Strangeness spin, magnetic moment, and strangeness configurations of the proton

C. S. An,[1,*] D. O. Riska,[2,†] and B. S. Zou[1,3,‡]

[1] *Institute of High Energy Physics, CAS, P.O. Box 918, Beijing 100049, China*

[2] *Helsinki Institute of Physics and Department of Physical Sciences, P.O. Box 64, University of Helsinki, FIN-00014 Helsinki, Finland*

[3] *CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China*

(Received 17 November 2005; published 16 March 2006)

The implications of the empirical signatures for the positivity of the strangeness magnetic moment μ_s and the negativity of the strangeness contribution to the proton spin Δ_s on the possible $uuds\bar{s}$ configurations of five quarks in the proton are analyzed. The empirical signs for the values for these two observables can only be obtained in configurations where the *uuds* subsystem is orbitally excited and the $\bar{s}$ antiquark is in the ground state. The configurations in which the $\bar{s}$ is orbitally excited, including the conventional $K^+\Lambda^0$ configuration, with the exception of that in which the *uuds* component has spin 2, yield negative values for μ_s. Here, the strangeness spin Δ_s, strangeness magnetic moment μ_s, and axial coupling constant G_A^s are calculated for all possible configurations of the $uuds\bar{s}$ component of the proton. In the configuration with $[4]_{FS}[22]_F[22]_S$ flavor-spin symmetry, which is likely to have the lowest energy, μ_s is positive and $\Delta_s \simeq G_A^s \simeq -1/3\mu_s$.

DOI: 10.1103/PhysRevC.73.035207 PACS number(s): 14.20.Dh, 12.39.−x, 13.40.Em, 24.85.+p

I. INTRODUCTION

Four recent experiments on parity violation in electron-proton scattering suggest that the strangeness magnetic moment of the proton μ_s is positive [1–4]. In contrast, most theoretical calculations have led to negative values for this observable [5], with but few exceptions [6–11]. The implications of the empirical result for the configuration of the proton constituents is considered by a calculation of μ_s for all positive parity configurations of the $uuds\bar{s}$ system with one constituent in the first orbitally excited state, which may be contained in the proton, have recently been studied in Ref. [12]. The formalism summarized in that reference is elaborated here in more detail along with a few minor corrections. In addition, it is now applied to the strangeness contribution to the proton spin Δ_s and the corresponding strangeness axial form factor G_A^s.

In the case of the strangeness contribution to the proton spin Δ_s, the empirical indications are that it is very small [13] or small and negative (-0.10 ± 0.06) [14,15]. Extrapolation of the empirical values for the strangeness axial form factor G_A^s to low Q^2 indicates a nonzero negative value for that quantity [16].

Here it is noted that μ_s is positive and that both Δ_s and G_A^s are negative and smaller in magnitude than μ_s in the $uuds\bar{s}$ configuration, which is likely to have the lowest energy, where the $\bar{s}$ quark is in the ground state and the *uuds* system is in the *P* state. If, in contrast, the strange antiquark is in the *P* state and the four quarks are in their ground state, the μ_s has the opposite and empirically contraindicated sign. The latter configurations include that of a fluctuation of the proton into a kaon and a strange hyperon, which is well known to lead to a negative value for the strangeness magnetic moment [17–20].

This paper gives a detailed description of the wave functions of the $uuds\bar{s}$ configurations that may be contained in the proton. The paper has six sections. Section II contains the definitions of the strangeness observables and flavor wave functions for the $uuds\bar{s}$ components of the proton and discusses their expected hyperfine splittings. The configurations in which the *uuds* system is in the ground state are considered in Sec. III, and the corresponding configurations in which the *uuds* system is in the *P* state are considered in Sec. IV. The implications of the empirical values for μ_s and Δ_s on the probability of the $s\bar{s}$ being in the proton are considered in Sec. IV. A summarizing discussion is given in Sec. V.

II. STRANGENESS OBSERVABLES AND FLAVOR WAVE FUNCTIONS FOR $uuds\bar{s}$ COMPONENTS

In the nonrelativistic quark model, the strangeness magnetic moment μ_s and the strangeness contribution to the proton spin Δ_s are defined as the expectation value of the following two operators in the proton state:

$$\vec{\mu}_s = e\sum_i \frac{\hat{S}_i}{2m_s}(\hat{l}_i + \hat{\sigma}_i),$$
$$\vec{\sigma}_s = \hat{\sigma}_s + \hat{\sigma}_{\bar{s}}, \quad (1)$$

where $\hat{S}_i$ is the strangeness counting operator with eigenvalue $+1$ for s and -1 for $\bar{s}$ quark and m_s is the constituent mass of the strange quark. The strangeness axial form factor is in turn defined as the matrix element of the strangeness axial current operator

$$\vec{A}^s = \vec{\gamma}_i^s\gamma_5^s + \vec{\gamma}_i^{\bar{s}}\gamma_5^{\bar{s}}\,. \quad (2)$$

The matrix element of this operator in the proton is denoted as G_A^s. In the nonrelativistic limit and at $Q^2 = 0$, one has $G_A^s = \Delta_s$.

Key ingredients in the calculation of the matrix elements of these operators are the flavor wave functions for the $uuds\bar{s}$ components in the proton. These are usually constructed by coupling *uuds* flavor wave functions with the $\bar{s}$ flavor wave

*E-mail: ancs@ihep.ac.cn

†E-mail: riska@pcu.helsinki.fi

‡E-mail: zoubs@ihep.ac.cn

function. There are four possible flavor symmetry patterns for the *uuds* system: $[4]_F$, $[31]_F$, $[22]_F$, and $[211]_F$ in the Weyl tableaux of the SU(3) group [21,22]. Combining of these with the antiquark with flavor symmetry $[1]^*_F$ leads to the following pentaquark multiplet representations of SU(3):

$$[4]_F \otimes [1]^*_F = \mathbf{10} \oplus \mathbf{35}, \tag{3}$$

$$[31]_F \otimes [1]^*_F = \mathbf{8} \oplus \mathbf{10} \oplus \mathbf{27}, \tag{4}$$

$$[22]_F \otimes [1]^*_F = \mathbf{8} \oplus \bar{\mathbf{10}}, \tag{5}$$

$$[211]_F \otimes [1]^*_F = \mathbf{1} \oplus \mathbf{8}. \tag{6}$$

Here, the numbers in boldface indicate the dimensions of the pentaquark representations. For example, the tentative θ^+ pentaquark belongs to the baryon antidecuplet $\bar{\mathbf{10}}$ representation. Since the proton belongs to the baryon octet representation, the possible flavor symmetry patterns for the *uuds* in the proton are limited to $[22]_F$, $[211]_F$, and $[31]_F$. The corresponding flavor wave functions can be obtained as in Refs. [21,22]. For convenience, we list the relevant ones here explicitly.

For the mixed flavor symmetry representations $[22]_F$ and $[211]_F$, there are two and three independent flavor wave functions, respectively,

$$\begin{aligned}|[22]_{F_1}\rangle = {}& \frac{1}{\sqrt{24}}[2|uuds\rangle + 2|uusd\rangle + 2|dsuu\rangle + 2|sduu\rangle \\ & - |duus\rangle - |udus\rangle - |sudu\rangle - |usdu\rangle \\ & - |suud\rangle - |dusu\rangle - |usud\rangle - |udsu\rangle],\end{aligned} \tag{7}$$

$$\begin{aligned}|[22]_{F_2}\rangle = {}& \frac{1}{\sqrt{8}}[|udus\rangle + |sudu\rangle + |dusu\rangle + |usud\rangle \\ & - |duus\rangle - |usdu\rangle - |suud\rangle - |udsu\rangle],\end{aligned} \tag{8}$$

$$\begin{aligned}|[211]_{F_1}\rangle = {}& \frac{1}{\sqrt{16}}[2|uuds\rangle - 2|uusd\rangle - |duus\rangle - |udus\rangle \\ & - |sudu\rangle - |usdu\rangle + |suud\rangle + |dusu\rangle \\ & + |usud\rangle + |udsu\rangle],\end{aligned} \tag{9}$$

$$\begin{aligned}|[211]_{F_2}\rangle = {}& \frac{1}{\sqrt{48}}[3|udus\rangle - 3|duus\rangle + 3|suud\rangle - 3|usud\rangle \\ & + 2|dsuu\rangle - 2|sduu\rangle - |sudu\rangle + |usdu\rangle \\ & + |dusu\rangle - |udsu\rangle],\end{aligned} \tag{10}$$

$$\begin{aligned}|[211]_{F_3}\rangle = {}& \frac{1}{\sqrt{6}}[|sudu\rangle + |udsu\rangle + |dsuu\rangle \\ & - |usdu\rangle - |dusu\rangle - |sduu\rangle].\end{aligned} \tag{11}$$

In the case of the mixed flavor symmetry $[31]_F$ for the *uuds* system, there is a need to separate the isospin 1/2 and 3/2 states, both of which are listed in Ref. [21]. The reason for the presence of two separate classes of wave functions with the flavor $[31]_F$ is the following. In the case of flavor SU(3) model, the Δ^+ resonance is composed of *uud* valence quarks with the flavor symmetry $[3]_F$. In the case of the $uuds\bar{s}$ component of this baryon and its *uuds* subsystem, the latter may also have the mixed flavor symmetry $[31]_F$. This flavor state of the *uuds* subsystem is therefore a combination of a Δ^+ and a proton component. There are six independent flavor wave functions for *uuds* of the flavor symmetry $[31]_F$. Among them, by using the weight diagram method [22], the three wave functions labeled as ψ^θ in Ref. [21] are found to correspond to isospin 1/2. These are explicitly

$$\begin{aligned}|[31]_{F_1}\rangle = {}& \frac{1}{\sqrt{18}}[2|uusd\rangle + 2|suud\rangle + 2|usud\rangle \\ & - |sudu\rangle - |usdu\rangle - |dusu\rangle - |udsu\rangle \\ & - |dsuu\rangle - |sduu\rangle],\end{aligned} \tag{12}$$

$$\begin{aligned}|[31]_{F_2}\rangle = {}& \frac{1}{12}[6|uuds\rangle - 3|duus\rangle - 3|udus\rangle - 4|dsuu\rangle \\ & - 4|sduu\rangle + 5|sudu\rangle + 5|usdu\rangle + 2|uusd\rangle \\ & - |suud\rangle - |dusu\rangle - |usud\rangle - |udsu\rangle],\end{aligned} \tag{13}$$

$$\begin{aligned}|[31]_{F_3}\rangle = {}& \frac{1}{\sqrt{48}}[-3|duus\rangle + 3|udus\rangle - 3|dusu\rangle + 3|udsu\rangle \\ & - 2|dsuu\rangle + 2|sduu\rangle - |sudu\rangle + |usdu\rangle \\ & - |suud\rangle + |usud\rangle].\end{aligned} \tag{14}$$

The color symmetry of all the *uuds* configurations is limited to $[211]_C$ in order to combine with the $\bar{s}$ antiquark to form a color singlet [cf. (6)]. The Pauli principle requires that corresponding orbital-flavor-spin states have the mixed symmetry $[31]_{XFS}$ so as to combine with the mixed color symmetry state $[211]_C$ to form the required completely antisymmetric four-quark state [1111]. Since the intrinsic parity is positive for a quark and negative for an antiquark, the $uuds\bar{s}$ component in a proton, which has positive parity, requires that either the $\bar{s}$ is in the *P* state with the *uuds* system in the ground state with spatial symmetry $[4]_X$ or that one of the quarks is in the *P* state so that the *uuds* system has mixed spatial symmetry $[31]_X$.

There are several configurations of the $uuds\bar{s}$ system that have positive parity, isospin 1/2, spin 1/2, and one unit of orbital angular momentum. The spin-dependent hyperfine interaction between the quarks splits these states, so that the configurations with the lowest energy may be expected to be those with the highest probability for admixture in the proton.

In most models, the hyperfine interaction between quarks in the baryon is spin dependent. In the common color magnetic hyperfine interaction model, the spin dependence is such that the spin singlet state has lower energy than the spin triplet state [23]. This is also the case for the instanton-induced interaction model [24]. Finally, the schematic flavor and spin-dependent interaction model

$$H' = -C \sum_{i<j} \vec{\lambda}_i \cdot \vec{\lambda}_j\, \vec{\sigma}_i \cdot \vec{\sigma}_j, \tag{15}$$

which gives the qualitative description with correct ordering of the states in the low lying part of the baryon spectrum in all flavor sectors [25,26], implies that antisymmetric flavor and spin configurations have the lowest energy. In this interaction, C is a constant that represents an average of the spin-spin part of the interaction expected to be mediated by pseudoscalar meson exchange [27]. Phenomenologically $C \sim 20-30$ MeV.

In the next two sections, the strangeness spin Δ_s and magnetic moment μ_s are calculated for all possible configurations of the $uuds\bar{s}$ system that have the quantum numbers of the proton.

TABLE I. Flavor and spin configurations of *uuds* quark states in the ground state [28] and the corresponding operator matrix elements.

uuds ground state	$\bar{\sigma}_s$	$\Delta_s(P_{s\bar{s}})$	$\mu_s(\frac{m_p}{m_s}P_{s\bar{s}})$	$-CC_2^{(6)}$
$[31]_{FS}[211]_F[22]_S$	–	$-1/3$	$-1/3$	$-16C$
$[31]_{FS}[211]_F[31]_S$	13/36	85/216	$-95/216$	$-13\frac{1}{3}C$
$[31]_{FS}[22]_F[31]_S$	1/2	5/12	$-5/12$	$-9\frac{1}{3}C$
$[31]_{FS}[31]_F[22]_S$	–	$-1/3$	$-1/3$	$-8C$
$[31]_{FS}[31]_F[31]_S$	65/108	281/648	$-259/648$	$-5\frac{1}{3}C$
$[31]_{FS}[31]_F[4]_S$	–	1/6	7/6	0

III. CONFIGURATIONS WITH THE *uuds* SYSTEM IN ITS GROUND STATE

In configurations where the *uuds* quarks are in their ground state, the spatial state has complete symmetry $[4]_X$, and accordingly their flavor-spin state has to have the mixed symmetry $[31]_{FS}$. All the different flavor and spin state symmetry configurations that can combine to the required $[31]_{FS}$ mixed symmetry combination have been listed in Table I [28]. In the table, the matrix elements of the quadratic Casimir operator of SU(6), $C_2^{(6)}$, multiplied by the constant C in the flavor-spin interaction (15) are also listed so as to give an indication of the energy splitting between these configurations. The requirement of positive parity requires that for these configurations, the strange antiquark in the $uuds\bar{s}$ system has to be in the P state.

The wave functions of the proton state with spin +1/2 that have any one of these symmetries may be written in the general form

$$\left|p,+\frac{1}{2}\right\rangle = A_{s\bar{s}}\sum_{abc}\sum_{s_z mm's'_z} C^{\frac{1}{2}\frac{1}{2}}_{1s_z,jm}C^{jm}_{1m',\frac{1}{2}s'_z}C^{[1^4]}_{[31]_a[211]_a}C^{[31]_a}_{[F]_b[S]_c} \times [F]_b[S]_c[211]_{C,a}\bar{Y}_{1m'}\bar{\chi}_{s_{z'}}\varphi(\{\vec{r}_i\}). \quad (16)$$

Here $[F]_b$, $[S]_c$, and $[211]_{C,a}$ represent the flavor, spin, and color state wave functions, denoted by their Weyl tableaux. The sums over a, b, and c run over the configurations of the $[31][F][S]$ representations of the S_4 permutation group for which the corresponding Clebsch-Gordan (CG) coefficients $C^a_{b,c}$ do not vanish. The value of the first of these CG coefficients are $C^{[1^4]}_{[31]_a[211]_a} = \pm\frac{1}{\sqrt{3}}$ [21]. Finally $\bar{Y}_{1m'}$ and $\bar{\chi}_{s'_z}$ denote the orbital and spin states of the antistrange quark, respectively. In Eq. (16), $A_{s\bar{s}}$ is the amplitude of the $s\bar{s}$ component in the proton and $\varphi(\{\vec{r}_i\})$ is a symmetric function of the coordinates of the $uuds\bar{s}$.

The wave functions of the *uuds* subsystem may be organized in groups according to their spin states. Consider first the states with the spin symmetry $[22]_S$. To these belong the configuration $[211]_F[22]_S$, which is expected to have the lowest energy among all the configurations with the mixed flavor-spin symmetry $[31]_{FS}$. Because the total spin of the *uuds* system with this symmetry is $S = 0$ and the angular momentum space is isotropic, it gives no contribution to μ_s and σ_s. In these configurations, only the $\bar{s}$ quark contributes to μ_s and Δ_s (and G^s_A), that is,

$$\mu_s = -\frac{1}{3}\frac{e}{2m_s}P_{s\bar{s}}, \quad (17)$$

$$\Delta_s = -\frac{1}{3}P_{s\bar{s}}. \quad (18)$$

Here, $P_{s\bar{s}}$ is the probability of this configuration. In units of nuclear magnetons, μ_s takes the value

$$\mu_s = -\frac{1}{3}\frac{m_p}{m_s}P_{s\bar{s}}. \quad (19)$$

In this configuration, the numerical value of μ_s is negative and $\sim$2 times Δ_s, as $m_p \simeq 2m_s$.

The spin of the *uuds* system in the states with spin symmetry $[31]_S$ is $S = 1$. If we neglect the interaction of quarks, then the wave function of the proton in the angular momentum space may be written in the general form

$$\begin{aligned}\left|p,+\frac{1}{2}\right\rangle = \frac{A_{s\bar{s}}}{\sqrt{2}}\Big\{ & C^{\frac{1}{2}\frac{1}{2}}_{11,\frac{1}{2}-\frac{1}{2}}\big[C^{11}_{11,10}|11\rangle_{A_S}|10\rangle_{B_X} \\ & + C^{11}_{10,11}|10\rangle_{A_S}|11\rangle_{B_X}\big]\left|\frac{1}{2}\frac{1}{2}\right\rangle_{B_S} + C^{\frac{1}{2}\frac{1}{2}}_{10,\frac{1}{2}\frac{1}{2}} \\ & \times\big[C^{10}_{11,1-1}|11\rangle_{A_S}|1-1\rangle_{B_X} + C^{10}_{1-1,11}|1-1\rangle_{A_S}|11\rangle_{B_X}\big] \\ & \times\left|\frac{1}{2}\frac{1}{2}\right\rangle_{B_S} + C^{\frac{1}{2}\frac{1}{2}}_{00,\frac{1}{2}\frac{1}{2}}\big[C^{00}_{11,1-1}|11\rangle_{A_S}|1-1\rangle_{B_X} \\ & + C^{00}_{10,10}|10\rangle_{A_S}|10\rangle_{B_X} + C^{00}_{1-1,11}|1-1\rangle_{A_S}|11\rangle_{B_X}\big] \\ & + \left|\frac{1}{2}\frac{1}{2}\right\rangle_{B_S}\Big\}. \end{aligned} \quad (20)$$

For the sake of brevity, the subindica A_S and B_X represent the spin state of the *uuds* system and the orbital state of the $\bar{s}$ quark, respectively. Explicit evaluation of the matrix elements with these wave functions for μ_s and Δ_s leads to

$$\mu_s = -\frac{1}{2}\frac{m_p}{m_s}\left(1 - \frac{1}{3}\bar{\sigma}_s\right)P_{s\bar{s}}. \quad (21)$$

$$\Delta_s = \frac{1}{3}\left(1 + \frac{1}{2}\bar{\sigma}_s\right)P_{s\bar{s}}. \quad (22)$$

Here, the $\bar{\sigma}_s$ is the expectation value of the z component of the spin of the s quark in the configuration where $s_z = 1$. [Note that our Eq. (21)] here corrects Eq. (4) in Ref. [12]. The numerical value of $\bar{\sigma}_s$ depends on the detailed configuration as shown below. The results in all cases satisfy the inequality $\bar{\sigma}_s < 1$ as shown in Table I. This result implies that μ_s in all these configurations is negative and that Δ_s is positive, in contradiction with experiment.

The final possibility is that the spin state of the *uuds* system is completely symmetric: $[4]_S$. In this case, it has spin $S = 2$, and as a consequence, the total angular momentum of the $\bar{s}$ quark has to be $j = \frac{3}{2}$ in order to combine with the *uuds* system to form the proton state with spin +1/2. The wave function then may be expressed in a way analogous to Eq. (20). The relevant matrix elements for these configurations are found to be

$$\mu_s = \frac{7}{6}\frac{m_p}{m_s} P_{s\bar{s}}, \tag{23}$$

$$\Delta_s = \frac{1}{6} P_{s\bar{s}}. \tag{24}$$

This configuration thus yields positive values for both μ_s and Δ_s as well as G_A^s. It is very unlikely that this configuration has a large probability of being in the proton, as it is expected to have an energy 100–150 MeV above all the other configurations with the mixed flavor-spin symmetry $[31]_{FS}$.

The (tedious) calculation of the average spin value $\bar{\sigma}_s$ in Eq. (22) may be illustrated by the following explicit calculation for the case of the configuration $[31]_{FS}[22]_F[31]_S$. The results for this and the other configurations are listed in Table I.

There are three combinations of the mixed symmetry states $[22]_F$ and $[31]_S$ that combine to form the mixed symmetry state $[31]_{FS}$. The explicit expressions for these are [21]

$$\left|[31]_{FS_1}\right\rangle = \frac{1}{\sqrt{2}}\left\{\left|[22]_{F_1}\right\rangle\left|[31]_{S_2}\right\rangle + \left|[22]_{F_2}\right|[31]_{S_3}\rangle\right\}, \tag{25}$$

$$\left|[31]_{FS_2}\right\rangle = \frac{1}{2}\left\{\sqrt{2}\left|[22]_{F_1}\right|[31]_{S_1}\rangle + \left|[22]_{F_1}\right\rangle\left|[31]_{S_2}\right\rangle - \left|[22]_{F_2}\right\rangle\left|[31]_{S_3}\right\rangle\right\}, \tag{26}$$

$$\left|[31]_{FS_3}\right\rangle = \frac{1}{2}\left\{\sqrt{2}\left|[22]_{F_2}\right\rangle\left|[31]_{S_1}\right\rangle - \left|[22]_{F_2}\right\rangle\left|[31]_{S_2}\right\rangle - \left|[22]_{F_1}\right\rangle\left|[31]_{S_3}\right\rangle\right\}, \tag{27}$$

where the $[22]_F$ flavor functions are given by Eqs. (7) and (8) and the three spin wave functions are

$$|[31]_{S_1}\rangle = \frac{1}{\sqrt{12}}[3|\uparrow\uparrow\uparrow\downarrow\rangle - |\uparrow\uparrow\downarrow\uparrow\rangle - |\uparrow\downarrow\uparrow\uparrow\rangle - |\downarrow\uparrow\uparrow\uparrow\rangle], \tag{28}$$

$$|[31]_{S_2}\rangle = \frac{1}{\sqrt{6}}[2|\uparrow\uparrow\downarrow\uparrow\rangle - |\uparrow\downarrow\uparrow\uparrow\rangle - |\downarrow\uparrow\uparrow\uparrow\rangle], \tag{29}$$

$$|[31]_{S_3}\rangle = \frac{1}{\sqrt{2}}[|\uparrow\downarrow\uparrow\uparrow\rangle - |\downarrow\uparrow\uparrow\uparrow\rangle]. \tag{30}$$

From Eqs. (25)–(27), one obtains

$$\bar{\sigma}_s = \tfrac{1}{3}\left(\bar{\sigma}_{FS_1} + \bar{\sigma}_{FS_2} + \bar{\sigma}_{FS_3}\right). \tag{31}$$

It then follows from Eqs. (7), (8), (28)–(30) that

$$\bar{\sigma}_s = \tfrac{1}{2}, \tag{32}$$

and at last, from Eqs. (21) and (22),

$$\mu_s = -\frac{5}{12}\frac{m_p}{m_s} P_{s\bar{s}}, \tag{33}$$

$$\Delta_s = \frac{5}{12} P_{s\bar{s}}. \tag{34}$$

The matrix elements needed for the values of Δ_s and μ_s in all other configurations may be calculated by the same method.

IV. CONFIGURATIONS WITH $\bar{s}$ IN ITS GROUND STATE

In configurations where the $\bar{s}$ antiquark is in the ground state, the lowest possible orbital configuration allowed by the requirement of positive parity for the *uuds* state is that with an orbital angular momentum of $L = 1$. The corresponding spatial state has the mixed symmetry labeled by the Weyl tableau $[31]_X$. The possible symmetries of the flavor-spin state are then either complete symmetry $[4]_{FS}$ or the mixed symmetries $[31]_{FS}$, $[22]_{FS}$, and $[211]_{FS}$. All the flavor and spin symmetry configurations that can combine to these configurations, and which can be a component of the proton, are listed in Table II. The wave functions of the proton with spin $+1/2$ may for all of these symmetry configurations be expressed in the general form

$$\left|p, +\frac{1}{2}\right\rangle = \sum_{abcde}\sum_{Ms'_z m s_z} C^{\frac{1}{2}\frac{1}{2}}_{\mathrm{JM},\frac{1}{2}s'_z} C^{\mathrm{JM}}_{1m,Ss_z} C^{[1^4]}_{[31]_a[211]_a} \times C^{[31]_a}_{[31]_b[FS]_c} C^{[FS]_c}_{[F]_d[S]_e} [31]_{X,m}(b)[F]_d[S]_{s_z}(e) \times [211]_C(a)\bar{\chi}_{s'_z}\varphi(\{r_i\}). \tag{35}$$

Here, J is the total angular momentum of the *uuds* system, and M the corresponding z component. The orbital angular momentum of these *uuds* states is $L = 1$, with the z component m.

These configurations may also be organized in groups according to their spin symmetry. The method described explicitly in the previous section may then be applied to evaluate the matrix elements that are required for μ_s and Δ_s.

Of these configurations, the configuration $[4]_{FS}[22]_F[22]_S$ is expected to have the lowest energy. For this and all the configurations that have the mixed spin symmetry $[22]_S$, the desired matrix elements for μ_s and Δ_s are

$$\mu_s = \frac{m_p}{3m_s}(1 + 2\bar{l}_s)P_{s\bar{s}}, \tag{36}$$

$$\Delta_s = -\frac{1}{3} P_{s\bar{s}}. \tag{37}$$

Here, $\bar{l}_s$ is the average value of the z component of the orbital angular momentum of the s quark in the *uuds* system for $m = 1$. The numerical value for $\bar{l}_s$ depends on the detailed configuration and may be calculated by the same method as that described for $\bar{\sigma}_s$ in the previous section. The results of $\bar{l}_s$ for various $[31]_X$ configurations with *uuds* total angular momentum $J = 1$ are listed in Table II along with the corresponding values of $\bar{\sigma}_s$. The values for $\bar{l}_s$ are in every case smaller than 1/2, that is $\bar{l}_s < 1/2$. Note that the values of $\bar{l}_s$ and $\bar{\sigma}_s$ given in Ref. [12] for the mixed $[31]_F$ flavor symmetry are not correct due to the neglect of the isospin decomposition and are corrected in Table II.

In the case of the lowest energy configuration $[4]_{FS}[22]_F[22]_S$, $\bar{l}_s = 1/4$ and consequently

$$\mu_s = \frac{m_p}{2m_s} P_{s\bar{s}} . \tag{38}$$

The value of μ_s in this configuration is consequently positive, and since $m_p/m_s \simeq 2$, it is approximately equal to the probability of that configuration occurring in the proton. Moreover, for this configuration, $\mu_s \simeq -3\Delta_s$, a relation that agrees with the present experimental values within their uncertainty range.

Because of its low energy, this is the most likely $s\bar{s}$ component in the proton. The $[4]_{FS}[22]_F[22]_S$ *uuds* wave

TABLE II. Flavor and spin configurations of the *uuds* quark states in the first orbitally excited state [28] with total angular momentum $J = 1$ and the corresponding operator matrix elements.

uuds P state	$\bar{l}_s$	$\bar{\sigma}_s$	$\Delta_s(P_{s\bar{s}})$	$\mu_s(\frac{m_p}{m_s}P_{s\bar{s}})$	$-CC_2^{(6)}$
$[4]_{FS}[22]_F[22]_S$	1/4	–	−1/3	1/2	$-28C$
$[4]_{FS}[31]_F[31]_S$	1/4	7/9	−2/27	73/108	$-21\frac{1}{3}C$
$[31]_{FS}[211]_F[22]_S$	13/48	–	−1/3	37/72	$-16C$
$[31]_{FS}[211]_F[31]_S$	119/432	13/36	−23/108	707/1296	$-13\frac{1}{3}C$
$[31]_{FS}[22]_F[31]_S$	1/4	1/2	−1/6	7/12	$-7\frac{1}{3}C$
$[31]_{FS}[31]_F[22]_S$	13/48	–	−1/3	37/72	$-8C$
$[22]_{FS}[211]_F[31]_S$	1/4	5/12	−7/36	5/9	$-5\frac{1}{3}$
$[31]_{FS}[31]_F[31]_S$	295/1296	65/108	−43/324	2371/3888	$-5\frac{1}{3}$
$[22]_{FS}[22]_F[22]_S$	1/4	–	−1/3	1/2	$-4C$
$[211]_{FS}[211]_F[22]_S$	11/48	–	−1/3	35/72	0
$[31]_{FS}[31]_F[4]_S$	43/216	–	1/6	497/648	0
$[211]_{FS}[211]_F[31]_S$	119/432	65/108	−43/324	811/1296	$2\frac{1}{3}C$
$[22]_{FS}[31]_F[31]_S$	13/54	5/12	−7/36	251/324	$2\frac{2}{3}C$
$[22]_{FS}[22]_F[4]_S$	1/4	–	1/6	3/4	$4C$
$[211]_{FS}[22]_F[31]_S$	1/4	1/2	−1/6	7/12	$6\frac{2}{3}$
$[211]_{FS}[211]_F[4]_S$	23/72	–	1/6	157/216	$8C$
$[211]_{FS}[31]_F[22]_S$	11/48	–	−1/3	35/72	$8C$
$[211]_{FS}[31]_F[31]_S$	31/144	13/36	−23/108	227/432	$10\frac{2}{3}C$

function has the simple form

$$|[4]_{FS}\rangle = \frac{1}{\sqrt{2}}\left\{[22]_{F_1}[22]_{S_1} + [22]_{F_2}[22]_{S_2}\right\}. \quad (39)$$

The explicit form for the two flavor components $[22]_{F1}$ and $[22]_{F_2}$ are given in Eqs. (7) and (8). The two corresponding spin functions are readily constructed by the substitutions $u \leftrightarrow\uparrow$ and $d, s \leftrightarrow\downarrow$ with an additional $1/\sqrt{2}$ in the normalization factor. To complete the wave function for this configuration, one needs the explicit antisymmetric color space wave function

$$|[1111]_{CX}[211]_C[31]_X; m\rangle = \sum_{a=1}^{3} C^{[1^4]}_{[31]_a[211]_a}[211]^a_C[31]^a_{Xm}. \quad (40)$$

As the operators that are considered here do not depend on color, the explicit color wave functions with the mixed symmetry $[211]_C$ are not required. The explicit spatial wave functions may be constructed by reference to Eqs. (28)–(30), with the substitution of ground-state wave functions for the three constituents denoted ↑ and a *P*-state wave function, multiplied by the spherical harmonic Y_{1m}, for the constituent denoted ↓.

For the states which have spin symmetry $[31]_S$, the total angular momentum of the *uuds* system *J* may take the values 0 and 1. For $J = 0$ the results are

$$\mu_s = -\frac{m_p}{m_s}P_{s\bar{s}}, \quad (41)$$

$$\Delta_s = P_{s\bar{s}}. \quad (42)$$

These are the only configurations with the *uuds* system in an orbitally excited state, which lead to negative values for the strangeness magnetic moment. As the lowest one of these configurations lies $\sim$ 140–200 MeV above the configuration with $[4]_{FS}[22]_F[22]_S$ symmetry, it is unlikely to have a large probability of being a component of the proton.

In the case when $J = 1$, the corresponding results are

$$\mu_s = \frac{m_p}{3m_s}(1 + \bar{l}_s + \bar{\sigma}_s)P_{s\bar{s}}, \quad (43)$$

$$\Delta_s = -\frac{1}{3}(1 - \bar{\sigma}_s)P_{s\bar{s}}. \quad (44)$$

Finally, for the states which have spin symmetry $[4]_S$, the results are

$$\mu_s = \frac{1}{3}\frac{m_p}{m_s}\left(\frac{5}{2} - \bar{l}_s\right)P_{s\bar{s}}, \quad (45)$$

$$\Delta_s = \frac{1}{6}P_{s\bar{s}}. \quad (46)$$

Since these configurations give positive strangeness contributions to the proton spin in apparent conflict with the experimental situation and are expected to require a relatively high energy of excitation, they are not likely to have any large probability of being in the proton.

V. PROBABILITY OF $uuds\bar{s}$ COMPONENT

Above, the possibility of there being transition matrix elements between the $uuds\bar{s}$ and the *uud* components of the proton was not considered, as only diagonal matrix elements in configurations with $s\bar{s}$ components were calculated. The calculation of such transition matrix elements demands an explicit model for the spatial wave functions for determining the overlap. These transition matrix elements modify the proportionality coefficients between the strangeness magnetic moment and the probability of the $s\bar{s}$ components in the proton. Referring to the analogous situation in the case of decay of the $\Delta(1232)$ considered in Ref. [29], these contributions are expected to allow smaller values for $P_{s\bar{s}}$ than, e.g., Eq. (38) with the present empirical values for μ_s. For example, in the case of the $[31]_X[4]_{FS}[22]_F[22]_S$ configuration, the effect of the transition matrix elements would lead to modification of the expression (38) for μ_s by an additional factor such that

$$\mu_s = \frac{m_p}{2m_s}(1+\delta)P_{s\bar{s}}. \tag{47}$$

Here, the relative modification caused by transition matrix elements is contained in the term δ. Because the *uuds* configuration is a P state, this term would have the general form

$$\delta \sim C_0 m_s \sqrt{\langle r^2\rangle}\sqrt{\frac{P_{uud}}{P_{s\bar{s}}}}, \tag{48}$$

where $P_{uud} + P_{s\bar{s}} = 1$. Here, C_0 is a factor that takes into account the overlap between the wave function for the $uud\gamma$ state and the $uuds\bar{s}$ component of the proton, and $\langle r^2\rangle$ is the mean square radius of the $uuds\bar{s}$ component. The mass factor m_s arises from the $s\bar{s} \to \gamma$ vertex. Provided that these wave functions have the same phase, it follows from Eq. (47) that the empirical value for μ_s sets an upper limit for $P_{s\bar{s}}$. For example, in the harmonic oscillator model employed in Ref. [29], in which $\sqrt{\langle r^2\rangle} = 1/\omega$, the coefficient $C_0 = \sim 0.9$. Since $2m_s \sim m_p$ and in the oscillator model $m_s\sqrt{\langle r^2\rangle} = m_s/\omega \sim 0.5 - 1$, the following approximate relation emerges for μ_s in the $[31]_X[4]_{FS}[22]_F[22]_S$ configuration:

$$\mu_s \simeq P_{s\bar{s}} + (0.45 - 0.90)\sqrt{P_{s\bar{s}} - P_{s\bar{s}}^2}. \tag{49}$$

If this equation is solved for the $s\bar{s}$ configuration probability $P_{s\bar{s}}$ with the mean result for μ_s given by the SAMPLE experiment ($\mu_s = 0.37 \pm 0.2 \pm 0.26 \pm 0.07$) [1] as input, one obtains $P_{s\bar{s}} \simeq 0.10 - 0.20$. By (37), this probability for the $s\bar{s}$ configuration gives $\Delta_s = -(0.03 - 0.07)$, which values fall well within the range of values for this quantity (-0.10 ± 0.06) as determined from the recently enlarged set of inclusive and semi-inclusive polarized deep inelastic scattering data [15]. Note that in the case of Δ_s or G_A^s, the contribution from transition matrix elements relative to the diagonal matrix elements is much smaller than in the case of μ_s.

The inclusion of pentaquark components with the $[31]_X[4]_{FS}[22]_F[22]_S$ configuration not only reproduces well the strangeness magnetic moment and spin of the proton, but also is consistent with the observed excess of $\bar{d}$ over $\bar{u}$ [30] and the quark spin contribution [15] in the proton. In this configuration, only $udud\bar{d}$ and $udus\bar{s}$ pentaquark components are allowed for the proton but no $uduu\bar{u}$ component. The quark wave function for the proton may then be expressed as

$$|p\rangle = A_{3q}|uud\rangle + A_{d\bar{d}}|[ud][ud]\bar{d}\rangle + A_{s\bar{s}}|[ud][us]\bar{s}\rangle, \tag{50}$$

with the normalization condition $|A_{3q}|^2 + |A_{d\bar{d}}|^2 + |A_{s\bar{s}}^2|^2 = 1$. To reproduce the observed [30] light flavor sea quark asymmetry in the proton, $\bar{d} - \bar{u} = 0.12$, one then needs $P_{d\bar{d}} \equiv |A_{d\bar{d}}|^2 = 12\%$; while to reproduce the observed strangeness spin of the proton, $\Delta_s = -0.10 \pm 0.06$, one needs $P_{s\bar{s}} = 12{-}48\%$. With more than 24% of this kind of pentaquark component in the proton, this can reproduce the observed [15] u-quark spin contribution in the proton $\Delta_u = 0.85 \pm 0.17$ as well, since the pentaquark components give no contribution to the Δ_u and $\Delta_u = \frac{4}{3}|A_{3q}|^2$. Instead, they give the contribution $\Delta L_q = \frac{4}{3}(P_{d\bar{d}} + P_{s\bar{s}})$ of the proton spin through the orbital angular momentum of quarks. In Ref. [15], the spin contributions from both $\bar{d}$ and $\bar{s}$ antiquarks were found to be negative, while that from $\bar{u}$ is very uncertain even as to its sign and could be far less polarized than $\bar{d}$ and $\bar{s}$. The pentaquark configuration considered here gives such quark spin contributions, with $-\frac{1}{3}P_{d\bar{d}}$ from $\bar{d}$ and $-\frac{1}{3}P_{s\bar{s}}$ from $\bar{s}$ and zero from $\bar{u}$ antiquarks. The total d-quark spin contribution is predicted to be $\Delta_d = -\frac{1}{3}(1 - P_{s\bar{s}})$, which is only slightly smaller than the lower end of the observed value $\Delta_d = -(0.33 \sim 0.56)$. In Ref. [15], it was also found that the available data do not require the gluon to be polarized.

VI. DISCUSSION

The complete analysis given above of all the positive parity configurations of the form $uuds\bar{s}$ with spin 1/2 and at most one unit of orbital angular momentum reveals that the present empirical signs for μ_s and Δ_s imply that the *uuds* subsystem has to be orbitally excited and that the $\bar{s}$ antiquark is in its ground state. The configurations of the $uuds\bar{s}$ system that can agree with the empirical indications that μ_s is positive and that Δ_s is negative and smaller in magnitude than μ_s are the configurations with $[31]_X[22]_S$ and $[31]_X[31]_S$ that have $J = 1$. These configurations do not include the conventional $K\Lambda^0$ and $K\Sigma^0$ configurations, which yield negative signs for μ_s [17–20].

The configuration $[31]_X[4]_{FS}[22]_F[22]_S$ stands out by the fact that its energy is some 140–200 MeV lower than any other in the flavor-spin interaction model of Eq. (15). Moreover, its energy is more than 240 MeV lower than the lowest energy configuration with the $\bar{s}$ in the P state, which would correspond to the $K\Lambda^0$ loop fluctuations. The lower excitation energy should increase the probability of this configuration as a component in the proton. Two recent diquark cluster models [31,32] proposed for the tentative θ^+ pentaquark correspond to similar *uuds* configurations as $[31]_X[4]_{FS}[22]_F[22]_S$ for the $uuds\bar{s}$ component in the proton with the common feature that the $\bar{s}$ quark is in its ground state and, hence, similar values for the strangeness spin and magnetic moment for the proton [12].

Another interesting point worth noting is that a negative value for the strange electric form factor G_E^s is hinted at by the data [4]. This indicates that strange quarks have a larger rms radius than antistrange quarks in the proton. The $uuds\bar{s}$ configurations that give the empirical signs for the strangeness spin and magnetic moment do have that feature, as the strange antiquark is in the ground state and the *uuds* component is in the orbitally excited state.

The possible smaller-than-expected role of the long-range $K\Lambda^0$ and $K\Sigma^0$ fluctuations, which lead to negative values for μ_s, has recently been analyzed in Ref. [33]. The conclusion drawn in that work is that the main contribution to μ_s from these "chiral loops" arises from their high momentum components, the determination of which *a priori* falls outside of the convergence range of the effective chiral field theory approach. As a consequence, the loop expansion as calculated by effective field theory methods may not give realistic results. Recent numerical QCD lattice results that have been obtained in the quenched approximation have given both positive [10] and negative [34] results for μ_s and, therefore, have not settled the issue of the sign of μ_s.

The diquark cluster configuration or similar $[31]_X[4]_{FS}[22]_F[22]_S$ $uuds$ configurations for the $uuds\bar{s}$ system all lead to positive values for the strangeness magnetic moment, negative values for the strangeness spin or axial form factor, and negative values for the strangeness electric form factor of the proton at low values of momentum transfer. Moreover, they also give a natural explanation for the mass ordering of the $N^*(1440)1/2^+$, $N^*(1535)1/2^-$, and $\Lambda^*(1405)1/2^-$ resonances by admixture of large pentaquark components [28], which is otherwise very puzzling in the conventional $3q$ constituent quark model. In the diquark cluster pentaquark configuration [31,35], the $N^*(1440)1/2^+$ is composed of the $[ud][ud]\bar{d}$ with two $[ud]$ scalar diquarks in the relative P wave, the $N^*(1535)1/2^-$ is composed of the $[ud][us]\bar{s}$ with diquarks in the ground state, and $\Lambda^*(1405)1/2^-$ is composed of the $[ud][us]\bar{u}$ and $[ud][ds]\bar{d}$. The large admixture of $[ud][us]\bar{s}$ in the $N^*(1535)1/2^-$ resonance makes it heavier than the $N^*(1440)1/2^+$ and the $\Lambda^*(1405)1/2^-$ [28] along with a large coupling to the channels involving strangeness, such as $N\eta$ and $K\Lambda$ [36].

All these facts and discussions suggest that there are significant $qqqq\bar{q}$ components in baryons and they may be mainly in colored quark cluster configurations rather than in "meson cloud" configurations.

ACKNOWLEDGMENTS

We thank K. T. Chao, B. Q. Ma, and S. Pate for the suggestion to consider the strangeness spin of the proton and useful discussions. C. S. An thanks S. Zhou for helpful discussions. B. S. Zou acknowledges the hospitality of the Helsinki Institute of Physics during the course of this work. Research was supported in part by the Academy of Finland Grant No. 54038 and the National Natural Science Foundation of China under Grants Nos. 10225525 and 10435080.

[1] D. T. Spayde *et al.*, Phys. Lett. **B583**, 79 (2004).
[2] K. Aniol *et al.*, Phys. Rev. C **69**, 065501 (2004).
[3] F. Maas *et al.*, Phys. Rev. Lett. **94**, 152001 (2005).
[4] D. S. Armstrong *et al.*, Phys. Rev. Lett. **95**, 092001 (2005).
[5] D. H. Beck and R. D. McKeown, Annu. Rev. Nucl. Part. Sci. **51**, 189 (2001).
[6] H. Weigel *et al.*, Phys. Lett. **B353**, 20 (1995).
[7] P. Geiger and N. Isgur, Phys. Rev. D **55**, 299 (1997).
[8] S.-T. Hong, B.-Y. Park, and D. P. Min, Phys. Lett. **B414**, 229 (1997).
[9] V. E. Lyubovitskij, P. Wang, T. Gutsche, and A. Faessler, Phys. Rev. C **66**, 055204 (2002).
[10] R. Lewis, W. Wilcox, and R. M. Woloshyn, Phys. Rev. D **67**, 013003 (2003).
[11] A. Silva, H. C. Kim, and K. Goeke, Eur. Phys. J. A **22**, 481 (2004).
[12] B. S. Zou and D. O. Riska, Phys. Rev. Lett. **95**, 072001 (2005).
[13] A. Airapetian *et al.* (HERMES Collaboration), Phys. Rev. D **71**, 012003 (2005).
[14] B. W. Filippone and X.-D. Ji, Adv. Nucl. Phys. **26**, 1 (2001).
[15] D. de Florian, G. A. Navarro, and R. Sassot, Phys. Rev. D **71**, 094018 (2005).
[16] S. F. Pate, Phys. Rev. Lett. **92**, 082002 (2004).
[17] M. Musolf and M. Burkhardt, Z. Phys. C **61**, 433 (1984).
[18] H. Forkel, F. S. Navarra, and M. Nielsen, Phys. Rev. C **61**, 055206 (2000).
[19] L. Hannelius and D. O. Riska, Phys. Rev. C **62**, 045204 (2000).
[20] X. S. Chen, R. G. E. Timmermans, W. M. Sun, H. S. Zong, and F. Wang, Phys. Rev. C **70**, 015201 (2004).
[21] J. Q. Chen, *Group Representation Theory for Physicists* (World Scientific, Singapore, 1989).
[22] Zhong-Qi Ma, *Group Theory in Physics* (Science Press, Beijing, 1998) in Chinese.
[23] S. Capstick and N. Isgur, Phys. Rev. D **34**, 2809 (1986).
[24] U. Loring, B. Metsch, and H. Petry, Eur. Phys. J. A **10**, 395 (2001); **10**, 447 (2001).
[25] L. Ya. Glozman and D. O. Riska, Phys. Rep. **268**, 263 (1996).
[26] L. Ya. Glozman and D. O. Riska, Nucl. Phys. **A603**, 326 (1996); **A620**, 510 (1997).
[27] D. O. Riska and G. E. Brown, Nucl. Phys. **A653**, 251 (1999).
[28] C. Helminen and D. O. Riska, Nucl. Phys. **A699**, 624 (2002).
[29] Q. B. Li and D. O. Riska, nucl-th/0507008.
[30] G. T. Garvey and J. C. Peng, Prog. Part. Nucl. Phys. **47**, 203 (2001), and references therein.
[31] R. Jaffe and F. Wilczek, Phys. Rev. Lett. **91**, 232003 (2003).
[32] E. Shuryak and I. Zahed, Phys. Lett. **B589**, 21 (2004).
[33] J. Donoghue *et al.*, Fizika **14**, 217 (2005).
[34] D. B. Leinweber, S. Boinepalli, I. C. Cloet, A. W. Thomas, A. G. Williams, R. D. Young, J. M. Zanotti, and J. B. Zhang, Phys. Rev. Lett. **94**, 212001 (2005).
[35] A. Zhang *et al.*, High Energy Phys. Nucl. Phys. **29**, 250 (2005).
[36] B. C. Liu and B. S. Zou, Phys. Rev. Lett. **96**, 042002 (2006).