PHYSICAL REVIEW C **73**, 015205 (2006)

η mesons in nuclear matter

X. H. Zhong,[1,*] G. X. Peng,[2,3,†] Lei Li,[1] and P. Z. Ning[1,‡]

[1]*Department of Physics, Nankai University, Tianjin 300071, China*

[2]*Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100039, China*

[3]*China Center of Advanced Science and Technology (World Lab.), Beijing 100080, China*

(Received 29 June 2005; published 31 January 2006)

The η-nucleon (ηN) interactions are deduced from the heavy-baryon chiral perturbation theory to the next-to-leading-order terms. Combining the relativistic mean-field theory for nucleon system, we have studied the in-medium properties of the η meson. We find that all the elastic-scattering ηN interactions come from the next-to-leading-order terms. The ηN sigma term is found to be about 280 ± 130 MeV. The off-shell terms are also important to the in-medium properties of the η meson. On application of the latest determination of the ηN scattering length, the ratio of the η-meson effective mass to its vacuum value is near 0.84 ± 0.015, whereas the optical potential is about $-(83 \pm 5)$ MeV, at the normal nuclear density.

DOI: 10.1103/PhysRevC.73.015205 PACS number(s): 21.65.+f, 21.30.Fe

I. INTRODUCTION

The studies of meson-baryon interactions and the meson properties in nuclear medium are interesting subjects in nuclear physics. The pion-nucleon/pion-nucleus and kaon-nucleon/kaon-nucleus interactions have been much studied, both theoretically and experimentally. Because of the lack of η beams, the η-nucleon/η-nucleus interaction is still not as clear as that of the pion-nucleon/pion-nucleus and kaon-nucleon/kaon-nucleus. Because the η-nucleus quasibound states were first predicted by Haider and Liu [1] and Li *et al*. [2], when it was realized that the η-nucleon interaction is attractive, the study of the η-nucleus bound states has been one of the focuses in nuclear physics [3–11].

The key point for the study of η-nucleus bound states is the η-nuclear optical potential. There have been some works in this field. Waas and Weise studied the *s*-wave interactions of η mesons in nuclear medium and obtained a potential $U_\eta \simeq -20$ MeV [12]. Chiang *et al*. [13] obtained $U_\eta \simeq -34$ MeV by assuming that the mass of the $N^*(1535)$ did not change in the medium. Tsushima *et al*. predicted that the η-meson potential was typically -60 MeV using the quark-meson coupling (QMC) model [14]. Inoue and Oset also obtained $U_\eta \simeq -54$ MeV with their model [15]. Obviously, there are model dependencies in describing the in-medium properties of the η meson. Therefore, further studies are needed. In this article, first, we deduce the ηN interactions from chiral perturbation theory and then, second, using the relativistic mean-field theory for the nucleon system, we study the properties of the η meson in uniform nuclear matter. The relativistic mean-field theory (RMF) is one of the most popular methods in modern nuclear physics. It has been successful in describing the properties of ordinary nuclei/nuclear matter and hypernuclei/nuclear matter [16,17].

However, the chiral perturbation theory (ChPT) was first applied by Kaplan and Nelson to investigate the in-medium properties of (anti-)kaons [18]. Some years later, an effective chiral Lagrangian in heavy-fermion formalism [19] was also introduced to study the kaon-nuclear/nucleon interactions or kaon condensation [20–22]. The advantage of using the heavy-fermion Lagrangian for chiral perturbation theory was clearly pointed out in Ref. [19]. Compared with the previous chiral perturbation theory [18], the outstanding point in Refs. [20–22] is that additional next-to-leading-order terms, i.e., off-shell terms, are added to the Lagrangian. The additional terms are essential for a correct description of the *KN* interactions. The chiral perturbation theory also had been used in the study of η-meson in-medium properties in Ref. [12,15], where only the leading-order terms were kept in the calculations. Given that the higher-order terms, e.g., off-shell terms, are important to the ηN interactions, and they have not been included in the previous studies for the ηN interactions with chiral perturbation theory, we have, in the present work, studied the ηN interactions with the heavy-baryon chiral perturbation theory up to the next-to-leading-order terms. Combining the RMF for nuclear matter, we obtain the in-medium properties of the η meson. Comparing our results with the previous results (with only leading-order terms), we find that the next-to-leading-order terms are important to the calculations indeed. The η-nucleon sigma term is found to be 280 ± 130 MeV. The ratio of the η-meson effective mass to its vacuum value is 0.84 ± 0.015, whereas the depth of the optical potential is $-(83 \pm 5)$ MeV at the normal nuclear density. The large uncertainty in the sigma term $\Sigma_{\eta N}$ does not affect the results significantly in low-density region, varying by about 8 MeV at normal nuclear density.

This article is organized as follows. In the subsequent section, the effective chiral Lagrangian density we used is given, the effective Lagrangian for ηN interactions is derived, and the coefficients for the sigma and off-shell terms are determined. Then, in Sec. III, combining the RMF for nucleons, we obtain the η-meson energy, effective mass, and optical potential in nuclear matter. We present our results and

*E-mail: zhongxianhui@mail.nankai.edu.cn

†E-mail: gxpeng@ihep.ac.cn

‡E-mail: ningpz@nankai.edu.cn

discussion of the η-meson in-medium properties in Sec. IV. Finally a summary is given in Sec. V.

II. THE ηN INTERACTIONS IN CHIRAL PERTURBATION THEORY

A. The theory framework

The interactions between pseudoscalar mesons (pion, kaon, and eta mesons) and baryons (nucleons and hyperons) are described by the $SU(3)_L \times SU(3)_R$ chiral Lagrangian, which can be written as

$$\mathcal{L}_{\rm chiral} = \mathcal{L}_\phi + \mathcal{L}_{\phi B}. \quad (1)$$

$\mathcal{L}_\phi$ is the mesonic term to the second chiral order [18],

$$\mathcal{L}_\phi = \frac{1}{4} f^2 {\rm Tr}\partial^\mu \Sigma \partial_\mu \Sigma^\dagger + \frac{1}{2} f^2 B_0 [{\rm Tr} M_q (\Sigma - 1) + {\rm h.c.}]. \quad (2)$$

The second piece of the Lagrangian in Eq. (1), $\mathcal{L}_{\phi B}$, describes the meson-baryon interactions and reads at lowest order [18]

$$\mathcal{L}_{\phi B}^{(1)} = {\rm Tr}\bar{B}(i\gamma^\mu \partial_\mu - m_B)B + i{\rm Tr}\bar{B}\gamma^\mu [V_\mu, B] + D{\rm Tr}\bar{B}\gamma^\mu \gamma^5 \{A_\mu, B\} + F{\rm Tr}\bar{B}\gamma^\mu\gamma^5 [A_\mu, B], \quad (3)$$

The next-to-leading order chiral Lagrangian for s-wave meson-baryon interactions reads [20]

$$\begin{aligned}\mathcal{L}_{\phi B}^{(2)} = {} & a_1 {\rm Tr}\bar{B}(\xi M_q \xi + {\rm h.c.})B + a_2 {\rm Tr}\bar{B}B(\xi M_q \xi + {\rm h.c.}) \\ & + a_3 {\rm Tr}\bar{B}B{\rm Tr}(M_q\Sigma + {\rm h.c.}) + d_1 {\rm Tr}\bar{B}A^2 B \\ & + d_2 {\rm Tr}\bar{B}(vA)^2 B + d_3 {\rm Tr}\bar{B}BA^2 \\ & + d_4 {\rm Tr}\bar{B}B(vA)^2 + d_5 {\rm Tr}\bar{B}B{\rm Tr}A^2 \\ & + d_6 {\rm Tr}\bar{B}B{\rm Tr}(vA)^2 + d_7 {\rm Tr}\bar{B}A_\mu {\rm Tr}A^\mu B \\ & + d_8 {\rm Tr}\bar{B}(vA){\rm Tr}(vA)B + d_9 {\rm Tr}\bar{B}A_\mu BA^\mu \\ & + d_{10}{\rm Tr}\bar{B}(vA)B(vA). \end{aligned} \quad (4)$$

In the above equations, $M_q = {\rm diag}\{m_q, m_q, m_s\}$ is the current quark mass matrix, B_0 relates to the order parameter of spontaneously broken chiral symmetry, the constants D and F are the axial vector couplings whose values can be extracted from the empirical semileptonic hyperon decays, the pseudoscalar meson decay constants are equal in the $SU(3)_V$ limit and denoted by $f = f_\pi \simeq$ 93 MeV, v_μ is the four-velocity of the heavy baryon (with $v^2 = 1$), and $\Sigma = \xi^2 = \exp(i\sqrt{2}\Phi/f)$, $V^\mu = (\xi\partial^\mu\xi^\dagger + \xi^\dagger\partial^\mu\xi)/2$, and $A^\mu = (\xi\partial^\mu\xi^\dagger - \xi^\dagger\partial^\mu\xi)/(2i)$. The 3×3 matrix B is the ground-state baryon octet, m_B is the common baryon octet mass in the chiral limit, and Φ collects the pseudoscalar meson octet.

The next-to-leading-order terms in Eq. (4) have been developed for heavy baryons by Jenkins and Manohar [19]. The heavy-baryon chiral theory is similar to the nonrelativistic formulation of baryon chiral perturbation theory [23]. However, the heavy-baryon theory has the advantage of manifest Lorentz invariance, and quantum corrections can be computed in a straightforward manner by the ordinary Feynman graphs rather than the time-ordered perturbation theory [24]. The Lagrangian has been shown to be suitable for describing the chiral properties of nuclear system in Ref. [25], where one can also find detailed discussions on how to systematically compute the higher-order terms of this Lagrangian. In this article, we limit our calculations to the squared characteristic small momentum scale Q^2 (involving no loops) for s-wave ηN scattering, because the corrections from the higher-order coupling are suppressed, at low energy, by powers of Q/Λ_χ, with $\Lambda_\chi \sim 1$ GeV being the chiral symmetry breaking scale. Hence no loops need to be calculated in this article. If the loop corrections are included, the higher-order terms, i.e., next-to-next-to-leading order, should be added. We will consider it in our later work.

Expanding Σ up to the order of $1/f^2$, and using the heavy-baryon approximation, i.e.,

$$v = \frac{p}{m} = \left(\sqrt{1 + \frac{\mathbf{p}^2}{m^2}}, v_x, v_y, v_z\right) \approx (1, 0, 0, 0) \quad (5)$$

(because v_x, v_y, and v_z are very small), we easily obtain the Lagrangian for ηN interactions:

$$\mathcal{L}_\eta = \frac{1}{2}\partial^\mu\eta\partial_\mu\eta - \frac{1}{2}\left(m_\eta^2 - \frac{\Sigma_N}{f^2}\bar{\Psi}_N\Psi_N\right)\eta^2 + \frac{1}{2}\frac{\kappa}{f^2}\bar{\Psi}_N\Psi_N\partial^\mu\eta\partial_\mu\eta, \quad (6)$$

where m_η corresponds to the mass of the η meson, which is determined by $m_\eta^2 = \frac{2}{3}B_0(m_q + 2m_s)$. $\Sigma_{\eta N}$ is the ηN sigma term, which is determined by

$$\Sigma_{\eta N} = -\tfrac{2}{3}[a_1 m_q + 4a_2 m_s + 2a_3(m_q + 2m_s)]. \quad (7)$$

From Eq. (6), we can see that the last three terms of Eq. (3) do not contribute to the ηN interactions. The $\Sigma_{\eta N}/f^2$ term in Eq. (6) is deduced from the first three terms of Eq. (4), which corresponds to the chiral breaking and shifts the effective mass of the η meson in the nuclear medium. The last term of Eq. (6) is the contribution from the last 10 terms of Eq. (4), which is called the "off-shell" term. κ is a constant relevant to d_is (i = 1–10). Its value is to be determined from the ηN scattering length.

B. The determination of the ηN sigma term and κ

To calculate $\Sigma_{\eta N}$, we should know the parameters on the right-hand side of Eq. (7). In fact these parameters have been previously discussed in Refs. [26–29] and are used in Ref. [18].

As is well known, the KN sigma term can be written as [21]

$$\Sigma_{KN} = -(m_s + m_q)(a_1 + 2a_2 + 4a_3)/2. \quad (8)$$

Solving a_3 from this equation and then substituting the corresponding expression into Eq. (7) leads to

$$\Sigma_{\eta N} = \frac{2}{3}\left[\frac{2+r}{1+r}\Sigma_{KN} + a_1 m_s\left(1 - \frac{r}{2}\right) - a_2 m_s(2 - r)\right], \quad (9)$$

where $r = m_q/m_s \ll 1$. Expanding the right-hand side of

Eq. (9) to a Taylor series with respect to r, we have

$$\Sigma_{\eta N} = \tfrac{2}{3}(2\Sigma_{KN} + a_1 m_s - 2a_2 m_s) - \tfrac{1}{3}(2\Sigma_{KN} + a_1 m_s - 2a_2 m_s)\, r + \text{higher-order terms in } r. \qquad (10)$$

Because of the extreme smallness of r, and also because of the fact that our formulas are valid merely up to the next-to-leading order, we take only the first two terms, i.e., $\Sigma_{\eta N} = (1/3)(2\Sigma_{KN} + a_1 m_s - 2a_2 m_s)(2 - r)$. Usually, r is in the range of (1/24, 1/26) [30–33], and we use the modest value $r = 1/25$. In fact, the concrete value does not matter significantly because of the extreme smallness of r. The values for $a_1 m_s$ and $a_2 m_s$ can be well determined by Gell-Mann Okubo mass formulas, giving the result $a_1 m_s = -67$ MeV. For $a_2 m_s$, one has 125 MeV [20] or a little bigger value 134 MeV [29], and we take the average $a_2 m_s = 130$ MeV. The value for KN sigma term has some uncertainties. The latest result is $\Sigma_{KN} = 312 \pm 37$ MeV in the perturbative chiral quark model [34]. The lattice gauge simulation gave $\Sigma_{KN} = 450 \pm 30$ MeV [35]. The result of lattice quantum chromodynamics is $\Sigma_{KN} = 362 \pm 13$ MeV [36], and the prediction using the Nambu-Jona-Lasinio model is $\Sigma_{KN} = 425$ (with an error bar of 10–15%) [37]. Thus, in our calculations, we use $\Sigma_{KN} = 380 \pm 100$ MeV in its possible range. Equipped with the above parameters, we finally obtain $\Sigma_{\eta N} = 283 \pm 131$ MeV, where ± 131 MeV reflects the uncertainty ± 100 MeV in Σ_{KN}. Naturally, if one uses a smaller Σ_{KN} value, e.g., $\Sigma_{KN} = 2m_\pi$ [20], $\Sigma_{\eta N}$ would also be smaller.

For the other parameter κ, it is not too difficult, from the Lagrangian in Eq. (6), to derive the ηN scattering length (on-shell constraints):

$$a^{\eta N} = \frac{1}{4\pi f^2 (1 + m_\eta / M_N)} \left(\Sigma_{\eta N} + \kappa m_\eta^2\right). \qquad (11)$$

So we can determine κ with a given $\Sigma_{\eta N}$ and $a^{\eta N}$ via the relation

$$\kappa = 4\pi f^2 \left(\frac{1}{m_\eta^2} + \frac{1}{m_\eta M_N}\right) a^{\eta N} - \frac{\Sigma_{\eta N}}{m_\eta^2}. \qquad (12)$$

Recently, Green *et al.* [38] analyzed the new experimental data from GRAAL [39] and gave the real part of ηN scattering length $a^{\eta N} = 0.91$ fm, which agrees with their previous result [40]. With the similar method, Arndt *et al.* [41] also predicted $a^{\eta N} = 1.03 - 1.14$ fm, comparable to that found by Green *et al.* So one can assume that $a^{\eta N}$ is in the range of $0.91 \sim 1.14$ fm. Using the central value $a^{\eta N} = 1.02$ fm leads to $\kappa = 0.40 \pm 0.08$ fm. For the η and nucleon masses, we use $m_\eta = 547.311$ MeV [42] and $M_N = 939$ MeV.

It should be pointed out that the ηN interactions in the present model come from the term of $\Sigma_{\eta N}/f^2$ and the off-shell term, whereas the leading Tomozawa-Weinberg term simply vanishes. We do not consider any other nondiagonal coupled channel, which was investigated with the chiral coupled channel model by Waas and Weise [12]. According to their calculations, the contribution of nondiagonal coupled channel to the ηN optical potential is on the order of $\sim$20 MeV at normal nuclear density.

III. IN-MEDIUM PROPERTIES OF η MESONS

The Lagrangian for one η meson in nuclear matter is given by

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_\eta, \qquad (13)$$

where $\mathcal{L}_0$ is the Lagrangian for the nucleon system. In this article, we adopt the standard Lagrangian, $\mathcal{L}_0$, for the nucleon system in relativistic mean-field theory (given in the appendix). $\mathcal{L}_\eta$ is the Lagrangian for the η meson, which is given in Eq. (6). On application of the Lagrangian in Eq. (13), we immediately have the equation of motion for the η-meson field

$$\left(\partial_\mu \partial^\mu + m_\eta^2 - \frac{\Sigma_{\eta N}}{f^2} \bar{\Psi}_N \Psi_N + \frac{\kappa}{f^2} \bar{\Psi}_N \Psi_N \partial_\mu \partial^\mu\right) \eta = 0. \qquad (14)$$

Defining the $\bar{\Psi}_N \Psi_N$ fluctuation δ as

$$\bar{\Psi}_N \Psi_N = \langle \bar{\Psi}_N \Psi_N \rangle + \delta, \qquad (15)$$

where $\langle \bar{\Psi}_N \Psi_N \rangle$ is the vacuum expectation value. Because the mean-field approximation is a very familiar method that has already been used in studying the in-medium properties of kaons with a similar chiral approach [43,44], we adopt it in our present calculations.

At the mean-field level, we neglect the fluctuation δ. Then the equation of motion for the η-meson field is simplified to

$$\left(\partial_\mu \partial^\mu + m_\eta^2 - \frac{\Sigma_{\eta N}}{f^2} \rho_s + \frac{\kappa}{f^2} \rho_s \partial_\mu \partial^\mu\right) \eta = 0, \qquad (16)$$

where $\rho_s \equiv \langle \bar{\Psi}_N \Psi_N \rangle$ is the scalar density.

Plane wave decomposition of Eq. (16) yields

$$-\omega^2 + \vec{k}^2 + m_\eta^2 - \frac{\Sigma_{\eta N}}{f^2} \rho_s + \frac{\kappa}{f^2} \rho_s (-\omega^2 + \vec{k}^2) = 0. \qquad (17)$$

The η-meson effective mass, m_η^*, in the nuclear medium is defined by

$$\omega = \sqrt{m_\eta^{*2} + \vec{k}^2}. \qquad (18)$$

Substituting this equation into Eq. (17) leads to the following definition:

$$m_\eta^* = \sqrt{\left(m_\eta^2 - \frac{\Sigma_{\eta N}}{f^2} \rho_s\right) \Big/ \left(1 + \frac{\kappa}{f^2} \rho_s\right)}. \qquad (19)$$

Simultaneously, the last two terms on the right-hand side of Eq. (17) is the η-meson self-energy, i.e.,

$$\Pi(\omega, \vec{k}; \rho_s) = -\frac{\Sigma_{\eta N}}{f^2} \rho_s + \frac{\kappa}{f^2} \rho_s (-\omega^2 + \vec{k}^2), \qquad (20)$$

which is a function of the η-meson single-particle energy ω and the momentum $\vec{k}$. Accordingly, the optical potential for η-meson in the nuclear matter is given by the following:

$$U_\eta = \frac{1}{2m_\eta} \Pi(\omega, \vec{k} = 0; \rho_s) = \frac{m_\eta^{*2} - m_\eta^2}{2m_\eta}. \qquad (21)$$

To obtain the η-meson in-medium properties, we need a relation between the scalar density ρ_s and the nucleon density $\rho_N = \langle \Psi_N^\dagger \Psi_N \rangle$. Because there is only one single η meson in the nuclear matter, its effect on the nuclear matter is negligible. According to the relativistic mean-field theory, we have the following relation between ρ_s, ρ_N, and the σ mean-field value σ_0:

$$\rho_s = \left(M_N + g_\sigma^N \sigma_0\right)^3 f(x), \tag{22}$$

where the function $f(x)$ is defined to be

$$f(x) \equiv [x\sqrt{1+x^2} - \ln(1+\sqrt{1+x^2})]/\pi^2 \tag{23}$$

with x being the ratio of the nucleon's Fermi momentum to its effective mass, i.e.,

$$x \equiv \frac{k_F}{M_N^*} = \left(\frac{3}{2}\pi^2\rho_N\right)^{1/3} \Big/ \left(M_N + g_\sigma^N \sigma_0\right). \tag{24}$$

The mean-field value σ_0 is connected to the scalar density ρ_s by

$$\rho_s = -\left(m_\sigma^2 \sigma_0 + g_2\sigma_0^2 + g_3\sigma_0^3\right)/g_\sigma^N. \tag{25}$$

Therefore, for a given nucleon density ρ, we can first solve σ_0 from

$$m_\sigma^2 \sigma_0 + g_2\sigma_0^2 + g_3\sigma_0^3 = -g_\sigma^N \left(M_N + g_\sigma^N \sigma_0\right)^3 f(x), \tag{26}$$

and then calculate the scalar density ρ_s from Eq. (25) or (22).

The detailed derivation of the Eqs. (22)–(26) can be seen in Ref. [45]. To be self-contained, we also attach a brief derivation in the appendix. In numerical calculations, we adopt the NL3 parameter set [17], i.e., $m_\sigma = 508.194$ MeV, $m_\omega = 782.501$ MeV, $g_\sigma^N = 10.217$, $g_\omega^N = 12.868$, $g_2 = -10.434\,\text{fm}^{-1}$, and $g_3 = -28.885$. The numerical results for ρ_s-ρ_N are given in Fig. 1, where one can see clearly that ρ_s is an increasing function of the nuclear density. When the density is about 1.5 times lower than the nuclear saturation density, ρ_s is nearly proportional to ρ_N. However, when the density is about 2 times higher than the normal nuclear density, ρ_s is nearly a constant. The mean-field value of the sigma filled is also given in Fig. 1 with a dotted curve. Its density behavior is similar to that of ρ_s.

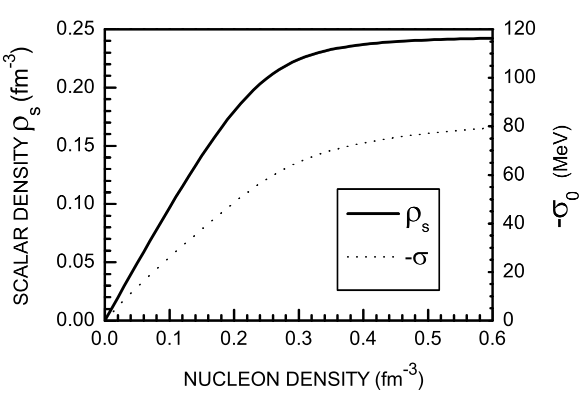

FIG. 1. The scalar density (full curve) and the negative sigma mean-filled value (dotted curve) as functions of the nucleon density. They both are increasing functions, but the increasing speed is getting slower and, finally, when the density is higher than about 2 times the nuclear saturation density, they are nearly constant.

TABLE I. A selection of the real part of the ηN-scattering length in literature. m_η^*/m_η and U_η are effective mass and the potential depth at normal nuclear density calculated with the scattering lengths. Where we use $\Sigma_{\eta N} = 280$ MeV in calculations.

Reaction or method	$a_{\eta N}$ (fm)	m_η^*/m_η	$-U_\eta$ (MeV)
[46]	0.25	0.952	26
[47]	0.27	0.946	27
$pn \longrightarrow d\eta$ [48]	$\leqslant$0.3	$\geqslant$0.94	$\leqslant$30
[49]	0.46(9)	0.915	44
[50]	0.487	0.91	46
[51]	0.51	0.905	49
[52]	0.55	0.902	51
[50]	0.577	0.90	54
[53]	0.621	0.89	57
[54]	0.68	0.88	61
[55]	0.717	0.88	63
Coupled K matrices [56]	0.75	0.875	67
$\eta d \longrightarrow \eta d$ [57]	$\geqslant$0.75	$\leqslant$0.875	$\geqslant$67
Coupled K matrices [40]	0.87	0.86	76
[38,58]	0.91	0.853	77
[59]	0.98	0.846	79
[60]	0.991	0.845	80
Coupled K matrices [40]	1.05	0.82	82
[41]	1.14	0.825	88

IV. RESULTS AND DISCUSSIONS

In this section, we discuss the effective mass, optical potential in nuclear medium, and the off-shell behavior of the η meson, respectively. For zero-momentum η mesons, we can see, from Eq. (18), that the energy ω is equal to its effective mass. Therefore, we no longer mention the η-meson energies in the following discussions.

In the calculation, the precision of the η-meson effective mass and optical potential are determined by the two parameters $\Sigma_{\eta N}$ and κ. Equation (12) connects the parameter κ to the scattering length $a^{\eta N}$, whose possible values are collected in Table I. To reflect uncertainties in the two quantities $\Sigma_{\eta N}$ and $a^{\eta N}$, we use the sigma term $\Sigma_{\eta N} = 150$, 280, and 410 MeV and the scattering length $a^{\eta N} = 0.91$ [38] and 1.04 [41] fm in numerical calculations.

Figures 2 and 3 show the η-meson effective mass and nuclear optical potential of the η meson as functions of the nuclear density. The results from Ref. [12] (straight line) is also shown in Fig. 2 for comparison. The curves in Figs. 2 and 3 are obviously divided into three groups that correspond to different scattering lengths $a^{\eta N} = 0.91$, 1.14 fm, and $\kappa = 0$, respectively. The dotted, solid, and dash-dotted curves in each group correspond to $\Sigma_{\eta N} = 150$, 280, and 410 MeV, respectively.

A. Effective mass

It is obvious from Fig. 2 that the η-meson effective mass decreases almost linearly in the region $\rho < \rho_0$. In this region, the results of Ref. [12] also show a linear relation for the effective mass with nuclear density. At higher densities,

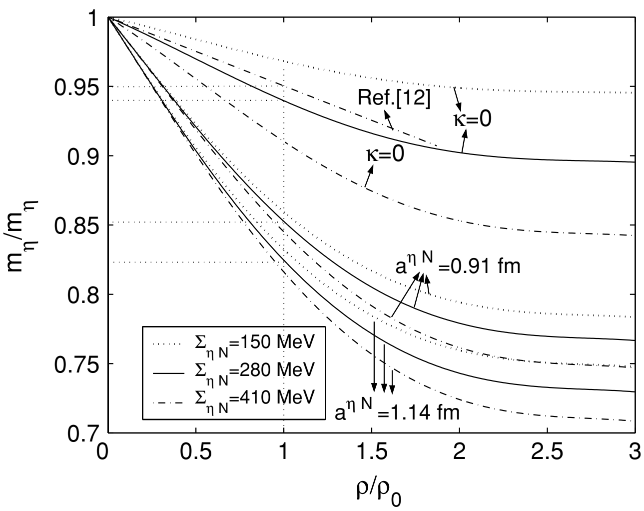

FIG. 2. The effective mass of the η meson as a function of nuclear density. The straight line is obtained from Ref. [12].

however, the effective mass decreases nonlinearly, and the decreasing speed becomes smaller and smaller and at last nearly constant in the range $\rho > 2\rho_0$. The reason is that when the density is higher than about 2 times the normal nuclear saturation density, ρ_s nearly is a constant (see Fig. 1).

For the same scattering length, we find that, in the low-density region $\rho \leqslant 0.5\rho_0$, the effective mass is nearly independent of the sigma term $\Sigma_{\eta N}$. When we set $\Sigma_{\eta N} = 280 \pm 130$ MeV, which changes in a large range, the variation of the effective mass is within ± 4 MeV at $\rho = \rho_0$. At high nuclear density, say $\rho = 3\rho_0$, the variation is within ± 10 MeV compared with that at the central value of $\Sigma_{\eta N} = 280$ MeV. Thus, we can conclude that the effective mass of η mesons is insensitive to the concrete value of $\Sigma_{\eta N}$ in the low-density region.

Although the latest predictions [38,41] give large scattering lengths $a^{\eta N} = 0.91 \sim 1.14$ fm, there are other different predictions [40,46–60]. To see the effects of different scattering length values on the η effective mass, we show, in Fig. 2, the results for $a^{\eta N} = 0.91$ and 1.14 fm, respectively. However, in Table I, we give, at normal nuclear density, the effective mass corresponding to the respective ηN scattering length in the literature [38,40,41,46–60]. From Fig. 2, we find that, with the same sigma term $\Sigma_{\eta N}$, the effective mass depends strongly on the scattering length $a^{\eta N}$. At $\rho = \rho_0$, the effective mass (with $\Sigma_{\eta N} = 280$ MeV) is $m^*_\eta/m_\eta = 0.85$ for $a^{\eta N} = 0.91$ and $m^*_\eta/m_\eta = 0.825$ for $a^{\eta N} = 1.14$ fm. When the scattering length varies from 0.25 to 1.14 fm, the effective mass will run from 0.95 to $0.825m_\eta$. Corresponding to $a^{\eta N} = 0.91 \sim 1.14$ fm, which are favored by recent works, and $\Sigma_{\eta N}$, which is predicted in Sec. II, the effective mass is $(0.84 \pm 0.015)m_\eta$.

At normal nuclear density, the effective mass in Ref. [12] is $0.95m_\eta$, which agrees with the result of the small scattering length $a^{\eta N} = 0.25$ fm. As pointed out in the above, the effective mass changes nonlinearly with increasing densities in the region $\rho_0 < \rho < 2\rho_0$. This behavior agrees with the predictions by Tsushima *et al.* [14] with quark-meson coupling model. The effective mass at $\rho = \rho_0$ predicted by them is about $0.88m_\eta$, which corresponds only to the result with scattering length $a^{\eta N} = 0.68$ fm. This can be clearly seen from Table I. The outstanding characteristic of our results is that the present calculations give much smaller effective mass than the others when we adopt the larger scattering length.

It should be mentioned that the chiral coupled channel model [12] gives much larger in-medium effective mass for η mesons than our predictions. The main reason is as such. In the chiral coupled channel model, there are only the leading-order terms, and so the contributions to the effective mass come only from the nondiagonal coupled channel. Whereas in our model, the leading-order terms do not contribute to the calculations. All the contributions to the results come from the next-to-leading-order terms.

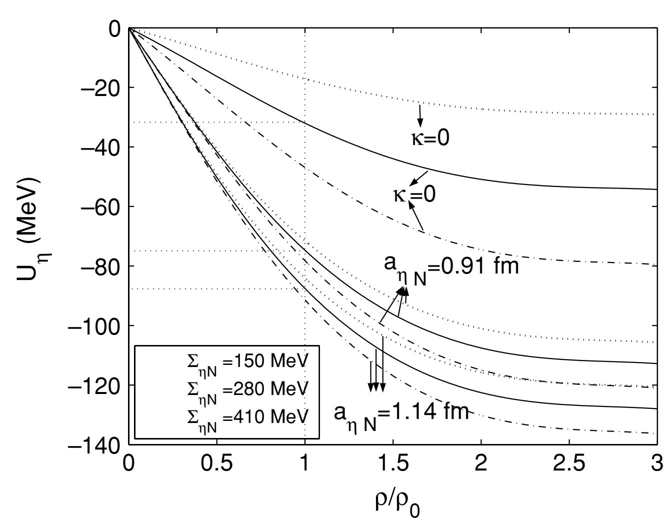

FIG. 3. The optical potential of η mesons as a function of the nuclear density.

B. Optical potential

The optical potential U_η as a function of nuclear density is plotted in Fig. 3. We find that the density behavior of U_η is quite similar to the effective mass in Fig. 3. The reason is that the optical potential has a relation $U_\eta \simeq m^*_\eta - m_\eta$ as an approximation, which varies linearly with the effective mass m^*_η of the η meson.

Similarly, it is also seen that the effect from the uncertainties of sigma term $\Sigma_{\eta N}$ are quite limited in its possible range, and the optical potential depends strongly on the value of the scattering length. At normal nuclear density, the upper limit of the uncertainties from the sigma term $\Sigma_{\eta N}$ is no more than 8 MeV. However, the optical potential can change from -78 to -88 MeV when we modify the scattering length $a^{\eta N}$ from 0.91 to 1.14 fm. Because there are still uncertainties for the ηN scattering length, we listed the possible potential depths corresponding to the possible scattering lengths appearing in the literature cited in Table I. From the table, we can see that the potential depth at normal nuclear density ranges from 26 to 88 MeV, because of the uncertainties of scattering lengths. According to the newest predictions, i.e., $a^{\eta N} = 0.91 \sim 1.14$ fm [38,41], the potential depth is about

83 ± 5 MeV. This is a very strong attractive potential that was never predicted by the previous models.

There have been some predictions for the nuclear potential of η mesons in other references. According to the SU(3) chiral dynamics with coupled channels, the optical potential depth at normal nuclear density is $U_\eta \simeq -20$ MeV [12], which is close to our formulas with a smaller scattering length $a_{\eta N} < 0.25$ fm. In Ref. [13], by assuming that the mass of the $N^*(1535)$ did not change in the medium, the optical potential $U_\eta = -34$ MeV was obtained, which is close to our calculation with $a_{\eta N} \sim 0.30$ fm. The η potential from the QMC model by Tsushima *et al.* and chiral unitary approach by Inoue *et al.* are typically -60 and -54 MeV, which are comparable to our formulas with $a_{\eta N} = 0.55$–0.68 fm. Therefore, if we want to obtain shallower optical potential, we need to use a smaller scattering length. Because recent works favor the bigger scattering length, our formulas give much deeper optical potential.

C. The effect of off-shell term

Finally, we discuss the role of the off-shell term in our calculation. In present model, the off-shell term κ is determined by the scattering length $a^{\eta N}$. From the analysis of Secs. IV A and IV B, we know that the scattering length $a^{\eta N}$ strongly affects the calculations. The importance of the off-shell behavior for low-energy scattering had been pointed out in many Refs. [20,43,61].

To clarify the effect of off-shell term on our calculation thoroughly, we turn off the off-shell term ($\kappa = 0$) and show the results in Fig. 2 and 3. At $\rho = \rho_0$, without the off-shell terms, the effective mass is $m^*_\eta/m_\eta \simeq 0.94 \pm 0.03$ and the optical potential is $-(32 \pm 16)$ MeV, corresponding to $\Sigma_{\eta N} = 280 \pm 130$ MeV. Thus, without the off-shell terms, we no longer have strong attractive potential for the η meson in a nuclear medium. The calculations are independent of the scattering length. Also in this case, the calculations depend strongly on the quantity of $\Sigma_{\eta N}$. Without the off-shell terms, the variation of the optical potential from the uncertainties of $\Sigma_{\eta N}$ can reach about 30 MeV at normal nuclear density. However, it is no more than 8 MeV, when the off-shell behavior is considered. Thus, the off-shell terms can dramatically depress the effects from the uncertainties of $\Sigma_{\eta N}$.

V. SUMMARY

In this article, we have derived an effective Lagrangian for ηN s-wave interaction from the effective meson-baryon chiral Lagrangian, including the next-to-leading-order terms. Up to $1/f^2$ terms for s-wave ηN interaction, only the sigma term and off-shell term survive. It is found that the ηN sigma term is $\Sigma_{\eta N} = 280 \pm 130$ MeV according to the KN sigma term. The off-shell term κ is determined by the scattering length. If we adopt the newest predictions $a_{\eta N} \approx 0.91$–1.14 fm for the scattering lengths [38,41], we obtain the value $\kappa = 0.40 \pm 0.08$ fm.

Using the relativistic mean-field theory for the nucleon system, we calculate the effective mass and optical potential of η mesons in uniform nuclear medium in the mean-field approximation. According to the latest predictions $a_{\eta N} \approx 0.91$–1.14 fm for the scattering lengths [38,41], at normal nuclear density the effective mass is about $(0.84 \pm 0.015)m_\eta$ and the depth of optical potential is $U_\eta \simeq -(83 \pm 5)$ MeV.

Finally, we should reiterate the importance of the next-to-leading-order terms of the chiral Lagrangian. In fact, the leading-order terms do not contribute to the ηN interactions. All contribution comes from the next-to-leading order terms. It indicates that the next-to-leading order terms should be included in the study of the ηN interaction. In the present article, we do not consider corrections from the nondiagonal coupled channel. According the study of Waas and Weise, the correction may be on the order of 20 MeV for the optical potential.

ACKNOWLEDGMENTS

We thank Professor X. B. Zhang for helpful discussions. This work was supported by the Natural Science Foundation of China (10275037, 10375074, 10575054, and 90203004), the Doctoral Programme Foundation of the China Institution of Higher Education (20010055012) and the Science and Technology Innovation Foundation of Nankai University, China.

APPENDIX A: RELATION BETWEEN THE SCALAR DENSITY AND NUCLEON DENSITY IN THE RELATIVISTIC MEAN-FIELD APPROACH

In this appendix, we give a short derivation of the relation between the scalar density and nucleon density in the RMF approach.

In RMF, the effective Lagrangian density [16] can be written as

$$\begin{aligned}\mathcal{L}_0 = {} & \bar{\Psi}_N(i\gamma^\mu\partial_\mu - M_N)\Psi_N - g^N_\sigma\bar{\Psi}_N\sigma\Psi_N \\ & - g^N_\omega\bar{\Psi}_N\gamma^\mu\omega_\mu\Psi_N - g^N_\rho\bar{\Psi}_N\gamma^\mu\rho^a_\mu\frac{\tau_a}{2}\Psi_N \\ & + \tfrac{1}{2}\partial^\mu\sigma\partial_\mu\sigma - \tfrac{1}{2}m^2_\sigma\sigma^2 - \tfrac{1}{3}g^2_2\sigma^3 - \tfrac{1}{4}g^2_3\sigma^4 \\ & - \tfrac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} + \tfrac{1}{2}m^2_\omega\omega^\mu\omega_\mu - \tfrac{1}{4}R^{a\mu\nu}R^a_{\mu\nu} \\ & + \tfrac{1}{2}m^2_\rho\rho^{a\mu}\rho^a_\mu - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} \\ & - e\bar{\Psi}_N\gamma^\mu A^\mu\tfrac{1}{2}(1+\tau_3)\Psi_N, \end{aligned} \tag{A1}$$

with $\Omega^{\mu\nu} = \partial^\mu\omega^\nu - \partial^\nu\omega^\mu$, $R^{a\mu\nu} = \partial^\mu\rho^{a\nu} - \partial^\nu\rho^{a\mu}$, $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. On application of the mean-field approximation, we have the equation of motion for nucleons:

$$\left(\gamma_\mu k^\mu - M_N - g^N_\sigma\sigma_0 - g^N_\omega\gamma^0\omega_0 - g^N_\rho\gamma^0\tau^3\rho_{03}\right)\Psi_N = 0, \tag{A2}$$

where the σ, ω, and ρ fields are replaced with their mean-field values σ_0, ω_0 and ρ_0. σ_0 and ω_0 satisfy

$$m^2_\sigma\sigma_0 + g_2\sigma^2_0 + g_3\sigma^3_0 = -g^N_\sigma\rho_s, \tag{A3}$$

$$m^2_\omega\omega_0 = g^N_\omega\rho_N, \tag{A4}$$

with $\rho_s \equiv \langle\bar{\Psi}_N\Psi_N\rangle$ and $\rho_N \equiv \langle\Psi^\dagger_N\Psi_N\rangle$. Therefore, at the mean-field level, the energy density of nuclear matter is

$$\varepsilon = \frac{1}{2}m_\sigma^2\sigma_0^2 + \frac{1}{3}g_2\sigma_0^3 + \frac{1}{4}g_3\sigma_0^4 + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{4}{(2\pi)^3}\int_0^{k_F}\left(\vec{k}^2 + {M_N^*}^2\right)^{1/2} d\vec{k}, \tag{A5}$$

where $M_N^* = M_N + g_\sigma^N\sigma_0$ is the effective mass of nucleons.

In Eq. (A5), the energy density has been expressed as an explicit function of σ_0. Because σ_0 should minimize ε, i.e., $\partial\varepsilon(\sigma_0)/\partial\sigma_0$, we immediately have

$$m_\sigma^2\sigma_0 + g_2\sigma_0^2 + g_3\sigma_0^3 = -\frac{4g_\sigma^N}{(2\pi)^3}\int_0^{k_F}\frac{M_N^*\, d\vec{k}}{\left(\vec{k}^2 + {M_N^*}^2\right)^{1/2}}, \tag{A6}$$

which is nothing but Eq. (26). Equation (A3) corresponds to Eq. (25). From combining Eq. (A6) with Eq. (A3), Eq. (22) is derived.

[1] Q. Haider and L. C. Liu, Phys. Lett. **B172**, 257 (1986); L. C. Liu and Q. Haider, Phys. Rev. C **34**, 1845 (1986).
[2] G. L. Li, W. K. Cheung, and T. T. Kuo, Phys. Lett. **B195**, 515 (1987).
[3] C. García-Recio, T. Inoue, J. Nieves, and E. Oset, Phys. Lett. **B550**, 47 (2002).
[4] H. Nagahiro, D. Jido, and S. Hirenzaki, Nucl. Phys. **A761**, 92 (2005).
[5] H. Nagahiro, D. Jido, and S. Hirenzaki, Phys. Rev. C **68**, 035205 (2003).
[6] A. Sibirtsev, J. Haidenbauer, J. A. Niskanen, and Ulf-G. Meißner, Phys. Rev. C **70**, 047001 (2004).
[7] S. Wycech, A. M. Green, and J. A. Niskanen, Phys. Rev. C **52**, 544 (1995).
[8] J. D. Johnson *et al.*, Phys. Rev. C **47**, 2571 (1993).
[9] S. A. Rakityansky, S. A. Sofianos, M. Braun, V. B. Belyaev, and W. Sandhas, Phys. Rev. C **53**, R2043 (1996).
[10] Q. Haider and L. C. Liu, Phys. Rev. C **66**, 045208 (2002).
[11] D. Jido, H. Nagahiro, and S. Hirenzaki, Phys. Rev. C **66**, 045202 (2002).
[12] T. Waas and W. Weise, Nucl. Phys. **A625**, 287 (1997).
[13] H. C. Chiang, E. Oset, and L. C. Liu, Phys. Rev. C **44**, 738 (1991).
[14] K. Tsushima, D. H. Lu, A. W. Thomas, and K. Saito, Phys. Lett. **B443**, 26 (1998).
[15] T. Inoue and E. Oset, Nucl. Phys. **A710**, 354 (2002).
[16] B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. **16**, 1 (1986);P.-G. Reinhard, Rep. Prog. Phys. **52**, 439 (1989); M. Rufa, P. -G. Reinhard, J. A. Maruhn, W. Greiner, and M. R. Strayer, Phys. Rev. C **38**, 390 (1988); D. Hirata, H. Toki, T. Watabe, I. Tanihata, and B. V. Carlson, *ibid.* **44**, 1467 (1991); R. Brockmann and H. Toki, Phys. Rev. Lett. **68**, 3408 (1991); M. Rufa, J. Schaffner, J. Maruhn, H. Stöcker, W. Greiner, and P. G. Reinhard, Phys. Rev. C **42**, 2469 (1990); J. Mareš and B. K. Jennings, *ibid.* **49**, 2472 (1994); Jürgen Schaffner and I. N. Mishustin, *ibid.* **53**, 1416 (1996); J. Meng and P. Ring, Phys. Rev. Lett. **77**, 3963 (1996); H. Shen, H. Toki, K. Oyamatsu, and K. Sumiyoshi, Nucl. Phys. **A637**, 435 (1998); L. S. Geng, H. Toki, and J. Meng, Prog. Theor. Phys. **112**, 603 (2004); L. S. Geng, H. Toki, and J. Meng, J. Phys. G: Nucl. Part. Phys. **30**, 1915 (2004); Y. H. Tan, Y. A. Luo, P. Z. Ning, and Z. Y. Ma, Chin. Phys. Lett. **18**, 1030 (2001); Y. H. Tan, X. H. Zhong, C. H. Cai, and P. Z. Ning, Phys. Rev. C **70**, 054306 (2004); X. H. Zhong, Y. H. Tan, G. X. Peng, L. Li, and P. Z. Ning, *ibid.* **71**, 015206 (2005); T. Nikšić, D. Vretenar, P. Finelli, and P. Ring, *ibid.* **66**, 024306 (2002); G. A. Lalazissis, T. Nikšić, D. Vretenar, and P. Ring, *ibid.* **71**, 024312 (2005); S. F. Ban, J. Li, S. Q. Zhang, H. Y. Jia, J. P. Sang, J. Meng, *ibid.* **69**, 045805 (2004).
[17] G. A. Lalazissis, J. König, and P. Ring, Phys. Rev. C **55**, 540 (1997).
[18] D. B. Kaplan and A. E. Nelson, Phys. Lett. **B175**, 57 (1986).
[19] E. Jenkins and A. Manohar, Phys. Lett. **B255**, 558 (1991); **B259**, 353 (1991).
[20] G. E. Brown, Chang-Hwan Lee *et al.*, Nucl. Phys. **A567**, 937 (1994).
[21] C. H. Lee, G. E. Brown, D. P. Min, and M. Rho, Nucl. Phys. **A585**, 401 (1995).
[22] N. Kaiser, P. B. Siegel, and W. Weise, Nucl. Phys. **A594**, 325 (1995).
[23] S. Weinberg, Phys. Lett. **B251**, 288 (1990); Nucl. Phys. **B363**, 3 (1991).
[24] E. Jenkins and A. V. Manohar, in *Proceedings of the Workshop on Effective Field Theories of the Standard Model, Debogoko, Hungary, Aug. 22–26 (1991)*, edited by U.-G. Meißner, World Scientific.
[25] T.-S. Park, D.-P. Min, and M. Rho, Phys. Rep. **233**, 341 (1993).
[26] H. Georgi, *Weak Interactions and Modern Particle Theory*, Benjamin/Cummings, Menlo Park, California 1984.
[27] R. Shrock and L. Wang, Phys. Lett. **41**, 1692 (1978).
[28] W. Langbein, Nuovo Cimento **51**, 219 (1979).
[29] H. D. Politzer and M. B Wise, Phys. Lett. **B273**, 156 (1991).
[30] J. Gasser and H. Leutwyler, Phys. Rep. **87**, 77 (1982).
[31] S. Weinberg, Trans. New York Acad. Sci. **38**, 185 (1977).
[32] H. Leutwyler, Phys. Lett. **B378**, 313 (1996).
[33] Eur. Phys. J. A **22**, 89 (2004).
[34] V. E. Lyubovitskij, Th. Gutsche, Amand Faessler, and E. G. Drukarev, Phys. Rev. D **63**, 054026 (2001).
[35] G. E. Brown and M. Rho, Phys. Rep. **269**, 333 (1996).
[36] S. J. Dong, J.-F. Lagaë, and K. F. Liu, Phys. Rev. D **54**, 5496 (1996).
[37] T. Hatsuda and T. Kunihiro, Phys. Rep. **247**, 221 (1994).
[38] A. M. Green and S. Wycech, Phys. Rev. C **71**, 014001 (2005).
[39] F. Renard *et al.*, Phys. Lett. **B528**, 215 (2002).
[40] A. M. Green and S. Wycech, Phys. Rev. C **60**, 035208 (1999).
[41] R. A. Arndt, W. J. Briscoe, T. W. Morrison, I. I. Strakovsky, R. L. Workman, and A. B. Gridnev, Phys. Rev. C **72**, 045202 (2005).
[42] M. Abdel-Bary *et al.* (GEM Collaboration), Phys. Lett. **B619**, 281 (2005).
[43] Jürgen Schaffner, I. N. Mishustin, and T. Bondorf, Nucl. Phys. **A625**, 325 (1997).
[44] G. Q. Li, C.-H. Lee, and G. E. Brown, Nucl. Phys. **A625**, 372 (1997).
[45] X. H. Zhong, G. X. Peng and P. Z. Ning, arXiv:nucl-th/0501064.
[46] C. Bennhold and H. Tanabe, Nucl. Phys. **A530**, 625 (1991).
[47] R. S. Bhalerao and L. C. Liu, Phys. Rev. Lett. **54**, 865 (1985).

[48] V. Yu. Grishina, L. A. Kondratyuk, M. Buscher, C. Hanhart, J. Haidenbauer, and J. Speth, Phys. Lett. **B475**, 9 (2000).
[49] W. Briscoe, T. Morrison, I. Strakovsky, and A. B. Gridniev, PiN Newslett. **16**, 391 (2002).
[50] T. Feuster and U. Mosel, Phys. Rev. C **58**, 457 (1998).
[51] Ch. Sauerman, B. L. Friman, and W. Nörenberg, Phys. Lett. **B341**, 261 (1995); Ch. Deutsch-Sauerman, B. L. Friman, and W. Nörenberg, *ibid.* **B409**, 51 (1997).
[52] C. Wilkin, Phys. Rev. C **47**, R938 (1993).
[53] V. V. Abaev and B. M. K. Nefkens, Phys. Rev. C **53**, 385 (1996).
[54] N. Kaiser, P. B. Siegel, and W. Weise, Phys. Lett. **B362**, 23 (1995).
[55] M. Batinić, I. Dadić, I. Šlaus, A. Švarc, B. M. K. Nefkens, and T.-S. H. Lee, arXiv:nucl-th/9703023.
[56] A. M. Green and S. Wycech, Phys. Rev. C **55** , R2167 (1997).
[57] S. A. Rakityansky, S. A. Sofianos, N. V. Shevchenko, V. B. Belyaev, and W. Sandhas, Nucl. Phys. **A684**, 383 (2001).
[58] M. Batinić, I. Šlaus, A. Švarc, and B. M. K. Nefkens, Phys. Rev. C **51**, 2310 (1995); Erratum, *ibid.* **57**, 1004 (1998).
[59] M. Arima, K. Shimizu, and K. Yazaki, Nucl. Phys. **A543**, 613 (1992).
[60] G. Penner and U. Mosel, Phys. Rev. C **66** , 055212 (2002).
[61] H. Yabu, S. Nakamura, and Kubodera, Phys. Lett. **B317**, 269 (1993); J. Delorme, M. Ericson, and T. E. O. Ericson, *ibid.* **B291**, 379 (1993).