

## Probing surface diffuseness of nucleus-nucleus potential with quasielastic scattering at deep sub-barrier energies

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(Received 5 December 2005; published 17 March 2006)

We perform a systematic study on the surface property of nucleus-nucleus potential in heavy-ion reactions using large-angle quasielastic scattering at energies well below the Coulomb barrier. At these energies, the quasielastic scattering can be well described by a single-channel potential model. Exploiting this fact, we point out that systems which involve spherical nuclei require the diffuseness parameter of around 0.60 fm in order to fit the experimental data, while systems with a deformed target between 0.8 and 1.1 fm.

DOI: [10.1103/PhysRevC.73.034607](https://doi.org/10.1103/PhysRevC.73.034607)

PACS number(s): 25.70.Bc, 25.70.Jj, 24.10.Eq

### I. INTRODUCTION

The Woods-Saxon form, which is characterized by the depth, radius and diffuseness parameters, has often been used for the internuclear potential for heavy-ion reactions. Conventionally, the diffuseness parameter of around 0.63 fm has been employed for calculations of elastic and inelastic scattering, which are sensitive only to the surface region of the nuclear potential [1,2]. This value of surface diffuseness parameter has been well accepted, partly because it is consistent with a double folding potential [3]. In contrast, a recent systematic study has shown that experimental data for heavy-ion fusion reactions at energies close to the Coulomb barrier require a larger value of the diffuseness parameter, ranging between 0.75 and 1.5 fm, as long as the Woods-Saxon parametrization is used as a nuclear potential [4]. The origin of the discrepancy in the surface diffuseness parameter between the scattering and fusion processes has not yet been understood.

Large-angle quasielastic scattering at deep sub-barrier energies provides an alternative way to look at this problem. Quasielastic scattering and fusion are both inclusive processes and are complimentary to each other. The former is related to the reflection probability at the Coulomb barrier, while the latter to the penetration probability. In heavy-ion reactions at energies near the Coulomb barrier, it is well known that the channel coupling effects caused by the collective inelastic excitations of the colliding nuclei strongly affect the reaction dynamics [5,6]. At deep sub-barrier energies, however, the channel coupling effects on quasielastic scattering can be disregarded, since the reflection probability is almost unity at these energies irrespective of the presence of channel couplings, even though inelastic channels themselves may be strongly populated [7]. This is similar to fusion at energies well above the Coulomb barrier, where the penetrability is almost unity [4].

The above concept was recently applied to the experimentally measured quasielastic scattering cross sections for the  $^{16}\text{O} + ^{154}\text{Sm}$  system at deep sub-barrier energies [7]. It was found that the larger surface diffuseness parameter of around 1.0 fm is required for this system in order to fit the data. This value is consistent with the one required for fusion.

It is apparent that a more systematic study is necessary, in order to clarify whether the quasielastic scattering around the Coulomb barrier generally requires a larger value of surface diffuseness parameter than the conventional value of around 0.63 fm. The aim of this paper is to carry out such systematic study on quasielastic scattering at deep sub-barrier energies. To this end, we calculate the excitation function of the quasielastic cross sections for systems involving both spherical and deformed nuclei. The reactions  $^{32,34}\text{S} + ^{197}\text{Au}$ ,  $^{32,34}\text{S} + ^{208}\text{Pb}$ ,  $^{16}\text{O} + ^{154}\text{Sm}$ ,  $^{186}\text{W}$ ,  $^{208}\text{Pb}$ , for which experimental data exist at deep sub-barrier energies, are studied. We show that a surface diffuseness parameter of around 0.6 fm is favored by the data for reactions involving spherical nuclei, whilst those involving deformed nuclei require a larger value of the diffuseness parameter.

The paper is organized as follows. In the next section, we briefly review the large-angle quasielastic scattering at deep sub-barrier energies. We also explain the procedure of our analyses which use a one dimensional ion-ion potential, including our definition of deep sub-barrier energies. In Sec. III, we present our results for the  $\chi^2$  fitting and discuss its sensitivity to the barrier height energy and to the channel coupling effects. We summarize the paper in Sec. IV.

### II. METHOD OF ANALYSES

#### A. Large-angle quasielastic scattering at deep sub-barrier energies

Our purpose in this paper is to study the surface property of ion-ion potential using heavy-ion quasielastic scattering. Before we explain the method of our analyses, let us first discuss briefly the advantage of exploiting large-angle quasielastic scattering at deep sub-barrier energies.

At energies well below the Coulomb barrier, the cross sections of (quasi)elastic scattering are close to the Rutherford cross sections, with small deviations caused by the effect of nuclear interaction. This effect can be taken into account by the semiclassical perturbation theory. The ratio of elastic scattering  $\sigma_{el}$  to Rutherford cross sections  $\sigma_R$  at a backward

angle  $\theta$  is given by Refs. [8,9]

$$\frac{d\sigma_{\text{el}}(E_{\text{c.m.}}, \theta)}{d\sigma_R(E_{\text{c.m.}}, \theta)} \sim 1 + \frac{V_N(r_c)}{ka} \frac{\sqrt{2a\pi k\eta}}{E_{\text{c.m.}}}, \quad (1)$$

where  $E_{\text{c.m.}}$  is the center-of-mass energy,  $k = \sqrt{2\mu E_{\text{c.m.}}}/\hbar$ ,  $\mu$  being the reduced mass, and  $\eta$  is the Sommerfeld parameter. This formula is obtained by assuming that the nuclear potential  $V_N(r)$  has an exponential form,  $\exp(-r/a)$ , around the classical turning point  $r_c = (\eta + \sqrt{\eta^2 + \lambda_c^2})/k$ , where  $\lambda_c = \eta \cot(\theta/2)$  is the classical angular momentum for the Rutherford scattering. We see from this formula that the deviation of the elastic cross sections from the Rutherford ones is sensitive to the surface region of the nuclear potential, especially to the surface diffuseness parameter  $a$ . Notice that, for small scattering angles, the Fresnel oscillation may complicate the formula. Also, as mentioned in the previous section, the channel coupling effects on the quasielastic cross sections are negligible at deep sub-barrier energies. We can thus study the effect of the surface diffuseness parameter in a transparent and unambiguous way using the large-angle quasielastic scattering at deep sub-barrier energies.

### B. Procedure

In order to compare with the experimental data for the quasielastic cross sections at deep sub-barrier energies, we use a one-dimensional optical potential with the Woods-Saxon form. Absorption following transmission through the barrier is simulated by an imaginary potential with  $W = 30$  MeV,  $a_w = 0.4$  fm, and  $r_w = 1.0$  fm. This model calculates the elastic and fusion cross sections, in which the elastic cross sections can be considered as quasielastic cross sections to a good approximation at these deep sub-barrier energies [10]. Note that the results are insensitive to the parameters of the imaginary part as long as it is well localized inside the Coulomb barrier.

In order to carry out a systematic study, we calculate the Coulomb barrier height using the Akyüz-Winther potential [11]. We examine several potentials with different values of surface diffuseness parameter, which give the same calculated barrier height. To this end, we vary the radius parameter  $r_0$  while keeping the depth parameter  $V_0$  to be 100 MeV. This is possible because the effect of variation in  $V_0$  and  $r_0$  on the Coulomb barrier height compensates with each other at the surface region.

We define the region of “deep sub-barrier energies” in the following way. In heavy-ion collisions at energies near the Coulomb barrier, collective inelastic excitations of the colliding nuclei and transfer reactions are strongly coupled to the relative motion. This causes the splitting of the Coulomb barrier into several distributed barriers [5,12]. We define the deep sub-barrier energies as around 3 MeV below the lowest barrier height or smaller. For this purpose, we first use the computer code CCFULL [13] in order to explicitly construct the coupling matrix (which includes the excitation energy for the diagonal components) for the coupled-channels equations for each system by including known low-lying

collective excitations. We then diagonalize it to obtain the lowest eigenbarrier.

We find that the deep sub-barrier region defined in this way corresponds to the region where the experimental value of the ratio of the quasielastic to the Rutherford cross sections is larger than around 0.94. We therefore include only those experimental data which satisfy  $d\sigma_{\text{qe}}/d\sigma_R \geq 0.94$  in the  $\chi^2$  fitting. A few experimental data points with values exceeding unity were excluded while performing the fits, but are shown in the figures below.

We apply this procedure to the  $^{32,34}\text{S} + ^{208}\text{Pb}$ ,  $^{32,34}\text{S} + ^{197}\text{Au}$  [14], and  $^{16}\text{O} + ^{208}\text{Pb}$  [15] reactions which involve spherical nuclei, as well as the  $^{16}\text{O} + ^{154}\text{Sm}$  and  $^{16}\text{O} + ^{186}\text{W}$  reactions [16] which involve a deformed target. For the deformed systems the scarcity of data points at deep sub-barrier energies led us to extend the fitting region to somewhat higher energies. This meant that the calculations had to take account of deformation effects as explained in Sec. III B.

## III. RESULTS AND DISCUSSION

### A. Spherical systems

We first present the results for systems involving spherical nuclei. Figure 1 compares the experimental data with the calculated cross sections obtained with different values of the surface diffuseness parameter in the Woods-Saxon potential for the  $^{32}\text{S} + ^{197}\text{Au}$  system (the upper panel) and the  $^{34}\text{S} + ^{197}\text{Au}$  system (the lower panel). The Coulomb barrier

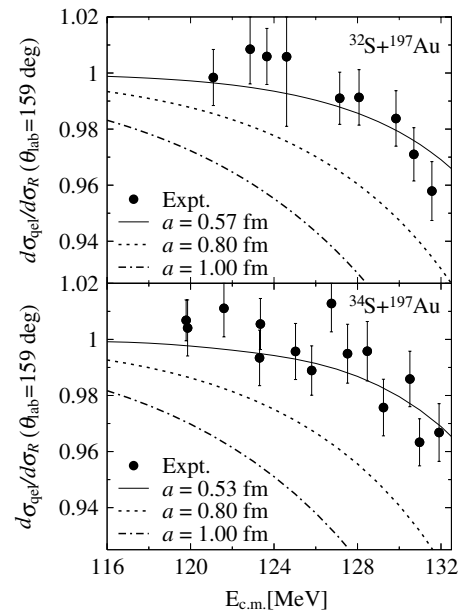


FIG. 1. The ratio of the quasielastic to the Rutherford cross sections at  $\theta_{\text{lab}} = 159^\circ$  for the  $^{32}\text{S} + ^{197}\text{Au}$  (the upper panel) reaction and for the  $^{34}\text{S} + ^{197}\text{Au}$  (the lower panel) reaction. The experimental data are taken from Ref. [14]. The solid line results from using a diffuseness parameter obtained by performing a least-square fit to the data. The dotted and the dot-dashed lines are obtained with the diffuseness parameter of  $a = 0.80$  fm and  $a = 1.00$  fm, respectively.

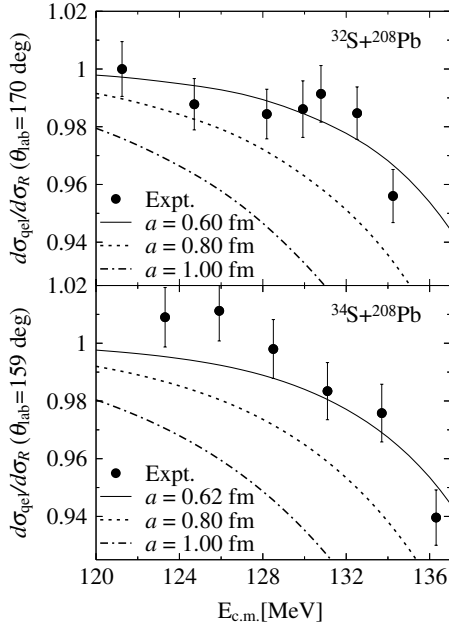


FIG. 2. The ratio of the quasielastic to the Rutherford cross sections for the  $^{32}\text{S} + ^{208}\text{Pb}$  (the upper panel) reaction at  $\theta_{\text{lab}} = 170^\circ$  and for the  $^{34}\text{S} + ^{208}\text{Pb}$  (the lower panel) reaction at  $\theta_{\text{lab}} = 159^\circ$ . The experimental data are taken from Ref. [14]. The meaning of each line is the same as in Fig. 1.

height is 141.2 MeV for the  $^{32}\text{S} + ^{197}\text{Au}$  reaction and is 140.2 MeV for the  $^{34}\text{S} + ^{197}\text{Au}$  reaction. The best fitted values for the surface diffuseness parameter are  $a = 0.57 \pm 0.04$  fm and  $a = 0.53 \pm 0.03$  fm for the  $^{32}\text{S}$  and  $^{34}\text{S} + ^{197}\text{Au}$  reactions, respectively. The cross sections obtained with these surface diffuseness parameters are denoted by the solid line in the figure. The dotted and the dot-dashed lines are calculated with the diffuseness parameter of  $a = 0.80$  fm and  $a = 1.00$  fm, respectively. Figure 2 shows the results for the  $^{32}\text{S} + ^{208}\text{Pb}$  (the upper panel) and the  $^{34}\text{S} + ^{208}\text{Pb}$  (the lower panel) reactions. The Coulomb barrier height is 145.1 and 144.1 MeV for the  $^{32}\text{S}$  and  $^{34}\text{S} + ^{208}\text{Pb}$  reactions, respectively. The best fitted values for the surface diffuseness parameter are  $a = 0.60 \pm 0.04$  fm and  $a = 0.63 \pm 0.04$  fm for the  $^{32}\text{S}$  and  $^{34}\text{S} + ^{208}\text{Pb}$  reactions, respectively.

It is evident from Figs. 1 and 2 that these spherical systems favor the standard value of the surface diffuseness parameter, around  $a = 0.60$  fm. The calculations with the larger diffuseness parameters,  $a = 0.80$  and  $1.00$  fm, underestimate the quasielastic cross sections and are not consistent with the energy dependence of the experimental data. We obtain a similar conclusion for the  $^{16}\text{O} + ^{208}\text{Pb}$  system, where the best fitted value for the surface diffuseness parameter is  $a = 0.59 \pm 0.10$  fm with the Coulomb barrier height of 76.1 MeV. The result for this system is shown in Fig. 3.

The conclusions are not sensitively dependent on the choice of barrier height energy  $V_B$ . In order to demonstrate this, we vary the barrier height by 1%, and repeat the same analyses. The result for the  $^{32}\text{S} + ^{208}\text{Pb}$  system is shown in Fig. 4. The solid line denotes the result obtained with the Akyüz-Winther

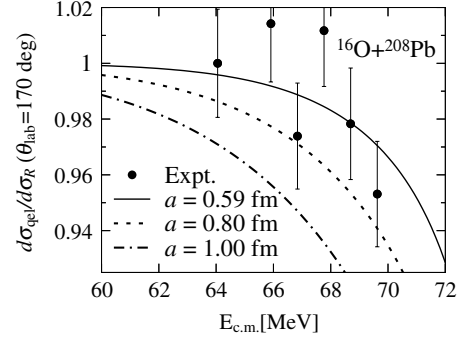


FIG. 3. The ratio of the quasielastic to the Rutherford cross sections for the  $^{16}\text{O} + ^{208}\text{Pb}$  reaction at  $\theta_{\text{lab}} = 170^\circ$ . The experimental data are taken from Ref. [15]. The meaning of each line is the same as in Fig. 1.

potential, as a reference, which is the same as the solid line in the upper panel of Fig. 2. The best fits and the resulting  $a$  values using  $V_B = 143.6$  MeV and  $V_B = 146.5$  MeV are also shown in Fig. 4. The  $a$  value changes by  $\pm 0.04$  fm for a  $\pm 1\%$  change in the barrier energy. The cross sections obtained with these potentials are shown in the figure by the dotted and the dot-dashed lines, respectively. One clearly sees that the effect of the variation of the Coulomb barrier height on the surface diffuseness parameter is small. The barrier energy obtained from the analysis of the above-barrier fusion cross sections is 144.03 MeV [4], which is within the range of  $V_B$  used in the calculations. Thus, the diffuseness parameter extracted in this work will not change significantly if  $V_B$  determined from fusion data, instead of the Akyüz-Winther prescription, is used. We have confirmed a similar behavior of the surface diffuseness parameter  $a$  for the other systems as well.

## B. Deformed systems

Let us next discuss the systems with a deformed target, that is,  $^{16}\text{O} + ^{154}\text{Sm}$ ,  $^{186}\text{W}$  reactions. For these systems, only a few data points are available at deep sub-barrier energies. We therefore include the experimental data at energies not only well below but also around the lowest barrier in the

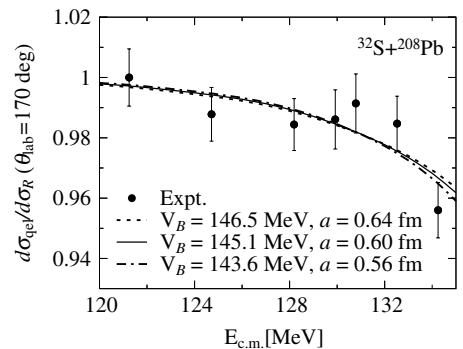


FIG. 4. Comparison of quasielastic cross sections obtained for three different values of the Coulomb barrier height for the  $^{32}\text{S} + ^{208}\text{Pb}$  reaction. The surface diffuseness parameter is determined for each barrier energy by fitting the experimental data.

$\chi^2$  fitting procedure. At these energies, the channel coupling effects start playing an important role in quasielastic reactions, and we include the effect of deformation of the target nucleus in our calculations. Therefore, our analyses for the deformed systems are somewhat more model dependent than those for the spherical systems presented in the previous subsection.

In order to account for the deformation effect on the quasielastic scattering, we use the orientation average formula [8,17], in which we neglect the finite excitation energy of the ground state rotational band. With this formula, the quasielastic cross section is given by

$$\sigma_{\text{qel}}(E_{\text{c.m.}}, \theta) = \int_0^1 d(\cos \theta_T) \sigma_{\text{el}}(E_{\text{c.m.}}, \theta; \theta_T), \quad (2)$$

where  $\theta_T$  is the angle between the symmetry axis of the deformed target and the direction of the projectile from the target. In the calculation for both the systems, we take six different orientation angles into account [18]. The results change only marginally even if we include the larger number of orientation angles.

The best fitted value for the surface diffuseness parameter obtained in this way is  $a = 1.14 \pm 0.03$  and  $0.79 \pm 0.04$  fm for the  $^{16}\text{O} + ^{154}\text{Sm}$  and  $^{16}\text{O} + ^{186}\text{W}$  reactions, respectively. The deformation parameters which we use in the calculations are  $\beta_2 = 0.306$  and  $\beta_4 = 0.05$  for  $^{154}\text{Sm}$  and  $\beta_2 = 0.29$  and  $\beta_4 = -0.03$  for  $^{186}\text{W}$ . Figures 5 and 6 compare the calculated cross sections with the experimental data. The solid line in each figure is obtained using the best fitted value of the diffuseness parameter. The dotted line shows the cross section obtained with the diffuseness parameter of  $a = 0.60$  fm as a reference. We find that the larger values of the surface diffuseness parameter,  $a = 1.14$  and  $0.79$  fm, in the nuclear potential are favored for these deformed system, in accordance with our previous conclusion in Ref. [7]. For the  $^{16}\text{O} + ^{154}\text{Sm}$  reaction, the calculated cross sections with the standard value of the surface diffuseness parameter around  $0.60$  fm are clearly in disagreement with the experimental data.

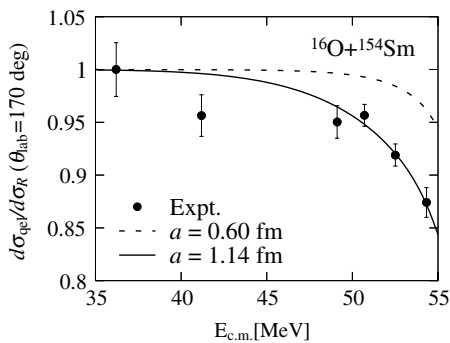


FIG. 5. The ratio of the quasielastic to the Rutherford cross sections for the  $^{16}\text{O} + ^{154}\text{Sm}$  reaction at  $\theta_{\text{lab}} = 170^\circ$ . The solid line is obtained using the best fitted value of the surface diffuseness parameter,  $a = 1.14$  fm. The dotted line denotes the cross sections obtained with the diffuseness parameter of  $a = 0.60$  fm. The experimental data are taken from Ref. [16].

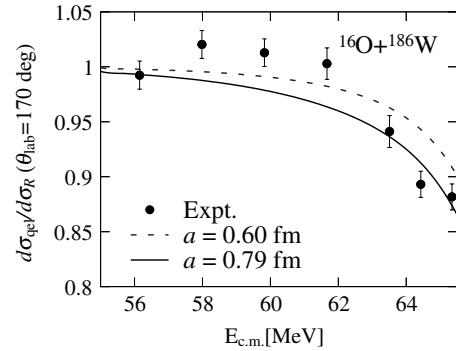


FIG. 6. The ratio of the quasielastic to the Rutherford cross sections for the  $^{16}\text{O} + ^{186}\text{W}$  reaction at  $\theta_{\text{lab}} = 170^\circ$ . The meaning of each line is the same as in Fig. 5. The experimental data are taken from Ref. [16].

### C. Discussion

Figure 7 summarizes the results for our systematic study for the surface diffuseness parameter. It shows the best fitted value of diffuseness parameter as a function of the charge product of the projectile and target nuclei for each system. The results for the spherical systems are denoted by the filled circles, while those for the deformed systems the filled triangles. One clearly sees the trend that the best fitted value of the diffuseness parameter is around  $0.60$  fm for the former, while it is much larger than that for the latter. Also, one sees that the surface diffuseness is almost constant for the spherical systems.

The value of the surface diffuseness parameter obtained in this study for the spherical systems agrees well with the conventionally used value  $a \sim 0.63$  fm. This suggests that the double folding potential is valid at least in the surface region and for systems which do not involve a deformed target. For these systems, the discrepancy between the values of the diffuseness parameter determined from fusion data (open circles and triangles in Fig. 7) and those from quasielastic data must be related with the dynamics inside the Coulomb barrier [4].

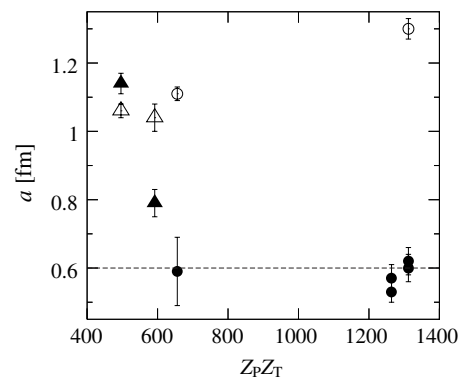


FIG. 7. The best fitted values of the surface diffuseness parameter  $a$  as a function of the charge product of the projectile and target nuclei,  $Z_P Z_T$ . The filled circles and triangles are for the spherical and the deformed systems, respectively. The open circles and triangles are the surface diffuseness parameters deduced from the analyses of fusion cross sections [4].

For the deformed systems studied here, the diffuseness parameter extracted from the quasielastic scattering is much larger than the conventional value of  $a \sim 0.63$  fm. Although this value is consistent with that extracted from fusion, the origin of the difference between the spherical and the deformed systems is not clear. One should bear in mind, however, that our analyses for the deformed systems are somewhat model dependent. This is due to the fact that the experimental data in the deep sub-barrier region are sparse for the deformed systems, and we need to include the deformation effect in the calculations in order to reproduce the strong energy dependence of the quasielastic cross sections at energies around the lowest barrier where the data exist. We have checked that the extracted values of the diffuseness parameter for the spherical systems remained almost the same even if we considered the data points around the Coulomb barrier, in addition to those at deep sub-barrier energies, in the fitting procedure like for the deformed systems. In order to clarify the difference in the diffuseness parameter between the spherical and the deformed systems, further precision measurements for large-angle quasielastic scattering at deep sub-barrier energies will be necessary, especially for deformed systems.

#### IV. SUMMARY

Large-angle quasielastic scattering provides a powerful tool not only for the analysis of the barrier distribution around the Coulomb barrier but also for the study of the surface property of the nuclear potential. This is due to the fact that

channel coupling effects play a minor role in quasielastic scattering at deep sub-barrier energies, that enables a relatively model independent analysis of ion-ion potential. Using this fact, we have systematically analyzed experimental data for quasielastic scattering at deep sub-barrier energies, with the aim of extracting the surface diffuseness parameter of internuclear potential. We obtained the diffuseness parameter that is consistent with the standard value of around  $a = 0.63$  fm for the systems involving spherical nuclei. In contrast, fits to the data for systems involving deformed nuclei require diffuseness parameter to be in the range of 0.8 to 1.1 fm, similar to that obtained from analyses of the fusion data at above-barrier energies.

The origin of the difference between the spherical and the deformed systems is not clear at the moment. In order to clarify this and confirm the systematics found in this paper, more experimental investigations on large-angle quasielastic scattering at deep sub-barrier energies will be certainly helpful, especially for deformed targets.

#### ACKNOWLEDGMENTS

We thank discussions with the members of the Japan-Australia Cooperative Scientific Program “Dynamics of Nuclear Fusion: Evolution Through a Complex Multi-Dimensional Landscape.” This work was supported by the Grant-in-Aid for Scientific Research, Contract No. 16740139 from the Japanese Ministry of Education, Culture, Sports, Science, and Technology. M.D. acknowledges the support of the Australian Research Council.

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