamma \to \pi^+\pi^-$

The process $\gamma\gamma \to \pi^+\pi^-$ is described by the following invariant variables

$$t = (k_1 + k_2), \quad s = (p_1 - k_1)^2, \quad u = (p_1 - k_2)^2, \quad (4)$$

where $p_1(p_2)$ and $k_1(k_2)$ are the pion and photon four-momenta.

We will consider the helicity amplitudes M_{++} and M_{+-} [27]. These amplitudes have no kinematical singularities or zeros and define the cross section of the process $\gamma\gamma \to \pi^+\pi^-$ as follows:

$$\frac{d\sigma_{\gamma\gamma\to\pi^+\pi^-}}{d\Omega} = \frac{1}{128\pi^2}\sqrt{\frac{(t-4\mu^2)}{t^3}}\left[t^2|M_{++}|^2 + \frac{1}{16}t^2(t-4\mu^2)^2\sin^4\theta^*|M_{+-}|^2\right], \quad (5)$$

where θ^* is the angle between the photon and the pion in the c.m. system of the process $\gamma\gamma \to \pi^+\pi^-$ and μ is the $\pi^\pm$ meson mass.

By constructing the DRs at fixed t with one subtraction at $s = \mu^2$ for the amplitude M_{++} we have

$$\mathrm{Re}M_{++}(s,t) = \mathrm{Re}\overline{M}_{++}(s=\mu^2,t) + B_{++}(s,t) + \frac{(s-\mu^2)}{\pi} \times \int_{4\mu^2}^{\infty} ds'\,\mathrm{Im}M_{++}(s',t)\left[\frac{1}{(s'-s)(s'-\mu^2)} - \frac{1}{(s'-u)(s'-\mu^2+t)}\right], \quad (6)$$

where B_{++} is the Born term equal to

$$B_{++}(s,t) = \frac{2e^2\mu^2}{(s-\mu^2)(u-\mu^2)} \quad (7)$$

and

$$\overline{M}_{++}(s=\mu^2,t) = M_{++}(s=\mu^2,t) - B_{++}(s=\mu^2,t). \quad (8)$$

Via crossing symmetry these DRs are identical to the DRs with two subtractions.

We determine the subtraction function $\mathrm{Re}\overline{M}_{++}(s=\mu^2,t)$ with the help of the DRs at fixed $s=\mu^2$ with two subtractions using crossing symmetry between the s and u channels:

$$\begin{aligned}\mathrm{Re}\overline{M}_{++}(s=\mu^2,t) &= \overline{M}_{++}(s=\mu^2,0)+t\left.\frac{d\overline{M}_{++}(s=\mu^2,t)}{dt}\right|_{t=0} \\ &+\frac{t^2}{\pi}\left\{P\int_{4\mu^2}^{\infty}\frac{\mathrm{Im}M_{++}(t',s=\mu^2)dt'}{t'^2(t'-t)}\right. \\ &\left.+\int_{4\mu^2}^{\infty}\frac{\mathrm{Im}M_{++}(s',u=\mu^2)ds'}{(s'-\mu^2)^2(s'-\mu^2+t)}\right\}, \end{aligned} \tag{9}$$

where P denotes a principal value integral.

The subtraction constants $\overline{M}_{++}(s=\mu^2,t=0)$ and $d\overline{M}_{++}(s=\mu^2,t)/dt\big|_{t=0}$ are determined in terms of differences of the dipole $[(\alpha_1-\beta_1)_{\pi^{\pm}}]$ and quadrupole $[(\alpha_2-\beta_2)_{\pi^{\pm}}]$ polarizabilities by taking into account the expressions of the sum and the difference of the generalized electric and magnetic polarizabilities of any multipole order through invariant amplitudes [28],

$$\begin{aligned}\overline{M}_{++}(s=\mu^2,t=0)&=2\pi\mu(\alpha_1-\beta_1)_{\pi^{\pm}},\\ \left.\frac{d\overline{M}_{++}(s=\mu^2,t)}{dt}\right|_{t=0}&=\frac{\pi\mu}{6}(\alpha_2-\beta_2)_{\pi^{\pm}}.\end{aligned} \tag{10}$$

The DRs for the amplitude $M_{+-}(s,t)$ have the same form as expressions (6) and (9) but with the substitutions $M_{++}\to M_{+-}$, $\mathrm{Im}M_{++}\to\mathrm{Im}M_{+-}$, and $B_{++}\to B_{+-}=B_{++}/\mu^2$. The subtraction constants are equal in this case to

$$\begin{aligned}\overline{M}_{+-}(s=\mu^2,t=0)&=\frac{2\pi}{\mu}(\alpha_1+\beta_1)_{\pi^{\pm}},\\ \left.\frac{d\overline{M}_{+-}(s=\mu^2,t)}{dt}\right|_{t=0}&=\frac{\pi}{6\mu}(\alpha_2+\beta_2)_{\pi^{\pm}}.\end{aligned} \tag{11}$$

To calculate a principal value integral in Eq. (9) we use the following expression:

$$\begin{aligned}t^2P\int_{4\mu^2}^{\Lambda}\frac{F(t')dt'}{t'^2(t'-t)}&=t^2\int_{4\mu^2}^{\Lambda}\frac{(F(t')-F(t))dt'}{t'^2(t'-t)}\\ &+F(t)\left[\ln\frac{4\mu^2(\Lambda-t)}{\Lambda(t-4\mu^2)}-t\frac{\Lambda-4\mu^2}{4\mu^2\Lambda}\right].\end{aligned} \tag{12}$$

III. DETERMINATION OF THE CHARGED PION POLARIZABILITIES

The DRs for the charged pions are saturated by the contributions of the $\rho(770)$, $b_1(1235)$, $a_1(1260)$, and $a_2(1320)$ mesons in the s channel and σ, $f_0(980)$, $f_0(1370)$, $f_2(1270)$, and $f_2(1525)$ in the t channel.

The parameters of the ρ, b_1, a_2, $f_2(1270)$, and $f_2(1525)$ mesons are given by the Particle Data Group [29]. The parameters of the $f_0(980)$, $f_0(1370)$, and a_1 mesons are taken as follows:

$f_0(980)$: $m_{f_0}=980$ MeV [29], $\Gamma_{f_0}=70$ MeV (the average of the PDG [29] estimate), $\Gamma_{f_0\to\gamma\gamma}=0.39\times10^{-3}$ MeV [29], $\Gamma_{f_0\to\pi\pi}=0.84\,\Gamma_{f_0}$ [30];

$f_0(1370)$: $m_{f_0(1370)}=1434$ MeV [31], $\Gamma_{f_0(1379)}=173$ MeV [31], $\Gamma_{f_0(1370)\to\gamma\gamma}=0.54\times10^{-5}$ MeV [32], $\Gamma_{f_0(1370)\to\pi\pi}=0.26\,\Gamma_{f_0(1370)}$ [33];

$a_1(1260)$: $m_{a_1}=1230$ MeV [29], Γ_{a_1} =425 MeV (the average value of the PDG estimate [29]), $\Gamma_{a_1\to\gamma\pi^{\pm}}=0.64$ MeV [34].

The values of the effective radius of the $f_2(1270)$ meson and its decay width into two photons are taken from our analysis of the reaction $\gamma\gamma\to\pi^0\pi^0$ [20]: $r_f=0.96$ fm and $\Gamma_{f_2(1270)\to\gamma\gamma}=3.05$ keV.

For the σ meson we use the values of the mass and the decay widths found in Ref. [18]: $m_\sigma=547$ MeV, $\Gamma_\sigma=1204$ MeV, and $\Gamma_{\sigma\to\gamma\gamma}=0.62$ keV. It is worth noting that by regarding the decay widths Γ_σ and $\Gamma_{\sigma\to\gamma\gamma}$ as free parameters in the present work, we have obtained practically the same values for them as in Ref. [18]. This value of $\Gamma_{\sigma\to\gamma\gamma}$ differs strongly from the result of Ref. [16] [$\Gamma_{\sigma\to\gamma\gamma}=(3.8\pm1.5)$ keV]. Using the latter magnitude of the decay width in our fit led to essential contradictions with the experimental data. However, the value $\Gamma_{\sigma\to\gamma\gamma}=0.62$ keV allowed Schumacher [35] to obtain a reasonable value of the σ meson contribution to the difference of the nucleon dipole polarizabilities.

Expressions for the imaginary parts of the resonances under consideration are given in Ref. [20].

The influence of the upper integration limit (Λ) in the DRs on results of the calculations was investigated. They are not changed for Λ more than (5 GeV)^2. In the present work we performed the integrations up to (12.5 GeV)^2.

The results of calculations are sensitive to a step length of integration at $t'<(4\text{ GeV})^2$. We took the minimal integration step length in this region that provided the stable result of the calculations.

The main contribution to the total cross section of the process $\gamma\gamma\to\pi^+\pi^-$ is given by the t-channel resonances and the pion polarizabilities. For s-channel resonances, the largest contribution is given by the ρ meson. Its contribution is important in the energy region of $\sqrt{t}>1$ GeV.

The best result of our fit to the experimental data for the total cross section [13,22–26] in the energy region from threshold to 2.5 GeV is presented in Fig. 1 by the solid curve.

We took into account all available data in this energy region and integrated them over $|\cos\theta^*|<0.6$. The error bars in this figure are the quadratic sum of statistical and systematic errors. This solid curve well describes the experimental data in the whole energy region under investigation. As a result of this fit, we have found the following values for the charged pion polarizabilities:

$$(\alpha_1-\beta_1)_{\pi^{\pm}}=13.0^{+2.6}_{-1.9}, \tag{13}$$

$$(\alpha_1+\beta_1)_{\pi^{\pm}}=0.18^{+0.11}_{-0.02}, \tag{14}$$

$$(\alpha_2-\beta_2)_{\pi^{\pm}}=25.0^{+0.8}_{-0.3}, \tag{15}$$

$$(\alpha_2+\beta_2)_{\pi^{\pm}}=0.133\pm0.015. \tag{16}$$

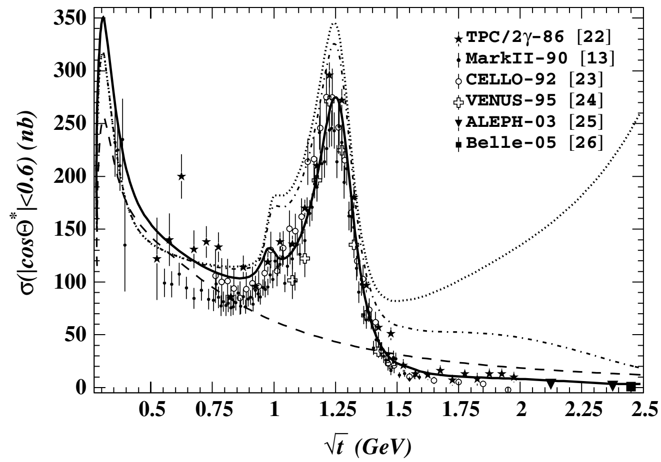

FIG. 1. The total cross section for the reaction $\gamma\gamma \to \pi^+\pi^-$ (with $|\cos\theta^*| < 0.6$). The solid curve is the best fit of the present work. The dashed curve is the Born contribution. The dotted and dashed-dotted curves are the results of a our calculation with $(\alpha_1 - \beta_1)_{\pi^\pm}$ and $(\alpha_1 + \beta_1)_{\pi^\pm}$ from ChPT and the fit of the experimental data with $(\alpha_1 - \beta_1)_{\pi^\pm}$ and $(\alpha_1 + \beta_1)_{\pi^\pm}$ fixed from ChPT, respectively.

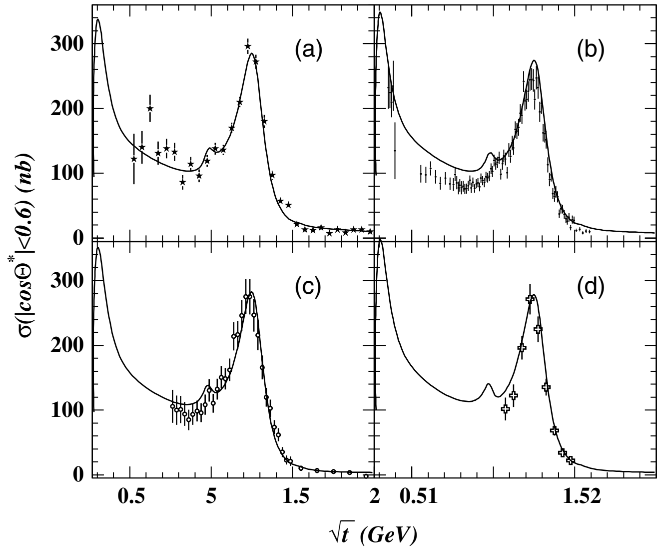

FIG. 2. Fits to the total cross section of the individual experimental works: (a) Ref. [22], (b) Ref. [13], (c) Ref. [23], and (d) Ref. [24].

It should be noted that the values of the sum of the dipole polarizabilities from Ref. [36] are equal to 0.22 ± 0.06 and 0.30 ± 0.04 from the analysis of MarkII and CELLO data, respectively.

The dashed curve in Fig. 1 is the Born term contribution. The dotted curve is a result of calculations using the DRs when $(\alpha_2 - \beta_2)_{\pi^\pm}$ and $(\alpha_2 + \beta_2)_{\pi^\pm}$ are equal to the values (15) and (16) but the dipole polarizabilities are taken from the ChPT calculations [5] as $(\alpha_1 - \beta_1)_{\pi^\pm} = 4.4$ and $(\alpha_1 + \beta_1)_{\pi^\pm} = 0.3$. The dashed-dotted curve presents a result of the fit of the experimental data when the quadrupole polarizabilities are the free parameters and the values of the dipole polarizabilities are fixed by the ChPT calculations [5]. This fit has given $(\alpha_2 - \beta_2)_{\pi^\pm} = 25.7$ and $(\alpha_2 + \beta_2)_{\pi^\pm} = 0.124$. Both the last curves are close to the results of the calculations in Ref. [8] in the energy region up to 700 MeV; however, they differ strongly from all experimental data on the total cross section at higher energies.

The fits of the data to the total cross section for the separate works [13,22–24] are presented in Fig. 2. These fits have given lower and upper values of the charged pion polarizabilities and were used to estimate errors in Eqs. (13)–(16) when the data of all works [13,22–26] were considered.

The angular distributions of the differential cross section of the process $\gamma\gamma \to \pi^+\pi^-$ at different energies are shown in Fig. 3. Presented in this figure are the experimental data of the MarkII, CELLO, and VENUS Collaborations. The differential cross sections of the CELLO and VENUS Collaborations were measured with 50-MeV energy intervals. To compare these data with the MarkII ones we averaged them for 100-MeV intervals. The solid and dashed curves are the results of calculations using our fit and that of ChPT (the latter when the values of the dipole polarizabilities are fixed by ChPT [5]) to the total cross sections in Fig. 1, respectively. This figure demonstrates a good description of the angular distributions by the solid curves with the polarizability values (13)–(16) found in the present work. However, the calculations with the dipole polarizabilities from ChPT [5] contradict these experimental data, particularly at higher energies.

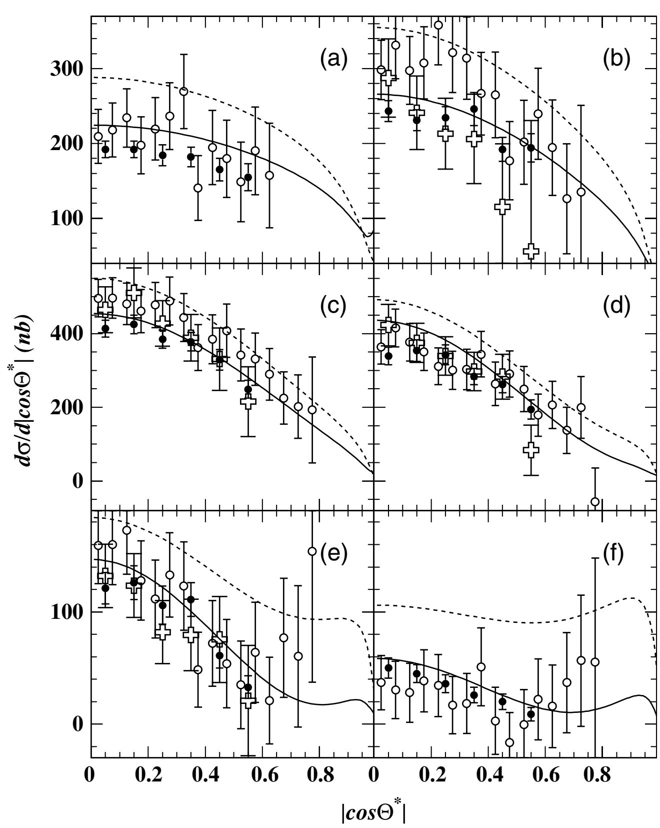

FIG. 3. Angular distributions of the differential cross sections for the following energy intervals: (a) 0.95–1.05 GeV, (b) 1.05–1.15 GeV, (c) 1.15–1.25 GeV, (d) 1.25–1.35 GeV, (e) 1.35–1.45 GeV, and (f) 1.45–1.55 GeV. The designations of the experimental data are the same as in Fig. 1.

TABLE II. The dipole and quadrupole polarizabilities of the charged pions in units 10^{-4} fm^3 and 10^{-4} fm^5, respectively.

	Present analysis	DSRs [20]	ChPT [5]
$(\alpha_1 - \beta_1)_{\pi^\pm}$	$13.0^{+2.6}_{-1.9}$	13.60 ± 2.15	4.4 ± 1.0
$(\alpha_1 + \beta_1)_{\pi^\pm}$	$0.18^{+0.11}_{-0.02}$	0.166 ± 0.024	0.3 ± 0.1
$(\alpha_2 - \beta_2)_{\pi^\pm}$	$25.0^{+0.8}_{-0.3}$	25.75 ± 7.03	
$(\alpha_2 + \beta_2)_{\pi^\pm}$	0.133 ± 0.015	0.121 ± 0.064	

IV. DISCUSSION

The experimental results (13)–(16) for the charged pion dipole and quadrupole polarizabilities and the predictions for these parameters from the DSRs [20] and the results of a two-loop analysis [$\mathcal{O}(p^6)$] [5] for the dipole polarizabilities are listed in Table II. As seen from this table, the values of the polarizabilities found are in a good agreement with the DSR predictions [20]. The difference of the dipole polarizabilities of the charged pions agrees very well with the results obtained from the radiative photoproduction of π^+ from the proton at MAMI [3] and from the scattering of high-energy π^- mesons off the Coulomb field of heavy nuclei [4]. However, this difference deviates from the recent predictions of ChPT [5].

The approximate $SU(2)_L \times SU(2)_R \times U(1)_V$ chiral symmetry in the two-flavor sector of QCD results in a Ward identity, which relates Compton scattering on a charged pion, $\gamma\pi^+ \to \gamma\pi^+$, to radiative charged-pion beta decay, $\pi^+ \to e^+\nu_e\gamma$. The result obtained for the charged pion dipole polarizabilities using ChPT at leading nontrivial order [$\mathcal{O}(p^4)$] can be written in the form

$$\alpha_{1\pi^\pm} = -\beta_{1\pi^\pm} = 2\frac{e^2}{4\pi}\frac{1}{(4\pi F_\pi)^2\mu}\frac{\bar{l}_6 - \bar{l}_5}{6}, \qquad (17)$$

where $F_\pi = 92.4$ MeV is the pion decay constant and $(\bar{l}_6 - \bar{l}_5)$ is a linear combination of scale-independent parameters of the Gasser and Leutwyler Lagrangian [37]. At the lowest nontrivial order [$\mathcal{O}(p^4)$] this difference is related to the ratio $\gamma = F_A/F_V$ of the pion axial-vector form factor F_A and the vector form factor F_V of the radiative pion beta decay [37]: $\gamma = (\bar{l}_6 - \bar{l}_5)/6$. Using $(\bar{l}_6 - \bar{l}_5) = 2.7 \pm 0.4$ Bürgi [5] had obtained at $\mathcal{O}(p^4)$

$$\alpha_{1\pi^+} = 2.7 \pm 0.4. \qquad (18)$$

When this value is used in a two-loop analysis [$\mathcal{O}(p^6)$] in Ref. [5], the result is

$$(\alpha_1 + \beta_1)_{\pi^\pm} = 0.3 \pm 0.1, \qquad (19)$$

$$(\alpha_1 - \beta_1)_{\pi^\pm} = 4.4 \pm 1.0. \qquad (20)$$

It is worth noting that the degeneracy $\alpha_{1\pi^+} = -\beta_{1\pi^+}$ has been removed at $\mathcal{O}(p^6)$. The corresponding corrections amount to an 11% (22%) change of the $\mathcal{O}(p^4)$ of the result for $\alpha_{1\pi^+}$ ($\beta_{1\pi^+}$). The effect of the new low-energy constants appearing at $\mathcal{O}(p^6)$ on the pion polarizabilities was estimated via resonance saturation by including vector and axial-vector mesons. Their contribution was found to be about 50% of the two-loop result.

TABLE III. The dipole and quadrupole polarizabilities of the π^0 meson in units 10^{-4} fm^3 and 10^{-4} fm^5, respectively. The ChPT data are from Refs. [40,41].

	Experiment	DSRs [20]	ChPT
$(\alpha_1 - \beta_1)_{\pi^0}$	-1.6 ± 2.2 [18] -0.6 ± 1.8 [9]	-3.49 ± 2.13	-1.9 ± 0.2
$(\alpha_1 + \beta_1)_{\pi^0}$	0.98 ± 0.03 [18] 1.00 ± 0.05 [36]	0.802 ± 0.035	1.1 ± 0.3
$(\alpha_2 - \beta_2)_{\pi^0}$	39.70 ± 0.02 [20]	39.72 ± 8.01	37.6 ± 3.3
$(\alpha_2 + \beta_2)_{\pi^0}$	-0.181 ± 0.004 [20]	-0.171 ± 0.067	0. 04

It should be noted that by taking into account higher orders in the quark mass expansion, Bijnens and Talavera have obtained $(\bar{l}_6 - \bar{l}_5 = 2.98 \pm 0.33)$ [38], which would lead to $(\alpha_1 - \beta_1)_{\pi^\pm}$ equal to 4.9 instead of 4.4, whereas the sum would remain the same as in Eq. (19). Nevertheless, the experimental results of Refs. [3,4] and the present ones differ from this prediction of ChPT for $(\alpha_1 - \beta_1)_{\pi^\pm}$ and further theoretical and experimental work is needed.

One of the reasons for such a deviation could be the neglect of the contribution of the wide σ meson in the ChPT calculations. As has been shown in Ref. [20], this resonance gives a main contribution to DSRs for $(\alpha_1 - \beta_1)_{\pi^\pm}$. This contribution essentially exceeds the contribution of the vector and axial-vector mesons.

Moreover, the analysis of the recent PIBEPA result [39] suggests an inadequacy of the present vector–axial-vector $(V - A)$ description of the radiative β decay, which would also reflect an inadequacy of the ChPT description in its present form.

To compare with the situation for the π^0 mesons, we present here Table III, where the experimental values of the π^0 meson dipole and quadrupole polarizabilities are confronted with predictions of DSRs [18,20] and two-loop calculations in the frame of ChPT.

The experimental values of the sum and the difference of the dipole polarizabilities of π^0 and the difference of its quadrupole polarizabilities do not conflict within the errors with the predictions of DSRs and ChPT. However, there are very big errors in the experimental values for the difference of the dipole polarizabilities. Therefore, it is difficult to draw a more unambiguous conclusion. For the sum of the quadrupole polarizabilities of π^0, the DSR result agrees well with the experimental value, but ChPT predicts a positive value, in contrast to the experimental result. However, as was noted in Ref. [41], this quantity was obtained in a two-loop approximation, which is a leading order result for this sum, and one expects substantial corrections to it from three-loop calculations.

V. CONCLUSION

The dispersion relations at fixed t with one subtraction at $s = \mu^2$ have been constructed for the invariant helicity amplitudes of the reaction $\gamma\gamma \to \pi^+\pi^-$. The subtraction functions were determined with the help of the DRs at fixed

$s = \mu^2$ with two subtractions at $t = 0$, where the subtraction constants were expressed through the dipole and quadrupole polarizabilities of the charged pion. These DRs, where the sum and the difference of the dipole and the quadrupole charged pion polarizabilities were free parameters, were used to fit the experimental data to the total cross section of the process $\gamma\gamma \to \pi^+\pi^-$ in the energy region from threshold to 2.5 GeV.

As a result, the dipole and the quadrupole polarizabilities have been found. The quadrupole charged pion polarizabilities have been determined for the first time. The value of the difference of the dipole polarizabilities $(\alpha_1 - \beta_1)_{\pi^\pm} = 13.0^{+2.6}_{-1.9}$ found agrees well with results obtained at MAMI [3] and in Serpukhov [4] and with the DSR predictions whereas it is at variance with the recent calculations in the framework of ChPT [5].

The results of the calculations of the angular distributions for the process under consideration, using the DRs constructed in the present work and the values of the dipole and the quadrupole polarizabilities (13)–(16), are in a good agreement with the experimental data.

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