Charge and parity projected relativistic mean field model with pion for finite nuclei

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We construct a new relativistic mean field model by explicitly introducing a π-meson mean field with charge number and parity projection. We call this model the charge and parity projected relativistic mean field (CPPRMF) model. We take the chiral σ model Lagrangian for the construction of finite nuclei. We apply this framework first for the ^{4}He nucleus as a pilot case and study the role of the π-meson field on the structure of nuclei. We demonstrate that it is essential to solve the mean field equation with the variation introduced after the projection in order to take the pionic correlations into account explicitly. We study the ground-state properties of ^{4}He by varying several parameters, such as the σ-meson mass and the ω-meson coupling constant. We are able to construct a good ground state for ^{4}He. A depression appears in the central region of the density distribution, and the second maximum and the position of the dip in the form factor of ^{4}He are naturally obtained in the CPPRMF model.

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I. INTRODUCTION

One of the fundamental goals of nuclear physics is to understand the mutual relation between the nuclear structure and the nuclear force. The π meson was introduced by Yukawa as a mediator of the nucleon-nucleon interaction [1]. The π-meson exchange interaction appears mainly as the tensor force in the nonrelativistic framework. It is considered that the tensor force plays an important role for the saturation property of nuclear matter [2]. The reaction matrix theory showed that the density dependence of the central component of the G matrix rests on the tensor force [2–4]. The pioneering work of the variational calculation of the ^{4}He nucleus by ATMS (amalgamation of two-body correlations into a multiple scattering process) showed the importance of the tensor force in nuclei. In this calculation we can see that almost half of the attraction originates from the tensor force [5]. Recently, variational calculations based on the realistic nuclear force in real space by the Argonne-Illinois Group were found successful in describing light nuclei ($A \leqslant 10$) and showed that the π meson plays a crucially important role in determining the nuclear structure [6]. The contribution of the π-meson exchange interaction to the total binding energy is about 70%–80% of the net two-body interaction. This *ab initio* calculation is the main motivation for our study. Since the importance of the π-meson exchange for the nuclear property has been demonstrated, it seems natural that we treat the π-meson exchange interaction and other meson exchange interactions equally based on the mean field theory. The difficulty of the treatment of the π-meson exchange originates from its pseudoscalar character. The source term of the π meson demands parity mixing [7]. This framework is, however, theoretically simple and faithful to the meson theory by Yukawa [1]. We hope, therefore, to include this feature of the importance of the π meson for the description of nuclei even for medium and heavy nuclei, where the mean field approximation is essential. We employ the relativistic framework by using the Lagrangian density in order to see more directly the coupling between the π meson and the nucleon.

A. Relativistic mean field model and pions

From the point of view of phenomenology and application for other subjects, the relativistic mean field model [8–13] is quite successful in the prediction of the structure of nuclei and the saturation property of nuclear matter [14]. The usual relativistic mean field framework is based on the major premise that the nuclear ground state is constructed by parity- and charge-number-conserved single-particle states. Under the mean field approximation, the π-meson source term is zero, and there is no space where the π-nucleon interaction can explicitly contribute to nuclear structure. This feature originates from the unique character of the π meson, whose pseudoscalar and isovector character leads to coupling with the nucleon by spin-flip and changes in the parity and the charge number. For this reason, σ, ω, and ρ mesons, which develop finite mean fields with parity- and charge-conserved single-particle states, are used as mediators of the nuclear force to construct relativistic mean field models. Several parameters of this framework, the meson-nucleon coupling constants and the strength of the nonlinear coupling terms, have been

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determined so as to reproduce the empirical binding energies and the r.m.s. radii of nuclei in a wide mass region. At that moment, the effect of the π-nucleon interaction is considered to be renormalized in those parameters and are supposed to be contained in the mean field models provided by the other mesons.

Since the π-meson exchange interaction is a very important constituent of the nucleon-nucleon interaction, it is necessary for the theoretical framework to reflect the unique character of the π meson in the nuclear structure. Furthermore, in order to proceed with the study of the relation of the nuclear force and the structure of medium and heavy nuclei, we would like to construct a relativistic framework based on the mean field theory in which the effect of the π-nucleon interaction is taken into account explicitly on the same footing as for other mesons. As mentioned above, since the π meson has a unique character, we have to select an improved model space by using single-particle states as basis states, which are obtained by solving the mixed parity and charge mean field. Recently a parity symmetry-breaking relativistic mean field theory was proposed in Ref. [15]. The use of the TM1 Lagrangian [12] supported by a relativistic Brueckner-Hartree-Fock calculation [16,17], plus the π-nucleon interaction using the free space π-nucleon coupling constant, demonstrated that mixed-parity self-consistent solutions were obtained for $N = Z$ closed-shell nuclei in the wide mass region. This effect appears to be large for *jj* closed-shell nuclei, and the contribution from the π meson increases with the nuclear surface area. On the other hand, for the LS closed-shell nuclei the pionic effect turned out to be very small. The amount of π-meson energy is clearly different in the two groups. When the effect of the π-nucleon coupling is strong, in the case of the *jj* closed-shell nuclei, the framework gives a good approximate description for π-meson energy, but when the effect of the π-nucleon coupling is small, as in the case of the LS closed-shell nuclei, the π-meson energy cannot be taken into account properly [15].

To solve these problems, it is very important to combine the parity projection procedure with this mean field framework. In this paper the relativistic framework involving the projection procedure based on the mean field theory will be derived and discussed. The positive-parity projected wave function involves the 2p-2h states as the major correction terms. It is expected that the admixture of 2p-2h states and 0p-0h states through the π-nucleon interaction will lead to large amount of π-meson energy. This admixture corresponds to the *D*-state probability for the ^{4}He nucleus. In the strong coupling case the parity projection does not change the energies of the ground state very much, while for the weak coupling case, as in the previous relativistic mean field framework [15], it is especially important whether the projection is performed before the energy variation. The finite π-meson mean field is obtained by the delicate balance between the energy loss due to the kinetic energy and the profit due to the pionic correlation in the total energy. The parity projection is performed before variation and provides the optimized variational model space for the admixture of the 2p-2h and 0p-0h states. Furthermore, we introduce the charged π mesons, $\pi^{\pm}$, for the construction of the single-particle states, which break not only the parity but also the charge number, and therefore we have to perform charge number projection. In the previous framework [15] the charged π-nucleon interaction does not give further energy gain because of the isospin symmetry for $N = Z$ nuclei [18,19]. It is expected that charge number projection before the variation is able to take into account the effect of the charged π-meson exchange interaction between the proton and the neutron and to enhance the pionic effects.

B. Chiral symmetries

We adopt the linear σ model Lagrangian [20] for the description of nuclei. The chiral symmetry is known to be the most important character in hadron physics. The π mesons emerge as Nambu-Goldstone bosons from the spontaneous SU(2) chiral symmetry breaking [21]. The chiral symmetry and the generation of the hadron mass are described clearly in the Nambu-Jona-Lasinio model with the Fermion fields [21]. At the hadron level, the chiral symmetry is well described by using the linear σ model of Gell-Mann and Levy [22]. The application of the linear σ model for the description of the nuclear system is demonstrated by several groups in the relativistic mean field approximation [23–26]. The chiral σ model in its original form, however, is not able to describe nuclear matter. This problem is removed by introducing the dynamic generation of the ω-meson mass in the same manner as the nucleon mass. It was suggested by Boguta to be one of the solutions for this problem [23]. The extended chiral σ model can reproduce the saturation property of infinite matter. The effective mass of the nucleon is relatively large, around 0.8 of the free value, as compared with that obtaining in the Walecka model of about 0.6 [8].

This model gives very large incompressibility, however. Further applications of this model for the finite nuclei were demonstrated by Savushkin *et al.* [25,26]. The predicted binding energies are reasonable, but the spin-orbit splitting is too small, reflecting the large effective mass of the nucleon. In our previous work we employed this Lagrangian in the parity symmetry-breaking relativistic mean field framework, introducing the π-meson mean field, and applied it to finite nuclei [27]. The role of the π meson in the nuclear structure, especially the properties of single-particle spectra, was carefully studied. It was found that energy splitting between the spin-orbit partners clearly appears for *jj* closed-shell nuclei. However, for LS closed-shell nuclei, since the effect of the π-nucleon coupling is very weak, there is no improvement for the single-particle spectra. As one possible solution for this problem, it is necessary to combine the projection scheme with the mixed parity relativistic mean field model based on the method of variation after projection. We can expect a sufficiently large π-meson mean field to also yield the energy splitting between the spin-orbit partners for the LS closed-shell nuclei.

In this paper we apply the extended chiral σ model with the charge and parity projected relativistic mean field for ^{4}He as a pilot case. We present the formalism of the scheme with charge and parity projected relativistic mean field framework and discuss the importance of the scheme with variation after projection to take the π-meson mean field into account properly.

We construct the framework based on the mean field theory, but we take the Slater determinant of the mixed-parity and -charge number single-particle wave function. In the present study we concentrate on the π-meson degrees of freedom. We do not include the tensor coupling term of the ρ meson or the contribution of the coupling of the π meson with the Δ state. The contents of this paper are follows. In Sec. II we present the formulation of the charge and parity projected relativistic mean field model. In Sec. III we apply this framework for ^{4}He as the simplest example. Results and discussions are given in Sec. III. A summary and outlook are given in Sec. IV.

II. FORMULATION

In this section we derive the charge and parity projected relativistic mean field (CPPRMF) model. This framework is composed of the mixed-parity and -charge number relativistic mean field with the scheme of strong π-nucleon interaction combined with the scheme of variation after projection. This framework goes beyond the mean field theory because of the variation after projection, but the mean field is the fundamental concept behind the projection method.

A. Lagrangian

We start with the linear σ model with the ω meson, which has chiral symmetry. In the relativistic approach, the pseudoscalar coupling between the π meson and the nucleon leads to an unrealistically large attractive contribution from the negative-energy states, because γ_5 involve strong coupling between positive- and negative-energy states. We thus employ the nonlinear realization of the Lagrangian density, which is obtained by the Weinberg transformation of the linear σ model [28], because pseudovector coupling appears in the π-nucleon interaction and the $\gamma_5\gamma_\mu$ operator decouples the positive- and negative-energy states. We take only the lowest-order term in the π-meson field, and the Lagrangian we use is [27]

$$\begin{aligned}\mathcal{L} = {} & \bar{\psi}\left(i\gamma_\mu\partial^\mu - M - g_\sigma\sigma - \frac{g_A}{2f_\pi}\gamma_5\gamma_\mu\tau\cdot\partial^\mu\pi - g_\omega\gamma_\mu\omega^\mu\right)\psi \\ & + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma{}^2\sigma^2 - \lambda f_\pi\sigma^3 - \frac{\lambda}{4}\sigma^4 + \frac{1}{2}\partial_\mu\pi\partial^\mu\pi \\ & - \tfrac{1}{2}m_\pi{}^2\pi^2 - \tfrac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \tfrac{1}{2}m_\omega{}^2\omega_\mu\omega^\mu \\ & + \widetilde{g_\omega}{}^2 f_\pi\sigma\omega_\mu\omega^\mu + \tfrac{1}{2}\widetilde{g_\omega}{}^2\sigma^2\omega_\mu\omega^\mu, \end{aligned} \tag{1}$$

where the masses and coupling constants are set at $M = g_\sigma f_\pi$, $m_\pi^2 = \mu^2 + \lambda f_\pi^2$, $m_\sigma^2 = \mu^2 + 3\lambda f_\pi^2$, and $m_\omega = \widetilde{g_\omega} f_\pi$. We take the empirical values for the masses and the π decay constant as $M = 939$ MeV, $m_\omega = 783$ MeV, $m_\pi = 139$ MeV, and $f_\pi = 93$ MeV. The σ-nucleon coupling constant g_σ and the $\sigma\omega$-coupling constant $\widetilde{g_\omega}$ are automatically fixed by the relations $g_\sigma = M/f_\pi = 10.1$ and $\widetilde{g_\omega} = m_\omega/f_\pi = 8.42$, respectively. The strength of the σ-meson self-energy terms depends on the σ-meson mass, m_σ, through the relation $\lambda = (m_\sigma^2 - m_\pi^2)/2f_\pi^2$. The σ-meson mass and ω-nucleon coupling constant, g_ω, are the free parameters. We introduce the π-nucleon coupling constant g_A into this Lagrangian. In the linear σ model $g_A = 1$. In the nonlinear realization, it is set to $g_A = 1.25$, which is related to the coupling strength in the free-space πNN scattering by the Goldberger-Treiman relation [29]. The characteristic feature of Lagrangian (1) is the Higgs mechanism, where not only the nucleons but also the ω mesons obtain their masses by σ-meson condensation in vacuum. The effective nucleon mass, $M^* = M + g_\sigma\sigma$, is given to be around 0.8 of the free value at the nuclear matter density. The spin-orbit force for this Lagrangian is about half of that of the TM1 case, reflecting the amount of nucleon effective mass.

B. Parity and isospin mixed single-particle wave function

We stipulate that the single-particle state have the mixed-parity and -charge number to extend the model space so that the π meson is able to contribute to the finite mean field as to other mesons. Since the π-meson mean field requires a change of parity and of charge number of the nuceon state as shown in Fig. 1, the nucleon Dirac spinor is given as

$$\begin{aligned}\psi_{n\tau jm} = {} & \begin{pmatrix} iG_{n\tau\kappa(p)}\mathcal{Y}_{\kappa m}\zeta(p) \\ F_{n\tau\kappa(p)}\mathcal{Y}_{\bar{\kappa}m}\zeta(p)\end{pmatrix} + \begin{pmatrix} iG_{n\tau\kappa(n)}\mathcal{Y}_{\kappa m}\zeta(n) \\ F_{n\tau\kappa(n)}\mathcal{Y}_{\bar{\kappa}m}\zeta(n)\end{pmatrix} \\ & + \begin{pmatrix} iG_{n\tau\bar{\kappa}(p)}\mathcal{Y}_{\bar{\kappa}m}\zeta(p) \\ F_{n\tau\bar{\kappa}(p)}\mathcal{Y}_{\kappa m}\zeta(p)\end{pmatrix} + \begin{pmatrix} iG_{n\tau\bar{\kappa}(n)}\mathcal{Y}_{\bar{\kappa}m}\zeta(n) \\ F_{n\tau\bar{\kappa}(n)}\mathcal{Y}_{\kappa m}\zeta(n)\end{pmatrix}, \end{aligned} \tag{2}$$

where $\zeta(t)$ represents an isospin state, and the orbital angular momentum and the spin state are given as $\mathcal{Y}_{\kappa m} = \sum_{m_l,m_s}(1/2m_s; l_\kappa m_l|jm)Y_{l_\kappa m_l}\chi_{1/2,m_s}$. The single-particle state (2) is composed of four parts. The first term is the proton normal parity state, the second is the neutron normal parity, the third is the proton abnormal parity state, and the forth is the neutron abnormal parity state. The label n distinguishes radial wave functions of states that have the same τjm quantum number. The label $\tau = 1$ or 2 distinguishes a proton-dominant or neutron-dominant state, and the κ represents the parity state. Thus $\kappa = -(l+1)$ for $l = j - 1/2$, $\kappa = l$ for $l = j + 1/2$, and $\bar{\kappa} = -\kappa$ represents the opposite (abnormal) parity state. The radial parts of the wave function, G and F, represent the upper and the lower components of the Dirac spinor, respectively. The intrinsic mixed parity and charge number total wave function is defined by a Slater determinant of a set $\{\psi_{n\tau jm}\}$,

$$\Psi = \prod_{i=1}^{A} a_i^\dagger|0\rangle, \qquad i = n\tau jm. \tag{3}$$

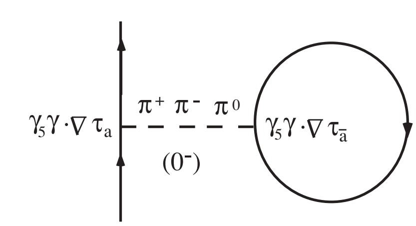

FIG. 1. Pion mean field diagram combined in the new relativistic framework.

Here the creation operator $a_i^\dagger$ creates a nucleon state with the quantum number i. The orthonormalization condition is $\langle\psi_i|\psi_j\rangle = \delta_{ij}$.

C. Parity and charge number projection

Since the nuclear state is a good eigenstate of parity and charge number, it is necessary to restore the parity and charge number of the total wave function. In order to obtain the total wave function of the definite parity and charge number state, the intrinsic total wave function has to be projected out into the good eigenstate of the parity and charge number by using the following charge number, $\mathcal{P}^c(Z)$, and parity, $\mathcal{P}^p(\pm)$, projection operators, as

$$\mathcal{P}^c(Z) = \frac{1}{2\pi}\int_0^{2\pi} d\theta e^{i(\hat{Z}-Z)\theta}, \quad \hat{Z} = \sum_{i=1}^{A}\frac{1+\tau_i^3}{2}, \tag{4}$$

$$\mathcal{P}^p(\pm) = \frac{1\pm\hat{P}}{2}, \quad \hat{P} = \prod_{i=1}^{A}\hat{p}_i, \quad \hat{p}_i\psi_i(\boldsymbol{r},\xi) = \gamma_0\psi_i(-\boldsymbol{r},\xi). \tag{5}$$

The charge number and parity projected total wave function is written as

$$\begin{aligned}\Psi^{[Z,\pm]} &= \mathcal{P}^c(Z)\mathcal{P}^p(\pm)\Psi,\\ &= \frac{1}{4\pi}\int_0^{2\pi} d\theta e^{-iZ\theta}[\Psi(\theta)\pm\Psi^p(\theta)],\end{aligned} \tag{6}$$

where

$$\Psi^q(\theta) = \prod_{i=1}^{A} e^{i\theta(1+\tau_i^3)/2}\Psi^q, \quad \psi_i(\theta) = e^{i\theta(1+\tau_i^3)/2}\psi_i. \tag{7}$$

The $\hat{Z}$ operates on the isospin state and makes the phase $e^{i\theta}$ change only for the proton state, that is, $e^{i\theta(1+\tau_i^3)/2}\zeta(p) = e^{i\theta}\zeta(p)$ and $e^{i\theta(1+\tau_i^3)/2}\zeta(n) = \zeta(n)$. We define an overlap matrix, $B^q(\theta)$,

$$\begin{aligned}\langle\Psi|\Psi^q(\theta)\rangle &= \det\left[\langle\psi_i|\psi_j^q(\theta)\rangle\right]\\ &= \det[B^q(\theta)]_{ij}.\end{aligned} \tag{8}$$

Hereafter if we use a symbol, $q = \{1, p\}$, as superscript, it represents both cases, that is, that the expectation value of various operators are taken by the mixed-parity and -charge number intrinsic wave function, $\Psi(\theta)$, when $q = 1$, and by the parity operated wave function, $\hat{p}\Psi(\theta) = \Psi^p(\theta)$, when $q = p$, respectively.

D. Total energy

We write the representation of the total energy of the A-nucleon system from Lagrangian density (1) to obtain a condition of total energy minimization. The relation between the Hamiltonian density and the Lagrangian density is written as

$$\mathcal{H} = \sum_\phi \frac{\partial\mathcal{L}}{\partial\dot{\phi}}\dot{\phi} - \mathcal{L}, \tag{9}$$

where ϕ denotes the nucleon field ψ and the π-, σ-, and ω-meson fields. According to this equation, the Hamiltonian density of one nucleon moving in the potential that is created by the meson field is written as (notation as in Ref. [30]),

$$\begin{aligned}\mathcal{H} = {}&\bar{\psi}\left(-i\boldsymbol{\gamma}\cdot\nabla + M + g_\sigma\sigma + \frac{g_A}{2f_\pi}\gamma_5\boldsymbol{\gamma}\tau^a\nabla\pi^a + g_\omega\gamma_\mu\omega^\mu\right)\psi\\ &+\frac{1}{2}\nabla\sigma\cdot\nabla\sigma + \frac{1}{2}m_\sigma^2\sigma^2 + \lambda f_\pi\sigma^3 + \frac{\lambda}{4}\sigma^4 + \frac{1}{2}\nabla\pi^a\cdot\nabla\pi^a\\ &+\frac{1}{2}m_\pi^2\pi^{a2} - \frac{1}{2}\nabla\omega^0\cdot\nabla\omega^0 + \frac{1}{2}(\nabla\times\boldsymbol{\omega})\cdot(\nabla\times\boldsymbol{\omega})\\ &-\frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \tilde{g}_\omega^2 f_\pi\sigma\omega_\mu\omega^\mu - \frac{1}{2}\tilde{g}_\omega^2\sigma^2\omega_\mu\omega^\mu,\end{aligned} \tag{10}$$

where we consider the static case and the time derivative terms, $\partial_0\phi$, vanish. The total Hamiltonian is written as

$$\hat{H} = \int d^3x\,\mathcal{H}. \tag{11}$$

The total energy representation is given, by using the Hamiltonian, as [8]

$$E^{[Z,\pm]} = \frac{\langle\Psi^{[Z,\pm]}|\hat{H}|\Psi^{[Z,\pm]}\rangle}{\langle\Psi^{[Z,\pm]}|\Psi^{[Z,\pm]}\rangle}. \tag{12}$$

The ψ is a nucleon field operator that operates the total Fermion system, $|\Psi^{[Z,\pm]}\rangle$, the charge number and parity projected wave function. When the source terms are large, the meson field operators can be approximated by their expectation values, which are the classical fields,

$$\sigma \longrightarrow \langle\mathcal{C}_\sigma|\sigma\left|\mathcal{C}_\sigma^q(\theta)\right\rangle = \sigma^q(\theta), \tag{13}$$

$$\omega_\mu \longrightarrow \langle\mathcal{C}_\omega|\omega_\mu\left|\mathcal{C}_\omega^q(\theta)\right\rangle = \delta_{\mu,0}\omega_0^q(\theta), \tag{14}$$

$$\pi^a \longrightarrow \langle\mathcal{C}_{\pi^a}|\pi^a\left|\mathcal{C}_{\pi^a}^q(\theta)\right\rangle = \pi^{aq}(\theta) \quad (a = 0,\pm), \tag{15}$$

where $\mathcal{C}_\phi$ are the coherent states for meson ϕ. The rotational invariance implies that the expectation value of the spatial component, $\langle\boldsymbol{\omega}\rangle$, vanishes. We explicitly write the meson coherent states, which have θ dependence. We have to distinguish two classical fields, $\langle\mathcal{C}|\phi|\mathcal{C}\rangle$ and $\langle\mathcal{C}|\phi|\mathcal{C}^p\rangle$. This is a critically important point to reduce the minimum energy condition, Eqs. (21)–(26) and (28). We extend the mean field approximation for the general nth order meson fields as

$$\langle\mathcal{C}|\phi(x)^n|\mathcal{C}^q\rangle = \langle\mathcal{C}|\phi(x)|\mathcal{C}^q\rangle^n. \tag{16}$$

We apply the above approximation to the cubic and quadratic self-interaction terms of the σ meson, which means that we approximate then as follows:

$$\begin{aligned}\langle\mathcal{C}|\sigma^3|\mathcal{C}^q(\theta)\rangle &= \langle\mathcal{C}|\sigma|\mathcal{C}^q(\theta)\rangle^3 = \sigma^q(\theta)^3,\\ \langle\mathcal{C}|\sigma^4|\mathcal{C}^q(\theta)\rangle &= \langle\mathcal{C}|\sigma|\mathcal{C}^q(\theta)\rangle^4 = \sigma^q(\theta)^4.\end{aligned} \tag{17}$$

This approximation agrees with the parity projected Hartree-Fock method for the linear mean field case without the nonlinear terms. Finally the total energy is given in the mean field approximation and for the static system as

$$E^{[Z,\pm]} = \int d^3x \frac{\langle\Psi^{[Z,\pm]}|\mathcal{H}|\Psi^{[Z,\pm]}\rangle}{\langle\Psi^{[Z,\pm]}|\Psi^{[Z,\pm]}\rangle} = \frac{1}{N^{[Z,\pm]}}\frac{1}{4\pi}\int_0^{2\pi} d\theta \mathrm{e}^{-iZ\theta}\int d^3x[\langle\Psi|\mathcal{H}(\theta)|\Psi(\theta)\rangle \pm \langle\Psi|\mathcal{H}^p(\theta)|\Psi^p(\theta)\rangle]. \quad (18)$$

The mean field Hamiltonian density is given as

$$\mathcal{H}^q(\theta) = \psi^\dagger \hat{h}_D^q(\theta)\psi + \mathcal{E}^q_{\text{meson}}(\theta), \quad (19)$$

where the Dirac Hamiltonian, $\hat{h}_D^q(\theta)$, and meson energy part, $\mathcal{E}^q_{\text{meson}}(\theta)$, are

$$\hat{h}_D^q(\theta) = -i\boldsymbol{\alpha}\cdot\nabla + \gamma_0(M + g_\sigma\sigma^q(\theta)) + g_\omega\omega^q(\theta) + g_\pi\gamma_0\gamma_5\boldsymbol{\gamma}\cdot\tau^a\nabla\pi^{aq}(\theta),$$

$$\mathcal{E}^q_{\text{meson}}(\theta) = \frac{1}{2}\left(-\nabla^2 + m_\sigma^2\right)\sigma^q(\theta)^2 + \lambda f_\pi\sigma^q(\theta)^3 + \frac{\lambda}{4}\sigma^q(\theta)^4 + \frac{1}{2}\left(-\nabla^2 + m_\pi^2\right)\pi^{aq}(\theta)^2 - \frac{1}{2}\left(-\nabla^2 + m_\omega^2\right)\omega^q(\theta)^2 - \widetilde{g}_\omega^2 f_\pi\sigma^q(\theta)\omega^q(\theta)^2 - \frac{1}{2}\widetilde{g}_\omega^2\sigma^q(\theta)^2\omega^q(\theta)^2. \quad (20)$$

E. Charge and parity projected relativistic mean field equations

Using the representation of the total energy, $E^{[Z,\pm]}$, defined in Eq. (18), we obtain the condition of energy minimization. To optimize the variational space, we adopt the variation after projection scheme. We employ the parity and charge number projected total wave function defined in Eq. (6) as a trial function. The unknown functions included in the total energy, $E^{[Z,\pm]}$, are the meson fields $\{\phi^q\}$ and occupied single-particle state $\{\psi_i, i = 1, \ldots, A\}$. The variation with respect to each meson, $\delta E^{[Z,\pm]}/\delta\phi^q = 0$, leads to the meson field equations:

$$\left(-\nabla^2 + m_\sigma^2\right)\sigma^q(\theta) = -g_\sigma\rho_s^q(\theta) - 3\lambda f_\pi\sigma^q(\theta)^2 - \lambda\sigma^q(\theta)^3 + \widetilde{g}_\omega{}^2 f_\pi\omega^q(\theta)^2 + \widetilde{g}_\omega{}^2\sigma^q(\theta)\omega^q(\theta)^2, \quad (21)$$

$$\left(-\nabla^2 + m_\omega^2\right)\omega^q(\theta) = g_\omega\rho_v^q(\theta) - 2\widetilde{g}_\omega{}^2 f_\pi\sigma^q(\theta)\omega^q(\theta) - \widetilde{g}_\omega{}^2\sigma^q(\theta)^2\omega^q(\theta), \quad (22)$$

$$\left(-\nabla^2 + m_\pi^2\right)\pi^{aq}(\theta) = g_\pi\rho^{aq}{}_{ps}(\theta). \quad (23)$$

The source terms of the meson fields can be calculated as

$$\rho_s^q(\theta) = \sum_{i=1}^A \psi_i^\dagger\gamma_0\tilde{\psi}_i^q(\theta), \quad (24)$$

$$\rho_v^q(\theta) = \sum_{i=1}^A \psi_i^\dagger\tilde{\psi}_i^q(\theta), \quad (25)$$

$$\rho^{aq}{}_{ps}(\theta) = \sum_{i=1}^A \nabla\cdot\psi_i^\dagger\gamma_0\gamma_5\boldsymbol{\gamma}\tau^a\tilde{\psi}_i^q(\theta) \quad (a = 0, \pm). \quad (26)$$

The field equation for σ and ω are coupled, and two meson fields are strongly correlated.

The variation with respect to each single-particle orbit under the orthonormalization condition, $\langle\psi_i|\psi_j\rangle = \delta_{ij}$,

$$\frac{\delta}{\delta\psi_i^*}\left\{E^{[Z,\pm]} - \sum_{i,j=1}^A \varepsilon_{ij}\langle\psi_i|\psi_j\rangle\right\} = 0, \quad (27)$$

leads to the charge number and parity projected relativistic mean field equations,

$$\frac{1}{4\pi}\int_0^{2\pi} d\theta e^{-iZ\theta}\Bigg(N(\theta)\Bigg\{\hat{h}_D(\theta)\tilde{\psi}_i(\theta) - [E^{[Z,\pm]} - E(\theta)]\tilde{\psi}_i(\theta) - \sum_{j=1}^A \eta_{ij}(\theta)\tilde{\psi}_j(\theta)\Bigg\} \pm N^p(\theta)\Bigg\{\hat{h}_D^p(\theta)\tilde{\psi}_i^p(\theta) - [E^{[Z,\pm]} - E^p(\theta)]\tilde{\psi}_i^p(\theta) - \sum_{j=1}^A \eta_{ij}^p(\theta)\tilde{\psi}_j^p(\theta)\Bigg\}\Bigg) = N^{[Z,\pm]}\sum_{j=1}^A \varepsilon_{ij}\psi_j, \quad (28)$$

where $\tilde{\psi}_i^q(\theta) = \sum_{k=1}^A \psi_k(B^q(\theta)^{-1})_{ki}$ and ε_{ij} represents the Lagrange multiplier. This is the variational equation for the mixed-parity and -charge number single-particle state, ψ_i. $E^q(\theta)$ represents the expectation value of the total Hamiltonian, $\hat{H}$, as $\Psi^q(\theta)$, and $\eta_{ij}^q(\theta)$ is an off-diagonal component of the single-particle state energy for the Dirac Hamiltonian. They are defined as

$$E^q(\theta) = \frac{\langle\Psi|\hat{H}|\Psi^q(\theta)\rangle}{\langle\Psi|\Psi^q(\theta)\rangle} = \sum_{i=1}^A\langle\psi_i|\hat{h}_D^q(\theta)\big|\tilde{\psi}_i^q(\theta)\rangle + E^q_{\text{meson}}(\theta), \quad (29)$$

$$\eta_{ij}^q(\theta) = \langle\psi_j|\hat{h}_D^q(\theta)\big|\tilde{\psi}_i^q(\theta)\rangle. \quad (30)$$

With the field equations for mesons, Eqs. (21)–(23), the contribution from the meson part to the total energy in Eq. (29) can be written as

$$E^q_{\text{meson}}(\theta) = \int d^3x\Bigg[-\frac{1}{2}g_\sigma\rho_s^q\sigma^q(\theta) - \frac{1}{2}g_\omega\rho_v^q\omega^q(\theta) - \frac{1}{2}(-g_\pi)\rho^{aq}_\pi\pi^{aq}(\theta) - \frac{1}{2}\lambda f_\pi\sigma^q(\theta)^3 - \frac{1}{4}\lambda\sigma^q(\theta)^4 + \frac{1}{2}\widetilde{g}_\omega f_\pi\sigma^q(\theta)\omega^q(\theta)^2 + \frac{1}{2}\widetilde{g}_\omega{}^2\sigma^q(\theta)^2\omega^q(\theta)^2\Bigg]. \quad (31)$$

The total norm is defined as

$$N^{[Z,\pm]} = \frac{1}{4\pi}\int_0^{2\pi} d\theta e^{-iZ\theta}[N(\theta) \pm N^p(\theta)], \quad (32)$$

where $N^q(\theta)$ denotes the norm between Ψ and $\Psi^q(\theta)\cdot$ $\{B^q(\theta)\}_{ij}$ represents the overlap matrix elements. The total energy can be written as

$$E^{[Z,\pm]} = \frac{1}{N^{[Z,\pm]}}\frac{1}{4\pi}\int_0^{2\pi} d\theta e^{-iZ\theta}[N(\theta)E(\theta) \pm N^p(\theta)E^p(\theta)]. \tag{33}$$

If the single-particle state has a definite parity and charge number, Eq. (28) results in the usual relativistic mean field equation, $\hat{h}_D\psi_i = \varepsilon_i\psi_i$.

We solve Eqs. (21)–(26) and (28) self-consistently. First we solve the normal Dirac equations, $\hat{h}_D\psi_i = \varepsilon\psi_i$, by using the prepared, certain initial potential and the obtained intrinsic mixed parity and charge wave function, Ψ. We then obtain the parity and isospin projected wave function as a trial wave function, following Eq. (6). Using this trial wave function, we calculate the source term for each meson according to Eqs. (24)–(26). Using these meson sources, we calculate the meson field by using Eqs. (21)–(23). Each potential is given by the meson field and by returning to the equation of motion for one nucleon, Eq. (28). We obtain the mixed-parity and -charge number single-particle state again. This is the procedure for one iteration loop. We continue this operation until convergence is achieved.

F. Numerical calculation

The wave functions for the nucleon and the meson fields are expanded in the Gaussian basis. For the nucleon, we write

$$G_{n\tau\kappa m_\tau} = \sum_{\mu=1} C^u_{n\tau\kappa m_\tau\mu}\phi_{l_\kappa\mu},$$
$$G_{n\tau\bar{\kappa} m_\tau} = \sum_{\mu=1} C^u_{n\tau\bar{\kappa} m_\tau\mu}\phi_{l_{\bar{\kappa}}\mu}, \tag{34}$$

$$F_{n\tau\kappa m_\tau} = \sum_{\mu=1} C^l_{n\tau\kappa m_\tau\mu}\phi_{l_{\bar{\kappa}}\mu},$$
$$F_{n\tau\bar{\kappa} m_\tau} = \sum_{\mu=1} C^l_{n\tau\bar{\kappa} m_\tau\mu}\phi_{l_\kappa\mu}. \tag{35}$$

The width of the Gaussian basis is chosen by means of the geometrical progression,

$$\phi_{l\mu} = N_F(l,\mu)r^l \exp\left(-\frac{r^2}{2\alpha_\mu^2}\right)$$
$$\left(\alpha_\mu = a_0 R_F^{\mu-1}, \mu = 1, 2, \ldots, n^F_{\max}\right), \tag{36}$$

where $a_0 = 0.5$ fm, $R_F = 1.2$, and $n^F_{\max} = 20$. For the σ-, ω-, and π^a-meson fields, we expand them in the same way as for the nucleon by means of the Gaussian functions,

$$\phi_\mu = N_M \exp\left(-\frac{r^2}{\beta_\mu^2}\right)\left(\beta_\mu = b_0 R_M^{\mu-1}, \quad \mu = 1, 2, \ldots, n^M_{\max}\right), \tag{37}$$

where $b_0 = 1.0$ fm, $R_M = 1.2, n^M_{\max} = 20$. The merits of the Gaussian (nonorthogonal) basis are the following. The Gaussian basis makes the calculation of matrix elements easy, and we are able to describe various types of wave functions well so as to take into account the long-range character of the π-nucleon interaction. As for the iteration, we take the imaginary time step method [31] and eliminate the small component in the Dirac equation so as not to pick up the solution for the negative energy state. We use a tolerance of 10^{-6}. The number of iterations is around 1000–10000.

III. RESULTS AND DISCUSSION

We apply the new relativistic mean field framework (CP-PRMF) constructed in the previous section to the ^{4}He nucleus. The ^{4}He nucleus is suitable as the simplest example, and it has been extensively studied with various theoretical methods. We assume that the intrinsic ground state is fully occupied, as $\{n\tau jm\} = \{0, 1, 1/2, \pm 1/2\}$ and $\{0, 2, 1/2, \pm 1/2\}$. The intrinsic total wave function (3) is a state of mixed charge numbers, $Z = 0$–4, and positive- and negative-parity states. The total wave function of the ^{4}He ground state (0^+, $Z = 2$) is obtained by projecting the positive-parity and charge state, $Z = 2$, according to Eq. (6).

In this Lagrangian we have two free parameters. One is the mass of the σ meson, m_σ, and another is the ω-nucleon coupling constant. In addition to these free parameters, we sometimes change the strength of π-nucleon coupling, namely, the axial vector coupling constant g_A, although we know it empirically by the Goldberger-Treiman relation [29], to test the critical point where the π-meson mean field becomes finite. We adjust these parameters under some constraint in the calculation.

A. Parity projection and variational method

Let us consider the positive- and negative-parity states, which are obtained by using the parity projection into the intrinsic mixed parity state to understand the mechanism of the π-nucleon interaction. We represent the mixed parity single-particle state as

$$|jm\rangle = \alpha_j|jm,\kappa\rangle + \beta_j|jm,\bar{\kappa}\rangle, \quad |\alpha_j|^2 + |\beta_j|^2 = 1. \tag{38}$$

The intrinsic mixed parity total wave function, which is fully occupied up to the Fermi level, defined as just multiples of the single-particle states for simplicity,

$$\begin{aligned}|\Psi\rangle &= \prod_{jm}(\alpha_j|jm,\kappa\rangle + \beta_j|jm,\bar{\kappa}\rangle)\\ &= \prod_{jm}\alpha_j|jm,\kappa\rangle + \sum_{j_1m_1}\beta_{j_1}|j_1m_1,\bar{\kappa}\rangle\prod_{jm\neq j_1m_1}\alpha_j|jm,\kappa\rangle\\ &\quad + \sum_{j_1m_1}\sum_{j_2m_2}\beta_{j_1}|j_1m_1,\bar{\kappa}\rangle\beta_{j_2}|j_2m_2,\bar{\kappa}\rangle\\ &\quad \times \prod_{jm\neq j_1m_1,j_2m_2}\alpha_j|jm,\kappa\rangle + \cdots.\end{aligned} \tag{39}$$

The first term corresponds to the state in which all the single-particle states are occupied by the normal parity state, and it is the $|0p-0h\rangle$ state. This state has the 0^+ state. In the second term, the $|j_1m_1,\kappa\rangle$ state is replaced with the opposite parity state, $|j_1m_1,\bar{\kappa}\rangle$. This means that in the $|0p-0h\rangle$ ground state

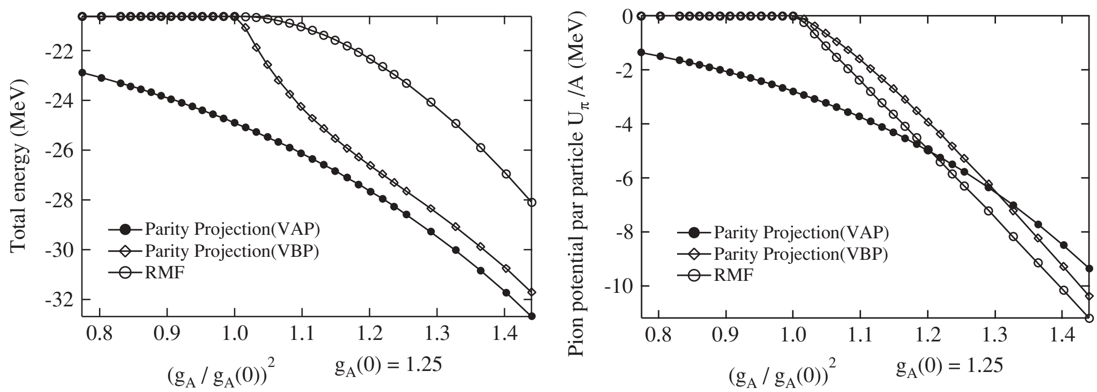

FIG. 2. Total energy (left) and the π-meson energy per particle (right) as a function of the π-nucleon coupling constant for the cases of variation before projection (diamonds), variation after projection (solid circles) and mixed parity RMF (open circles). $g_A(0)$ is the axial vector coupling constant in the free-space πNN scattering.

a particle moves from the normal parity single-particle state, $|j_1 m_1, \kappa\rangle$, to the abnormal parity state, $|j_1 m_1, \bar{\kappa}\rangle$. Thus the second term refers to the sum of $|1p - 1h\rangle$ states, which has 0^- spin-parity due to the $\pi(0^-)$-nucleon coupling. In the same manner, the third term means two 0^- $|1p - 1h\rangle$ states, namely, the $|2p - 2h\rangle$ states which have 0^+ parity. Therefore the wave functions that are projected to the positive and the negative parity states, respectively, are written as

$$\mathcal{P}^c(+)|\Psi\rangle = |(0p - 0h)\rangle + |(2p - 2h)\rangle + |(4p - 4h)\rangle + \cdots, \tag{40}$$

$$\mathcal{P}^c(-)|\Psi\rangle = |(1p - 1h)\rangle + |(3p - 3h)\rangle + \cdots. \tag{41}$$

The character of the parity projected wave function is that the positive-parity state consists of an even number of 1p-1h pairs with 0^-, which means that the positive-parity projection provides 2p-2h states as major correction terms. The matrix element of the Hamiltonian,

$$\langle 0p - 0h|\hat{h}_D|2p - 2h\rangle, \tag{42}$$

gives the dominant component for the π^a-nucleon interaction, while for other mesons, σ-, and ω-nucleon interactions $\langle 0p - 0h|\hat{h}_D|0p - 0h\rangle$ and $\langle 2p - 2h|\hat{h}_D|2p - 2h\rangle$, are the dominant components.

Figure 2 shows the total energy and the π-meson energy per particle as a function of the π-nucleon coupling constant squared. In the mixed parity relativistic mean field framework there is a critical coupling constant where the π-meson mean field starts to become finite as shown in Fig. 2. In the case of strong π-nucleon coupling, the variation before projection (VBP) method gives approximately the same results as the variation after projection (VAP) method. In the weak coupling region at around $(g_A/g_A(0))^2 \leqslant 1$, we do not get any energy gain by the VBP method. On the other hand, we obtain a large energy gain by the VAP method in this region. It is very important that the critical coupling constant is sufficiently small as compared with that of the free-space πNN coupling, $g_A = 1.25$, and this means that any state where the π-meson mean field becomes finite exists as more stable state. In the mixed parity relativistic mean field framework, there are two groups: one is the *jj* closed-shell nuclei, which is favorable for coupling with the π meson; the other is the LS closed-shell nuclei, which have small contributions from the π meson. In the parity projected relativistic mean field framework based on the VAP scheme, we can take into account the effect of π-nucleon interaction, namely 2p-2h correlations. The LS closed-shell nuclei also show a sufficiently large effect of the π-nucleon interaction. It is indispensable to solve the finite π-meson mean field based on the VAP scheme, especially when the π-nucleon coupling has a small effect.

B. Effect of charge projection

We perform now not only the parity projection but also the charge number projection. We compare the results obtained by using various methods in Table I. Here we fixed the σ-meson mass to be 850 MeV and the π-nucleon coupling g_A to be 1.15 in all methods. The ω-nucleon coupling constant g_ω is adjusted so as to reproduce roughly the empirical total energy and matter r.m.s. radius simultaneously in the charge and parity projected relativistic mean field (CPPRMF) calculation. The second column shows the results in the RMF calculation without parity and charge number projection. The π-meson mean field dose not appear yet, because the effect of the π-nucleon interaction cannot be taken into account sufficiently in this framework. The third column shows the results in the PPRMF method, in which only the parity projection is performed. The π-meson mean field becomes finite and contributes to the energy gain due to the coupling of 0p-0h state to the 2p-2h state through the π-nucleon interaction. For the even $N = Z$ nucleus, however, the single-particle state with all the isospin components is fully occupied, and the interaction term is invariant under the rotation in the isospin space. Thus the z axis can be taken as the direction of the isospin space without loss of generality [18,19]; that is, we choose $\boldsymbol{\sigma} \cdot \tau^a \nabla \pi^a \longrightarrow \boldsymbol{\sigma} \cdot \tau^0 \nabla \pi^0$. Therefore the only $\boldsymbol{\sigma} \cdot \tau^0 \nabla \pi^0$ type interaction is active owing to the isospin symmetry.

TABLE I. Constituents of the total energy (E_{total}) for the ground state with 0^+ of ^{4}He for various methods. The expectation values of the kinetic energy (E_{kin}), the π-meson potential (U_π), the σ-meson potential (U_σ), the ω-meson potential (U_ω), and the sum of the nonlinear and $\sigma\omega$ coupling term (E_{nl}) (MeV) are listed. The matter r.m.s. radius (R_m) (fm) is also listed. RMF denotes the usual relativistic mean field approach, but the mixed parity and isospin state is used as a trial wave function. PPRMF denotes the parity projected relativistic mean field method and CPPRMF denotes the charge and parity projected relativistic mean field method; g_A represents the strength of the π-nucleon coupling, and g_ω is the strength of the ω-nucleon coupling, which adjusts the strength of the repulsive force. m_σ is fixed to be 850 MeV.

	RMF	RMF	PPRMF	CPPRMF
g_A	0.0	1.15	1.15	1.15
g_ω	7.042	7.267	7.267	7.267
E_{total}	-28.22	-16.27	-20.47	-28.72
E_{kin}	65.08	44.38	56.76	83.93
U_π	0.00	0.00	-10.83	-43.16
$U_\sigma+U_\omega$	-58.76	-35.86	-37.51	-32.54
	$(=-237.55+178.79)$	$(=-133.33+97.47)$	$(=-166.26+128.75)$	$(=-212.13+179.59)$
E_{nl}	-15.17	-5.41	-9.52	-17.57
R_{m} (fm)	1.563	2.045	1.778	1.515

In the CPPRMF method, not only the $\boldsymbol{\sigma}\cdot\tau^0\nabla\pi^0$ type but also the $\boldsymbol{\sigma}\cdot\tau^-\nabla\pi^+$ and $\boldsymbol{\sigma}\cdot\tau^+\nabla\pi^-$ type interactions are active, and we can take this effect into account by method of variation after charge number projection. Thus the expected value of the π-meson energy, U_π, is around 3 times as large as that obtained in the case of the parity projected relativistic mean field method. This fact shows that the critical point where the π-meson mean field arises is sufficiently reduced, and a more stable state is realized where the π-meson mean field becomes finite. Therefore the variation after projection method is important for constructing the mean field framework with mixed parity and charge number to take into account the effect of the π-nucleon interaction properly. For the sake of comparison, we show the results of the relativistic mean field calculations in the first column. We adjust the ω-nucleon coupling g_ω to reproduce the empirical total energy of the ^{4}He nucleus when the π-meson mean field is switched off, $g_A = 0$.

C. Effect of π-meson mean field

We study the constituents of the total energy for the case of a finite π-meson mean field in the CPPRMF method. The σ-meson mass is set to several values, 777, 800, 840, and 850 MeV, and for each mass we adjust the ω-nucleon coupling constant g_ω and π-nucleon coupling constant g_A to reproduce the total energy. A series of results are shown in Table II. The sum of scalar and vector potentials corresponds to the central potential in the nonrelativistic framework. The components are shown in parentheses. In general, as the π-meson mean field becomes larger, the kinetic energy becomes larger, and the central potential, $U_\sigma + U_\omega$, becomes smaller. This is the general tendency when the π-meson mean field arises. The mechanism of the energy gain due to the π-nucleon interaction is shown in Eq. (42). To reach the 2p-2h state for the ^{4}He nucleus, for example, because two nucleons jump from a $0s_{1/2}$ orbital to the $0p_{1/2}$ orbital across the major shell in

TABLE II. Constituents of the total energy (E_{total}) for the ground state with 0^+ of ^{4}He in the CPPRMF method. The expectation values of the kinetic energy (E_{kin}), the π-meson potential (U_π), the σ-meson potential (U_σ), the ω-meson potential (U_ω), and the sum of the nonlinear and the $\sigma\omega$ coupling term (E_{nl}) (MeV) are listed. The matter r.m.s. radius (R_m) (fm) is also listed. We adjust the π-nucleon coupling g_A and the ω-nucleon coupling g_ω to reproduce the total energy for various σ-meson masses.

m_σ (MeV)	g_A	g_ω	E_{total}	E_{kin}	U_π	$U_\sigma+U_\omega$	E_{nl}	R_m (fm)
	1.15	7.753	-28.74	62.60	-25.07	$-32.28(=-169.49+137.22)$	-14.63	1.788
777	1.20	7.806	-28.52	65.79	-30.99	$-28.77(=-168.17+139.40)$	-15.17	1.775
	1.25	7.861	-28.55	69.67	-37.93	$-24.90(=-167.90+143.00)$	-16.02	1.755
	1.15	7.591	-28.53	68.63	-29.83	$-32.53(=-181.23+148.68)$	-15.43	1.702
800	1.20	7.642	-28.60	72.79	-37.22	$-28.53(=-180.74+152.21)$	-16.26	1.682
	1.25	7.703	-28.61	77.00	-45.22	$-23.87(=-179.66+155.79)$	-17.16	1.663
	1.15	7.329	-28.63	80.68	-40.18	$-32.63(=-205.43+172.80)$	-17.13	1.552
840	1.20	7.385	-28.74	85.72	-49.83	$-27.16(=-203.51+176.35)$	-18.09	1.532
	1.25	7.455	-28.59	90.25	-59.53	$-20.95(=-199.99+179.04)$	-18.99	1.519
	1.15	7.267	-28.72	83.93	-43.16	$-32.54(=-212.13+179.59)$	-17.57	1.515
850	1.20	7.327	-28.66	88.88	-53.13	$-26.58(=-208.81+182.22)$	-18.45	1.499
	1.25	7.398	-28.54	93.61	-63.38	$-20.00(=-204.95+184.95)$	-19.40	1.487

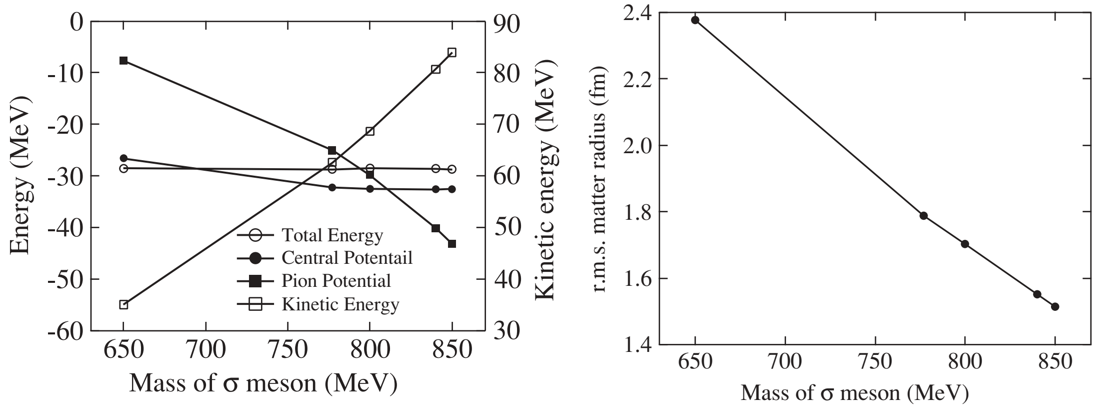

FIG. 3. The σ-meson mass dependence of the constituents of the total energy of ^{4}He under the constraint of the total energy in CPPRMF calculation. We adjust the ω-nucleon coupling to reproduce the total energy. For each m_σ, we take the following coupling constants: $g_\omega =$ 8.929 (650 MeV), 7.753 (777 MeV), 7.591 (800 MeV), 7.329 (840 MeV), 7.267 (850 MeV), respectively. On the left are the π-meson potential (solid squares), $\sigma + \omega$-meson potential (solid circles) and the kinetic energy (open squares, the scale is shown on the right axis). The total energy is represented by open circles. The matter r.m.s. radii are shown on the right.

shell model language, requires a large kinetic energy. The matter r.m.s. radius becomes small as the π-meson mean field increases, which is also understandable according to Eq. (42). It is feasible to include overlap between the $0p_{1/2}$ orbital and the $0s_{1/2}$ orbital so as to make the matrix element larger. Next let us compare the results in the same π-nucleon coupling constant lines for each σ-meson mass. The amount of π-meson potential becomes larger as the σ-meson mass becomes heavier. Of course, the kinetic energy increases accompanied by an increase in the π-meson attraction. In contrast, the σ-meson mass dependence of the central potential almost never changes, although the absolute values of the scalar and vector potentials increase. The potential becomes deeper with m_σ, and the matter r.m.s. radius becomes smaller. At around 850 MeV we can reproduce the total energy and matter r.m.s. radius simultaneously.

Figure 3 shows the mass dependence of the π-meson potential energy, central potential, and kinetic energy. The omega-nucleon coupling constant is adjusted to reproduce the empirical total binding energy for each mass. The π-nucleon coupling constant is set to be 1.15. In this mass region, at around 650 MeV, the π-meson potential dose not contribute so much. The matter r.m.s. radius is quite large. The contribution of the π-meson potential to total energy gain becomes larger as the σ-meson mass becomes heavier. There is a strong correlation between the kinetic energy and the π-meson potential. The right-hand part of Fig. 3 shows the matter r.m.s. radius. The matter r.m.s. radius becomes smaller with m_σ.

D. Intrinsic single-particle components

Figure 4 shows the square of the intrinsic single-particle wave function in the CPPRMF method. The proton-dominant single-particle wave functions ($\tau = 1$) are shown in the left-hand panel. The dominant component is the positive-parity

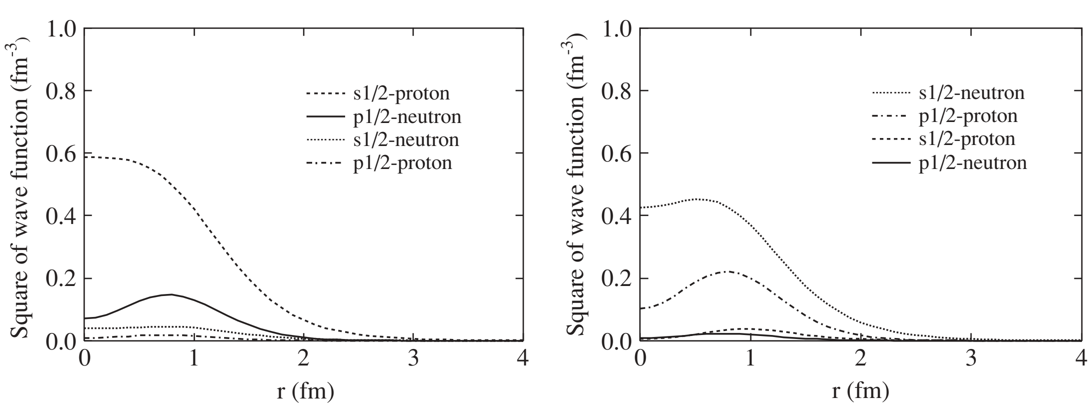

FIG. 4. Square of components of the single-particle wave function for the case with $m_\sigma = 850$ MeV, $g_\omega = 7.267$, and $g_A = 1.15$. The constituents of the total energy are listed in Table II, where the total energy and matter r.m.s. radius are reproduced simultaneously. The dashed curve represents the positive-parity ($0s_{1/2}$) proton state, the solid curve the negative-parity ($0p_{1/2}$) neutron state, the dotted curve the positive-parity neutron state, and the dotted-dashed curve the negative-parity proton state. The left-hand panel shows the intrinsic single-particle wave function of the lowest state, and the right-hand figure that of the second state.

($s_{1/2}$) proton state. This state couples with the negative-parity ($p_{1/2}$) neutron state through the π-nucleon interaction. The positive-parity ($s_{1/2}$) neutron and negative-parity ($p_{1/2}$) proton component also appear, but they are very tiny. The negative-parity ($p_{1/2}$) neutron component has its peak at around 0.8 fm. This component becomes quite compact compared with that of the normal harmonic-oscillator p-shell wave function, which has its peak at around 1.5 fm. We calculate the overlap between the ($p_{1/2}$) proton component (upper part) in the CPPRMF method and the 0p shell state of the harmonic-oscillator wave function with various widths, then the oscillator length at $b =$ 0.85 fm gives the maximum amount of overlap. This tendency has been clearly shown in the nonrelativistic treatment for the case of the tensor force [32]. The 2p-2h state in the CPPRMF method is not to be expressed in terms of the usual simple $0p$ state. This is because the mechanism of the energy gain is due to the π-nucleon interaction according to Eq. (42). If the overlap between the 0p-0h state [$(0s)^4$ configuration] and the 2p-2h state [$(0s)^{-2}(0p)^2$ configuration] becomes larger, the $0p$ state tries to shrink compared with the kinetic energy; then the matrix element of the π-nucleon interaction increases. Thus the compact distribution of the negative component is related to the increase of the kinetic energy as shown in Tables I and II. The negative-parity ($p_{1/2}$) neutron component has some value at the origin of the radius. This is because the lower part of this component has the opposite parity (s wave) and has the a value comparable with that of the upper part.

There is one more intrinsic single-particle state ($\tau = 2$) shown in the right-hand part of Fig. 4. This component has the positive-parity ($s_{1/2}$) neutron component as the dominant one. The negative-parity($p_{1/2}$) proton state also has some value at the origin for the same reason mentioned above. The positive-parity neutron component is no longer the normal 0s wave function not only due to the lower component, which has the opposite parity, but also due to the out-of-phase admixture of the $1s_{1/2}$ state.

E. Density distribution and form factor

The point proton density distribution of the ^{4}He ground state is shown in Fig. 5. The density distribution in the CPPRMF method is depressed at the central part. Its peak corresponds to that of the negative-parity ($p_{1/2}$) proton component in Fig. 4. This is because the π-nucleon interaction induces the admixture of $p_{1/2}$ and $s_{1/2}$ components. There is no depression at the central part unless the π-nucleon interaction is at work. The form factor of ^{4}He is obtained by Fourier transformation and is shown in Fig. 6 [33]. The form factor obtained in the CPPRMF method has a dip near the momentum transfer squared, $q^2 = 10$ fm^{-2}. This position is related to the depression of the density distribution in Fig. 5. Without the π-nucleon interaction, the form factor has the dip at a higher-momentum region, around $q^2 = 16$ fm^2. As the π-meson mean field becomes stronger, the dip position gradually approaches $q^2 = 10$ fm^{-2}. Another critical feature of the form factor in the CPPRMF method has a large amount of second maximum at the high-momentum region. It is related to the increase of the kinetic energy as the π-meson mean field works strongly. This fact means that the pionic correlation needs higher-momentum components. In this calculation the amount of the second maximum significantly grows up from the case without π-meson mean field. The dip position and the second maximum at higher momentum clearly indicate the π-meson effect in the nucleus.

We discuss the meson-exchange current in the Z diagram that corresponds to the three-body force in the nonrelativistic framework. As for the depression in the central region of the

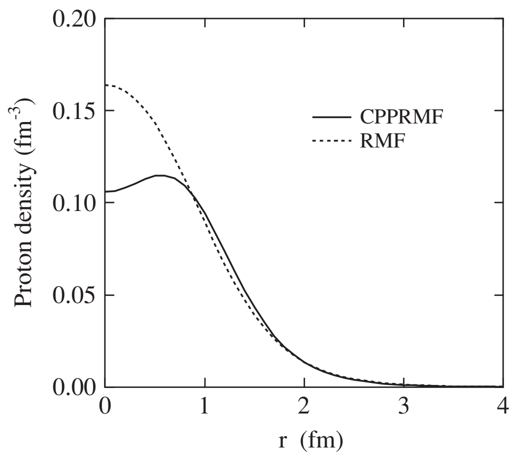

FIG. 5. Density distribution for the ^{4}He ground state in the case with $m_\sigma = 850$ MeV, $g_\omega = 7.267$, and $g_A = 1.15$. The constituents of the total energy are listed in Table II, where the total energy and matter r.m.s. radius are reproduced simultaneously (solid curve). For comparison the density distribution obtained by the usual relativistic mean field calculation is also shown by the dashed curve, which corresponds to the $(0s1/2)^4$ configuration. The parameters are taken to be $m_\sigma = 850$ MeV, $g_\omega = 7.042$, and $g_A = 0$, respectively.

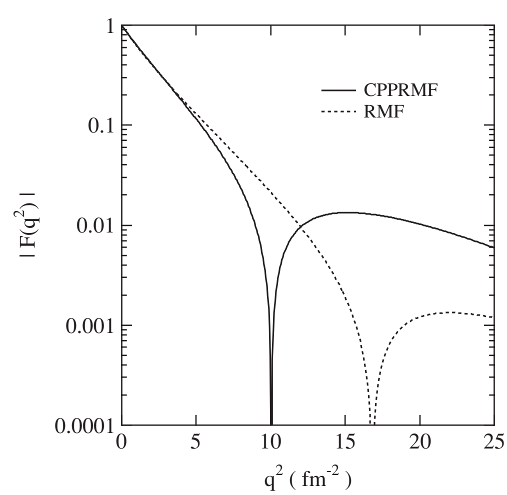

FIG. 6. Form factor for the ^{4}He ground state in the case with $m_\sigma = 850$ MeV, $g_\omega = 7.267$, and $g_A = 1.15$. The constituents of the total energy are listed in Table II, where the total energy and matter r.m.s. radius are reproduced simultaneously (solid curve). For comparison the form factor obtained by usual relativistic mean field calculation is also shown by dashed curve, which corresponds to the $(0s)^4$ configuration. The parameters are taken to be $m_\sigma = 850$ MeV, $g_\omega = 7.042$, $g_A = 0$, respectively.

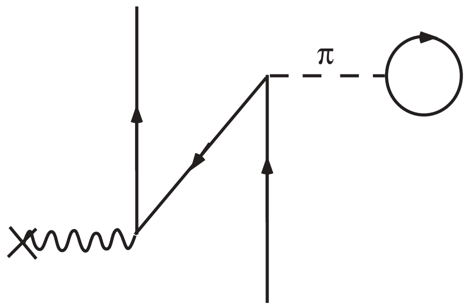

FIG. 7. Z diagram, which is known as the relativistic effect for the pionic pair current.

density distribution and the second maximum of the form factor, it is well known that the meson-exchange current that is enhanced by the two-pion exchange three-body force dominantly contributes to the central depression, as shown in the ATMS calculations [34]. The effect of the meson-exchange current is included naturally as a relativistic effect in the CPPRMF method. This is to be contrasted with any nonrelativistic models. Figure 7 shows the diagram of meson-exchange current, the so-called Z diagram [14].

IV. SUMMARY

We have constructed a new relativistic mean field basis model, where the parity and charge number projection is performed after the introduction of the finite π-meson mean field. We have developed the variation after the projection of the parity and the charge number. We call this model the charge and parity projected relativistic mean field (CPPRMF) model. The π-meson property of the pseudoscalar and isovector character requires the single-particle state with mixed-parity and -charge number as a basis in the mean field framework. To restore the symmetry, we perform the parity and charge number projection. We have used the chiral σ model in the nonlinear realization.

We have made the energy variational after projection. This procedure is very important sufficiently to take into account the effect of π-nucleon interaction. As is shown in Sec. III, variation before the projection procedure cannot sufficiently take into account the π-meson effect in the weak coupling region. For the π-nucleon interaction, the matrix element $\langle 0p - 0h|\hat{h}_D|2p - 2h\rangle$ gives the dominant contribution; it is better to use the projected wave function as a energy variational function so as to remove the negative-parity states (odd-number particle-hole states) in the ground state.

We have applied this CPPRMF framework for even $N = Z$ nuclei, especially for the ground state of the ^{4}He nucleus, to study the effect of the π meson in the ground-state structure. We have compared the amount of π-meson potential among the three methods, the relativistic mean field (RMF) without parity and charge number projection, the parity projected relativistic mean field (PPRMF), and the charge and parity projected relativistic mean field (CPPRMF). In the RMF without the projection method, when the π-nucleon coupling constant is weak there is no π-meson mean field. In the PPRMF method, the π-meson mean field appears, but the π-meson mean field effect is not large enough [4,32]. In the CPPRMF method, the amount of π-meson potential is around 3 times larger than that of PPRMF. This is because not only the π^0 but also the $\pi^\pm$ mesons contribute. Therefore the critical coupling constant where the π-meson mean field starts to become finite is much smaller than that of free space, g_A=1.25.

We have compared the constituents of the total energy in various cases of the strength of π-meson mean field. In general, as the π-meson mean field becomes stronger, the central potential (the sum of the scalar and vector potentials) becomes shallow, and the kinetic energy increases. Observing each component of the single-particle wave function, the negative-parity (p) proton state has quite a compact distribution compared with the usual p-shell harmonic-oscillator wave function. This fact is related to the increase in the kinetic energy. The π-nucleon interaction is accompanied by the higher-momentum components. Further, the amount of 0s state is depressed due to the parity admixture. It is seen as the depression at the central part of the density distribution in Fig. 5. We can naturally reproduce the feature of the density distribution and form factor of ^{4}He in the CPPRMF method as shown in Figs. 5 and 6. The results we obtained in this paper are qualitatively consistent with those of the case of the nonrelativistic framework (CPPHF) [32].

It is very important to perform the CPPRMF model calculations for medium and heavy nuclei in order to take into account the effect of π mesons in the mean field approximation. Since it is conceptually a simple method, but heavy in actual numerical calculations, it would be very interesting to study the medium and heavy nuclei in this method. Work in this direction is in progress.

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