Soliton with a pion field in the global color symmetry model

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We calculate the property of the global color symmetry soliton with the pion field being included explicitly. The calculated results indicate that the pion field provides a strong attraction so that the eigenenergy of a quark and the mass of a soliton reduce drastically, in contrast to those with only the sigma field.

I. INTRODUCTION

Quantum chromodynamics (QCD) has been accepted as the most successful theory to describe the strong interaction. However, the low-energy property, such as the property of baryon, is hard to be described with the precise QCD. Lattice QCD is then developed to calculate the property of baryons from the full QCD Lagrangian (see, for example, Ref. [1]). Meanwhile, on the side of continuous-field theoretical method, there are lots of phenomenological models to try to give the picture of the baryons. Soliton models, including the two most successful models, Skyrme model (or chiral soliton model, see, for example, Ref. [2]), and the chiral quark soliton model (or Nambu–Jona-Lasinio soliton (NJL) model, see, for example, Ref. [3]), are one kind of these models to give a vivid picture of baryons. However, the Skyrme model and the chiral quark soliton model give two contrary pictures. In Skyrme model, baryons are described as a soliton of mesons with no valence quark degrees of freedom. In contrast to Skyrme model, baryons are described as valence quarks bounded in the meson soliton background in the chiral quark soliton model. However, only local interactions are taken into account in NJL model.

To search for the solid QCD foundation of the soliton models and implement the nature of nonlocality of the quarks in mesons, the global color symmetry model (GCM) has been developed since the middle of 1980s [4–8]. In the GCM, the gauge symmetry was discarded and then QCD is reduced to a finite-range current-current interaction theory. One can get a quark-meson interaction model or a quark-diquark interaction model after bosonization [5] of the current-current interaction theory. Because there are many kinds of meson fields to interact with quarks and it is hard to deal with all of them, one usually takes the low-energy effective mode, the Goldstone mode pion and sigma, and discards the others. Then light baryons are modeled as solitons in the chiral meson background with Goldstone mesons [9–11]. With the GCM soliton, one can also discuss the dynamical confinement and, in particular, the effect of the nonlocality. Furthermore, the dependence of baryon properties on hadron matter medium has also been studied in

the GCM [12–18]. However, in the practical calculations, one usually takes the chiral circle constraint, i.e., solves the coupled equations of quark and σ-meson field explicitly, but discards the pion fields. Then in this article, we extend the GCM soliton to beyond the chiral circle constraint, i.e., deal with the scalar field σ and the pseudoscalar field π simultaneously. We can recognize that there is much difference between the results with and beyond the chiral circle constraint.

Below, we give a concise description of the GCM model and of the soliton in GCM model in Sec. II (an extensive review of GCM model can be found in series of articles [19,20]). In Sec. III, we describe the method of the numerical calculation and give our numerical results and compare the results with those with the chiral circle constraint. Finally, we summarize our work and give a brief remark in Sec. IV.

II. BRIEF REVIEW OF GCM

The GCM is constructed by the the generation functional in Euclidean space as [4,5]

$$
Z = \int \mathcal{D}\bar{q}(x)\mathcal{D}q(y) \exp\left[-\int d^4x\, \bar{q}(x)\, \not{\partial} q(x) - \frac{g^2}{2} \right.
$$
$$
\left. \times \iint d^4x d^4y\, \bar{q}(x)\gamma_\mu \frac{\lambda^a}{2} q(x) D(x-y) \bar{q}(y)\gamma_\mu \frac{\lambda^a}{2} q(y) \right].
$$
(1)

This functional is invariant under global color SU(3) transformation rather than the gauge color SU(3) transformation. $D(x - y)$ is the effective gluon propagator, which is a parameterized function to model the low-energy property and dynamics such as those of hadrons. It has been shown that the infrared enhancement of $D(x - y)$ is closely related to the chiral symmetry breaking (see for example Refs. [21,22]). Such a character should be embedded in the model.

It is convenient to calculate the Grassmann integration after bosonization [4]. First, one can reorder the quark fields through the Fierz transformation and get

$$
Z = \int \mathcal{D}\bar{q}\mathcal{D}q \exp\left[-\int \bar{q}\, \not{\partial} q - \frac{g^2}{2} \right.
$$
$$
\left. \times \iint \bar{q}(x)\frac{M^\theta}{2} q(y) D(x-y) \bar{q}(y)\frac{M^\theta}{2} q(x) \right], \quad (2)
$$

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where $M^\theta = K^a \otimes C^b \otimes F^c$, with $\{K^a\} = \{I, i\gamma_5, i\gamma_\mu/\sqrt{2},$ $i\gamma_5\gamma_\mu/\sqrt{2}\}$, $\{C^b\} = \{4I/\sqrt{3}, i\lambda/\sqrt{3}\}$, $\{F^c\} = \{I/\sqrt{2}, \vec{\tau}/\sqrt{2}\}$, are the direct product of spin, color SU(3), and flavor SU(2) matrices resulting from Fierz transformation. Then, one introduces bilocal Bose fields $B^\theta(x, y)$ that transform like $(\bar{q}(x)M^\theta q(y))/2$. The bilocal fields have the quantum numbers as those of mesons, so the fluctuations on vacuum can be identified as mesons. One then obtains

$$Z = \int \mathcal{D}B^\theta(x, y)e^{-S[B^\theta(x,y)]}, \quad (3)$$

with an action given as

$$S[B^\theta(x, y)] = \int d^4x d^4y \bar{q}(x)\left[\gamma \cdot \partial\delta(x-y) + \frac{M^\theta}{2}B^\theta(x, y)\right]$$
$$\times q(y) + \int d^4x d^4y \frac{B^\theta(x, y)B^\theta(y, x)}{2g^2D(x - y)}. \quad (4)$$

Introducing the quark chemical potential through the transformation

$$q(x) \rightarrow e^{\mu x_4}q(x) \quad (5)$$

and integrating the quark fields, one gets the action

$$S[\mu, B^\theta] = -\text{Tr} \ln G^{-1}[\mu, B^\theta]$$
$$+ \int d^4x d^4y \frac{B^\theta(x, y)B^\theta(y, x)}{2g^2D(x - y)}, \quad (6)$$

where the inverse of the quark propagator can be written as

$$G^{-1}(\mu; x, y) = e^{\mu x_4}G^{-1}(x, y)e^{-\mu y_4}$$
$$= (\gamma\partial - \mu\gamma_4)\delta(x-y) + e^{\mu x_4}\frac{M^\theta}{2}B^\theta(x, y)e^{-\mu y_4}. \quad (7)$$

Generally, the bilocal fields can be written as

$$B^\theta(x, y) = B_0^\theta(x, y) + \sum_i \Gamma_i^\theta(x, y)\phi_i^\theta\left(\frac{x+y}{2}\right), \quad (8)$$

where the first term is the translation invariant vacuum configuration of the bilocal fields. The second term is the fluctuation over the vacuum which can be identified as effective meson fields. Γ_i^θ is the form factor of the meson fields, which we take as the form of B_0^θ with appropriate Lorentz structure since the excitation of the inner degrees of freedom needs much energy and can be frozen in low energy range [8]. There are many meson fields, including scalar, pseudoscalar, vector, axial vector, and tenser field. In the lowest order, we can only consider the Goldstone mode, $\phi_i^\theta = \{\sigma, \vec{\pi}\}$, which was thought of as the most important low energy degrees of freedom.

The vacuum configuration can be determined by the saddle point condition $\delta S/\delta B_0^\theta = 0$, which induces a translation invariant quark self-energy $\Sigma(x - y) = (M^\theta/2)B_0^\theta(x, y) = (M^\theta/2)B_0^\theta(x - y)$. The equation of $\Sigma(x - y)$ in momentum space is, in fact, a truncated Dyson-Schwinger equation, which reads as follows:

$$\Sigma(p) = i\gamma p[A(p^2) - 1] + B(p^2)$$
$$= g^2 \int \frac{d^4q}{(2\pi)^4}D(p-q)\frac{\lambda^a}{2}\gamma_\mu \frac{1}{i\gamma q + \Sigma(q)}\gamma_\mu \frac{\lambda^a}{2}, \quad (9)$$

where $D(p)$ is the Fourier transformation of $D(x)$.

It is evident that the quark meson coupling is in Yukawa form while the meson self-interaction is nontrivial. The dynamical chiral symmetry breaking is because of $B(p^2) \neq 0$, which causes quark a dynamical mass $B(p^2)/A(p^2)$. One can model the low-energy property through a certain form of $A(p^2)$ and $B(p^2)$ determined by solving Eq. (9) with an effective gluon propagator or derived from some other models, such as instanton model or lattice stimulation.

In the chiral limit, the bilocal fields can be written as

$$\frac{M^\theta}{2}\left[B^\theta(x, y) - B_0^\theta(x, y)\right] = \frac{B(r)}{f_\pi}\chi(R)e^{i\gamma_5\tau\phi(R)/f_\pi}, \quad (10)$$

where $r = x - y$ and $R = (x + y)/2$ can be considered as the relative coordinate and centroid coordinate of a quark and an antiquark in a meson, respectively.

Expanding the action to leading order in derivatives of $\sigma = \chi \cos[\phi/f_\pi]$ and $\vec{\pi} = \hat{\phi}\chi \sin[\phi/f_\pi]$ at the vacuum $\sigma = f_\pi, \vec{\pi} = 0$, one gets

$$S[\sigma, \vec{\pi}] - S[f_\pi, 0] = \int d^4R\left\{\frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + U[\chi^2(R)]\right\}, \quad (11)$$

where $U(\chi^2)$ is the effective potential of mesons with $\chi^2 = \sigma^2 + \vec{\pi}^2$ and can be given as

$$U(\chi^2) = -12 \int \frac{d^4q}{(2\pi)^4}\left\{\ln\left[\frac{q^2A^2(q^2) + B^2(q^2)(\chi/f_\pi)^2}{q^2A^2(q^2) + B^2(q^2)}\right]\right.$$
$$\left. - \frac{B^2(q^2)[(\chi/f_\pi)^2 - 1]}{q^2A^2(q^2) + B^2(q^2)}\right\}, \quad (12)$$

and f_π is the pion decay constant given by the following:

$$f_\pi = 12 \int \frac{d^4q}{(2\pi)^4}\left[\frac{B^2A^2}{[q^2A^2 + B^2]^2}\right.$$
$$\left. - \frac{\frac{1}{2}q^2[(B')^2 + BB''] + BB'}{q^2A^2 + B^2}\right]. \quad (13)$$

It should be mentioned that the above expression of f_π is correct only when the derivatives of $A(p^2)$ are neglected. However, this approximation can be fairly right according to the characteristic of the effective gluon propagator in the present calculation. The full formula to evaluate the f_π can be found in Ref. [7].

The meson masses can be obtained by differentiating the potential U twice with respect to σ and π. It can be found that the pion mass is zero and the σ mass is finite at the chiral limit.

Originally, baryons are regarded as solitons with bag constant [4]

$$\mathcal{B} = U(0) - U(f_\pi),$$

where $U(0)$ is the effective potential of mesons in the soliton with assumption $\sigma = 0, \vec{\pi} = \vec{0}$, and $U(f_\pi)$ that of the vacuum with $\sigma = f_\pi, \vec{\pi} = \vec{0}$. Even though some baryon properties and their dependence on baryon density have been obtained in such a model (see, for example, Refs. [4,18]), the advantage of the quark-quark interaction in the GCM has not yet been represented explicitly. Then a much more elaborate soliton model was developed in 1990s [10]. As in Ref. [10], the meson

fields in the soliton are written explicitly as

$$\frac{M^\theta}{2}\left[B^\theta(x, y) - B_0^\theta(x, y)\right]$$
$$= B(x - y)\left[\sigma\left(\frac{x + y}{2}\right) + i\gamma_5\vec{\tau} \cdot \vec{\pi}\left(\frac{x + y}{2}\right)\right]. \quad (14)$$

After introducing a chemical potential of quarks as the above, one expresses the action difference between the vacuum state and a system with nonzero chemical potential as

$$S[\mu, \sigma, \vec{\pi}] = \text{Tr}\ln G^{-1}[\mu = 0, \sigma, \vec{\pi}] - \text{Tr}\ln G^{-1}[\mu, \sigma, \vec{\pi}]$$
$$- \int d^4x\left[\frac{1}{2}(\partial_x\sigma)^2 + \frac{1}{2}(\partial_x\vec{\pi})^2 + U(\chi^2)\right], \quad (15)$$

where $\chi^2 = \sigma^2 + \vec{\pi}^2$. The inverse of quark propagator at $\mu = 0$ is given as follows:

$$G^{-1}(\mu = 0; x, y) = \gamma\partial_x\delta(x - y) + \frac{1}{f_\pi}B(x - y)$$
$$\times\left[\sigma\left(\frac{x + y}{2}\right) + i\gamma_5\vec{\pi}\left(\frac{x + y}{2}\right)\right], \quad (16)$$

and that at finite μ can be written as follows:

$$G^{-1}(\mu; x, y) = e^{\mu x_4}G^{-1}(\mu = 0; x, y)e^{-\mu y_4}. \quad (17)$$

In the vacuum, the fields σ is f_π and $\vec{\pi}$ is zero determined by the saddle point condition of $S[\sigma, \pi]$. When there are valence quarks, the meson fields will vary with the spatial coordinate to show the shape of the bound system, the hadron.

One can define an effective potential through the Legendre transformation. With the static meson field, the effective potential is proportional to the energy functional. At the Hartree level as in Ref. [9], the energy functional of the static meson fields at a set of fixed quark occupation number n is

$$E[n, \sigma, \vec{\pi}] = E_q[n, \sigma, \vec{\pi}] + E_m[n, \sigma, \vec{\pi}], \quad (18)$$

where E_q is the valence quark's contribution,

$$E_q[n, \sigma, \vec{\pi}]\left[-\int d^4x\right]$$
$$= \text{Tr}\ln G^{-1}[\mu, \chi] - \text{Tr}\ln G^{-1}[0, \chi] - \mu n, \quad (19)$$

and E_m the meson field's contribution

$$E_m[n, \sigma, \vec{\pi}] = \int d^3x\left[\frac{1}{2}(\nabla\sigma)^2 + \frac{1}{2}(\nabla\vec{\pi})^2 + U(\chi)\right]. \quad (20)$$

The quark chemical potential μ is a functional of the meson fields σ and π and the quark occupation number n, which can be deduced from the relation $n = \partial\text{Tr}\ln G^{-1}[\mu; \sigma, \vec{\pi}]/\partial\mu$.

It is convenient to calculate the above trace of the quark propagator in Eq. (19) at certain bases, and one can get the bases through the spectral decomposition of the quark propagator. With the static meson fields, $G^{-1}(x, y)$ is time translation invariant and allows stationary eigenvectors of the form $u_j(\vec{x})e^{i\omega x_4}$, which satisfies

$$\int d^3y\, G^{-1}(\omega; \vec{x}, \vec{y})u_j(\vec{y}) = i\gamma_4\lambda_j(\omega)u_j(\vec{x}), \quad (21)$$

where $G^{-1}(\omega; \vec{x}, \vec{y})$ is the Fourier transformation of $G^{-1}(x_4 - y_4; \vec{x}, \vec{y})$. The eigenvalues $\lambda_j(\omega)$ can be written as the form $\lambda_j(\omega) = \omega - i\epsilon_j(\omega)$, where ϵ_j is the quark eigenenergy of the

state j. Calculating the functional in Eq. (19), one can know that the total energy is the sum of the positive eigenenergy ϵ_j, which satisfies $\lambda(\epsilon_j) = 0$, up to the chemical potential μ. These positive energy states satisfy the Dirac-like equation, which can be given in momentum space as

$$[i\gamma\, pA(p) + B(p)]u_j(\vec{p}) + \frac{1}{f_\pi}\int\frac{d^3k}{(2\pi)^3}$$
$$\times B\left(\frac{p + k}{2}\right)[\hat{\sigma}(\vec{p} - \vec{k}) + i\gamma_5\vec{\tau}\cdot\vec{\pi}(\vec{p} - \vec{k})]u_j(\vec{k}) = 0, \quad (22)$$

where $\hat{\sigma} = \sigma - f_\pi$. Because of the interaction, there is a renormalization constant for the quark field, which can be derived from the residue at the pole of Green's function and given as [9]

$$Z_j = -\int d^3p\,d^3q\,\bar{u}_j(\vec{p})\frac{\partial G^{-1}(i\epsilon_j; \vec{p}, \vec{q})}{\partial\epsilon_j}u_j(\vec{q}). \quad (23)$$

The total soliton energy functional of the soliton is then

$$E = \sum_{j=1}^3\epsilon_j + \int d^3x\left[\frac{1}{2}(\nabla\sigma)^2 + \frac{1}{2}(\nabla\vec{\pi})^2 + U(\chi)\right]. \quad (24)$$

The static meson fields are the configuration which minimizes the energy functional, i.e., they are the solutions of the equations $\delta E[\sigma, \vec{\pi}]/\delta\sigma = 0$ and $\delta E[\sigma, \vec{\pi}]/\delta\vec{\pi} = 0$. After some derivation, one can write these equations explicitly as

$$-\vec{\nabla}^2\sigma(\vec{r}) + \frac{\delta U}{\delta\sigma(\vec{r})} + Q_\sigma(\vec{r}) = 0, \quad (25)$$

$$-\vec{\nabla}^2\vec{\pi}(\vec{r}) + \frac{\delta U}{\delta\vec{\pi}(\vec{r})} + Q_{\vec{\pi}}(\vec{r}) = 0, \quad (26)$$

where the Q_σ and $Q_{\vec{\pi}}$ are the source terms contributed from the valence quarks and can be written as

$$Q_\sigma(\vec{R}) = \sum_{j=1}^3\frac{1}{f_\pi Z_j}\int d^3x\,d^3y\,\bar{u}_j(\vec{x})B\left(-\epsilon_j^2, \vec{x} - \vec{y}\right)$$
$$\times\delta\left(\frac{\vec{x} + \vec{y}}{2} - \vec{R}\right)u_j(\vec{y}), \quad (27)$$

$$Q_{\vec{\pi}}(\vec{R}) = \sum_{j=1}^3\frac{1}{f_\pi Z_j}\int d^3x\,d^3y\,\bar{u}_j(\vec{x})B\left(-\epsilon_j^2, \vec{x} - \vec{y}\right)$$
$$\times i\gamma_5\vec{\tau}\delta\left(\frac{\vec{x} + \vec{y}}{2} - \vec{R}\right)u_j(\vec{y}). \quad (28)$$

The above Eqs. (22) and (24)–(28) can be solved self-consistently with numerical technique. When one wants to solve these equations with the chiral circle constraint, one needs only to discard the π degrees of freedom or set π as zero to solve the other equations. And it has been carried out in Refs. [9,10]. In this work, we intend to solve the equations beyond the chiral circle constraint.

III. NUMERICAL CALCULATION AND RESULTS

To solve the soliton equations, one needs at first the effective gluon propagator $D(x - y)$ to get the scalar functions $A(p^2)$ and $B(p^2)$ and fix the quark propagator. In the present work, we take the effective gluon propagator in the form of the

Munczek-Nominovsky model [23]

$$g^2 D(q) = (2\pi)^4 \frac{3}{16} \eta^2 \delta^4(q). \qquad (29)$$

Then the Dyson-Schwinger equation [Eq. (9)] yields the following solutions:

$$A(p^2) = \begin{cases} 2, & p^2 \leqslant \frac{\eta^2}{4}, \\ \frac{1}{2}\left(1 + \sqrt{1 + \frac{2\eta^2}{p^2}}\right), & p^2 > \frac{\eta^2}{4}, \end{cases} \qquad (30)$$

$$B(p^2) = \begin{cases} \sqrt{\eta^2 - 4p^2}, & p^2 \leqslant \frac{\eta^2}{4}, \\ 0, & p^2 > \frac{\eta^2}{4}. \end{cases} \qquad (31)$$

This long-range interaction has been reported to have the dynamical confinement property [24]. Although the effective gluon propagator can take a more realistic form, we would show that this simple model can give a good data to compare with the experiment.

From Eqs. (22) and (24)–(28), one knows that they are coupled Dirac-like equation in momentum space and Klein-Gorden-like equations in coordinate space. As mentioned above, this set of coupled equations has been solved with the chiral circle constraint, i.e., setting $\vec{\pi} \equiv 0$, in Refs. [9,10]. In this work, we solve the equations with and beyond the chiral circle constraint with the same methods as that in Refs. [9,10].

For the equation of quark field, we solve it in momentum space. For a nucleon, we consider only the states with orbital angular momentum $L = 0$ and express the wave function of a valence quark as

$$u_j(\vec{p}) = \begin{pmatrix} f_j(p) \\ i\vec{\sigma} \cdot \hat{p} g_j(p) \end{pmatrix}. \qquad (32)$$

With the numerical integration and differentiating technique, we transform the Dirac-like equation to an algebraic eigenvalue equation $H_{mn}(\epsilon_j)X_n = 0$, where $X_{2n} = f_j(nh)$ and $X_{2n+1} = g_j(nh)$ with h being the integration step of the momentum, ϵ_j is the eigenvalue. If there exists a nontrivial solution of X, $\det(H)$ should be zero. We search for the eigenvalue ϵ_j from zero to a definite value until $\det[H(\epsilon_j)] = 0$ and get the eigenvalue ϵ and eigenvector X with the singular vector decomposition method (SVD) [25].

For the equations of the meson fields, we solve them in coordinate space with Newton's functional iteration method. Because the left-hand side of Eqs. (25) and (26) is a functional of σ and $\vec{\pi}$, we can refer it to $F[\sigma, \vec{\pi}]$. The meson fields $(\sigma_s, \vec{\pi}_s)$ are the solutions of $F[\sigma_s, \vec{\pi}_s] = 0$. To solve these equations, we expand the $F[\sigma, \vec{\pi}]$ at an initial value $\sigma = \sigma_0$, $\vec{\pi} = \vec{\pi}_0$ and obtain

$$F[\sigma, \vec{\pi}] = F[\sigma_0, \vec{\pi}_0] + \int d^4x \left. \frac{\delta F[\sigma, \vec{\pi}]}{\delta \sigma} \right|_{\sigma=\sigma_0, \vec{\pi}=\vec{\pi}_0} \delta\sigma(x)$$
$$+ \int d^4x \left. \frac{\delta F[\sigma, \vec{\pi}]}{\delta \vec{\pi}} \right|_{\sigma=\sigma_0, \vec{\pi}=\vec{\pi}_0} \delta\vec{\pi}(x) + \cdots. \quad (33)$$

Neglecting the higher order and setting $F[\sigma, \vec{\pi}]$ as zero, we gain an iteration equation of the meson fields,

$$\begin{cases} \sigma_{n+1}(x) = \sigma_n(x) + \delta\sigma_n(x), \\ \vec{\pi}_{n+1}(x) = \vec{\pi}_n(x) + \delta\vec{\pi}_n(x), \end{cases} \qquad (34)$$

where $\sigma_n(x)$ and $\vec{\pi}_n(x)$ are the solutions after n-times iteration. It should be mentioned that, with the chiral circle constraint, i.e., $\vec{\pi} \equiv 0$, the coupled equations can be solved with less effort. However, when one solves the equations beyond the chiral circle constraint, there are four fields: the scalar field σ and the triplet π fields. It is then very difficult to solve these equations. To the convenience of numerical calculation, we take the Hedgehog form solution [26]

$$\sigma(\vec{R}) = \sigma(R), \qquad (35)$$

$$\pi_i(\vec{R}) = \hat{R}_i\pi(R) \ (i = 1, 2, 3), \qquad (36)$$

which is one of the solution of the coupled equations.

In the calculation, we take at first a set of trial meson fields to solve the Dirac-like equation and get the eigenenergy and wave function of the quark and the renormalization constant. With the source terms constructed from the quark field, we solve the meson field equations to get the new meson fields. Inserting the new meson fields in the quark equation and repeating the above steps until the meson fields and quark wave function converge to a desired precision, we obtain the final solutions.

One can recognize that f_π is in proportion to η from Eq. (13) when one takes the effective propagator in the form of delta function in Eq. (29) and gets the function A and B as Eq. (30). One can then infer that the Eqs. (22), (25), and (26) can be scaled with the dimensional constant η. All the results can thus be scaled to dimensionless data.

After solving the coupled equations, we obtain the eigenenergy and wave function of the quark and the meson fields (the obtained results of the rescaled quark field and meson fields are illustrated in Fig. 1), and further the potential energy E_p and kinetic energy E_k of the meson fields. The energy of the soliton can then be determined by $E = 3\varepsilon_j + E_p + E_k$. Furthermore, we obtain the mass of the soliton with two methods. One is the naive center mass reduction, $M_s = \sqrt{E^2 - 3\langle p^2 \rangle}$, where $\langle p^2 \rangle$
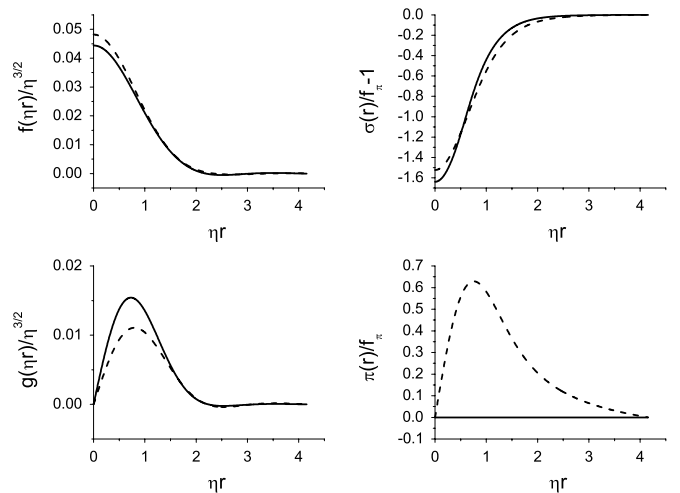

FIG. 1. Rescaled quark and meson fields in the soliton obtained in the numerical calculation. The solid and the dashed curves correspond to the results with and beyond chiral circle constraint, respectively. The left top (bottom) panel shows the upper (lower) component of the quark field. The right top (bottom) panel displays the σ (π) field which is scaled with the pion coupling constant f_π.

TABLE I. Calculated mass and root-mean-square radius of the soliton with and beyond the chiral circle constraint [with the strength parameter $\eta = 1.04$ in Eq. (29)].

	With	Beyond
Energy of valence quark ϵ (MeV)	339	129
Meson potential energy (MeV)	218	109
Meson kinetic energy (MeV)	267	697
Energy of soliton (MeV)	1502	1193
Mass (naive center of mass correction) (MeV)	1430	1124
Mass (recoil correction) (MeV)	1020	916
Radius (naive center of mass correction) (fm)	0.71	0.64
Radius (recoil correction) (fm)	0.59	0.56

is the expectation value of the square of the quark momentum. The other is the recoil correction [27].

To get a more impressive picture of the soliton, we adopt explicitly the results with $\eta = 1.04$ GeV, which is fixed by fitting the experiment data $f_\pi = 93$ MeV [10]. The obtained properties of the soliton are listed in Table I. The obtained quark wave function and the meson fields are illustrated in Fig 2.

From Table I and the Fig. 2, one can recognize evidently that, in addition to the quite extended distribution, the pion field provides a strong attractive interaction and reduces the energy of the valence quark from 339 to 129 MeV and further the whole energy of the soliton from 1430 to 1124 MeV. From Fig. 2, one can see that the σ field beyond the chiral circle constraint is a little weaker than that with the constraint. Then, the interaction between quarks with σ field is weaker. However, one can get a strong π field beyond the chiral circle constraint, which provides a strong attractive interaction between quarks. Thus, one can get a tighter bound and lighter soliton beyond the chiral circle constraint.
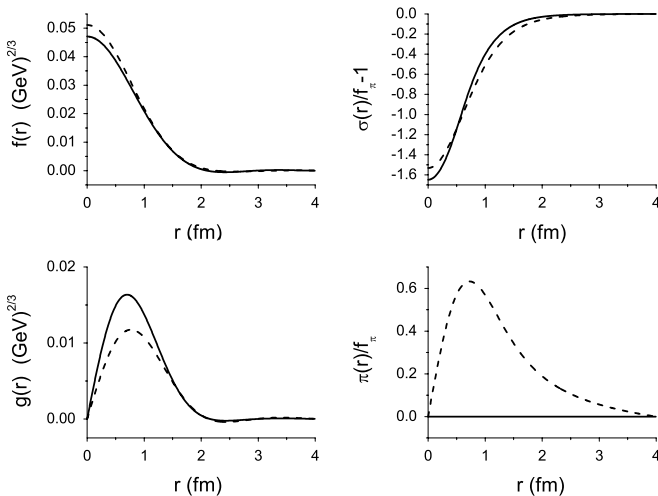

FIG. 2. Calcultated quark and meson fields in the soliton with and beyond chiral circle constraint in the case that the energy scale $\eta = 1.04$ GeV. The solid, dashed curves correspond to the results with and beyond chiral circle constraint, respectively. The left top (bottom) panel shows the upper (lower) component of the quark field. The right top (bottom) panel displays the σ (π) field which is scaled with the pion coupling constant f_π.

In the calculation with chiral circle constraint, we have a state with good quantum numbers, such as angular momentum and isospin. Then we can compare the result with the experiment data of a nucleon. The results listed in Table I show that the calculated result does not agree with experimental data well. In the case of beyond the chiral circle constraint, our calculation cannot give a state with good quantum numbers because of the inclusion of the pion field and the hedgehog approximation (the hedgehog state is invariant under the simultaneous Lorentz rotation and the isospin rotation. To derive a state with good quantum numbers, we should quantize the classical soliton). The state we obtained is then a mixture of a nucleon and a delta. The result listed in Table I indicates that the presently obtained mass of the soliton with the naive correction on the center of mass is comparable to the experimental data. However, in the case of the recoil correction on the center of mass, the obtained mass is smaller than the experimental data. Such a result is not strange because we have not quantized the soliton to a physical state. Meanwhile, it is promising to see that the soliton mass is close to the right. When we quantize the soliton, the nucleon mass is about tens of MeV heavier than the soliton mass. Then the mass of a nucleon is nearly equal to the experiment data. The table shows also that the radius of the soliton is smaller than the nucleon charge radius 0.87 fm observed in experiment. It is quite natural because we have taken the effective gluon propagator with infinite long-range interaction. The realistic interaction is surely not in infinite long range. If we take a more realistic form with long-range decreasing behavior, we expect that a better fit to the experiment data can be obtained.

IV. SUMMARY AND REMARKS

In this article, we studied the GCM soliton with and beyond the chiral circle constraint. The calculation shows that, when the pion field is taken into account in the scheme beyond the chiral circle constraint, the calculated mass of the GCM soliton is comparable with experimental data. The calculation indicates that the pion field provides a strong attraction, which binds the quarks in the soliton more tightly and decreases the energy of the soliton to a more realistic value.

In this calculation, we take the form of the effective gluon propagator as a delta function in momentum space. In such a case, the interaction between quarks is in infinite long range with a constant strength. The calculated root-mean-square of the radius of the soliton is then smaller than the experimental data. Because the realistic form of the effective gluon propagator should include the infrared enhancement, which is closely related to the chiral symmetry breaking, and the ultraviolet property of asymptotic freedom, which has a decrease on the interaction strength, more precise calculation with a realistic effective gluon propagator is needed to give a good description of the experimental data of nucleon. Moreover, the soliton we obtained with the pion field being included explicitly does not possesses the good quantum numbers of a nucleon. Then, a quantization on the classical soliton and calculations of more physical observables are necessary. The relevant works are under progress.

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