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Mach cones in an evolving medium

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The energy and momentum lost by a hard parton propagating through hot and dense matter has to be redistributed in the nuclear medium. Apart from heating the medium, there is the possibility that collective modes are excited. We outline a formalism that can be used to track the propagation of such a mode through the evolving medium if its dispersion relation is known. Under the assumption that a sound wave is created, we track the jet energy loss as a function of spacetime and follow the resulting mach cone throughout the fireball evolution. We compare with the angular correlation pattern of hard hadrons as obtained by the PHENIX Collaboration and find good agreement with the data provided that a substantial fraction of jet energy (∼90%) is deposited into a propagating mode and that the hot matter can be characterized by an equation of state with a soft point (not necessarily a mixed phase).

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Announcements have recently been made by the four detector collaborations at RHIC [1] that a new state of matter, distinct from ordinary hadronic matter, has been created in ultrarelativistic heavy-ion collisions (URHIC). A new and exciting challenge for both experiment and theory is now to study its properties. The energy loss of hard partons created in the first moments of the collision has long been regarded as a promising tool for this purpose [2–7]. However, before this tool can be fully exploited, the influence of the medium evolution on the energy loss has to be understood in a more quantitative way (cf. [8]).

Recently, measurements of two-particle correlations involving one hard trigger particle have shown a surprising splitting of the away-side peak for all centralities but peripheral collisions, qualitatively very different from a broadened away-side peak observed in p-p or d-Au collisions [9]. Interpretations in terms of energy lost to propagating colorless [10,11] and colored [12] sound modes have been suggested for this phenomenon. In [13], a Mach cone structure has been demonstrated to be visible in the angular pattern of photon emission if a source term for the medium energy is inserted in a hydro code, although this is at odds with the results of [10] where this form of source term does not yield a propagating shock wave. In addition, we note that energy densities more than 20 times above the value of the surrounding medium in the vicinity of 1 fm around the jet as shown in Fig. 1 of [13] do not seem to reflect the situation realized at RHIC, a possible reason for the discrepancy between the findings of Refs. [10] and [13].

In the following, we investigate whether such a mechanism can be in agreement with the measured hadronic data provided that a realistic evolution model of the hot and dense matter is used.

Throughout this manuscript we focus on central collisions only. Since we are interested in the deposition of lost jet energy into the medium, our first task is to determine the spacetime pattern of energy loss.

For the description of the evolution, we base our investigation on the fireball model outlined in Refs. [14] and [15]. The model is tuned to describe transverse mass spectra of hadrons, Hanbury-Brown-Twiss (HBT) correlation radii, and hadronic dN/dy distributions. The model has successfully predicted the photon yield [16–18] at RHIC and gives a fair description of R_{AA} in the high-p_T region without requiring unrealistically high gluon multiplicities or transport coefficients far from the perturbative estimate [8]. Therefore the model can be expected to give a fair representation of the relevant physics in both early and late stages of the evolution.

For the energy loss calculation we use results of Ref. [19] to obtain the probability $P(\Delta E)$ to lose the amount of energy ΔE from the two key quantities, plasma frequency

$$\omega_c(\mathbf{r_0}, \phi) = \int_0^\tau d\xi \xi \hat{q}(\xi) \tag{1}$$

and averaged momentum transfer

$$(\hat{q}L)(\mathbf{r_0}, \phi) = \int_0^\tau d\xi \hat{q}(\xi), \tag{2}$$

in a static equivalent scenario, calculated along the path of the hard parton through the medium. Here we take $\hat{q}(\xi) = c\epsilon(\xi)^{3/4}$ with ϵ the local energy density of the medium as determined by the fireball evolution [14] and $c = 3$ as required for the description of R_{AA} [8] and close to the perturbative $c \approx 2$ [20]. Since we are not interested in folding the result with a steeply falling spectrum but rather into the energy deposited on average in a given volume element, we focus on the average energy loss $\langle \Delta E \rangle = \int_0^\infty P(\Delta E)\Delta E d\Delta E$ in the following. For illustration of the resulting loss pattern, we show in Fig. 1 the energy deposition for a hard parton at midrapidity ($\eta = 0$) in the transverse (x, y) plane at $y = 0$ and propagating into the positive x direction as a function of initial position x.

The result shows the initial rise of the differential energy loss as an approximately linear increase reminiscent of the L^2 dependence of the energy loss in a static medium and a rather sudden drop in dE/dx once expansion significantly dilutes the medium. Once transverse expansion starts becoming important, the high power of the volume growth with time wins out over the advantage of having a longer decoherence time. It is the finite formation time of radiated quanta, which implies

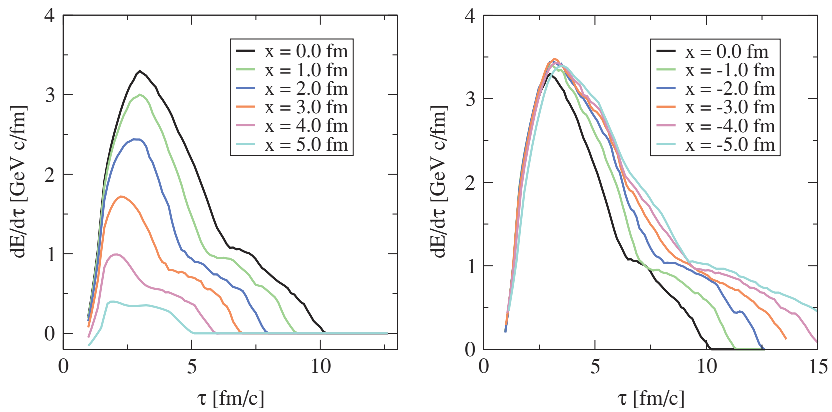

FIG. 1. (Color online) Energy deposition from hard quark at $\eta = 0$, $y = 0$ propagating into positive x as a function of proper time in the c.m. frame τ for different initial positions x. The shape of the energy-loss pattern for gluons is qualitatively similar but has a larger absolute normalization.

that lost energy does not appear in the medium immediately, that causes the curves to rise from zero.

Our further investigation relies now on the assumption that a fraction f of the energy lost to the medium excites a collective mode of the medium. Since our framework makes use of energy-momentum conservation laws, we do not have to address the important (though unsolved) question of what mechanism excites such a propagating mode. We considered the case where colorless sound characterized by a dispersion relation $E = c_s p$ is excited by energy fraction f. The remaining energy fraction $(1 - f)$ in essence heats the medium and leads to some amount of collective drift along the jet axis to conserve longitudinal momentum.

We calculate the speed of sound, c_s, locally from a quasiparticle description of the equation of state (EOS) as fitted to lattice results [21] as $c_s^2 = \partial p(T)/\partial \epsilon(T)$. This EOS shows a significant reduction of c_s as one approaches the phase transition but does not lead to a mixed phase. The dispersion relation along with the energy and momentum deposition determines the initial angle of propagation of the shock front with the jet axis (the "Mach angle") as $\phi = \arccos c_s$. We discretize the time into intervals $\Delta\tau$, calculate the energy deposited in that time interval $E(\tau)$, and propagate the part of the shock front remaining in the midrapidity slice (i.e., in the detector acceptance). Each piece of the front is propagated with the local speed of sound and the angle of propagation is continously adjusted as

$$\phi = \arccos \frac{\int_{\tau_E}^{\tau} c_s(\tau) d\tau}{(\tau - \tau_E)}, \tag{3}$$

where $c_s(\tau)$ is determined by the propagation history. Since sound propagates in the local rest frame, the shock front element is also carried away by the local flow velocity. Schematical calculations of this effect have already been presented in Ref. [22]. We illustrate this in Fig. 2 by showing the Mach cone in the transverse $\eta = 0$ plane for a jet traveling in the positive x direction, originating at $x = -6$ fm and either $y = 0$ (i.e., going through the fireball center) or $y = 3$ fm. Both the effect of the soft point in the EOS narrowing the cone at late times and the distortion of the cone in position space are clearly visible. Note that the distortion by flow is sizable in position space since flow velocities $v_T < 0.7$ are of the same order of magnitude as the speed of sound $c_s \sim 0.3$–0.5. However, since the effect of flow is already included in the standard calculation of m_T spectra, the direction of flow of excess momentum contained in the Mach cone in momentum space is hardly changed.

Once an element of the wave front reaches the freeze-out condition $T = T_F$, a hydrodynamical mode cannot propagate further. We assume that the energy contained in the shock wave is not used to produce hadrons but rather is converted into kinetic energy. In the local rest frame, we then have a matching condition for the dispersion relations

$$E = c_s p \quad \text{and} \quad E = \sqrt{M^2 + p^2} - M, \tag{4}$$

where $M = V\,[p(T_F) + \epsilon(T_F)]$ is the "mass" of a volume element at freeze-out temperature. Once we have calculated the additional boost a volume element receives from the shock wave using the matching conditions, we use the Cooper-Frye

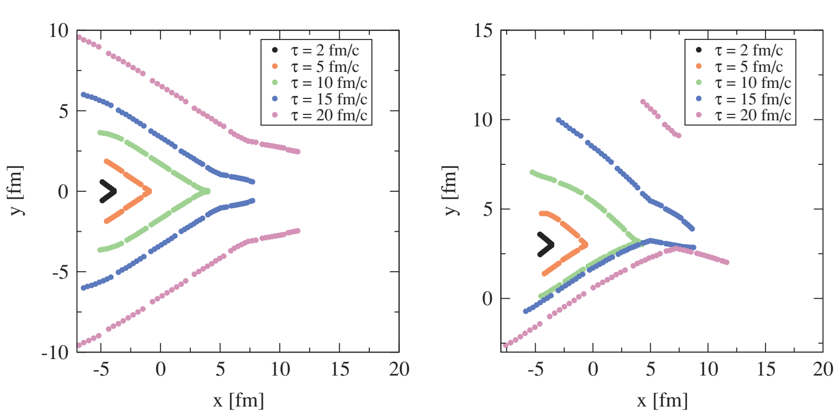

FIG. 2. (Color online) Mach cones excited by a jet traveling from $x = -5$ fm in the positive x direction [with (0,0) the fireball center] for $y = 0$ (left panel) and $y = 3$ fm (right panel).

formula

$$E\frac{d^3N}{d^3p} = \frac{g}{(2\pi)^3}\int d\sigma_\mu p^\mu \exp\left[\frac{p^\mu u_\mu - \mu_i}{T_f}\right] = d^4xS(x,p) \quad (5)$$

with $u^\mu = u^\mu_{\rm flow} + u^\mu_{\rm shock}$ to convert the fluid element into a hadronic distribution. The resulting momentum spectrum is thus a thermal two-component spectrum resulting from integration involving volume not part of the shock wave and volume receiving an additional boost from the shock wave.

From those considerations one can also calculate the typcial number of particles getting an additional boost from the shock front. An upper bound for c_s at T_F would be the speed of sound in a hadronic resonance gas, $c_s \approx \sqrt{0.2}$ [23], with a freeze-out temperature of $\approx$110 MeV [14]. From those considerations, for an energy in the collective mode of about 5 GeV, one finds $\approx$20 pions that are part of the shock wave. Assuming, for example, $c_s = 0.3$ one finds $\approx$50 pions. The mach cone would also contain a smaller number of additional kaons and nucleons. A 10-GeV jet can therefore potentially involve $\mathcal{O}(100)$ particles.

In the following we compare to the hard two-particle correlation data. For this purpose we simulate the PHENIX trigger conditions as closely as possible using a Monte Carlo approach. We start by generating vertices with a distribution weighted by the nuclear overlap

$$T_{AA}(\mathbf{b}) = \int dz\rho^2(\mathbf{b}, z). \quad (6)$$

We then determine the jet momentum and parton type by randomly sampling partonic transverse momentum spectra generated by the VNI/BMS parton cascade as described in Ref. [16]. Calculating the energy loss of the near-side parton, we decide if the experimental trigger condition is fulfilled. Since the experiment triggers on a hard hadron in the transition between the recombination and fragmentation regime, the model at this point cannot implement the trigger condition exactly. Instead, we require the trigger condition to be fulfilled by the parton and have checked that the model results do not change significantly when the near-side trigger threshold is increased by 2 GeV. We note that this procedure places the vertices fulfilling the trigger condition close to the surface of the produced matter; that is, in our model the medium is rather opaque, in agreement with the conclusions of Ref. [24].

Once the vertex and momentum of a near-side jet have passed the trigger condition, we determine the direction of the far-side parton in the transverse plane. To take into account intrinsic k_T, we do not propagate the away-side jet directly opposite the near-side jet but allow for a random angle. We have verified that this distribution, folded with the width of the near-side peak, reproduces the width of the far-side peak in the case of d-Au and 60–90% peripheral Au-Au collisions.

With vertex, energy, and direction of the away-side jet fixed, we calculate $dE/d\tau$ of the outgoing parton. We stop the calculation when a significant fraction of the energy is lost to the medium. We generate and propagate a shock wave by the previously outlined formalism and fold the resulting hadron spectra with the experimental acceptance cuts. For the remnants of the away-side jet, thermal physics and local boosts determine the spectral excess; therefore we have no ambiguity assoicated with the choice of recombination/fragmentation. In each event we assume that a fraction $(1 - f)$ of the energy lost from the hard parton heats the medium. We assume that because of momentum conservation this will lead to additional flow into the direction of the original away-side parton. Likewise, we account for the possibility of a punchthrough if the initial vertex is very peripheral and both near- and away-side partons propagate nearly tangential to the surface. (However, we note that these events are extremely rare and play almost no role in the calculation.)

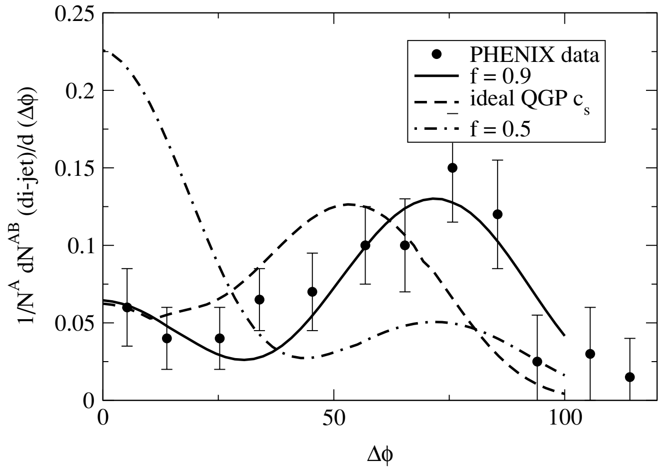

FIG. 3. Away-side correlation structure for different scenarios.

We present the resulting two-particle correlation structure on the away side in Fig. 3. Since the near-side associated particle distribution is not calculable in the framework outlined here we will only focus on the away-side region and make no attempt to describe near-side correlations; hence our results are normalized to the integral of the away-side correlation structure. We have chosen 0 degrees as the direction opposite to the near-side jet.

We present three different scenarios: Our best fit of the only model parameter f to the data (leading to $f \approx 0.9$), a scenario where f is set to 0.5, and one in which we do not determine c_s by the EOS but set it to the ideal QGP value $c_s \approx 1/\sqrt{3}$.

One can read off two conclusions immediately from the comparison with the data: First, a large fraction (about 90%) of the lost energy has to excite a shock wave to reproduce the data. Since most of the "Mach ring" in momentum space is outside the acceptance but any flow or punchthrough is predominantly inside, even a moderate fraction of jet energy heating the plasma would lead to a strong signal at zero degrees. To find a dip, the mechanism transporting energy away from this region has to be very efficient.

Second, the angular position of the peak is capable of measuring the speed of sound to some degree. We observe that on average it has to be significantly smaller than the ideal QGP value and that the data are in agreement with the situation as seen on the lattice (i.e., a soft point but no mixed phase). However, it would be premature to claim that this is the only possible description of the data.

In summary, we have demonstrated that the idea of shock waves being excited by the energy lost from a hard parton into the medium is able to account for the observed splitting of the away-side peak if a realistic model for the medium evolution is used. If this is the underlying mechanism, it has to dominate over simple heating of the plasma, as this would produce a significant peak in the forward direction. In this scenario, the away-side peaks essentially represent thermal physics, and we expect the momentum distribution at the peak between 1 and 2 GeV to be dominated by boosted thermal spectra and not by jet fragmentation or recombination physics.

We emphasize that the framework we have used is quite general and relies mainly on energy momentum conservation and the assumption of a deposition of a considerable fraction of the energy in a collective mode. We used a specific dispersion relation (colorless sound) to demonstrate that the two-particle correlations with hard trigger have some sensitivity to c_s in hot and dense matter. We see an average c_s far from the ideal QGP value and quite compatible with a crossover transition with a soft point.

Recently, measurements of three-particle correlations have been presented by both the PHENIX [25] and the STAR [26] Collaborations. Although we plan to address a calculation of three-particle correlations in the framework outlined here in a future publication, we remark at this point that the Mach cone is a collective phenomenon that can involve typically of the order of 20–50 particles for the given trigger conditions. Whereas all particles are strongly correlated with the original away-side parton (and hence with the observable near-side jet), once this correlation is subtracted only the momentum balance (with respect to the away-side parton momentum) $\sum p_L = p_{\text{away}}$ and $\sum \mathbf{p_T} = 0$ remains to account for residual correlation inside the cone. With momentum distributed across a significant number of particles, there is no a priori reason to expect strong correlations between two specific ones.

There are still many significant physics questions regarding jet energy deposition into the medium: Is there an appropiate theoretical framework to understand the jet energy deposition mechanism of a fraction f of the lost energy into collective modes in detail? Is sound the only collective mode compatible with the data? How can colorful and colorless sound collective modes and their interplay be consistently included? How sensitive is the correlation pattern to details of the fireball evolution or the EOS? Can we successfully calculate the p_T spectra of near and far side simultaneously? At the moment, the case for the excitation and propagation of a sound mode through the hot and dense system appears to be rather strong and offers exciting opportunities to study the evolving system from a new angle, and we plan to address those questions in forthcoming publications along with a systematic investigation of some of our model assumptions.

Note added in proof. After the acceptance of this paper, a new version of Ref. [13] has been made available in which the energy scaling between jet and medium has been reevaluated. The conclusions of the new version agree now with those in Ref. [10].

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