Time-dependent aspects of the semiclassical approach in the analysis of heavy ion reactions

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The semiclassical approach is one of the fundamental tools used to investigate and analyze heavy ion reaction processes. We show in this contribution that the evolution of the motion that emerges from the knowledge of the semiclassical amplitudes for the reaction channels as a function of time cannot be taken as a reliable representation for an actual wave function of the intrinsic states.

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I. INTRODUCTION

A large variety of nuclear reaction processes have been (and continue to be) studied with the aid of semiclassical time-dependent formalisms. In these, one blends a classical description of the relative motion of the colliding ions with a quantum-mechanical treatment of the excitation of the intrinsic degrees of freedom of the systems involved [1,2]. To deal with the intrinsic part of the formulation, one introduces specific reaction channels which can be put in correspondence with observables such as angular distributions and cross sections, that is, structural information that conforms to the particular nuclear model that one is applying and/or willing to test.

The microscopic construction of the coupling matrix elements between these two components of the scheme—relative and intrinsic motion—are calculated statically and yield form factors that are functions of the relative distance between the projectile and the target $\vec{R}$. It is then clear that an essential element of the semiclassical approach is to convert that spatial dependence into a function of time by the identification of a suitable "trajectory" of relative motion $\vec{R}(t)$.

With such framework in mind, one introduces the expansion of the time-dependent intrinsic wave function

$$|\Psi(t)\rangle = \sum_n a_n(t)|\Phi_n(t)\rangle \tag{1.1}$$

on the basis of the chosen, relevant eigenchannels $|\Phi_n(t)\rangle$ and arrives at the familiar semiclassical equations for the amplitudes $a_n(t)$

$$\frac{da_n(t)}{dt} = -\frac{i}{\hbar}\sum_m F_{nm}(t)\exp\left[\frac{i}{\hbar}(\epsilon_n - \epsilon_m)t\right]a_m(t), \tag{1.2}$$

where the form factors $F_{nm}(t)$ acquire their time dependence through the aforementioned trajectory of relative motion.

The function $\vec{R}(t)$ can be chosen, under especial circumstances, to be independent of the coupling to the internal degrees of freedom. For instance, at relativistic bombarding energies—where the energies of the intrinsic transitions are negligible compared to the beam energy—it is safely assumed that the trajectories are uniform, rectilinear, and essentially unaffected by the excitation process. At the other extreme of the scale, for Coulomb excitation at energies well below the barrier, one finds also many applications where the orbits are approximated by "unperturbed" Rutherford hyperbolae with the familiar correspondence between impact parameter and scattering angle.

There are occasions, however, when the average transfer of energy and angular momentum from the relative motion into the intrinsic degrees of freedom is large enough to prompt an improvement of the function $\vec{R}(t)$. The idea is to make use of conservation laws along the time and adjust, accordingly, the trajectory of relative motion by the introduction of effective damping forces. This procedure is crucial to the treatment of deep inelastic collisions in which the transfer of energy and angular moment can reach values of hundreds of MeV and dozens of , respectively. However, it is also done quite often in the case of ordinary Coulomb excitation mentioned earlier, where appropriate adjustments (symmetrization) of the hyperbolae take into account the finite energies and spins of the excited levels.

In the implementation of these procedures, it is tacitly implied that the expression (1.1) is valid not only asymptotically for $t \to \infty$ but also at all times in between. Actually, in most situations, *there is no way* to generate an alternative time evolution of the intrinsic wave function (1.1) since the problem is fully specified by the choice of Hamiltonian. This identification becomes crucial whenever theories are put forward for processes that occur in a scale of time comparable to that of the collision itself. In such case, a precise knowledge of the state of excitation at all times is a necessary ingredient of the scheme from either the classical or semiclassical points of view; to this end, the quantity $P_n(t) = |a_n(t)|^2$ is taken to represent the actual probability of the channel n to be populated at time t. It has assumed this physical meaning in preequilibrium theories [3,4], in the introduction of effective damping forces [5], and—not to be overlooked—in shaping our intuition about how this valuable tool for the analysis of nuclear reactions does work.

Another related topic that we would like to call attention to is the following. We know that it is possible to solve unambiguously the classical equations of motion associated with a given problem in terms of the acting forces. On the other hand, Schroedinger's quantal equations (which evolved from Hamilton's formulation of classical mechanics) are written down making use of the corresponding Hamiltonians. In most familiar circumstances, the potentials are specified only up to an arbitrary constant and, under these conditions, questions about the invariance of the time evolution of a reaction process generated by the semiclassical formalism can be conveniently sidestepped. Is it always that simple?

Keeping these considerations in mind, we chose to explore these fundamental issues in a deliberate context: The relativistic Coulomb excitation of the giant dipole resonance (GDR) in nuclei [6–12]. As we know, this regime brings into the construction of the couplings the scalar and vector potentials Φ and $\vec{A}$ [1]. Thus, contrary to most other situations, the formulation of this specific problem allows us to take a diversity of versions of the scalar and vector potentials as long as they are related by the gauge transformation

$$\begin{aligned} \Phi' &= \Phi - \frac{1}{c}\frac{\partial \Lambda}{\partial t}, \\ \vec{A}' &= \vec{A} + \vec{\nabla}\Lambda, \end{aligned} \tag{1.3}$$

where Λ is any arbitrary function of the space coordinates and time. It is clear that this freedom in the selection of the Hamiltonian for the Coulomb excitation problem has no counterpart at the level of classical forces since these are uniquely specified. To be concrete, the electromagnetic scalar and vector potentials may be selected in a large variety of ways, as it follows from Eqs. (1.3), but the Lorentz force acting on a charge q is given, for all choices, by the same expression, namely,

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}). \tag{1.4}$$

Two immediate questions that emerge are (i) is the time evolution of the wave function (1.1) independent of the choice of Λ, and (ii) does the average excitation energy derived from the semiclassical equations coincide at all times with the classical result, as one has been conditioned to expect?

In the following, we proceed to see what can be learned from posing these questions in the context of the problem at hand. We will limit ourselves to weak-coupling situations that allow for an appropriate truncation of the intrinsic space to the zero-, one- and two-phonon states. This limitation is quite important to stress. We are *only* interested in instances in which the results of the calculations emerge from a correct formulation and are numerically reliable throughout. A presentation on a breakdown of gauge invariance caused either by neglecting terms of the equations that should be there or by an excessive truncation of the space is, in our opinion, unnecessary.

In this contribution, we will show that there are indeed instances where the time-evolution of the intrinsic wave function *is not unique*-an outcome that goes beyond the introduction of an overall, trivial, multiplicative phase. Also, we will see that there are considerable differences between the classical motion and the quantal evolution of the system predicted by the semiclassical approximation. This realization challenges the common belief expressed earlier in the Introduction concerning the interpretation of the wave function (1.1). explicitly given as a function of time. When this expansion in the unperturbed channels is introduced in textbooks (cf., e.g. Ref. [13]) it is only natural to assume that it represents the evolution of the intrinsic state at all times.

We have just advanced a couple of reasons to be wary about using the semiclassical approximation uncritically. It seems reasonable, however, to expect some sort of stability in the physical predictions that emerge from fundamentally equivalent but apparently conflicting representations of the state vector at intermediate times. We will provide an answer to this puzzle but, nevertheless, show that some uncertainty in the interpretation of semiclassical solutions of the scattering problem does remain. An interesting discussion on the connection between quantal and semiclassical aspects of scattering processes has recently appeared in Ref. [14].

The paper is organized as follows. Section II introduces the set of semiclassical coupled equations that need be solved when $\Lambda \neq 0$. We devote Sec. III to a preliminary analysis of the situation in which we present simple arguments to prepare the reader about what to expect. Section IV illustrates, with selected examples, the way in which the gauge transformation affects the probabilities of excitation of the different channels as a function of time. This is done first for a purely harmonic mode and then, also, for a mode in which the energies of the levels and the coupling matrix elements deviate considerably from the harmonic limit. In Sec. V, we obtain the time dependence of the average excitation energy in the equivalent classical problem and compare it with the results obtained in the preceeding section. A discussion on the manifestation of these effects outside of the relativistic regime is also included. We close in Sec. VI with a brief summary of the work and our conclusions.

II. GAUGE TRANSFORMATION

We recall in this section the formalism used to solve—within the semiclassical approximation—the problem of relativistic Coulomb excitation of the giant dipole resonance. The calculation setup is, in many respects, the same one described in Ref. [15]. To the coupled-channel equations given there, we just append the additional terms that allow us to check the consequences of a gauge transformation such as the one shown in Eqs. (1.4).

The projectile moves in the y direction, and the classical trajectory of relative motion is contained in the (x, y) plane. It is then a good approximation to retain only the Cartesian components of the dipole mode along these orientations, leading to a coupling scheme, as indicated in Fig. 1. Here, the channels are labeled by the pair of integer numbers (n_x, n_y), the number of quanta excited in each direction in the harmonic limit. To explore the effects of possible anharmonicities, we have generalized the scheme of Ref. [15] and introduced a factor f (cf., Fig. 1) that will be allowed (in Sec. IV) to

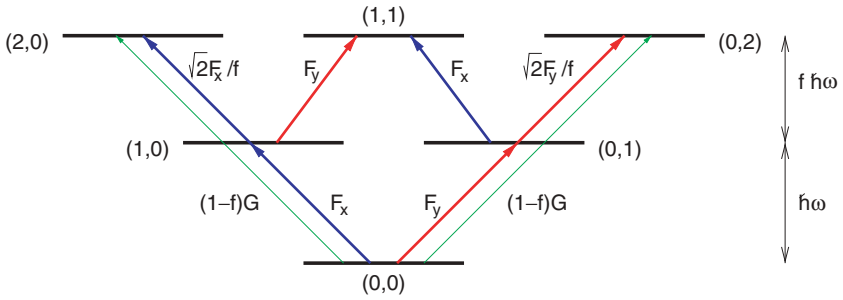

FIG. 1. (Color online) Schematic representation of the spectrum of levels retained for the semiclassical calculation of the reaction amplitudes. The channel labels are the pair of integer numbers (n_x, n_y) that correspond to the number of phonons in the x, y directions, i.e., those defining the reaction plane. Notice that for $f = 1$, the energies and matrix elements reduce to the case of a pure harmonic mode.

deviate slightly from 1. This changes the spectrum of excitation energies and the magnitude of the couplings sufficiently to invalidate the quite special properties that apply only to the case of a pure harmonic mode. The procedure thus mocks up anharmonic conditions and will be useful for checking the general character of our conclusions. It also emphasizes the advantage of solving the problem numerically instead of relying on analytic expressions of limited validity.

The model Hamiltonian describing the excitation of the intrinsic degrees of freedom by the electromagnetic field generated by the moving charge is (cf., e.g. [15,16])

$$H = \frac{1}{2D}\left(\vec{p} - \frac{q}{c}\vec{A}\right)^2 + q\Phi + \frac{1}{2}\,C\,r^2, \tag{2.1}$$

where C and D are the restoring force and mass parameters of the GDR and Φ and $\vec{A}$ are the Liènard-Wiechert scalar and vector potentials

$$\Phi(\vec{r}, t) = \frac{\gamma Z_p e}{\sqrt{(x-b)^2 + \gamma^2(y - vt)^2 + z^2}} \tag{2.2}$$

and

$$\vec{A}(\vec{r}, t) = \frac{v}{c}\,\Phi(\vec{r}, t)\frac{\vec{y}}{|y|}. \tag{2.3}$$

The Hamiltonian (2.1) was used in [15] to construct the coupling matrix elements between the intrinsic states and to set up the time-dependent semiclassical coupled-channel equations. Following the gauge transformation (1.3), the operator assumes the new form

$$H \to H' = \frac{1}{2D}\left(\vec{p} - \frac{q}{c}\vec{A} - \frac{q}{c}\vec{\nabla}\Lambda\right)^2 + q\Phi - \frac{q}{c}\frac{\partial\Lambda}{\partial t} + \frac{1}{2}\,Cr^2, \tag{2.4}$$

which can be recast in a form that explicitly separates the contributions associated with the gauge transformation, namely,

$$H' = H - \frac{q}{2mc}\left(\vec{p} - \frac{q}{c}\vec{A}\right)\vec{\nabla}\Lambda - \frac{q}{2mc}\vec{\nabla}\Lambda\left(\vec{p} - \frac{q}{c}\vec{A}\right) + \frac{q^2}{2mc^2}(\vec{\nabla}\Lambda)^2 - \frac{q}{c}\frac{\partial\Lambda}{\partial t}. \tag{2.5}$$

As in our previous paper, we can expand the Hamiltonian (2.5) to lowest order in powers of x, p_x, y, p_y. The explicit expressions are given in the appendix. From this expansion, one can easily obtain the radial matrix elements among the different harmonic oscillator states and then proceed to solve the coupled equations (1.2).

III. PRELIMINARY ANALYSIS

One expects that different choices of physically equivalent gauges should, in the solution of a given problem, lead to identical answers. But to *which* questions? This is an important point to determine. In this section, we will try to shed light on this issue by way of some theoretical considerations and within a specially simple context.

Let us start by noticing that in order to set appropriate boundary conditions for $t \pm \infty$ (or, equivalently, for the relative motion variable $|\vec{R}| \to \infty$), one must choose the function Λ so that it vanishes in those limits. Under these conditions, it is easy to anticipate that the final probability of populating the various channels will be independent of Λ. In fact, consider the state vector $|\Psi(t)\rangle$, a solution to the Schroedinger equation

$$H|\Psi(t)\rangle = i\hbar\frac{\partial|\Psi(t)\rangle}{\partial t}, \tag{3.1}$$

where H is the Hamiltonian (2.1). It is well known that the solution to the gauge-transformed Hamiltonian (2.5) can be obtained from the former solution by a simple operation, namely,

$$|\Psi'(t)\rangle = \exp\left[i\,\frac{q\Lambda(x, y, t)}{\hbar c}\right]|\Psi(t)\rangle. \tag{3.2}$$

It follows from this expression and the character of Λ that

$$|\Psi'(t = \infty)\rangle \equiv |\Psi(t = \infty)\rangle, \tag{3.3}$$

and, consequently, the asymptotic complex amplitudes for each and all the states of the GDR are identical.

We now turn to the question of time evolution. The state $|\Psi(t)\rangle$ may be expanded in the basis $|n_x, n_y\rangle$ of the dipole mode in Cartesian components as

$$|\Psi(t)\rangle = \sum_{n_x, n_y} a_{n_x, n_y}(t)\exp\left[-\frac{i}{\hbar}\epsilon_{n_x, n_y}t\right]|n_x, n_y\rangle, \tag{3.4}$$

where ϵ_{n_x, n_y} are the energy eigenvalues for the stationary states labeled by n_x, n_y. To make things easier, let us for the moment consider the particular case where Λ during the entire time evolution of the system fulfills $|q\Lambda/\hbar c| \ll 1$. Making use of

Eq. (3.2), the solution for the gauge-transformed potentials can be approximated by

$$|\Psi'(t)\rangle \approx \left[1 + i\frac{q\Lambda(x,y,t)}{\hbar c}\right]|\Psi(t)\rangle \approx \left[1 + i\frac{q}{\hbar c}(\Lambda(0,0,t) + \frac{\partial\Lambda}{\partial x}(0,0,t)x + \frac{\partial\Lambda}{\partial y}(0,0,t)y)\right]|\Psi(t)\rangle. \quad (3.5)$$

Here, we have expanded the operator to lowest order in x, y to be in tune with the situations covered in Ref. [15] and attentive to the fact that the expectation values and matrix elements of x, y are indeed sufficiently small to justify the dipole approximation (cf., e.g., Fig. 2 of [15]). This is, of course, also consistent with being close to the perturbation limit and having the configuration space properly truncated to the first three steps of the harmonic ladder.

By inspection of Eq. (3.5), one can see that the operators x and y have the effect of changing the states $|n_x, n_y\rangle$ in the right-hand side of (3.4) into combinations of $|n_x \pm 1, n_y\rangle$ and $|n_x, n_y \pm 1\rangle$, respectively. In a nonlinear case, the admixture of phonon numbers caused by x^2, y^2, xy terms, or higher order, would reach even farther from plus or minus 1. All of this amounts to a reshuffling of the amplitude labels leading to a situation in which the amplitude $a'_{n'_x,n'_y}$ turns out to be a combination of terms involving unprimed amplitudes a_{n_x,n_y} in several channels $|n_x, n_y\rangle$ close to but other than $|n'_x, n'_y\rangle$. To be concrete, in the harmonic and linear case and for the lowest three channels, one gets

$$\begin{aligned}
a'_{0,0}(t) &\approx \left(1 + \frac{iq}{\hbar c}\,\Lambda(0,0,t)\right)a_{0,0}(t) + \frac{iq}{\hbar c}\,\beta w \times \left[\frac{\partial\Lambda}{\partial x}(0,0,t)a_{1,0}(t) + \frac{\partial\Lambda}{\partial y}(0,0,t)a_{0,1}(t)\right],\\
a'_{1,0}(t) &\approx \left(1 + \frac{iq}{\hbar c}\Lambda(0,0,t)\right)a_{1,0}(t) + \frac{iq}{\hbar c}\,\beta\left[\frac{\partial\Lambda}{\partial x}(0,0,t)(a_{0,0}(t)/w + \sqrt{2}\,w a_{2,0}(t)) + \frac{\partial\Lambda}{\partial y}(0,0,t)a_{1,1}(t)\,w\right],\\
a'_{0,1}(t) &\approx \left(1 + \frac{iq}{\hbar c}\Lambda(0,0,t)\right)a_{0,1}(t) + \frac{iq}{\hbar c}\beta\left[\frac{\partial\Lambda}{\partial x}(0,0,t)a_{1,1}(t)\,w + \frac{\partial\Lambda}{\partial y}(0,0,t)(a_{0,0}(t)/w + \sqrt{2}\,w a_{0,2}(t))\right], \quad (3.6)
\end{aligned}$$

where β is the deformation parameter associated with the GDR, ω its characteristic frequency, and $w = \exp(-i\omega t)$.

Let us now look at the following interesting situation: Assume that there are no electric or magnetic fields at all and $\Phi = 0$, $\vec{A} = 0$ (for instance, set the projectile charge to zero). One can still consider a gauge transformation in this case and conclude that all scalar and vector potentials of the form

$$\begin{aligned}
\Phi' &= \frac{1}{c}\frac{\partial\Lambda}{\partial t},\\
\vec{A}' &= \vec{\nabla}\Lambda,
\end{aligned} \quad (3.7)$$

should be associated with the absence of any true, physically meaningful, electromagnetic excitation. The state vector $|\Psi(t)\rangle$ in the absence of any coupling between relative and intrinsic motion has coefficients

$$\begin{aligned}
a_{0,0}(t) &\equiv 1,\\
a_{n_x,n_y}(t) &\equiv 0 \text{ otherwise.}
\end{aligned} \quad (3.8)$$

Leaving aside the apparent complexity of Eqs. (3.6), notice that the semiclassical approximation applied to scalar and vector potentials of the form (3.7) would then yield, along the time t, nonzero probabilities to populate the states $|1,0\rangle$ and $|0,1\rangle$ (and also others)! The excitation probability should be zero for $t \to \infty$, as concluded earlier, but the time evolution of this quantity appears to be as arbitrary as the choice of Λ itself.

It is quite clear that the conclusions of this analysis do not depend on having exploited the small values of $|q\Lambda/\hbar c|$, the linear expansion in (3.2), or the peculiar form of the interaction potentials given in (3.7). Thus, one should be prepared to also find evidence of these intriguing aspects of the semiclassical approximation in the more general numerical examples that follow.

IV. RESULTS OF THE CALCULATIONS

The most frequently used choices of the electromagnetic potentials Φ, $\vec{A}$, belong to the so-called Coulomb or Lorentz gauges. Our choices (2.2) and (2.3) for the scalar and vector potentials, for example, satisfy the requirements of the Lorentz gauge. For the purpose of our presentation, there is nothing special about these and we do not need—or even want—to restrict the choices of the function Λ to keep ourselves within a determined family. This, of course, can be done. If we wanted to work within the Lorentz gauge, for instance, we would pick for Λ solutions of the equation $\nabla^2\Lambda - (1/c^2)\partial^2\Lambda/\partial t^2 = 0$. Similarly, starting from fields satisfying the Coulomb gauge, any choice of Λ as a solution of the equation $\nabla^2\Lambda = 0$ will keep us within the same gauge. Explicit expressions for Λ are known that allow the particular transformation from Lorentz to Coulomb gauges [17].

For the sake of generality, we prefer here to take advantage of the full freedom in the choice of Λ to explore effective Hamiltonians of different strength and interaction range. The idea is to infer, from the results of sample calculations, what is (or is not) preserved along the excitation process. In the course of this investigation, we used various analytic functions (each one defined with several adjustable parameters) to test the response of the formalism. We arrived at the same conclusions in all instances, indicating that the specific form of Λ is not crucial to the line of argument presented here.

In this section, we specifically show results obtained for

$$\Lambda(x,y,t) = \frac{R_g}{[(x - b_g)^2 + \gamma^2(y - vt)^2]^{\alpha_g}}. \quad (4.1)$$

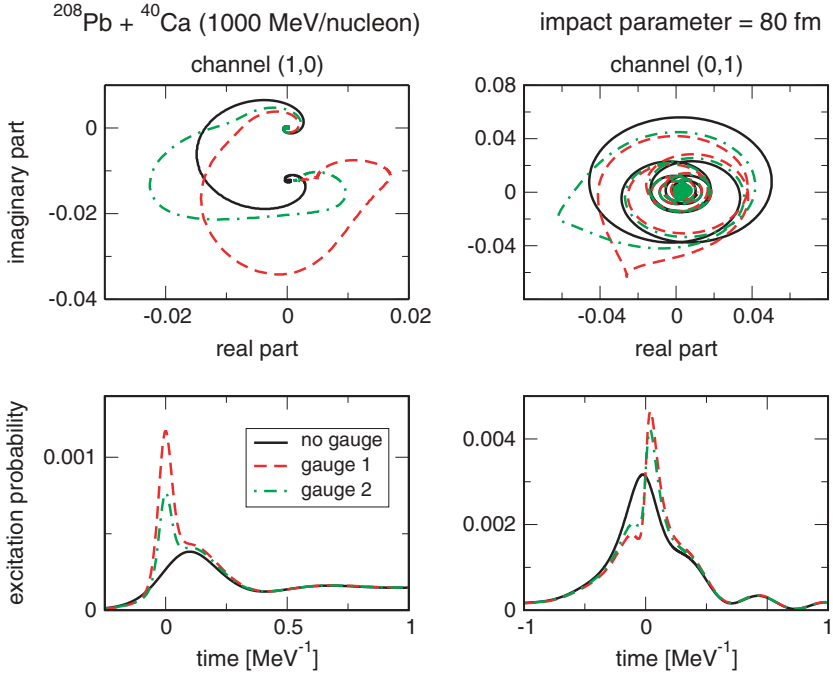

FIG. 2. (Color online) Top: time evolution of the coupled-channel amplitudes $a_{1,0}(t)$ (left) and $a_{0,1}(t)$ (right) represented parametrically in the complex plane. The amplitudes start from the origin for $t = -\infty$ and achieve their asymptotic value for $t = \infty$ after following a certain trajectory on the complex plane. The calculations are for a bombarding energy of 1000 MeV/nucleon and an impact parameter of 80 fm. Three situations are displayed: no gauge ($R_g = 0$), gauge 1 ($R_g = 100, b_g = 24$ fm, $\alpha_g = 0.4$), and gauge 2 ($R_g = 50, b_g = 24$ fm, $\alpha_g = 0.6$). Bottom: time evolution of the excitation probabilities $|a_{1,0}(t)|^2$ and $|a_{0,1}(t)|^2$. The key to all curves is given only once, in the lower-left frame. In all figures, time is expressed in terms of $t/\hbar$.

The particular form of Eq. (4.1), which is similar to that of other relevant couplings in our problem, is convenient because its magnitude at closest approach is controlled by a simple combination of R_g and b_g and its range by α_g. As mentioned above, the actual values of these parameters do not play a significant role in our analysis. Nevertheless they are quoted for each illustration, relegating the information to the corresponding figure caption. To keep the function Λ finite at large distances, we only considered positive values of α_g.

We show in Fig. 2 results for the Coulomb excitation of the GDR in the nucleus ^{40}Ca when bombarded by ^{208}Pb at 1000 MeV/nucleon. The excitation energy of the mode is taken to be $\hbar\omega_1 = 11.6$ MeV (see Refs. [12,15]). In the upper-left corner we display how the real and imaginary components of the complex number $a_{10}(t)$ evolve for an impact parameter $b = 80$ fm and Λ specified as indicated in the caption. The three different curves correspond to the solution obtained with the potentials Φ, $\vec{A}$ as in Ref. [15] (no gauge) and two different cases of gauge-transformed potentials (gauge 1, gauge 2). Since this is a parametric representation, the variable t is not observable in this frame. Notice the very different excursions that the amplitude of the channel (1, 0) performs in the complex plane depending on the choice of Λ. The end of the "promenade" is, however, in all cases identical (as we expected).

A similar situation is displayed in the upper-right frame for the amplitude $a_{01}(t)$. Here, the curves are quite complicated because of the predominance of the coupling term proportional to p_y, associated, as we shall see, with a longer interaction range. Although it is not possible to discern the asymptotic values in this picture, an examination of the numbers shows clearly that also for this channel the final state of excitation is exactly the same, regardless of the gauge employed.

The time evolution of the quantities $P_{1,0}(t) = |a_{1,0}(t)|^2$ and $P_{0,1}(t) = |a_{0,1}(t)|^2$ is shown in the lower half of the figure, just below the frames corresponding to the complex amplitudes for each channel. The time evolution for the excitation probability $P_{0,1}(t)$ is now easier to interpret than the information provided for the complex amplitude. The sudden character of the excitation mechanism for both channels at the chosen relativistic energy is quite evident.

The example displayed in Fig. 2, repeated for other sets of parameters R_g, b_g and α_g, many impact parameters, and various functional forms of Λ, leads to the following conclusion: Alternative calculations, independent of the gauge chosen, yield the same final probability of excitation for all the channels. Their time evolution, however, *is not unique*. Notice that it would have been impossible, by means of a few illustrations, to prove the opposite of this latest contention. A few counter-examples suffice, on the other hand, to sustain the claim.

It can be argued that the invariance of the asymptotic results with respect to gauge transformations is to be expected because of the pure harmonic character of the GDR mode that we have been assuming so far. This is represented in Fig. 1 as the coupling scheme that results from setting $f = 1$. Other values of f close to 1, remember, provide mock-up conditions of anharmonicity. For instance, $f = 0.9$ lowers somewhat the excitation energy for the transitions from the one- to two-phonon levels, increases the form factor between them, and introduces a small direct coupling between the ground state and the second layer of excited states.

The information contained in Fig. 3, which is in all respects the analog of Fig. 2 but for anharmonic conditions, shows that the conclusions drawn earlier did not rely on having a pure harmonic mode in the linear coupling limit. A value $f = 0.9$ was chosen for this illustration and the parameters b and b_g were changed to 100 and 30 fm, respectively. We have thus taken advantage of this new example to exhibit the general character of the results. (For simplicity, F and

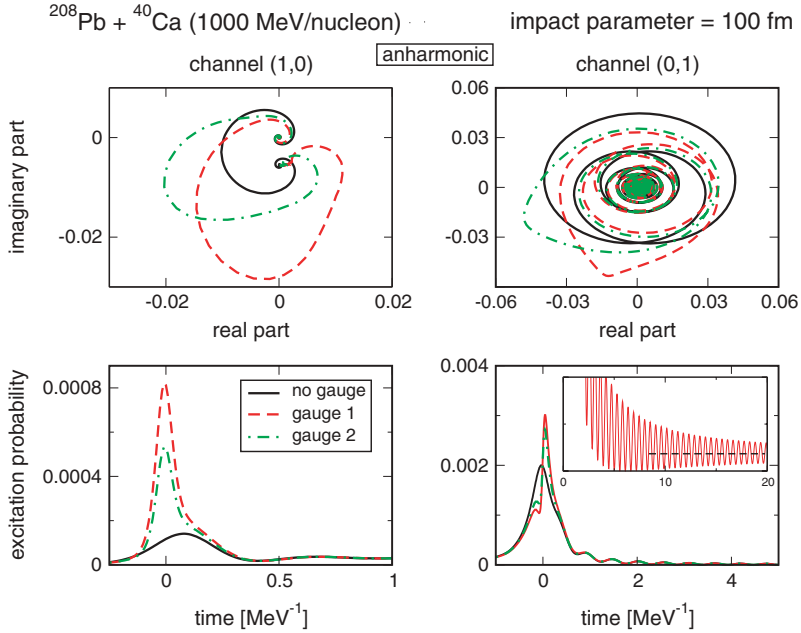

FIG. 3. (Color online) Analogous in all respects to Fig. 2, but for the case in which the presence of anharmonicities is simulated by a factor $f = 0.9$. The values of the parameters used in this illustration are identical to those quoted in Fig. 1. Sole exceptions are the impact parameter $b = 100$ fm and $b_g = 30$ fm. Inset shows in arbitrary scale the behavior of the function for a much larger interval of time as it approaches a finite, nonzero limit (dashed line).

G have been arbitrarily assigned the same radial dependence; a better formulation incorporates in the Hamiltonian (2.1) an anharmonic term and its effects are calculated consistently, as was done in Ref. [18]). There is indeed a finite final excitation energy in the channel (0,1) for which the sudden character of the collision is most favorable. To appreciate this, however, we have been forced to display the function multiplied by a large factor. This is shown in the inset, were the probability in arbitrary units is followed over a much larger interval of the time variable. The dashed line represents here the excitation probability after, waiting sufficiently long, it reaches a constant (nonzero) asymptotic value.

We show in Fig. 4 the excitation probabilities as a function of time that correspond to the case in which we take a projectile with the mass corresponding to lead but with zero charge. This recreates the hypothetical situation we used for the sake of argument in Sec. III. We can see that the resulting probabilities reflect the analytical expressions (3.6) derived above and help us understand the features of the probability curves resulting from the coupled-channel equations and displayed in Figs. 2–3.

Conditions of weak coupling—which can always be enforced by increasing sufficiently the impact parameter—are indeed prevalent for the values of b used in Figs. 2–3. In fact, the adequacy of the truncation of the basis of intrinsic states has been checked throughout (for instance, extending the set of channels to include all three-phonon levels).

V. CLASSICALLY ANALOGOUS PROBLEM

We now turn to a comparison between the time evolution of the system from a quantal point of view, as it was worked out in the previous section, and the one emerging from the solution of the classically equivalent problem. In Ref. [15], a very compact computer code RCE was distributed that does precisely this, exploiting the analytic expressions of the electric and magnetic fields generated by the relativistically moving projectile [16]. With such a tool, supplemented by the quantization rules already used in [15], we can calculate the classical probabilities of excitation of the giant dipole resonance, namely, $P_{\mathrm{GDR}}(t) = P_{10}(t) + P_{01}(t)$, and compare our results with what was presented earlier. This is done in Fig. 5. Only the "no-gauge" results are shown, but one should keep very much in mind that it is possible to produce an arbitrary number of different quantal curves by changing the function Λ. We appreciate in the figure that the classical and quantal values

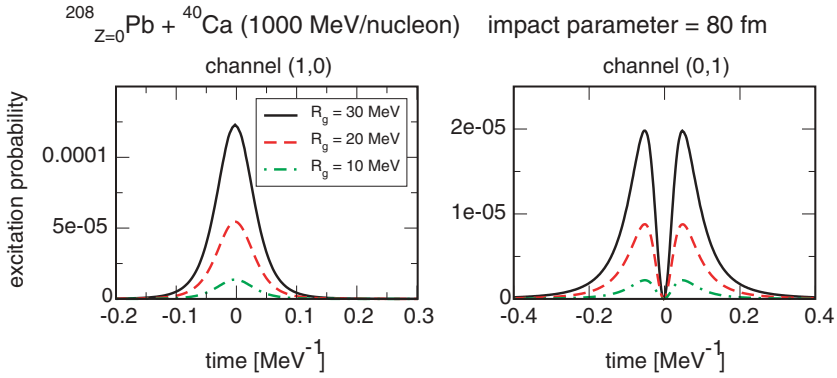

FIG. 4. (Color online) Probabilities as a function of time for the fictitious excitation of the two one-phonon channels, with the charge of lead arbitrarily set to zero. The characteristics of these curves, seen in the light of the analytic formulas for the amplitudes derived in Sec. III, allow us to understand the results found in Figs. 2–3.

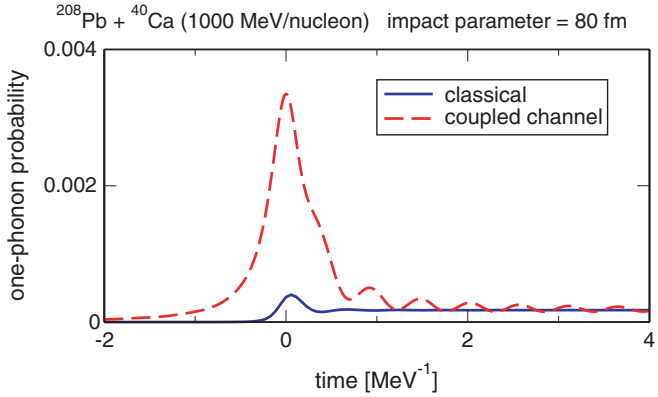

FIG. 5. (Color online) Time evolution of the probability of populating the giant dipole resonance [i.e., $P_{\rm GDR}(t) = P_{1,0}(t) + P_{0,1}(t)$] in the reaction ^{208}Pb + ^{40}Ca, impact parameter $b = 80$ fm. Solid line results from solving the classical equations of motion in the presence of the electric and magnetic fields created by the projectile moving with a relativistic velocity $v_p \approx c$. Dashed curve shows the outcome of the semiclassical approximation for the "no-gauge" situation already encountered in Figs. 2–3.

for $t \to \infty$ coincide, as we already knew from the analysis of Ref. [15]. However, the time dependence is quite different. Notice also that the asymptotic value of the probability is reached much more efficiently with the classical analysis than by making use of the semiclassical quantal amplitudes.

In retrospect, the time evolutions that came from the semiclassical formalism are somewhat odd. In fact, the oscillations in the probabilities for the one-phonon channels—especially the (01)—shown in Figs. 2 and 3 mean that there is a sustained "exchange" of probability between the ground state and the GDR (also observed by the authors of Ref. [17]) that keeps taking place long after there is any apparent justification for such trade-off to occur.

Let us expand on this a bit further. The electric operators that govern Coulomb excitation for a multipolarity λ have the familiar radial dependence $R^{-\lambda-1}$. For the GDR, one then expects a cutoff of the couplings for large distances that is R^{-2}. In the relativistic formulation, however, the presence of the vector potential in $\left(\vec{p} - (q/c)\vec{A}\right)^2$ ends up introducing, in our context, a coupling term of the form $\Phi\, p_y$. The radial dependence of such a term is R^{-1}, which is responsible for the much longer interaction range noted above. It also accounts for the rapid oscillations, because the frequency of the probability exchange shown in Figs. 2 and 3 corresponds precisely to that of the harmonic motion. Regardless of this "explanation," an indisputable fact is that the relativistic electric and magnetic fields $\vec{E}$, $\vec{B}$, have both an asymptotic radial dependence R^{-2} [16]. Beyond these considerations, the semiclassical time evolution of the probabilities displayed in Figs. 2, 3, and 5 as we have already stressed, is not even unique. Confronted with this situation, one is practically compelled to give up making sense out of any particular choice; we will present some other arguments to this effect in Sec. VI.

The classical evolution is, on the other hand, a well-defined problem. Actually, for "perturbative" conditions where $\langle E \rangle \ll \hbar\omega$, what we are plotting in Fig. 5 is just proportional to the average excitation energy of the mode, and it makes perfect sense that this quantity achieves its asymptotic constant value right after the intrinsic and relative components of the motion get effectively decoupled.

We should now mention that although we called the reference curves "no gauge" they *do* correspond to a specific choice. In fact, the potentials of Ref. [15] given in (2.2) and (2.3) are no other than the pure Lièнard-Wiechert expressions, belonging to the Lorentz gauge. Given the results shown in Fig. 5, one wonders if some special gauge transformations could be found to bring the coupled-channel probabilities as close as possible to the compact, classical predictions.

Let us finish this section with one last elaboration. We are accustomed to classifying as "relativistic collisions" those in which the energy per nucleon becomes comparable with the rest energy of either neutrons or protons. Thus, reaction formalisms are not normally adapted to conform to a relativistic regime until bombarding energies reach to about $\approx$400 MeV/nucleon.

The aspects of the semiclassical picture that we have brought into focus in this contribution, however, become relevant whenever magnetic effects are not negligible. But the usual criterion to ignore effects related to the vector potential $\vec{A}$ is that $v \ll c$. Notice that this condition *is not fulfilled even in what we call low-energy nuclear reactions*. For instance, in the reaction Ca + Pb at 50 MeV per nucleon, one can estimate $v/c \approx 0.3$. Thus, before even checking with an actual calculation, one would expect that the differences in range and the rapid oscillations in the time evolution predicted by the semiclassical approximation will be present also at traditional low-bombarding energies, i.e., *outside* of a strictly relativistic reaction regime.

The calculation shown in Fig. 6 confirms this anticipation. Of course, the uniform, rectilinear assumption concerning the trajectory of relative motion would be questionable in this limit, but it would not invalidate the lesson we draw. One

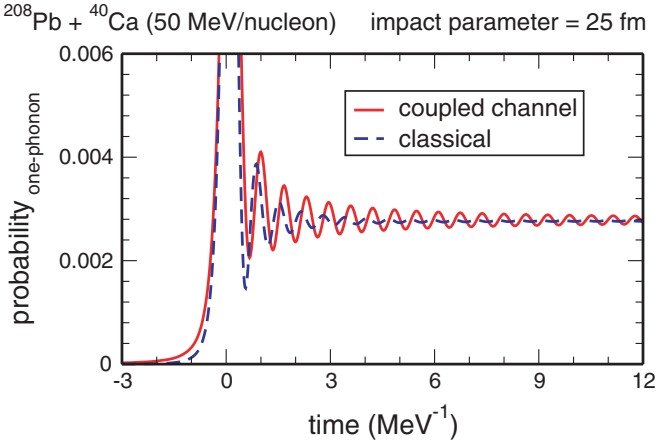

FIG. 6. (Color online) Analogous to Fig. 5, but at a relatively low bombarding energy of 50 MeV/nucleon and impact parameter $b = 25$ fm. The classical and coupled-channel results are in better overall agreement, but the persistence of long-range oscillations in the quantal probability remains in full evidence.

can entertain the idea of a hypothetical problem in which, by some combination of forces, the relative motion may indeed be constrained to stay on a straight line.

VI. SUMMARY AND CONCLUSIONS

A nuclear collision is easily visualized as a time-dependent process in which a projectile approaches a standing target and undergoes the particular sequence of steps which characterizes or defines the reaction in question. It was obvious from the very beginning, however, that enforceable initial conditions cannot conform to this intuitive idea. In fact, it is not feasible to send a single projectile to the awaiting target. The experimental situation corresponds rather to that generated by an accelerator beam whose section across is of human scale (i.e., infinite insofar as nuclei are concerned) and which carries an electrical current usually in the order of, say, microamperes. This amounts to an enormous number of impinging particles per unit time and per unit area.

These circumstances may appear at first inconvenient in terms of the original, simple, time-dependent visualization. But they actually made possible an extraordinary simplification in the formal treatment of the scattering problem. It is easy to realize that the boundary conditions we mentioned above are best embodied in an "incident" plane wave (whose normalization is tied to the incoming flux of particles) that represents the continuous, uniform, and uninterrupted arrival of projectiles over a very wide front perpendicular to the beam direction. The situation thus becomes, for all practical matters, *stationary*, and one searches for the scattering amplitudes $f_n(\theta, \varphi)$ by solving the Schroedinger equation *independent of time*. Notice, then, that in such a formulation, which the semiclassical treatment aims to approximate, nowhere is there a variable "time" to be considered. Neither ate there "impact parameters" to be concerned with, since the incident plane wave results from a coherent combination of all partial waves ℓ.

Why is it, then, that expressing the solution to our problem in terms of the time evolution of some set of equations for a given impact parameter does not make us uneasy? Let us just say that we are by now so accustomed to approaching the scattering problem with the concept of "test" particles that we no longer pause to reflect that neither of those physical quantities, time t or impact parameter b, are true observables from a quantal point of view. Yet, and as a consequence, no meaningful or unique answers should necessarily be expected from the calculational framework whenever questions involving these variables are posed.

The evolution time it takes for the classical formalism to yield the asymptotic values appears reasonable because it conforms to the collision time associated with the actual range of the Coulomb interaction. The semiclassical approximation, on the other hand, may generate a remarkable transfer of probability between the reaction channels that continues to take place over a much longer interval of time. It seems difficult to associate such rapidly oscillating behavior with a physically significant effect, and thus it may be judged an artifact of the approximation. (But, of course, ordinary intuition tends to be more classical than quantal.) One way to accept the fact that curves with a different time dependence can be generated at will is by accepting the absence of a concrete meaning attached to any them.

It is reassuring to see that our examples provide no evidence that questions the ability of the semiclassical approximation to predict the final values of definite observables, such as the excitation probability of the different reaction channels for a given impact parameter. This shores up our confidence in the calculation of total cross sections through the semiclassical approximation. In the case of angular distributions, however, the adopted practice of making use of the relationship $\vartheta \leftrightarrow b$ should be more carefully examined. In fact, the correspondence between impact parameter and scattering angle is loosely satisfied even by the heaviest systems in nature (see, for instance, Fig. 5 of Ref. [19]), and a given ϑ is fed by a substantial range of impact parameters. A combination of the different partial waves may, for instance, effectively reduce the long time it takes to stabilize the excitation probabilities.

To summarize, it follows from our present analysis that the answer to the two questions posed earlier, just below Eq. (1.4), are both negative. As a consequence, the identification $|a_n(t)|^2 \leftrightarrow P_n(t)$ as a meaningful quantity providing information about the state of excitation of the system at the time t [or at a relative motion distance $R(t)$] is not always appropriate. This, in turn, raises fundamental questions about the indiscriminate use of time-dependent semiclassical coupled-channel amplitudes in the formulation of theories where a detailed knowledge of the intrinsic state of excitation at all times is deemed necessary.

We do not claim to have a complete understanding of the aspects of the semiclassical formalism put in evidence by the theoretical considerations and model calculations performed in the course of this investigation. On the other hand, some of their characteristics and possible implications seem important enough to share with our colleagues, and we urge them to explore further these issues.

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APPENDIX

Let us consider the Hamiltonian (2.1). Expanding to leading order in the coordinates and the momenta, we have

$$H \approx H_0 + \mathcal{A}x + \mathcal{B}y + \mathcal{C}p_y, \tag{A1}$$

where

$$\begin{aligned}
\mathcal{A} &= q\left(\frac{\partial\Phi}{\partial x}\right)_0 + \frac{q^2v^2}{Dc^4}\Phi_0\left(\frac{\partial\Phi}{\partial x}\right)_0 + i\,\frac{q\hbar v}{2Dc^2}\left(\frac{\partial^2\Phi}{\partial y\partial x}\right)_0,\\
\mathcal{B} &= q\left(\frac{\partial\Phi}{\partial y}\right)_0 + \frac{q^2v^2}{Dc^4}\Phi_0\left(\frac{\partial\Phi}{\partial y}\right)_0 + i\,\frac{q\hbar v}{2Dc^2}\left(\frac{\partial^2\Phi}{\partial y^2}\right)_0,\\
\mathcal{C} &= -\frac{qv}{Dc^2}\Phi_0.
\end{aligned} \tag{A2}$$

Similarly, one can expand the Hamiltonian (2.4) obtained through the gauge transformation. In this case,

$$H' \approx H + \Delta H, \tag{A3}$$

with

$$\Delta H = (\Delta H)_0 + \mathcal{D}x + \mathcal{E}p_x + \mathcal{F}y + \mathcal{G}p_y. \tag{A4}$$

The coefficients of the linear terms in (A4) are now

$$\begin{aligned}
\mathcal{D} = &-\frac{q}{c}\left(\frac{\partial^2\Lambda}{\partial t\,\partial x}\right)_0 + \frac{q^2 v}{mD^3}\Phi_0\left(\frac{\partial^2\Lambda}{\partial y\partial x}\right)_0 \\
&+ q^2\frac{(\vec{\nabla}\Lambda)_0}{Dc^2}\left(\frac{\partial\vec{\nabla}\Lambda}{\partial x}\right)_0 + \frac{q^2 v}{Dc^3}\left(\frac{\partial\Phi}{\partial x}\right)_0\left(\frac{\partial\Lambda}{\partial y}\right)_0 \\
&+ i\frac{q\hbar}{2Dc}\left(\frac{\partial\vec{\nabla}^2\Lambda}{\partial x}\right)_0, \\
\mathcal{E} = &-\frac{q}{Mc}\left(\frac{\partial\Lambda}{\partial x}\right)_0, \\
\mathcal{F} = &-\frac{q}{c}\left(\frac{\partial^2\Lambda}{\partial t\,\partial y}\right)_0 + \frac{q^2 v}{Dc^3}\Phi_0\left(\frac{\partial^2\Lambda}{\partial y^2}\right)_0 \\
&+ q^2\frac{(\vec{\nabla}\Lambda)_0}{Dc^2}\left(\frac{\partial\vec{\nabla}\Lambda}{\partial y}\right)_0 + \frac{q^2 v}{Dc^3}\left(\frac{\partial\Phi}{\partial x}\right)_0\left(\frac{\partial\Lambda}{\partial y}\right)_0 \\
&+ i\frac{q\hbar}{2Dc}\left(\frac{\partial\vec{\nabla}^2\Lambda}{\partial y}\right)_0, \\
\mathcal{G} = &-\frac{q}{Mc}\left(\frac{\partial\Lambda}{\partial y}\right)_0.
\end{aligned} \tag{A5}$$

Notice that the expansion includes higher derivatives of the potential Φ than in Ref. [15], and therefore the form factors between channels in the coupling scheme that change the number n_x by ± 1 acquire also a small imaginary component.

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