# Thermal time scales in a color glass condensate

V. Parihar,[1,*] A. Widom,[1] and Y. N. Srivastava[1,2]
[1]*Physics Department, Northeastern University, Boston, Massachusetts 02115, USA*
[2]*Physics Department and INFN, University of Perugia, Perugia, Italy*
(Received 11 August 2005; published 25 January 2006)

In a model of relativistic heavy-ion collisions wherein the unconfined quark-gluon plasma is condensed into glass, we derive the Vogel-Fulcher-Tammann cooling law. This law is well known to hold true in condensed matter glasses. The high-energy plasma is initially created in a very hot negative temperature state and cools down to the Hagedorn glass temperature at an ever decreasing rate. The cooling rate is largely determined by the QCD string tension derived from hadronic Regge trajectories. The ultimately slow relaxation time is a defining characteristic of a color glass condensate.

DOI: 10.1103/PhysRevC.73.017901 PACS number(s): 25.75.Nq, 24.85.+p, 12.38.Mh

## I. INTRODUCTION

There has been considerable recent interest [1–5] in measurements of a possible quark-gluon plasma whose properties may be probed by relativistic heavy-ion collisions (RHICs). Central to the study of a possible unconfined color plasma is the time scale necessary to form this thermodynamic fluid phase. The central question concerns how the phase formation time compares with the collision time of the heavy-ion probes.

The apparent experimentally observed suppression of high transverse momentum jet production has led to the notion that thermal formation times may be longer than might be at first expected. The notion of an unconfined color fluid plasma was, for some RHIC energies, replaced by the notion of a color glass condensate [6–13] (CGC). In simple terms, thermal relaxation times in glasses are much longer than thermal relaxation times in fluids. If the thermal relaxation times were *not* much shorter than the RHIC collision times, then the observed heavy quark suppression would become understandable.

The thermal relaxation time in condensed matter glasses follows a well-known and long-established law due to Vogel [14], Fulcher [15], and Tammann [16] (VFT). The thermal relaxation time depends on temperature according to the VFT rule [17]

$$\tau \approx \tau_\infty \exp\left[\frac{\Phi}{k_B(T - T_g)}\right] (T > T_g), \tag{1}$$

wherein $\Phi$ is a thermal activation energy and $T_g$ is a dynamical glass temperature. Our purpose is to point out that the VFT equation (1) is expected to hold true for glass phases obtained from QCD-inspired models. The derivation of Eq. (1) will be given, and the method for determining the parameters will thereby be provided.

The notion that the perturbation theory QCD "vacuum" is in reality an excited state at a negative temperature [18] is introduced in Sec. II. In terms of the dynamic dielectric response function $\varepsilon$, the imaginary part is negative for the QCD vacuum and positive for the QED vacuum. It is the negative temperature feature of the perturbation theory QCD vacuum that allows for the avoidance of Landau ghosts [19,20]. The color screening $\varepsilon$ directly yields a QCD string potential between interacting quarks. In Sec. III we discuss the QCD strings whose tension $\sigma$ is empirically described by hadronic Regge trajectories; in Sec. IV, the dynamical glass temperature $T_g$ will be explored via the Hagedorn [21] entropy. We estimate a glass temperature and an activation energy, respectively, of

$$k_B T_g = \sqrt{\frac{3\hbar c\sigma}{4\pi}} \approx 207\,\text{MeV} \quad \text{and} \quad \Phi \approx 725\,\text{MeV}. \tag{2}$$

At its inception, the Hagedorn temperature was viewed as the largest possible temperature that could be achieved by smashing hadrons together at very high center-of-mass energy. Currently the use of QCD perturbation theory for very high energies implies, for short time scales, unconfined color, i.e., almost free quarks and gluons.

For high-energy RHIC events, the almost free initially produced quarks and gluons constitute a very hot plasma at negative temperature. The plasma cools at first to even more negative temperatures reaching $T \to -\infty$ and entering at $T \to +\infty$. In other words, it is the negative inverse temperature $\beta \equiv (k_B T)^{-1}$ (and not $T$) that passes through zero when you cool a system that starts at a negative temperature. Now the plasma further quickly cools from $\infty > T \gg T_g$, heading toward the Hagedorn temperature $T_g$ from above. But, in accordance with VFT glass asymptotic equation (1), the cooling relaxation time grows exponentially ever larger, $\tau \to \infty$ as $T \to T_g + 0^+$. The collision ends before the temperature can get below the Hagedorn temperature. The glass never fully hardens, as discussed in the concluding Sec. V.

## II. PERTURBATION THEORETICAL QCD

Recall the running coupling constant in QED, i.e.,

$$\alpha(Q^2) = \frac{e^2}{4\pi\hbar c\varepsilon(Q^2)}, \tag{3}$$

*Electronic address: parihar.v@neu.edu

 

wherein the dielectric response of the vacuum obeys the dispersion relation

$$\varepsilon(Q^2) = 1 - \frac{Q^2}{\pi}\int_0^\infty \left[\frac{\mathrm{Im}\,\varepsilon(-\nu - i0^+)}{\nu + Q^2}\right]\frac{d\nu}{\nu}. \tag{4}$$

For timelike wave vectors, the vacuum is dissipative:

$$\mathrm{Im}\,\varepsilon(-\nu - i0^+) \geqslant 0 \quad \text{if} \quad \nu = -Q^2 > 0. \tag{5}$$

Among the processes contributing to the dissipative vacuum equation (5) are the creation out of the vacuum of charged particle-antiparticle pairs produced by incident electromagnetic radiation. From Eqs. (4) and (5), it follows that for some spacelike $Q^2 > 0$, the real dielectric response will vanish. This constitutes the Landau ghost problem of QED. In particlular, the potential energy between two static charges, $z_1 e$ and $z_2 e$, is determined by the dielectric screening function $e^2 \to e^2/\varepsilon(Q^2)$ as discussed in standard works on quantum electrodynamics [22]:

$$V(r) = \left(\frac{z_1 z_2 e^2}{4\pi r}\right)\chi(r),$$
$$\chi(r) = \frac{2}{\pi}\int_0^\infty \sin(Qr)\left[\frac{dQ}{Q\varepsilon(Q^2)}\right]. \tag{6}$$

Note that $\lim_{r\to\infty}\chi(r) = 1$.

For the QCD case, the running coupling constant

$$\alpha_s(Q^2) = \frac{g^2}{4\pi\hbar c\varepsilon_s(Q^2)}, \tag{7}$$

wherein

$$\varepsilon_s(Q^2) = -\frac{Q^2}{\pi}\int_0^\infty \left[\frac{\mathrm{Im}\,\varepsilon_s(-\nu - i0^+)}{\nu + Q^2}\right]\frac{d\nu}{\nu}. \tag{8}$$

To one loop order in perturbation theory, one finds

$$\frac{\mathrm{Im}\,\varepsilon_s(-\nu - i0^+)}{\pi} = -\frac{g^2}{4\pi\hbar c}(\beta_0 + \cdots)$$
$$\beta_0 = \frac{1}{4\pi}\left(\frac{11}{3}N_c - \frac{2}{3}n_f\right), \tag{9}$$

wherein $N_c$ is the number of colors and $n_f$ is the number of flavors.

Note the condition

$$\mathrm{Im}\,\varepsilon_s(-\nu - i0^+) \leqslant 0 \quad \text{(color amplifier)}, \tag{10}$$

which implies that the perturbation theory vacuum is in reality an excited QCD state that decays into the true vacuum [18]. Unlike the perturbation theory QED vacuum, which requires external radiation energy to produce particle pairs, the perturbation theory QCD vacuum *spontaneously* decays into true vacuum, radiating bursts of hadrons. The excited state QCD perturbation theory vacuum is similar to an excited amplifying laser material with inverted excitation energy levels. The laser material spontaneously decays into the true ground state, radiating a photon pulse. Amplifying media may be described by a negative temperature. Initial high-energy particles blast the true vacuum into a QCD perturbation theory vacuum at a negative temperature. The resulting system then cools to true vacuum, radiating hadrons. When a very hot negative temperature cools down, the temperature keeps going down. The temperature goes down through $T = -\infty$, then appears at $T = +\infty$, further cooling through decreasing positive temperatures. From Eqs. (8) and (10) it follows for spacelike wave vectors that $\varepsilon_s(Q^2) > 0$, so there are no QCD Landau ghosts.

The quark potentials are described in terms of the color screening function

$$V_s(r) = \eta^{ab} t_{1a} t_{2b}\left(\frac{g^2}{4\pi r}\right)\chi_s(r),$$
$$\chi_s(r) = \frac{2}{\pi}\int_0^\infty \sin(Qr)\left[\frac{dQ}{Q\varepsilon_s(Q^2)}\right], \tag{11}$$

where the color matrices for the two quarks are $t_{1a}$ and $t_{2b}$ Taking two derivatives of Eq. (11) with respect to $r$ yields

$$\chi_s''(r) = -\frac{2}{\pi}\int_0^\infty \sin(Qr)\left[\frac{QdQ}{\varepsilon_s(Q^2)}\right]. \tag{12}$$

From the small wave-number dependence of the color dielectric response,

$$\frac{L^2}{2} = \lim_{Q\to 0}\frac{\varepsilon_s(Q^2)}{Q^2} = -\frac{1}{\pi}\int_0^\infty \mathrm{Im}\,\varepsilon_s(-\nu - i0^+)\frac{d\nu}{\nu^2}, \tag{13}$$

one finds via Eqs.(12) and (13) that

$$\lim_{r\to\infty}\chi_s''(r) = -\left(\frac{2}{L^2}\right) = \lim_{r\to\infty}\frac{\chi_s(r)}{2r^2}. \tag{14}$$

The central result of this section is a confining linear potential

$$V_s(r) = -\eta^{ab} t_{1a} t_{2b}\sigma r \quad \text{as} \quad r\to\infty, \tag{15}$$

with a string tension

$$\sigma = \frac{g^2}{4\pi L^2}. \tag{16}$$

Let us now consider in more detail the nature of the QCD string.

## III. QCD STRINGS

The QCD string may be physically pictured as follows [23]: (i) Between two quarks with color charges $t_{1a}g$ and $t_{2b}g$ is a string. (ii) Inside the string is a gluon condensate electric field

$$\mathcal{E} = \sqrt{\eta^{ab}\langle \mathbf{E}_a\cdot\mathbf{E}_b - \mathbf{B}_a\cdot\mathbf{B}_b\rangle} = \frac{g}{4\pi L^2}. \tag{17}$$

(iii) The tension in the string is

$$\sigma = g\mathcal{E} = \hbar c Q_s^2, \tag{18}$$

wherein $\hbar c Q_s$ is the saturation [2,3,5–10] energy scale. (iv) Let us suppose that two virtually zero mass quarks move with light speed $c$ on the ends of a linear string that extends along the straight line segment $-a < r < a$. In rigid body rotation,

$$
\begin{aligned}
\frac{|\mathbf{v}|}{c} &= \frac{|r|}{a} \quad \text{(speed)},\\
Mc^2 &= \int_{-a}^{a} \frac{\sigma dr}{\sqrt{1-|\mathbf{v}/c|^2}} = \pi\sigma a,\\
J &= \frac{1}{c^2}\int_{-a}^{a} \frac{\sigma r|\mathbf{v}|dr}{\sqrt{1-|\mathbf{v}/c|^2}} = \frac{\pi\sigma a^2}{2c},\\
J &= \frac{M^2c^3}{2\pi\sigma} \quad \text{(classical)}.
\end{aligned} \tag{19}
$$

The quantum relationship between angular momentum $J$ and mass $M$ is taken to be

$$J = \hbar\alpha_0 + \frac{M^2c^3}{2\pi\sigma}. \tag{20}$$

From experimental linear Regge trajectories

$$\hbar c\sigma \approx 0.18\,\mathrm{GeV}^2. \tag{21}$$

The question arises as to the nature of quark-antiquark pair creation, i.e., jet production, in a scattering experiment wherein the incoming particles supply the energy to create the gluon condensate within the string as well as supplying the quark pair energy ($mc^2$ per quark). Quark motion along the string obeys Newton's law for the rate of change of momentum,

$$\frac{dp}{dt} = g\mathcal{E} = \sigma. \tag{22}$$

The quark energy-momentum relation in a 1+1 dimensional QCD string follows from the partitioned Hamiltonian matrix

$$H = \begin{pmatrix} cp & mc^2 \\ mc^2 & -cp \end{pmatrix}, \tag{23}$$

where (from the quark-pair creation viewpoint) the transition rate per unit time to create a quark is determined by Fermi's golden rule,

$$\bar{\nu} = \frac{2\pi}{\hbar}|mc^2|^2\delta(2cp). \tag{24}$$

From the viewpoint of QCD string fragmentation, the probability that the string dissociates may be written as

$$
\begin{aligned}
d^2P &= \frac{dpdr}{2\pi\hbar}\exp\left(-\int \bar{\nu}dt\right),\\
\frac{d^2P}{drdt} &= \frac{g\mathcal{E}}{2\pi\hbar}\exp\left(-\int \bar{\nu}\frac{dp}{g\mathcal{E}}\right),
\end{aligned} \tag{25}
$$

where Eq. (22) has been invoked. The transition rate per unit time per unit string length for fragmentation follows from Eqs. (24) and (25); it is

$$\gamma = \frac{g\mathcal{E}}{2\pi\hbar}e^{-m^2c^3/\hbar g\mathcal{E}} = \frac{\sigma}{2\pi\hbar}e^{-m^2c^3/\hbar\sigma}. \tag{26}$$

From the above results, it appears that strings connecting high-mass quarks are less likely to fragment than strings connecting low-mass quarks. The QCD system starts out very hot, so that critical glass temperatures are approached from above. It is here that the notion of a string glassy state appears most useful.

## IV. HAGEDORN GLASS TEMPERATURE

We have shown in Eq. (19) for a classical QCD string model that the energy and angular momentum are related by

$$\frac{E^2}{2\pi c\sigma} = J. \tag{27}$$

In the quantum theory of the Boson (gluon) string, $J$ has the spectrum

$$J = \hbar N \quad \text{where} \quad N = 0, 1, 2, 3, \ldots. \tag{28}$$

If there are $n_{k,j} = 0, 1, 2, 3, \ldots$ string excitations with polarization $j$, each carrying angular momentum $\hbar k$ with $k = 1, 2, 3, \ldots$, then

$$N = \sum_{j=1}^{2}\sum_{k=1}^{\infty} k n_{kj}. \tag{29}$$

Equation (29) gives rise to a statistical mechanical entropy problem that involves analytical number theory. How many different ways $\Omega$ can you form the integer $N$ out of the smaller integers $\{n_{kj}\}$? From the asymptotic solution [24] of an earlier and similar partitioning problem, the solution to the QCD excitation string partitioning problem was found [25]. In the large $N \to \infty$ limit,

$$\ln\Omega(N) \approx 2\pi\sqrt{\frac{2N}{3}} + \ln[(6N)^{-7/4}\sqrt{3}]. \tag{30}$$

From the general definition of degeneracy and entropy,

$$S = k_B \ln\Omega, \tag{31}$$

along with Eqs. (27), (28), and (30), we find the Hagedorn string entropy

$$
\begin{aligned}
\frac{S}{k_B} &= \left(\frac{E}{k_BT_g}\right) - \frac{7}{2}\ln\left(\frac{E}{k_BT_g}\right) + \frac{\tilde{S}}{k_B},\\
\frac{\tilde{S}}{k_B} &= \frac{1}{2}\left[\ln(3) + 7\ln\left(\frac{2\pi}{3}\right)\right].
\end{aligned} \tag{32}
$$

The Hagedorn glass temperature is

$$k_BT_g = \sqrt{\frac{3\hbar c\sigma}{4\pi}} = \sqrt{\frac{3}{4\pi}}\hbar c Q_s \approx 207\,\mathrm{MeV}, \tag{33}$$

where Eq. (21) has been invoked. The thermal equations of state for a hot string now follow from

$$\frac{1}{T} = \frac{\partial S}{\partial E} = \frac{1}{T_g} - \frac{7k_B}{2E}. \tag{34}$$

Thus, the entropy as a function of temperature reads as

$$
\begin{aligned}
\frac{S}{k_B} &= \frac{7}{2}\left(\frac{T_g}{T-T_g}\right) - \left(\frac{7}{2}\right)\ln\left[\frac{T}{(T-T_g)}\right] + S_\infty,\\
\frac{S_\infty}{k_B} &= \frac{7}{2}\left[1 + \ln\left(\frac{2\pi}{3}\right) - \ln\left(\frac{7}{2}\right)\right] + \frac{1}{2}\ln 3.
\end{aligned} \tag{35}
$$

The role of the entropy in determining transition rates follows from the rule of averaging over initial states and summing over final states. Since the ratio of the number of final states to the number of initial states is given by

$$\frac{\Omega_f}{\Omega_i} = \exp\left(\frac{S_f - S_i}{k_B}\right), \tag{36}$$

the transition rates may be phase space dominated by the exponential entropy factors. The dominant entropy arises from bosonic gluon excitations of the QCD string. The Fermi or Bose nature of physical particles depends only on the number, respectively odd or even, of quarks tied together (in a polymerlike fashion) by the string. The thermal relaxation time for very hot $T \to \infty$ color unconfined states is related to the thermal relation times for finite temperature $T$ unconfined states via

$$\begin{aligned} \tau_\infty &= \tau \exp\left(\frac{S_\infty - S}{k_B}\right) (T > T_g), \\ \tau &= \tau_\infty \left(\frac{T - T_g}{T}\right)^{7/2} \exp\left[\frac{\Phi}{k_B(T - T_g)}\right]. \end{aligned} \tag{37}$$

Equation (37) is consistent with the VFT asymptotic equation (1) with

$$\Phi = \frac{7k_B T_g}{2} \approx 725\,\text{MeV}. \tag{38}$$

In Eq. (37) the dynamical prefactor attempt frequency $\tau_\infty^{-1} \sim cQ_s \sim 6.43 \times 10^{23}$ Hz, as defined in Eqs. (18) and (21). Equation (37) is the central result of this work.

## V. CONCLUSION

The true vacuum of QCD is completely nonperturbative owing to a currently poorly understood quark-gluon confinement mechanism. The success of perturbative QCD is due to a "vacuum" that is (in reality) an *excited* negative temperature state. The energy required for creating this excited state arises from the incident energy of colliding particles. Perturbative QCD is presumed at very short times after particle collisions. QCD perturbation theory describes almost free quarks and gluons. This viewpoint leads naturally toward the possibility of a weakly coupled quark-gluon plasma phase in high-energy heavy-ion collisions (RHICs). Our analysis of quark-gluon plasma formation devolves around a postcollision vacuum as an unstable state. The negative temperature feature of the perturbative vacuum is responsible for the avoidance of the Landau ghost [18,20]. The metastable plasma state goes into a glassy state wherein the temperature settles into positive, more stable value.

RHIC experiments regarding high transverse momentum jets led to the conjecture that the thermal quark-gluon plasma formation times may be longer than the collision times and hence to the notion of a color glass condensate. The glass relaxation times are long, and a collision ends before the glass fully hardens. We have thus demonstrated in the hadronic QCD string model that the unconfined quark-gluon plasma condenses into glass. Our computation invokes the fact that the hot plasma begins at a negative temperature. We have derived the VFT cooling equation (1) for glass with a glass transition temperature $T_g \approx 207$ MeV and an activation energy of $\Phi \approx 725$ MeV. We have interpreted the glass transition temperature as the Hagedorn temperature (approached from above) at which collisions cease as a result of rapidly growing thermal relaxation times. It appears quite satisfactory that asymptotic freedom from perturbative QCD, in concert with a hadronic string with its condensed color electric fields and a highly degenerate Regge spectrum, suffices to produce the VFT cooling law.


[1] L. Maiani, F. Piccinini, A. D. Polosa, and V. Riquer, Nucl. Phys. **A748**, 209 (2005).
[2] E. V. Shuryak and I. Zahed, Phys. Rev. D **70**, 054507 (2004).
[3] E. V. Shuryak, Nucl. Phys. **A702**, 83 (2002).
[4] S. Soff, S. A. Bass, and A. Dumitru, Phys. Rev. Lett. **86**, 3981 (2001).
[5] D. Teaney, J. Lauret, E. V. Shuryak, Phys. Rev. Lett. **86**, 4783 (2001).
[6] M. Gyulassy and L. McLerran, Nucl. Phys. **A750**, 30 (2005).
[7] D. Kharzeev and K. Tuchin, Nucl. Phys. **A753**, 316 (2005).
[8] E. Iancu and R. Venugopalan, in *QGP 3*, edited by R. Hwa and X.-N. Wang (World Scientific, Singapore, 2004).
[9] J. Jalilian-Marian, Mod. Phys. Lett. A **19**, 1251 (2004).
[10] R. Venugopalan, arXiv:hep-ph/0412396 (2004).
[11] E. Iancu, A. Leonidov, and L. McLerran, arXiv:hep-ph/0202270 (2002).
[12] R. Venugopalan, Comments Nucl. Part. Phys. **22**, 113 (1997).
[13] A. Ayala, J. Jalilian-Marian, L. McLerran, and R. Venugopalan, Phys. Rev. D **52**, 2935 (1995) .
[14] H. Vogel, Phys. Z. **22**, 645 (1921).
[15] G. S. Fulcher, J. Am. Ceram. Soc. **77**, 3701 (1925).
[16] G. Tammann and W. Hesse, Z. Anorg. Allg. Chem. **156**, 245 (1926).
[17] H. Tanaka, Phys. Rev. Lett. **90**, 055701 (2003).
[18] Y. Srivastava, S. Pacetti, G. Pancheri, and A. Widom, "Dispersive techniques for $\alpha_s$, $R_{had}$ and instability of the perturbative vacuum," Invited talk presented at "$e^+e^-$ Physics at Intermediate Energies Workshop," April 2001 (Paper no. T19), SLAC, Stanford, California, USA: eConf C010430 **T19** (2001).
[19] L. D. Landau, in *Neils Bohr and the Development of Physics*, edited by W. Pauli (Pergamon, London, 1955).
[20] S. Sivasubramanian, Y. Srivastava, and A. Widom, Mod. Phys. Lett. B **16**, 1201 (2002).
[21] R. Hagedorn, Nuovo Cimento Suppl. **3**, 147 (1965).
[22] V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii, *Quantum Electrodynamics* (Butterworth-Heinemann, Oxford, 1997), Sec. 114.
[23] Y. N. Srivastava and A. Widom, Phys. Rev. D **63**, 077502 (2001).
[24] G. H. Hardy and S. Ramanujan, Proc. London Math. Soc. **17**, 75 (1918).
[25] K. Huang and S. Weinberg, Phys. Rev. Lett. **25**, 895 (1970).