Triaxial superdeformed and normal-deformed high-spin band structures in ^{170}Hf

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The high-spin structure of ^{170}Hf was investigated using the EUROBALL spectrometer. The previously known level scheme was extended in the low-spin region as well as to higher spins, and several new bands were discovered. In particular, two bands were identified which show the characteristics of triaxial superdeformation. One of these bands is strongly populated, and its excitation energy and spins are established. Configuration assignments are made to the normal-deformed bands based on comparisons of their properties with cranked shell model calculations. The results for the very high spin states provide important input for such calculations.

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I. INTRODUCTION

A fascinating topic of nuclear-structure research is the investigation of stable triaxiality in nuclei. Triaxial shapes have been predicted theoretically for a long time, but unambiguous experimental evidence was only found in recent years. Compared to axially symmetric deformed nuclei, triaxial nuclei have additional degrees of freedom for rotational motion. The rotational angular momentum may be not only perpendicular to the symmetry axis, as in axially symmetric nuclei, but also distributed among the three different axes. The corresponding motion gives rise to families of "wobbling" bands which were predicted 30 years ago [1] but were discovered only recently in odd-*A* Lu isotopes [2]. Wobbling is a unique fingerprint of stable triaxiality. Another fingerprint of triaxiality is the occurrence of "chiral" bands [3]. If different nucleons align their spins along the short and long axis of a triaxial nucleus and the rotational angular momentum points along the third axis, it is possible to distinguish between left- and right-handed rotation which may lead to pairs of energetically degenerate bands. Several doublets of nearly degenerate bands have recently been suggested to result from nuclear chirality, for refs. see [4,5].

In the Lu-Hf mass region, calculations of potential-energy surfaces (PES) show local minima with large quadrupole deformation ($\epsilon \approx 0.4$) and with a substantial triaxiality ($\gamma \approx \pm 20^\circ$) for all combinations of parity and signature, (π, α). The minima result from gaps in the calculated energy-level spectrum for large γ in this mass region [6]. An example of such a calculation, performed with the ULTIMATE CRANKER (UC) code [7], is shown in Fig. 1. Several rotational bands in Lu and Hf isotopes have been associated with the minimum with positive γ [2]. The large deformation was demonstrated by in-band transition quadrupole moments deduced from lifetimes [8–11], and the stable triaxiality was proven by the discovery of wobbling excitations in the odd-*A* isotopes ^{161}Lu [12], ^{163}Lu, ^{165}Lu, and ^{167}Lu [2]. However, it is not understood why wobbling seems to occur in the odd-*A* Lu isotopes only, and not in any of the Hf or odd-odd Lu nuclei where triaxial superdeformed (TSD) bands were also found. Furthermore, the calculations predict the TSD minima for a wide range of nuclei in this mass region; however, experiments did not reveal evidence of bands with such properties in all of them. In particular, the PES calculations for the light Hf isotopes with $A \leqslant 166$ predict pronounced TSD minima, but no bands that can be associated with these shapes were discovered. In the heavier Hf isotopes with $A \geqslant 174$, the PES calculations show a local axially symmetric superdeformed (SD) minimum which is more pronounced than the TSD local minima. In this region, SD and TSD structures may coexist, but the experimentally observed bands cannot easily be associated with the different shapes [10,11,13].

The main motivation for the present work was to search for TSD structures in ^{170}Hf. The results on a weak TSD band in this nucleus, now named TSD2, have been published recently [14].

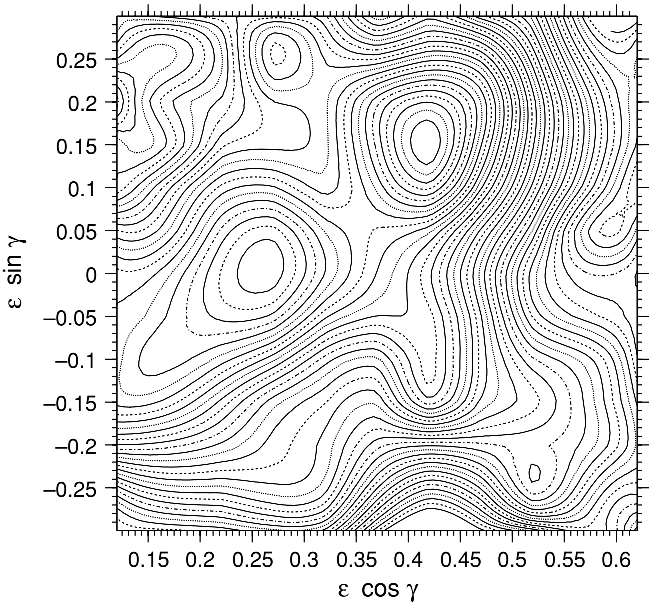

FIG. 1. Potential-energy surface for ^{170}Hf at $I = 51$ for parity and signature $(\pi, \alpha) = (-, 1)$ calculated with the UC code. Separation between contour lines is 200 keV.

Our discovery of a second TSD band, named TSD1 due to its higher intensity, which is connected to lower-lying levels in ^{170}Hf, allows us to study the development of the various PES minima along the isotopic chain. TSD bands have previously been observed in ^{168}Hf [15], $^{173-175}$Hf [10,11,13], and, most recently, ^{171,172}Hf [16]; however, only three of them have been linked to lower-lying states.

A second motivation was to identify the normal-deformed (ND) excitations up to the highest spins possible in ^{170}Hf. The high-spin structure of deformed nuclei shows a variety of interesting effects caused by the interplay of collective and single-particle excitations. The effects of the Coriolis and pairing forces can be compared with predictions of the cranked shell model (CSM) [3,17]. The CSM is very successful in reproducing crossing frequencies and aligned angular momenta for the first two or three band crossings at low and medium rotational frequencies. However, at higher frequencies where crossings with four- or more-quasiparticle bands occur, the agreement between experiment and theory is not as good. The reason for the discrepancies is probably a combination of effects caused by pairing-gap reductions, deformation changes, and mixing of quasiparticle states which influence the location of the active orbitals near the Fermi surface [6,18]. In this work, the previously known ND level scheme of ^{170}Hf [19–22] was extended appreciably to higher spins, and several new rotational bands were discovered.

Details on the experiment and the results are given in the following section. The results are discussed in Sec. III, followed by a short summary in the last section.

II. EXPERIMENTAL PROCEDURE AND RESULTS

High-spin states in ^{170}Hf were populated in the reaction ^{124}Sn(^{50}Ti,4n) at 216 MeV at the Vivitron accelerator at IReS, Strasbourg. Gamma-ray coincidences were measured with the EUROBALL spectrometer [23,24] which, at the time of the experiment, consisted of 29 tapered Ge detectors and 15 Cluster and 26 Clover composite detectors, each surrounded by a bismuth germanate (BGO) anti-Compton shield. The "inner ball" of 210 BGO detectors was used as a multiplicity filter. The target consisted of a stack of two ^{124}Sn foils with thicknesses of 600 and 460 μg/cm^2. Events were written to tape with the requirement that at least eight BGO detectors of the inner ball and four unsuppressed Ge detectors were in prompt coincidence. This resulted in a total of 2.3×10^9 events with a γ-ray coincidence fold $f \geqslant 3$ after Compton suppression.

In the off-line analysis, the γ-ray coincidence data were sorted into a three-dimensional coincidence array (cube) and further analyzed using the RADWARE program package [25]. Examples of the coincidence spectra are displayed in Fig. 2. To obtain information on the multipole order of γ-ray transitions, two angular distribution matrices were sorted. They contain events detected in all detectors on one axis and those detected at forward/backward (fb) angles and around 90° on the other axes. Gates were set in these two matrices on the axis with events detected in all detectors. The intensity ratios, $R_A = [I(\gamma_1^{\rm fb}, \gamma_2^{\rm all})]/[I(\gamma_1^{90^\circ}, \gamma_2^{\rm all})]$, of the transitions in the gated spectra can be used to distinguish between stretched dipole and stretched quadrupole transitions.

The energy-level scheme of ^{170}Hf based on previous work [19–22] and on the results of our measurements is presented in Fig. 3. Previously known bands have been extended in the low-spin region as well as to higher spins, and several new bands have been discovered. The bands are numbered according to their intensities, except for the strongly coupled bands 12 and 13, for which no reliable intensities could be derived because band 12 is built on the $I^\pi = 8^-$ isomer [22] and band 13 is not linked to other levels in the scheme. The intensities of the ND bands 2 to 11 vary between 12 and 3%, measured relative to the 321 keV $6^+ \rightarrow 4^+$ transition of band 1. In addition to the ND bands, two TSD bands were found in ^{170}Hf. The lower-energy transitions of TSD1, between 622 and 946 keV, were known previously [20]. It has an intensity of 8%, while TSD2 is about a factor of 10 weaker. Because of its low intensity, it was not possible to establish the links between TSD2 and lower-lying levels [14].

The band TSD1 is connected to several known levels in ^{170}Hf, and its excitation energy and spins are firmly established. We assign negative parity to this band because the 1085 keV transition linking TSD1 to band 1 (see Fig. 3) has probably $E1$ multipolarity. Its angular distribution ratio is compatible with a pure stretched dipole. An $M1$ transition of such high energy would be expected to show an $E2$ admixture. Negative parity for TSD1 is also consistent with the 597 keV stretched quadrupole, probably $E2$, transition to the negative-parity band 5 and with the two tentative $\Delta I = 2$, probably $E2$, links of 940 and 980 keV to the negative-parity band 2 (see Fig. 3). In the decay-out region, TSD1 is mixed with ND levels. This is seen in the irregular level spacing at the bottom of the band. A simple two-level mixing estimate for the $19^{(-)}$ states of TSD1 and band 5 results in an interaction energy of 28 keV. Another, much smaller mixing occurs between the $I = 31$ and 33 states with states of the same spins in

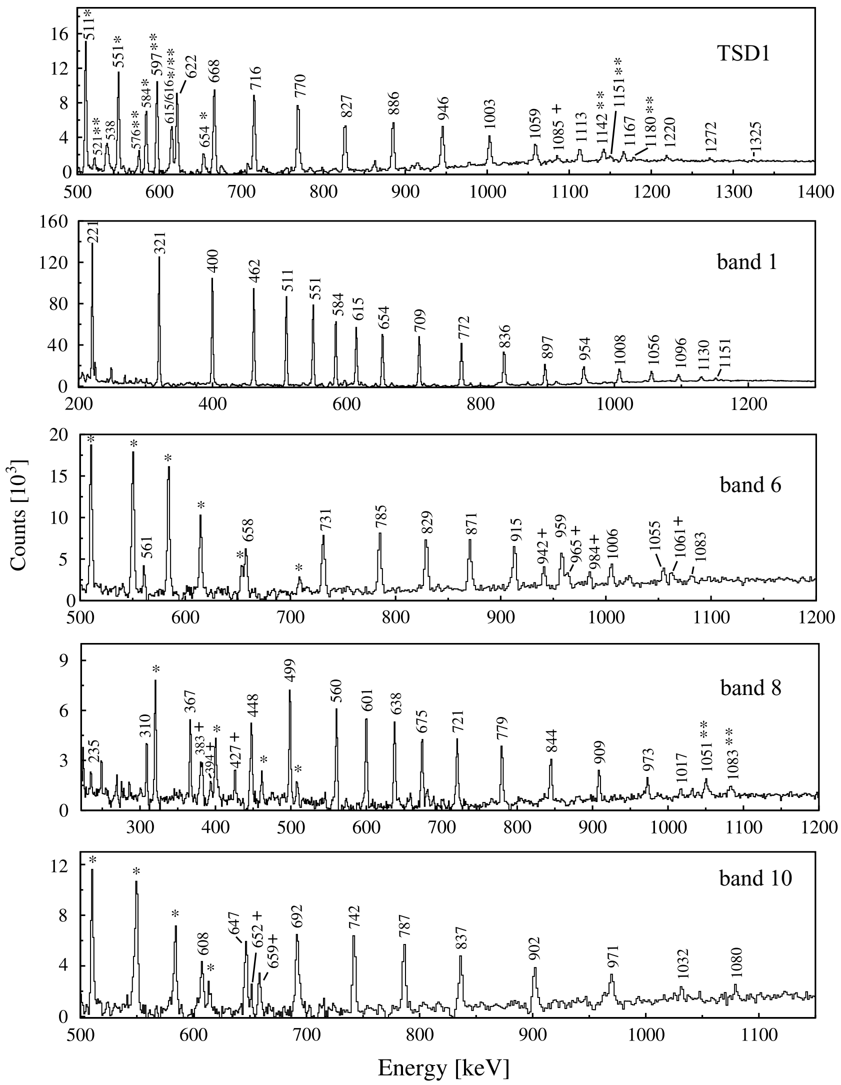

FIG. 2. Gamma-ray coincidence spectra of TSD1 and selected ND bands in ^{170}Hf. Ground-band transitions are marked by asterisks, linking transitions by plus signs, and transitions detected in connected bands by double asterisks.

the negative-parity band 2 where we estimate an interaction energy of $\leqslant$2.5 keV. Note that the occurrence of this mixing also supports our parity assignment.

III. DISCUSSION

A. TSD bands

The two TSD bands have average energy separations between consecutive transitions of 55 keV. This corresponds to dynamic moments of inertia $J^{(2)}$ of about 73 $\hbar^2$/MeV. In Fig. 4, the dynamic moments of inertia of TSD1 and TSD2 are compared to those of several TSD bands in other Hf and Lu nuclei which have been associated with the PES minima at $(\epsilon, \gamma) \approx (0.4, 20^\circ)$. The similarities are obvious and probably the two new TSD bands have a similar structure. Furthermore, the $J^{(2)}$ values of a recently discovered TSD band in ^{171}Hf [16] are very similar to those of TSD1 in ^{170}Hf.

A closer inspection of the moments of inertia of the TSD bands in Lu and Hf nuclei shows that they behave somewhat differently as a function of rotational frequency. While the $J^{(2)}$ values of most of the Lu bands increase slightly with frequency, most Hf bands show a small decrease with frequency and several bands are rather flat [2]. Lifetimes have not been measured in all cases, but there also seems to be a systematic difference in the transition quadrupole moments Q_t between the different groups of bands: the Lu bands and band 1 in ^{175}Hf have lower quadrupole moments, with values between 6 and 9b deduced from lifetime measurements [8–11]; while for the other Hf, bands, values between 11 and 14 b have been obtained [10,11,15]. These differences are most likely due to variations in the location of the strongly deformed PES minima in the (ϵ, γ) plane; however, it has not been possible to quantitatively reproduce the measured Q_t values in the UC calculations. The PES calculations for ^{175}Hf show two different strongly deformed minima at high spins: an axially symmetric one, which is lowest in energy and corresponds to a somewhat larger Q_t, and a triaxial minimum with a somewhat smaller Q_t. Therefore, it was suggested [10] that band 1 in that nucleus, which shows a smaller quadrupole moment, may be associated with the triaxial minimum; and band 2, with the larger Q_t, may belong to the axially symmetric strongly deformed minimum. However, neither of the experimental values are well reproduced by the calculations.

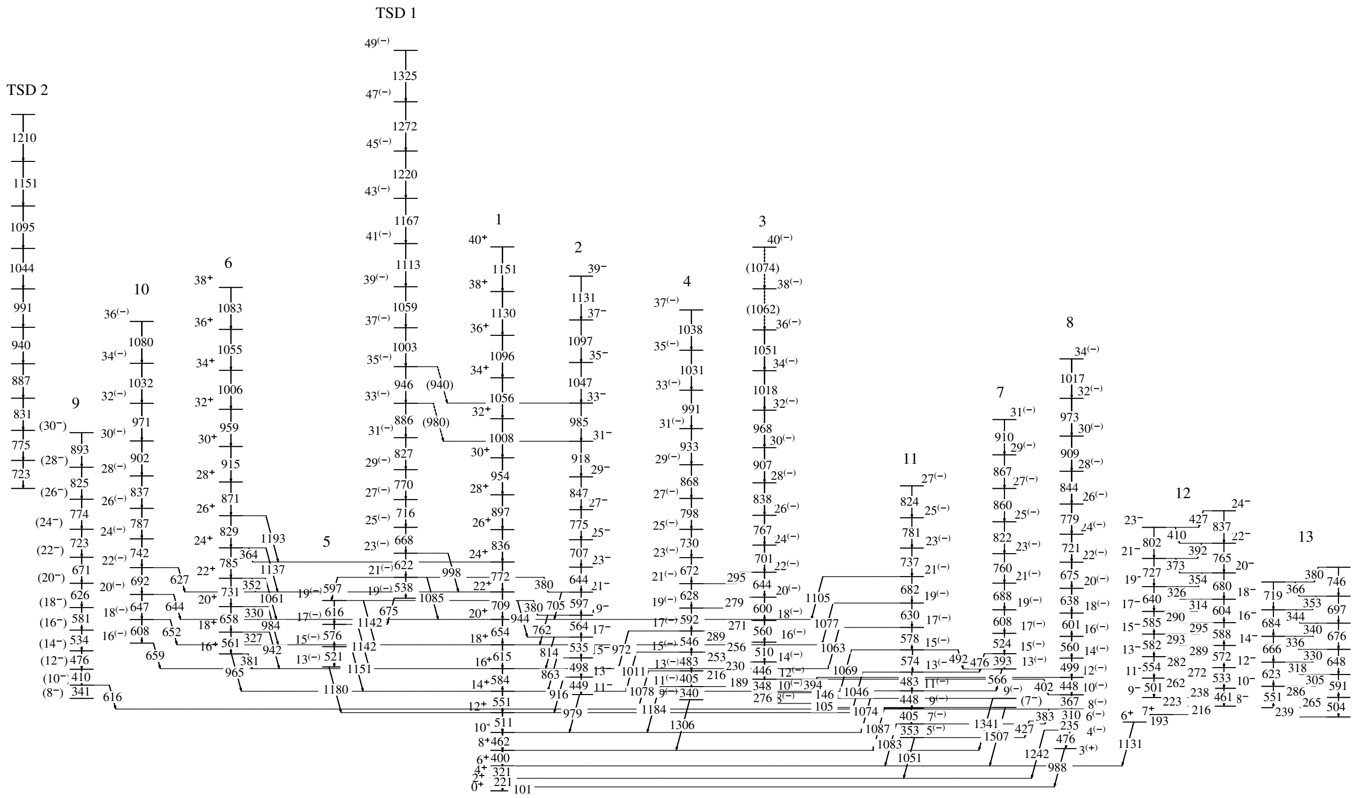

FIG. 3. Level scheme of ^{170}Hf.

The dynamic moments of inertia of TSD1 and TSD2 in ^{170}Hf show a mixed behavior as a function of frequency. Towards low and high frequencies, the $J^{(2)}$ values decrease; at intermediate frequencies, they increase. Therefore, the moments of inertia alone do not decide to which of the groups of bands they belong. A measurement of lifetimes is certainly necessary. The PES calculations for ^{170}Hf (see the example given in Fig. 1) show for all combinations of parity and signature only the TSD local minimum in addition to the ND minimum. Thus, the two new bands have to be associated with the triaxial minimum. However, a final proof of the triaxiality awaits the discovery of wobbling. From the present data, we have evidence that it is unlikely for the two bands to be members of the same wobbling family with similar alignments and with enhanced transitions between the two bands as found in the Lu isotopes [2].

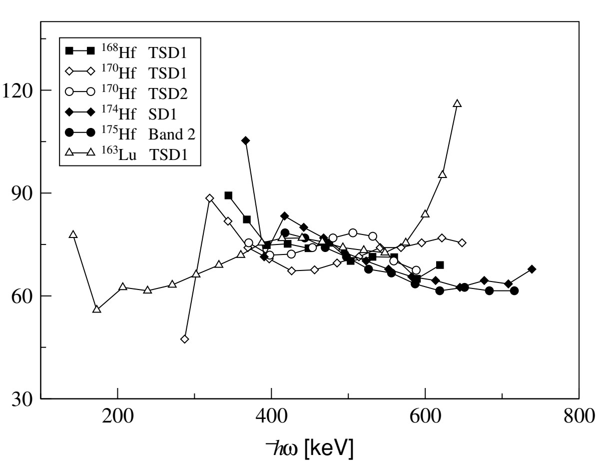

FIG. 4. Dynamic moments of inertia $J^{(2)}$ as a function of rotational frequency of TSD1 and TSD2 in ^{170}Hf compared to other TSD bands in Lu and Hf nuclei.

In our previous report on TSD2 [14], we argued on the basis of coincidences with ND transitions and similarities to TSD bands in the Lu isotopes that the spin range should be from $I \approx 22$ or 24 to 42 or 44. However, the new results on the high-spin band 2 in ^{175}Hf [10] could allow a different spin assignment to TSD2 in ^{170}Hf. If these two bands had a similar structure, the spins of TSD2 could be much higher; a spin range of about 40 to 60 would be possible. On the basis of the presently available data, it is not possible to firmly decide between the two alternatives. However, if it would be a very high-spin band, its alignment would be very large, $i_x \approx 30\,\hbar$. Such a band would be yrast at high spins and would be expected to be more strongly populated than observed experimentally. Furthermore, as mentioned above, band 2 in ^{175}Hf probably belongs to the axially symmetric minimum, which is not predicted by the PES calculations for ^{170}Hf (see Fig. 1).

The assignment of TSD1 and TSD2 to the PES minima with large triaxial deformation, $(\epsilon, \gamma) \approx (0.4, 20^{\circ})$, is also supported by their aligned angular momenta i_x, which are plotted as a function of rotational frequency in Fig. 5, upper panel. For TSD2, the lower spin range with $I_0 = 24$ was assumed. The alignments were obtained from the measured spins subtracting a reference core [26], $I_{\rm ref} = J_0\omega + J_1\omega^3$,

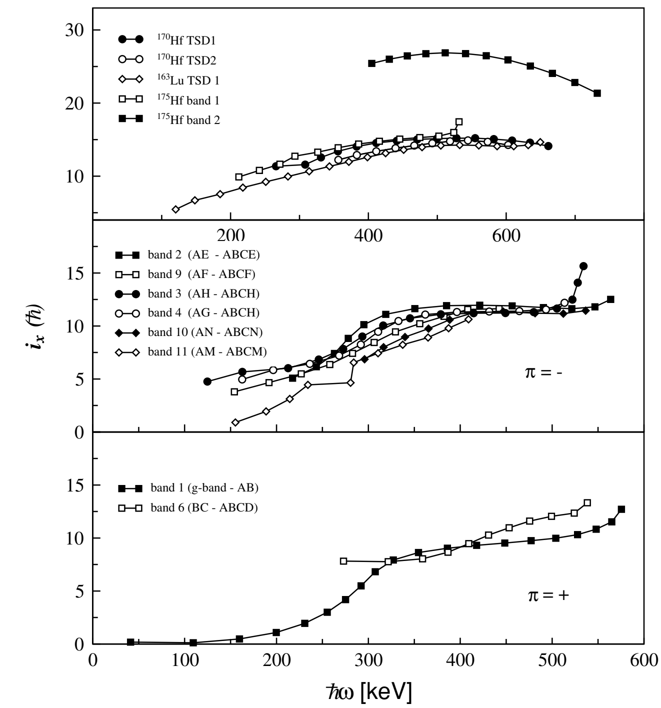

FIG. 5. Aligned angular momenta i_x as a function of rotational frequency of the TSD and ND bands in ^{170}Hf. Upper panel: TSD1 and TSD2 in ^{170}Hf compared to bands in ^{163}Lu and ^{175}Hf; center panel: most of the negative-parity ND bands; lower panel: positive-parity ND bands.

with the parameters $J_0 = 30\ \hbar^2$/MeV and $J_2 = 50\ \hbar^4$/MeV^3 from a global fit to the observed bands in ^{170}Hf. These alignments are compared to those for TSD bands with experimentally known spins in ^{163}Lu [27] and ^{175}Hf [10]. Note that for ^{163}Lu, triaxiality has been proven by the discovery of wobbling.

The TSD bands show large alignments already at low frequencies, typical for aligned high-j quasiparticle structures. They increase with increasing frequency and then saturate. In particular, they do not show the rather sudden increase caused by the decoupling of a pair of $i_{13/2}$ neutrons, AB, as observed in the ND bands (see lower panels of Fig. 5 and discussion below). The close similarity of the alignments of band 1 in ^{175}Hf and TSD1 in ^{170}Hf indicates a similar structure of the two bands. For ^{175}Hf, a $(\nu h_{9/2} i_{13/2}^2)(\pi i_{13/2}^2)$ configuration was suggested [10]. The negative-parity band TSD1 in ^{170}Hf may then have a corresponding configuration, however, with an even number of excited quasiparticles. The upbend in i_x observed at $\omega = 0.55$ MeV/$\hbar$ in the ^{175}Hf band has been suggested to be caused by the interaction with band 2 in that nucleus [10].

Our PES calculations show that the TSD minima at $I \approx 50$ are lowest for ^{170}Hf and increase in energy toward ^{168}Hf and ^{172,174}Hf. The relatively strong population of about 8% of TSD1 in ^{170}Hf compared to the neighboring isotopes, where the strongest bands have intensities in the order of 1–4% [13, 15,16], is consistent with these calculations.

TABLE I. Neutron Nilsson orbitals close to the Fermi surface and corresponding CSM [17] quasiparticle notations.

Nilsson config.	$\alpha = +1/2$	$\alpha = -1/2$
$[642]5/2^+$	A	B
$[651]3/2^+$	C	D
$[521]1/2^-$	E	F
$[512]5/2^-$	G	H
$[521]3/2^-$	M	N

B. ND bands

The aligned angular momenta of most of the ND bands of ^{170}Hf are displayed in the two lower panels of Fig. 5. The alignments increase in various discrete steps at rotational frequencies where crossings with bands with a larger number of aligned quasiparticles occur. The alignments at low and medium frequencies are caused by quasineutrons. Table I summarizes the neutron Nilsson orbitals close to the Fermi surface and the corresponding CSM quasiparticle notations [17]. The ground band, the quasiparticle vacuum, is crossed by the two-quasineutron band AB (see lower panel of Fig. 5). The crossing frequency of $\omega = 0.26$ MeV/$\hbar$ and the alignment gain of $i_x = 9\hbar$ are well reproduced by CSM calculations. The large interaction causing the gradual alignment is characteristic for the $N = 98$ isotones [28]. At higher frequencies, the alignment shows a second increase. It is not clear if it is caused by the crossing with the four-quasineutron (ABCD) or with the two-quasineutron-two-quasiproton (ABab) band, see below.

Band 6, the other positive-parity band, starts with a large alignment which then increases gradually over a wide frequency range. The configuration of this band starts with two quasineutrons, BC, and the gradual alignment gain is due to the crossing with a band with an additional pair of quasineutrons, AD, with a large interaction. In the region of the alignment increase, the dynamic moment of inertia is rather large. At the highest frequencies, an upbend is observed in the alignment which is probably caused by quasiprotons since the lowest-energy high-j quasineutron orbitals A, B, C, and D are occupied.

Most of the negative-parity bands start with an alignment of 4 to 5 $\hbar$ (see middle panel of Fig. 5), which then increases at a frequency around $\omega = 0.3$ MeV/$\hbar$. Their configurations contain already one quasineutron of $i_{13/2}$ origin, A, and the alignment gain is caused by the crossing with bands with the next possible pair of quasineutrons, BC, aligned. For example, band 2, which starts with the configuration AE [19], is crossed by the four-quasineutron band AEBC. The configurations for the other bands are similar; they are given in the figure.

The negative-parity bands which are observed to sufficiently high rotational frequencies show an upbend above $\omega = 0.5$ MeV/$\hbar$. Here, the configurations already contain three quasineutrons of $i_{13/2}$ origin, A, B, and C. Thus, the CD alignment is blocked, and the alignment gain is probably due to a crossing with a band with two additional quasiprotons. Arguments for proton alignment in this frequency range have also been presented for neighboring ^{169}Hf [29]. On the other hand, coming back to band 1 where orbital C is not occupied, the crossing at 0.55 MeV/$\hbar$ may be either due to neutron or proton alignment. The CSM calculations can presently not give a definitive answer to this question. However, as more and more data become available at the highest frequencies, systematic studies may provide information on the effects that influence the alignment of nucleons at very high spins.

IV. SUMMARY

Excited states up to very high spins were investigated in ^{170}Hf using the EUROBALL spectrometer. The level scheme was extended considerably; the previously known bands have been extended and several new bands were found. Two bands, TSD1 and TSD2, show the characteristics of the TSD bands observed in neighboring Hf and Lu nuclei. Their moments of inertia and aligned angular momenta are very similar to those of bands in the isotopes where triaxiality has been proven by the discovery of wobbling excitations. The new TSD bands fill a gap between the lighter and heavier Hf isotopes. For one of the bands, TSD1, excitation energy and spins are established. These results are important input to the refinement of future calculations for stable triaxial nuclei in this mass region. The properties of the ND bands are compared to CSM calculations, and configuration assignments are made. For several ND bands, the onset of a high-frequency crossing is observed for which the structure cannot definitively be predicted by the CSM.

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[1] A. Bohr and B. R. Mottelson, in *Nuclear Structure* (Benjamin, New York, 1975), Vol. 2.
[2] G. B. Hagemann, Eur. Phys. J. A **20**, 183 (2004), and references therein.
[3] S. Frauendorf, Rev. Mod. Phys. **73**, 463 (2001).
[4] K. Starosta, T. Koike, C. J. Chiara, D. B. Fossan, D. R. LaFosse, A. A. Hecht *et al*., Phys. Rev. Lett. **86**, 971 (2001).
[5] C. M. Petrache, Eur. Phys. J. A **20**, 39 (2004).
[6] R. Bengtsson and H. Ryde, Eur. Phys. J. A **22**, 355 (2004).
[7] T. Bengtsson, Nucl. Phys. **A512**, 124 (1990).
[8] G. Schönwaßer, H. Hübel, G. B. Hagemann, H. Amro, R. M. Clark, M. Cromaz *et al*., Eur. Phys. J. A **15**, 435 (2002).
[9] A. Görgen, R. M. Clark, M. Cromaz, P. Fallon, G. B. Hagemann, H. Hübel, I. Y. Lee, A. O. Macchiavelli, G. Sletten, D. Ward, and R. Bengtsson, Phys. Rev. C **69**, 031301(R) (2004).
[10] D. T. Scholes, D. M. Cullen, F. G. Kondev, R. V. F. Janssens, M. P. Carpenter, D. J. Hartley *et al*., Phys. Rev. C **70**, 054314 (2004).
[11] D. J. Hartley, M. K. Djongolov, L. L. Riedinger, G. B. Hagemann, R. V. F. Janssens, F. G. Kondev *et al*., Phys. Lett. **B608**, 31 (2005).
[12] P. Bringel, G. B. Hagemann, H. Hübel, A. Al-khatib, P. Bednarczyk, A. Bürger *et al*., Eur. Phys. J. A **24**, 167 (2005).
[13] M. K. Djongolov, D. J. Hartley, L. L. Riedinger, F. G. Kondev, R. V. F. Janssens, K. Abu Saleem *et al*., Phys. Lett. **B560**, 24 (2003).
[14] A. Neußer, H. Hübel, G. B. Hagemann, S. Bhattacharya, P. Bringel, D. Curien *et al*., Eur. Phys. J. A **15**, 439 (2002).
[15] H. Amro, P. G. Varmette, W. C. Ma, B. Herskind, G. B. Hagemann, G. Sletten *et al*., Phys. Lett. **B506**, 39 (2001).
[16] W. C. Ma, private communication.
[17] R. Bengtsson and S. Frauendorf, Nucl. Phys. **A327**, 139 (1979).
[18] A. V. Afanasjev, D. B. Fossan, G. J. Lane, and I. Ragnarsson, Phys. Rep. **322**, 1 (1999).
[19] J. C. Lisle, J. D. Garrett, G. B. Hagemann, B. Herskind, and S. Ogaza, Nucl. Phys. **A366**, 281 (1981).
[20] J. Irwin, J. C. Lisle, R. Chapman, D. Clarke, F. Khazaie, J. Mo, G. S. Li, G. J. Yuan, J. D. Garrett, and H. Ryde, Annual Rep., Daresbury Lab., 1988/89, p. 79.
[21] L. Guang-sheng, Chin. Phys. Lett. **16**, 796 (1999).
[22] D. M. Cullen, D. E. Appelbe, A. T. Reed, C. Baktash, I. Y. Lee, and A. O. Macchiavelli, Phys. Rev. C **60**, 057303 (1999).
[23] F. A. Beck, Prog. Part. Nucl. Phys. **28**, 443 (1992).
[24] J. Simpson, Z. Phys. A **358**, 139 (1997).
[25] D. C. Radford, Nucl. Instrum. Methods A **361**, 297 (1995).
[26] S. M. Harris, Phys. Rev. **138**, B509 (1965).
[27] D. R. Jensen, G. B. Hagemann, I. Hamamoto, B. Herskind, G. Sletten, J. N. Wilson *et al*., Eur. Phys. J. A **19**, 173 (2004).
[28] H. J. Jensen, R. A. Bark, R. Bengtsson, G. B. Hagemann, P. O. Tjom, S. Y. Araddad *et al*., Z. Phys. A **359**, 127 (1997).
[29] K. A. Schmidt, M. Bergström, G. B. Hagemann, B. Herskind, G. Sletten, P. G. Varmette, J. Domscheit, H. Hübel *et al*., Eur. Phys. J. A **12**, 15 (2001).