Neutrino emissivity under neutral kaon condensation

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Neutrino emissivity from neutron star matter with neutral kaon condensate is considered. It is shown that a new cooling channel is opened and that all previously known channels acquire greater emissivity reaching the level of the direct URCA cycle in normal matter.

I. INTRODUCTION

Neutral kaon $\bar{K}^0$ condensation treated jointly with K^- condensation has been considered lately by some authors [1,2]. They paid attention mainly to how the extension to the $\bar{K}^0$ condensate alters the equation of state and the composition of dense matter. Matter composition is a highly important issue for neutron star cooling during the first few million years after the neutron star's birth. In this period of a star's life, it is mainly cooled by neutrino emission from the dense core of the star. Generally, the presence of negative boson condensate makes matter more isospin symmetric with the proton fraction increasing quickly, with density, easily exceeding the threshold value for the direct URCA cycle [3], which is the most effective mechanism of cooling in the dense interior of a neutron star. Thus, in this way, kaon condensation favors fast cooling of neutron stars. But this conclusion should be taken carefully. It was shown in [4] that for some classes of nuclear models, negative kaons become so abundant that a very high proton fraction is required to maintain the matter neutrality. For a very high proton fraction, the direct URCA cycle is blocked again. However, in matter with kaon condensate, besides the direct URCA cycle, another type of reaction is permissible because the properties of nucleons in a dense medium change when kaoncondensate is formed. Nucleons appear to be dressed by kaons and become linear combinations of vacuum states [5,6], that is,

$$\tilde{n} = u\,n + v\,p, \tag{1}$$

$$\tilde{p} = -\bar{v}\,n + \bar{u}\,p, \tag{2}$$

where u and v depend on the condensate amplitude and obey the usual unitarity condition: $u\bar{u} + v\bar{v} = 1$. When condensate vanishes $\langle K \rangle \to 0$, the coefficients recover the pure nucleons, i.e., $u \to 1$, $v \to 0$. Such quasiparticles cease to be the eigenstate of charge. Hence, besides the usual direct URCA process (dURCA), which corresponds to neutron decay and its inversion, other reactions are also possible between nucleons dressed with kaons (kURCA):

$$\text{dURCA}: \quad \tilde{n} \quad \leftrightarrow \quad \tilde{p} + l + \nu_l. \tag{3}$$

$$\text{kURCA}: \quad \begin{aligned} \tilde{n} &\leftrightarrow \tilde{n} + l + \nu_l, \\ \tilde{p} &\leftrightarrow \tilde{p} + l + \nu_l. \end{aligned} \tag{4}$$

For the kURCA processes, it is easier to fulfill the kinematic condition (triangle condition) concerning the nucleon and lepton Fermi momenta. It may be shown that independently of the value of the proton fraction, even for almost pure neutron star matter, where $x \approx 0$, at least one of the above reactions takes place. However, the emissivities I of the three channels are not the same. Roughly speaking, they may be classified by means of the Cabibbo angle θ_C [6] as

$$I_{\text{dURCA}} \sim \cos^2 \theta_C, \qquad I_{\text{kURCA}} \sim \sin^2 \theta_C. \tag{5}$$

The emissivity in the dURCA cycle is proportional to $\cos^2 \theta_C$; whereas that for kURCA, to $\sin^2 \theta_C$. This means that the kURCA branch is about two orders of magnitude less effective than the dURCA branch. In order to obtain the value of neutrino emissivity, a concrete model of strong interaction must be used. As already mentioned, there are models of K^- condensation for which dURCA is switched off, and only less effective kURCA may cool the matter.

An interesting question is to what extent the picture changes with the inclusion of the $\bar{K}^0$ condensate. Such a component seems to be exotic, but as shown by Pal *et al.* [1], it is quite plausible to consider the matter with both K^- and $\bar{K}^0$. The addition of $\bar{K}^0$ to the model does not require additional parameters because the form of $\bar{K}^0 N$ couplings comes from the symmetry considerations, and we need only to know the $K^- N$ coupling constant. That is important because the constant is not a well-determined quantity, and we would like to avoid any further uncertainty. The critical density for $\bar{K}^0$ condensation is always higher than that for K^-. This comes from the fact that the $\bar{K}^0$ effective mass must drop to 0, whereas the K^- effective mass should drop only to the electron chemical potential. Pal *et al.* calculated the critical density for kaon condensation, which is $(2–3.5)n_0$ for K^- and $(3–5)n_0$ for $\bar{K}^0$, where the uncertainty comes from only the not-well-known kaon-nucleon coupling strength. After the $\bar{K}^0$ appearance, both condensates may coexist because such a state lowers the total energy of the system. This also makes the equation of state slightly softer than in the pure K^- case. The [1] authors also considered several different parametrizations of the nuclear models (with different stiffness) and showed that critical density for $\bar{K}^0$ production is available in the center of a neutron star with realistic mass. Even if the central part of the star with $\bar{K}^0$ is very tiny, it could have dramatic consequences for the cooling scenario. Page and Applegate [7] have shown that even if the central kernel, where the direct URCA process

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is allowed, occupies only a few percent of the total mass, it cools the star at the same rate as it would if it comprised more than half of the total mass. One may say that if only the central density of a star exceeds the threshold value for direct URCA, the star is cooled according to the fast scenario in which it reaches the temperature around 10^5 K on the scale of 10^2 years, instead of 10^6 years for the slow cooling driven by modified URCA processes. So, it is important to see whether the $\bar{K}^0$ presence leads to fast neutrino cooling. In this work, we focus on the details of weak interactions in matter with kaon condensate and show that the inclusion of $\bar{K}^0$ condensation leads to such a mixing between quasinucleons that it removes the difference between the dURCA and the kURCA cycles, placing them on the same footing.

Now we would like to refer to the issue of the presence of hyperons in neutron star matter. In general, kaon condensation should be considered in common with hyperons. Hyperons appear at lower density than the threshold for K^- condensation and may move it up to higher densities even to completely block the production of kaons for some sets of model parameters [8,9]. The same behavior of the threshold density was observed in the $\bar{K}^0$ case by Banik and Bandyopadhyay [2]. Axial coupling of kaons and hyperons opens the possibility of p-wave condensation [10] and certainly affects the URCA cycles by introducing momentum dependence into the expression for neutrino emissivity, much as in the pion condensation case [5]. However, the hyperonic star becomes more arguable in light of the recent observations of massive neutron stars in x-ray binary systems [11] and lately also for radio pulsars [12] which suggest masses above $2M_\odot$. Different models, those based on hypernuclear observables [13,14] or on relativistic mean field theory [9,15], conclude that maximal mass does not exceed $1.8M_\odot$. One may notice that hyperons make the equation of state (EOS) much softer for very fundamental reasons. Their production creates additional baryonic Fermi seas and lowers neutron chemical potential μ_n which contributes directly to the pressure $P = -\varepsilon + \mu_n n_B$, where ε is the energy density and n_B is the baryon number density. Kaons also make the EOS softer, but the scale of this effect is model dependent and we have some freedom to avoid an EOS that is too soft. The kaon condensate has zero pressure and may modify the stiffness of the EOS only indirectly through effective masses of nucleons or lepton abundance, so various details of the model are relevant. Nevertheless, the total effect mainly depends on the kaon-nucleon coupling, and previous works have shown that matter with charged kaons [9,16] and with neutral kaons [1] is still able to support a neutron star mass above $2M_\odot$, which is in agreement with observations.

II. CHIRAL MODEL AND WEAK NUCLEAR CURRENTS

In the context of kaon condensation, the SU$(3)_L \times$ SU$(3)_R$ chiral model proposed by Kaplan and Nelson [17] is commonly used. The current algebra is naturally built into this model, so it may be used to find the form of hadronic currents needed to get the matrix elements for semileptonic reactions in the presence of the K^- and $\bar{K}^0$ condensates. The chirally symmetric part

takes the form

$$\mathcal{L}_\chi = \frac{f^2}{4}\,\text{Tr}\,\partial_\mu U \partial^\mu U^+ + \text{Tr}\,\bar{B}(i\gamma^\mu D_\mu - m_B)B \\ + F\,\text{Tr}\,\bar{B}\gamma^\mu\gamma_5[\mathcal{A}_\mu, B] + D\,\text{Tr}\bar{B}\gamma^\mu\gamma_5\{\mathcal{A}_\mu, B\}. \quad (6)$$

Mesons are represented by the matrix

$$U = \xi^2 = \exp\left(i\sqrt{2}\frac{M}{f}\right), \quad (7)$$

where M and B include the meson and baryon octet (for notation details, see [18])

$$M = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta_8 \end{pmatrix},$$

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{pmatrix}. \quad (8)$$

In order to get proper expressions for β-type transitions for nucleons in the presence of K^- and $\bar{K}^0$, one needs to know the conserved currents coming from the Lagrangian (6). They may be found by employing the Noether theorem to the chiral transformation $U \to LUR^+$, $\xi \to L\xi h^+ = hR^+$, and $B \to hBh^+$, where L, R, and h are SU(3) matrices. It is convenient to decompose the currents into two parts: purely mesonic and baryonic, where the latter contains baryons coupled to meson fields,

$$V^\mu = V_M^\mu + V_B^\mu, \qquad A^\mu = A_M^\mu + A_B^\mu, \quad (9)$$

and

$$V_{M,a}^\mu = -i\frac{f^2}{4}\,\text{Tr}\,\lambda_a(U^+\partial^\mu U + U\partial^\mu U^+), \quad (10)$$

$$A_{M,a}^\mu = i\frac{f^2}{4}\,\text{Tr}\,\lambda_a(U^+\partial^\mu U - U\partial^\mu U^+), \quad (11)$$

$$V_{B,a}^\mu = \frac{1}{4}\,\text{Tr}\,\bar{B}\gamma^\mu[u_+^a, B] + \frac{F}{4}\,\text{Tr}\,\bar{B}\gamma^\mu\gamma_5[u_-^a, B] \\ + \frac{D}{4}\,\text{Tr}\,\bar{B}\gamma^\mu\gamma_5\{u_-^a, B\}, \quad (12)$$

$$A_{B,a}^\mu = \frac{1}{4}\,\text{Tr}\,\bar{B}\gamma^\mu[u_-^a, B] + \frac{F}{4}\,\text{Tr}\,\bar{B}\gamma^\mu\gamma_5[u_+^a, B] \\ + \frac{D}{4}\,\text{Tr}\,\bar{B}\gamma^\mu\gamma_5\{u_+^a, B\}, \quad (13)$$

where $u_\pm^a = \xi^+\lambda_a\xi \pm \xi\lambda_a\xi^+$. The expression for axial mesonic current A_M^μ allows one to identify the parameter f with the pion decay constant f_π, whereas the baryonic part is relevant for semileptonic decays of baryons. The above formulas are similar in their form to the weak nuclear currents presented in [19] for the SU(2) × SU(2) chiral model.

The ground state of matter with kaon condensate is described by Fermi seas of baryons and the nonvanishing

expectation value of kaon fields. According to the Baym theorem [20], the kaon mean field acquires time dependence

$$\langle K^\pm \rangle = \frac{f\theta}{\sqrt{2}} \exp(\pm i\mu t), \quad \langle K^0 \rangle = \langle \bar{K}^0 \rangle = \frac{f\phi}{\sqrt{2}}, \quad (14)$$

recalling that for neutral kaons, their mean field is independent of time, as the chemical potential for neutral particles vanishes. The quantities θ, ϕ are nondimensional condensate amplitudes, useful in parametrizing the condensate state. These condensate amplitudes acquire the meaning of rotation angles in chiral space if instead of the matrix element $\langle \tilde{p}, \theta, \phi | \mathcal{O} | \tilde{n}, \theta, \phi \rangle$ of any operator $\mathcal{O}$ between quasinucleons, one considers $\langle p | \mathcal{U}^+(\theta, \phi) \mathcal{O} \mathcal{U}(\theta, \phi) | n \rangle$, the matrix element between normal nucleons but with a rotated $\mathcal{O}$. In this way, for example, the isospin-raising operator V_{1+i2}, relevant to the β decay, in the condensate state becomes linear combinations of all currents from the octet V_a, leading to transitions forbidden in the normal state, as reactions shown in (4). In our approach, we do not need to know the explicit form of the $\mathcal{U}(\theta, \phi)$ because any nuclear current may be found directly by putting expectation values of kaon fields (14) into Eqs. (12) and (13). In this derivation, the important quantity is the kaon field matrix ξ, which now takes the form

$$\xi = \frac{1}{\chi^2} \begin{pmatrix} \phi^2 + \theta^2 \cos\frac{\chi}{2} & -2 e^{it\mu}\theta\phi \sin^2\frac{\chi}{4} & ie^{it\mu}\theta\chi \sin\frac{\chi}{2} \\ -2 e^{-it\mu}\theta\phi \sin^2\frac{\chi}{4} & \theta^2 + \phi^2 \cos\frac{\chi}{2} & i\phi\chi \sin\frac{\chi}{2} \\ ie^{-it\mu}\theta\chi \sin\frac{\chi}{2} & i\phi\chi \sin\frac{\chi}{2} & \chi^2 \cos\frac{\chi}{2} \end{pmatrix}, \quad (15)$$

where $\chi^2 = \theta^2 + \phi^2$. The weak hadronic current J_μ has the usual $V - A$ structure; but for the strange particle case, besides the isospin-raising part $V_{1+i2} - A_{1+i2}$, one must include the strangeness-changing part coming from the SU(3) octet. Following the Cabibbo theory, the hadronic current is

$$J_\mu = \cos\theta_C (V_{1+i2} - A_{1+i2})_\mu + \sin\theta_C (V_{4+i5} - A_{4+i5})_\mu. \quad (16)$$

The rates of the reactions which operate in the URCA cycles is described by the baryonic part V_B, A_B of the total SU(3) current (9). Knowing the form of the kaon field matrix ξ, we may calculate the full octet of currents. Putting them into (16), one gets the weak hadronic current J^μ in the presence of the K^- and $\bar{K}^0$ condensates, that is,

$$J^\mu = \frac{\cos\theta_C}{\chi^4} \left\{ \bar{p} \, \Gamma_{pn}^\mu n \left[(\theta^4 + \phi^4) \cos\frac{\chi}{2} + \frac{\theta^2\phi^2}{2} \left(3 + \cos\frac{\chi}{2} \right) \right] \right.$$
$$+ (\bar{n} \, \Gamma_{nn}^\mu n + \bar{p} \, \Gamma_{pp}^\mu p) \, 2\theta\phi \, e^{-i\mu t} \sin^2\frac{\chi}{4}$$
$$\left. + \bar{n} \, \Gamma_{np}^\mu p \left(2\theta\phi \, e^{-i\mu t} \sin^2\frac{\chi}{4} \right)^2 \right\}$$
$$+ \frac{i \sin\theta_C}{\chi^3} \sin\frac{\chi}{2} \left\{ \bar{p} \, \tilde{\Gamma}_{pn}^\mu n \left(\phi^2 + \theta^2 \cos\frac{\chi}{2} \right) \phi \right.$$
$$+ (\bar{n} \, \tilde{\Gamma}_{nn}^\mu n + \bar{p} \, \tilde{\Gamma}_{pp}^\mu p) \, \theta \, e^{-i\mu t}$$
$$\left. + \bar{n} \, \tilde{\Gamma}_{np}^\mu p \, 2e^{-2i\mu t}\theta^2\phi \sin^2\frac{\chi}{4} \right\}, \quad (17)$$

where the matrices Γ_{ij} and $\tilde{\Gamma}_{ij}$ are linear combinations of the Dirac matrices, which are explicitly given in the appendix.

III. NEUTRINO EMISSIVITY

At this point, we are ready to derive the transition rate for β processes in matter with the K^-, $\bar{K}^0$ condensates. Because the energy of nucleons and leptons is much smaller than the mass of $W^\pm$ particles, it is sufficient to use the Fermi theory of weak interactions, for which the Hamiltonian takes the usual form

$$H_{\text{weak}} = \frac{G_F}{\sqrt{2}} J^\mu l_\mu. \quad (18)$$

Where J^μ is hadronic and l^μ is leptonic weak current. In the case of charged-kaon (K^-) condensate only, J^μ derived in the previous section reduces to the simpler form

$$J_{\phi=0}^\mu = \cos\theta_C \, \bar{p}\gamma^\mu(1 - g_A\gamma_5)n \cos\frac{\theta}{2} + \sin\theta_C$$
$$\times [\bar{n}\gamma^\mu(1 + \Delta g \, \gamma_5)n + 2\bar{p}\gamma^\mu(1 - F\gamma_5)p]\frac{i}{2} \sin\theta, \quad (19)$$

where $g_A = D + F$ and $\Delta g = D - F$, and which is equivalent to the results already presented in [21]. Comparing (19) and (17), note that the extension to neutral kaons introduces an additional term $\sim \bar{n}\gamma^\mu p$, which is absent in the pure K^- condensate case. This term opens a new URCA channel, let us call it k0URCA.

$$\text{k0URCA}: \quad \tilde{p} \leftrightarrow \tilde{n} + e + \bar{\nu}_e. \quad (20)$$

Although slightly exotic, this kind of "proton decay" is possible in dense matter, as one must remember that quasinucleons represent mixed states of normal nucleons and do not possess a well-determined charge. Moreover, the kURCA transitions, i.e., transitions between the same type of quasinucleons (4), are now also present in the $\cos\theta_C$-dependent part of the weak current. This means that for the dense matter state, where the two kinds of condensates are simultaneously present ($\phi \neq 0, \theta \neq 0$), both the kURCA and dURCA cycles will take place at the same rate. This is an important result as it is easier to fulfill the triangle condition for the Fermi momenta of nucleons in the case of the kURCA channel.

The energy per unit volume and time released in one cycle due to neutrino emission is equal to the product of the neutrino energy and the doubled β-decay rate for a given quasiparticle $i = \tilde{p}$, or $\tilde{n}$, that is,

$$I_{\text{URCA}} = \frac{2}{(2\pi)^{12}} \int d^3 p_i d^3 p_f d^3 p_e d^3 p_\nu f_i$$
$$\times (1 - f_f)(1 - f_e)W_{if}\varepsilon_\nu. \quad (21)$$

The decay rate W_{ij} for a transition $i \to j + e + \nu_e$ is given by the expression

$$W_{if} = (2\pi)^4 \delta(\varepsilon_f - \varepsilon_i - \varepsilon_e - \varepsilon_\nu)\delta(\boldsymbol{p}_f - \boldsymbol{p}_i - \boldsymbol{p}_e - \boldsymbol{p}_\nu)$$
$$\times |\langle f \, e \, \nu_e | H_{\text{weak}} | i \rangle|^2. \quad (22)$$

The squared matrix element may be factorized in a standard manner

$$|\langle f \, e \, \nu_e | H_{\text{weak}} | i \rangle|^2 = H^{\mu\nu}L_{\mu\nu}, \quad (23)$$

where the leptonic tensor is

$$L^{\mu\nu} = \frac{1}{\varepsilon_e \varepsilon_\nu} \left(p_e^\mu p_{\bar\nu}^\nu + p_e^\nu p_{\bar\nu}^\mu - p_e \cdot p_{\bar\nu}\, g^{\mu\nu} + i\epsilon^{\mu\nu\rho\sigma} p_{e\rho} p_{\bar\nu\sigma} \right),$$
(24)

and the hadronic tensor includes the hadronic weak current J^μ with summation over the nucleon spin states

$$H^{\mu\nu} = \sum_{s,s'} \langle p_f, s'|J^\mu|p_i, s\rangle \langle p_i, s|J^{+\nu}|p_f, s'\rangle.$$
(25)

For nonrelativistic nucleons, the hadronic tensor becomes momentum independent $H^{\mu\nu} = 2(|v|^2\delta_{0\mu}\delta_{0\nu} + |a|^2\delta_{i\mu}\delta_{i\nu})$, where a, v are the axial and vector parts of the hadronic current (17). The neutrino momentum $|\mathbf{p}_\nu|$ is of the order of thermal energy T, which means it is much smaller than the momenta of nucleons and electrons, and may be neglected in the δ function in (22). Finally, the matrix element (23) may be treated as a constant, being independent of particle momenta. The phase space in (21) is weighted only by the three Fermi-Dirac distribution functions f_i for $\tilde{p}, \tilde{n}, e$, because the neutrino leaves the matter freely. In order to calculate the emissivity integral, one may use the so-called phase-space decomposition, a very useful technique to obtain the reaction rate for strongly degenerated systems [22,23]. After this calculation, neutrino emissivity may be finally written as

$$I_{\mathrm{URCA}} = \frac{457\,\pi}{20\,160} T^6 m_p^* m_n^* m_e^* |M_{ij}|^2 \Theta_{\mathrm{ife}},$$
(26)

where Θ_{ife} is a step function corresponding to the triangle condition. It is equal to 1 if vectors $\mathbf{p}_f, \mathbf{p}_i, \mathbf{p}_e$ form a closed triangle, and 0 otherwise. M_{ij} is the momentum-independent matrix element (23) corresponding to different types of reactions, and m_i^* are effective masses of nucleons and electrons. In the case of the K^- and $\bar{K}^0$ condensation, there are four channels for different URCA cycles: one in (3), two in (4), and one in (20). The last one is typical of the presence of neutral kaons, whereas the reactions (4) are connected with the charged-kaon condensate. Of course, the first one, i.e., the dURCA cycle, is possible in normal npe matter as well as in matter with kaons. All of them belong to the class of direct URCA processes, where only three degenerated fermions take part in the cycle, which can be seen in the temperature dependence $\sim T^6$. So, the four different channels are only distinguished by means of their matrix elements $|M_{ij}|^2$, which now depend on the amplitudes of the two condensates θ and ϕ:

dURCA : $$|M_{np}|^2 = 2\,G_F^2\left(1 + 3g_A^2\right)\frac{\left(\phi^2 + \theta^2 \cos\frac{\chi}{2}\right)^2}{\chi^8}$$
$$\times \left[\left(\theta^2 + \phi^2 \cos\frac{\chi}{2}\right)^2 \cos^2\theta_C \right.$$
$$\left. + \phi^2\chi^2 \sin^2\frac{\chi}{2} \sin^2\theta_C\right].$$
(27)

kURCA : $$|M_{nn}|^2 = 8\,G_F^2\,\cos^2\theta_C\,\frac{\theta^2\phi^2 \sin^4\frac{\chi}{4}}{\chi^8}$$
$$\times\left\{\left[\theta^2 + \chi^2 + (\phi^2 + \chi^2)\cos\frac{\chi}{2}\right]^2\right.$$

$$+ 3\left[2\,\Delta g\,\chi^2 \cos^2\frac{\chi}{4}\right.$$
$$\left. - g_A\left(\theta^2 + \phi^2 \cos\frac{\chi}{2}\right)\right]^2\Bigg\}$$
$$+ 2\,G_F^2\,\sin^2\theta_C\,\frac{\theta^2 \sin^2\frac{\chi}{2}}{\chi^6}$$
$$\times\left\{\left[\phi^2 - (\phi^2 + \chi^2)\cos\frac{\chi}{2}\right]^2\right.$$
$$\left. + 3\left[\Delta g\,\chi^2 \cos\frac{\chi}{2} + 2g_A\phi^2 \sin^2\frac{\chi}{4}\right]^2\right\}$$
(28)

kURCA : $$|M_{pp}|^2 = 8\,G_F^2\,\cos^2\theta_C\,\frac{\theta^2\phi^2 \sin^4\frac{\chi}{4}}{\chi^8}$$
$$\times\left\{\left[\phi^2 + \chi^2 + (\theta^2 + \chi^2)\cos\frac{\chi}{2}\right]^2\right.$$
$$+ 3\left[2\,\Delta g\,\chi^2 \cos^2\frac{\chi}{4} - g_A\right.$$
$$\left.\times\left(\phi^2 + \theta^2 \cos\frac{\chi}{2}\right)\right]^2\Bigg\} + 2\,G_F^2\,\sin^2\theta_C$$
$$\times\frac{\theta^2 \sin^2\frac{\chi}{2}}{\chi^6}\left\{\left[\phi^2 + (\theta^2 + \chi^2)\cos\frac{\chi}{2}\right]^2\right.$$
$$+ 3\left[\Delta g\,\chi^2 \cos\frac{\chi}{2}\right.$$
$$\left. - g_A\left(\phi^2 + \theta^2 \cos\frac{\chi}{2}\right)\right]^2\Bigg\}$$
(29)

k0URCA : $$|M_{pn}|^2 = 32\,G_F^2\left(1 + 3\,g_A^2\right)\frac{\theta^4\phi^2 \sin^6\frac{\chi}{2}}{\chi^8}$$
$$\times\left(\phi^2 \sin^2\frac{\chi}{4} \cos^2\theta_C\right.$$
$$\left. + \chi^2 \cos^2\frac{\chi}{4} \sin^2\theta_C\right).$$
(30)

As shown in [21], for the case of the K^- condensate only ($\phi \to 0$), these matrix elements take a much simpler form

$$|M_{np}|^2 = 2\,G_F^2\left(1 + 3\,g_A^2\right)\cos^2\theta_C \cos^2\frac{\theta}{2},$$
(31)

$$|M_{nn}|^2 = \frac{1}{2}G_F^2\left(1 + 3\,\Delta g^2\right)\sin^2\theta_C \sin^2\theta,$$
(32)

$$|M_{pp}|^2 = 2\,G_F^2\left(1 + 3\,F^2\right)\sin^2\theta_C \sin^2\theta,$$
(33)

$$|M_{pn}|^2 = 0,$$
(34)

where, for the transition $\tilde{p} \to \tilde{n} + e + \bar\nu$, which is not possible in the pure K^- condensate, the corresponding matrix element vanishes. By comparing expressions (28), (29) and (32), (33), one may see how the inclusion of $\bar{K}^0$ introduces the terms proportional to $\cos^2\theta_C$ and, in this way, makes the kaon-induced URCA processes at the same level of intensity as the direct URCA cycle in normal matter. This was already noted in the discussion of the weak hadronic current (17). The matrix elements M_{if} are also highly θ and ϕ dependent.
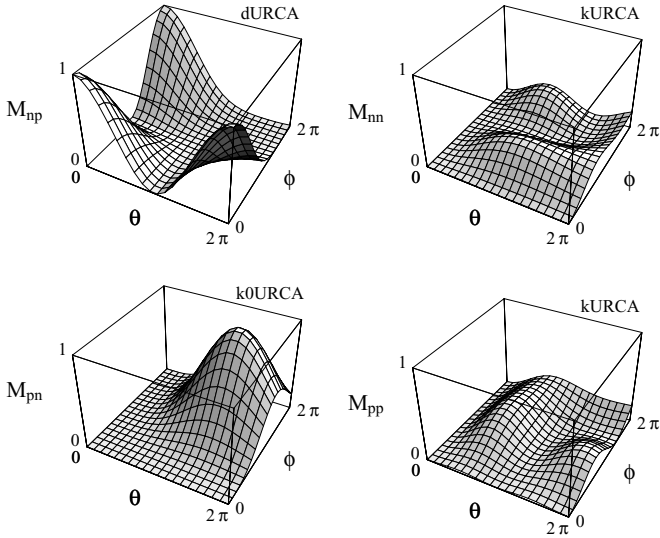
FIG. 1. Normalized matrix element for different URCA channels as a function of condensate amplitudes θ and ϕ.

This dependence is shown in Fig. 1, where the values of matrix elements were normalized to the maximal value for the dURCA cycle, which is $M_{np}^{\max} = 2G_F^2(1 + 3g_A^2)\cos^2\theta_C$. The behavior of θ and ϕ with matter density depends on the model of strong interactions used for dense matter description [4,24]. Both the unknown strength of the kaon-nucleon coupling and the details of interactions in the nonstrange sector affect the condensate behavior. However, roughly speaking, these papers showed that the typical value of the K^- condensate amplitude θ is around 1, and in some cases it may be somewhat greater than $\pi/2$. Therefore, one may suspect that the amplitude of the $\bar{K}^0$ condensate takes similar values. A careful look at the plots in Fig. 1 reveals that different cycles reach their maxima in different regions of the θ-ϕ plane, so one may conclude that almost independently of the concrete values of the K^- and $\bar{K}^0$ amplitudes, there always exists one cycle with emissivity approximately equal to the maximal value of M_{if}, i.e., $2G_F^2(1 + 3g_A^2)\cos^2\theta_C$. The given cycle works in neutron star matter when the corresponding triangle condition is satisfied:

$$\text{dURCA and k0URCA}: \quad |k_p - k_n| < k_e < k_p + k_n, \quad (35)$$

$$\text{kURCA for } \tilde{n}: \quad 2k_n > k_e, \quad (36)$$

$$\text{kURCA for } \tilde{p}: \quad 2k_p > k_e. \quad (37)$$

These inequalities show that independently of the proton and lepton abundances, at least one of the URCA channels is opened. Thus, the final conclusion is that the simultaneous presence of the K^- and $\bar{K}^0$ condensates leads to matter cooled very fast with the intensity of the fastest direct URCA cycle.

IV. CONCLUSIONS

The extension of the charged kaon to the neutral kaon condensate leads to such mixing between the components of currents from the SU(3) octet that two new features emerge. First, the $\bar{K}^0$ condensate opens an additional channel for the URCA process (quasiproton decay). Second, the $\bar{K}^0$ presence results in emissivity for the kaon-induced URCA processes scaling no longer with $\sin^2\theta_C$ but obtaining a contribution that scales as $\cos^2\theta_C$. This puts the kaon-induced URCA at the same level of importance as the normal direct URCA cycle. Moreover, the triangle condition says that different cycles are opened in different matter compositions, covering the whole range from pure neutron to pure proton matter ($0 < x < 1$). Therefore, one may finally conclude that matter with the K^- and $\bar{K}^0$ condensates is cooled at the level of the most effective (direct) URCA cycle, regardless of its detailed composition.

APPENDIX

Below are matrices Γ_{ij} and $\tilde{\Gamma}_{ij}$, which appear in the expression for the hadronic current (17), for the $\cos\theta_C$-dependent part

$$\Gamma_{pn}^{\mu} = \gamma^{\mu}(1 - g_A\gamma_5)$$

$$\Gamma_{pp}^{\mu} = \gamma^{\mu}\left\{-(\phi^2+\chi^2) - (\theta^2+\chi^2)\cos\frac{\chi}{2}\right.$$
$$\left. + \gamma_5\left[g_A\left(\phi^2+\theta^2\cos\frac{\chi}{2}\right) - 2\Delta g\,\chi^2\cos^2\frac{\chi}{4}\right]\right\}$$

$$\Gamma_{nn}^{\mu} = \Gamma_{pp}^{\mu}(\phi \leftrightarrow \theta)$$

$$\Gamma_{np}^{\mu} = \gamma^{\mu}(1 - g_A\gamma_5)$$

and for the $\sin\theta_C$-dependent part

$$\tilde{\Gamma}_{pn}^{\mu} = -\gamma^{\mu}(1 - g_A\gamma_5)$$

$$\tilde{\Gamma}_{pp}^{\mu} = \gamma^{\mu}\left\{-\phi^2 - (\theta^2+\chi^2)\cos\frac{\chi}{2} + \gamma_5\left[g_A\left(\phi^2+\theta^2\cos\frac{\chi}{2}\right)\right.\right.$$
$$\left.\left. - \Delta g\,\chi^2\cos\frac{\chi}{2}\right]\right\}$$

$$\tilde{\Gamma}_{nn}^{\mu} = \gamma^{\mu}\left[\phi^2 - (\phi^2+\chi^2)\cos\frac{\chi}{2}\right.$$
$$\left. + \gamma_5\left(2g_A\phi^2\sin^2\frac{\chi}{4} + \Delta g\,\chi^2\cos\frac{\chi}{2}\right)\right]$$

$$\tilde{\Gamma}_{np}^{\mu} = \gamma^{\mu}(1 - g_A\gamma_5).$$

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