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ΛN space-exchange correlation effects in the $^5_\Lambda$He hypernucleus

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A complete realistic study of the $^5_\Lambda$He hypernucleus is presented using a realistic Hamiltonian and a fully correlated wave function that takes into account all relevant dynamical correlations and ΛN space-exchange correlation (SEC). Results are sensitive to SEC, which significantly affects energy breakdown, Λ-separation energy, nuclear core polarization, point proton radius, and density profiles.

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Strangeness can be experimentally injected in a bound nuclear system through the (K^-, π^-) reaction, for example, causing subtle distortions in it. This introduces new symmetries to the system, replacing older ones [1]. Hypernuclei are unique laboratories for studying these interesting aspects owing to the presence of the strangeness degree of freedom. The $^5_\Lambda$He hypernucleus comes to mind first for an in-depth study because of the rich experimental statistics. We perform a variational Monte Carlo study using a realistic Hamiltonian and a fully correlated wave function (WF) that includes the ΛN space-exchange correlation (SEC). The effect of the SEC on energy breakdown, nuclear core polarization (NCP), point proton radius, and density profiles is presented. Realistic studies [2,3] have been performed on s-shell single hypernuclei using the realistic two-nucleon (*NN*) Argonne v_{18} potential [4] and the three-nucleon (*NNN*) Urbana model-IX potential [5,6] in the nonstrange sector in conjunction with the two-baryon (ΛN) Urbana-type charge symmetric potential [7,8] and the three-baryon (ΛNN) potential [9–11] in the strange sector. In addition, there are studies of $^5_\Lambda$He [12] and of $^{17}_{\Lambda}$O [13] using the truncated *NN* (v_6) potential, which is the first six terms of the Argonne v_{14} potential [14] and a Coulomb term. A couple of these studies aim at pinning down the strengths of the ΛNN force [3,12]. Despite the fact that the ΛN space-exchange potential arising from an equivalent ΛN interaction in the relative p state is not insignificant, the SEC has always been put aside when writing the WF in both the aforementioned studies as well as in several others [15,16]. In an alternative approach, Nemura *et al.* [17] have performed an *ab initio* calculation of s-shell single hypernuclei by explicitly including the Σ degree of freedom at the two-body level. Nogga *et al.* [18] have performed Faddeev-Yukubosvky calculations but have yet to extend these to $^5_\Lambda$He. In their calculations, the SEC effects cannot be extracted as the SEC is included naturally in the formalism.

The charge-symmetric ΛN potential is written as

$$v_{\Lambda N}(r) = v_0(r)(1-\varepsilon+\varepsilon P_x) + (v_\sigma/4)T_\pi^2(r)\boldsymbol{\sigma}_\Lambda\cdot\boldsymbol{\sigma}_N. \quad (1)$$

Here, the first term is the sum of the direct potential $[v_0(r) = v_c(r) - v_{2\pi}(r)]$ and the space-exchange potential $[\varepsilon v_0(r)(P_x-1)]$. Therefore, function ε, which determines the odd-state potential, is the strength of the space-exchange potential relative to the direct potential. It is quite poorly estimated from the Λp forward-backward asymmetry, whose value ranges from 0.1 to 0.38 [19]. In these expressions, $v_c(r) = W_c/[1+\exp(r-R)/ar]$ is the Woods-Saxon repulsive potential with $W_c = 2137$ MeV, $R = 0.5$ fm, and $a = 0.2$ fm, and $v_{2\pi} = \bar{v}T_\pi^2(r)$ is the two-pion attractive potential. The $\bar{v} = (v_s + 3v_t)/4$ and $v_\sigma = v_s - v_t$ terms are, respectively, the spin-average and spin-dependent strengths, with $v_{s(t)}$ the singlet(triplet) state depths. A value of $\bar{v} \approx 6.15(5)$ MeV is found to be consistent with low-energy Λp scattering data [10].

We solve the Schrödinger equation

$$\left[\frac{-\hbar^2}{2\mu}\nabla^2 + \tilde{v}^\ell_{s(t)}(r) + \theta(r) + \frac{\hbar^2\ell(\ell+1)}{2\mu r^2}\right] f^\ell_{s(t)}(r) = 0 \quad (2)$$

for the radial solutions $f^\ell_s(r)$ and $f^\ell_t(r)$ using quenched ΛN potentials in singlet and triplet states:

$$\tilde{v}^\ell_s(r) = \left[v_c(r) - \alpha_{2\pi}\bar{v}T_\pi^2(r)\right]\left(1-\varepsilon+\varepsilon P^\ell_x\right) + (3/4)\alpha_\sigma v_\sigma T_\pi^2(r), \quad (3)$$

$$\tilde{v}^\ell_t(r) = \left[v_c(r) - \alpha_{2\pi}\bar{v}T_\pi^2(r)\right]\left(1-\varepsilon+\varepsilon P^\ell_x\right) - (1/4)\alpha_\sigma v_\sigma T_\pi^2(r), \quad (4)$$

where $\alpha_{2\pi}$ and α_σ are quenching factors for the two-pion and spin-exchange parts, respectively, of the central and spin channels [13]; $P^\ell_x \equiv P_x$ is a Majorana space-exchange operator whose value is $1(-1)$ for $\ell = 0(1)$; the function $T_\pi(r)$ is the one-pion exchange tensor potential; and $\theta(r)$ is an auxiliary potential that ensures the asymptotic behavior of the long-range correlation functions ($f^\ell_{s(t)} \sim r^{-\nu}e^{-\kappa r}$) [2,10].

Using these radial solutions $f^\ell_s(r)$ and $f^\ell_t(r)$, we obtain the ℓ-dependent spin-averaged correlation function

$$f^\ell_{\Lambda N}(r) = \left[f^\ell_s(r) + 3f^\ell_t(r)\right]/4. \quad (5)$$

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$f^c_{\Lambda N}(r) = f^0_{\Lambda N}(r)$ is the ΛN repulsive central correlation function with no SEC. With SEC,

$$u^x_{\Lambda N}(r) = \left[f^0_{\Lambda N}(r) - f^1_{\Lambda N}(r)\right]/2, \quad (6)$$

and the correlation function $f^c_{\Lambda N}(r)$ is modified as

$$f^c_{\Lambda N}(r) = \left[f^0_{\Lambda N}(r) + f^1_{\Lambda N}(r)\right]/2 = f^0_{\Lambda N}(r) - u^x_{\Lambda N}(r). \quad (7)$$

The weak spin-spin correlation function is written as $u^\sigma_{\Lambda N}(r) = [f^0_s(r) - f^0_t(r)]/4$.

The WF of A-baryon s-shell hypernuclei with l number of Λ baryons and $A-l$ number of nucleons is written as

$$|\Psi\rangle = \left[1 + U^3 + \sum_{i<j}^{A-l} U^{LS}_{ij}\right]\left[\prod_{j=1}^{A-l}\left(1 + u^\sigma_{\Lambda j}\right)\right]\left[S\prod_{i<j}^{A-l}(1+U_{ij})\right]\Psi_J + \frac{\eta}{\Lambda_p}\sum_{\lambda=1}^{l}\sum_{n=1}^{A-l}[1+U^3]\left[S\prod_{i<j}^{A-l}(1+U_{ij})\right]\Psi_J u^x_{\lambda n}P_x, \quad (8)$$

where $U^3 = 1 + \sum_{\lambda=1}^{l}\sum_{j<k}^{A-l} U_{\lambda jk} + \sum_{i<j<k}^{A-l}(U_{ijk} + U^{TNI}_{ijk})$ and

$$\Psi_J = \left[\prod_{\lambda=1}^{l}\prod_{j<k}^{A-l} f^c_{\lambda jk}\right]\left[\prod_{\lambda=1}^{l}\prod_{j=1}^{A-l} f^c_{\lambda j}\right] \times \left[\prod_{\lambda=1}^{l-1} f^c_{\Lambda\lambda}\right]\left[\prod_{i<j<k}^{A-l} f^c_{ijk}\right]\left[\prod_{i<j}^{A-l} f^c_{ij}\right]\chi^\sigma_\Lambda \Psi_{JT} \quad (9)$$

is the Jastrow WF involving two- and three-baryon central correlations and the appropriate spin function (χ^σ_Λ) resulting from l number of Λ baryons. η is a variational parameter. In Eqs. (8) and (9), U_{ij}, U^{LS}_{ij}, U_{ijk}, U^{TNI}_{ijk}, and $U_{\Lambda jk}$ are the noncommuting two- and three-baryon correlation operators (where subscripts i, j, k, and n stand for nucleons and λ for Λ baryons). Functions $u^x_{\Lambda i}P_x$ and $u^\sigma_{\Lambda i}\boldsymbol{\sigma}_\Lambda \cdot \boldsymbol{\sigma}_N$ [13] are SEC and spin-spin ΛN correlations. S is the symmetrization operator. The second term in Eq. (8) is due to the SEC, where the P_x operation (exchange of space positions between Λ and N) runs over ΛN pairs: $\sum_{\lambda=1}^{l}\sum_{n=1}^{A-l} = \Lambda_p$. With $l = 0$ and without Λ correlation functions, these equations represent A nucleon s-shell nucleus wave function [20,21]. To make the WFs translationally invariant, all the positions of baryons are measured from the c.m. of the system ($\mathbf{R}_{\text{c.m.}} = [m_N \sum_{i=1}^{A-l}\mathbf{r}_i + m_\Lambda \sum_{\lambda=1}^{l}\mathbf{r}_\lambda]/[(A-l)m_N + lm_\Lambda]$),

$$\tilde{\mathbf{r}} = \mathbf{r} - \mathbf{R}_{\text{c.m.}}. \quad (10)$$

A P_x operation on a ΛN pair interchanges the positions of Λ and N. This results in a new set of configurations ($\mathbf{r}^x \equiv P_x\mathbf{r}$), which alters the c.m. to a new position $\mathbf{R}'_{\text{c.m.}}$ by a shift $\Delta\mathbf{R}_{\text{c.m.}} = \mathbf{R}'_{\text{c.m.}} - \mathbf{R}_{\text{c.m.}}$. To keep it unaltered, we make a translational shift in all the baryon positions:

$$\tilde{\mathbf{r}}' = \mathbf{r}^x - (\mathbf{R}_{\text{c.m.}} + \Delta\mathbf{R}_{\text{c.m.}}) = \mathbf{r}^x - \mathbf{R}'_{\text{c.m.}}. \quad (11)$$

The Λ-separation energy is expressed as

$$B_\Lambda = \frac{\langle\Psi_{A-1}|H_{NC}|\Psi_{A-1}\rangle}{\langle\Psi_{A-1}|\Psi_{A-1}\rangle} - \frac{\langle\Psi_A|H|\Psi_A\rangle}{\langle\Psi_A|\Psi_A\rangle}, \quad (12)$$

where, Ψ_A and Ψ_{A-1} refer to the WFs of the hypernucleus and of its isolated bound nuclear core (NC).

A nonrelativistic Hamiltonian H of the hypernucleus involving two- and three-body forces is written as a sum of the Hamiltonians due to the NC (H_{NC}) and due to the Λ (H_Λ):

$$H_{\text{NC}} = T_{\text{NC}} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk}, \quad (13)$$

$$H_\Lambda = T_\Lambda + \sum_i v_{\Lambda i} + \sum_{i<j} V_{\Lambda ij}. \quad (14)$$

$V_{\Lambda NN}$ is the ΛNN force written as a sum of two Wigner types of forces: $V_{\Lambda NN} = V^D_{\Lambda NN} + V^{2\pi}_{\Lambda NN}$. Here, $V^D_{\Lambda NN}$ is a dispersive force, suggested by the suppression mechanism owing to ΛN–ΣN coupling [19,22–24], which may be written with explicit spin dependence as [10]

$$V^D_{\Lambda ij} = W^D T^2_\pi(r_{\Lambda i})T^2_\pi(r_{\Lambda j})[1 + \boldsymbol{\sigma}_\Lambda \cdot (\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)/6]. \quad (15)$$

$V^{2\pi}_{\Lambda NN}$ is a sum of two terms resulting from p- and s-wave π–N scatterings, $V^{2\pi}_{\Lambda NN} = V^S_{\Lambda NN} + V^P_{\Lambda NN}$, written as

$$V^P_{\Lambda ij} = -(C^P/6)(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j)\{X_{i\Lambda}, X_{\Lambda j}\}, \quad (16)$$

$$V^S_{\Lambda ij} = C^S Z(m_\pi r_{i\Lambda})Z(m_\pi r_{j\Lambda})\boldsymbol{\sigma}_i \cdot \hat{\mathbf{r}}_{i\Lambda}\boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{j\Lambda}\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j, \quad (17)$$

with $X_{\Lambda i} = (\boldsymbol{\sigma}_\Lambda \cdot \boldsymbol{\sigma}_i)Y_\pi(r_{\Lambda i}) + S_{\Lambda i}T_\pi(r_{\Lambda i})$ and $Z(x) = \frac{x}{3}[Y_\pi(x) - T_\pi(x)]$. Here, C^P, C^S, and W^D are strengths, $S_{\Lambda i} = 3(\boldsymbol{\sigma}_\Lambda \cdot \hat{\mathbf{r}}_{\Lambda i})(\boldsymbol{\sigma}_i \cdot \hat{\mathbf{r}}_{\Lambda i}) - \boldsymbol{\sigma}_\Lambda \cdot \boldsymbol{\sigma}_i$ is a tensor operator, and Y_π is the Yukawa function. Charge asymmetry and other possibilities may improve the Hamiltonian. Correlations induced by ΛNN potentials are included using scaled pair distances ($\bar{r}$) and a variational parameter δ^m as in Ref. [2],

$$U_{\Lambda ij} = \sum_{m=D,P,S} \delta^m V^m(\bar{r}_{\Lambda i}, \bar{r}_{ij}, \bar{r}_{j\Lambda}). \quad (18)$$

We perform calculations for both choices of WF: with SEC and without SEC. We use three different sets of v_s and v_t, which give three different values of spin-average strength $\bar{v}$ and a constant spin-dependent strength v_σ as in Table I, referred to as $\bar{v}1$, $\bar{v}2$, and $\bar{v}3$. With each set, we use three values of ε: 0.1, 0.2, and 0.3. In principle, we tune the variational parameters of the WF every time we change a potential strength. In the absence of SEC, the WF remains constant with the variation of ε as its parameters stay tuned. However, SEC offsets some of the parameters, especially the quenching parameter $\alpha_{2\pi}$ and the asymptotic parameter κ, the latter of which is correlated with ε. Thus, even for $\ell = 0$, radial solutions $f^\ell_s(r)$ or $f^\ell_t(r)$ with SEC differ from those with no SEC. The optimized correlation functions, $f^c_{\Lambda N}(r) = f^0_{\Lambda N}(r)$ with no SEC and $f^c_{\Lambda N}(r) = f^0_{\Lambda N}(r) - u^x_{\Lambda N}(r)$ and $u^x_{\Lambda N}(r)$ with SEC, are plotted in Fig. 1. Results are presented in Table II.

We begin with the no-SEC case for the calculations with the strengths $\bar{v}1$, $\varepsilon = 0.3$, $C^P = 0.75$ MeV, and $C^S = 0.15$ MeV.

TABLE I. ΛN potential strengths in units of MeV.

	v_s	v_t	$\bar{v} = (v_s + 3v_t)/4$	$v_\sigma = v_s - v_t$
$\bar{v}1$	6.33	6.09	6.15	0.24
$\bar{v}2$	6.28	6.04	6.10	0.24
$\bar{v}3$	6.23	5.99	6.05	0.24

TABLE II. Energy breakdown for $^{5}_{\Lambda}$He. All quantities are in units of MeV except for ε. Subscripts i, j, and k refer to nucleons.

	$\varepsilon=0.1$			$\varepsilon=0.2$			$\varepsilon=0.3$		
	(SEC) A	(No SEC) B	A − B	(SEC) C	(No SEC) D	C − D	(SEC) E	(No SEC) F	E − F
				$\bar{v}1$ ($\bar{v}=6.15$ and $v_\sigma=0.24$)					
T_Λ	8.77(3)	9.00(3)	−0.23(4)	8.49(3)	9.00(3)	−0.51(4)	8.11(3)	9.00(3)	−0.89(4)
$v_0(r)(1-\varepsilon)$	−16.15(5)	−16.59(5)	0.44(7)	−13.92(5)	−14.75(5)	0.83(7)	−11.64(4)	−12.90(4)	1.26(6)
$v_0(r)\varepsilon P_x$	−1.56(1)	−1.61(1)	0.05(1)	−3.02(1)	−3.23(1)	0.21(1)	−4.30(2)	−4.84(2)	0.54(4)
$(\frac{1}{4})v_\sigma T_\pi^2(r)\boldsymbol{\sigma}_\Lambda\cdot\boldsymbol{\sigma}_i$	0.015(0)	0.040(0)	−0.025(0)	0.012(0)	0.040(0)	−0.028(0)	0.009(0)	0.040(0)	−0.031(0)
$v_{\Lambda i}$	−17.69(4)	−18.16(6)	0.47(7)	−16.93(4)	−17.94(6)	1.01(7)	−15.93(4)	−17.71(6)	1.78(7)
$V_{\Lambda ij}^D$	2.29(1)	2.42(1)	−0.13(1)	2.25(1)	2.42(1)	−0.17(1)	2.16(1)	2.42(2)	−0.26(2)
$V_{\Lambda ij}^P$	−2.88(2)	−2.71(2)	−0.17(3)	−2.68(2)	−2.71(2)	0.03(3)	−2.53(2)	−2.71(2)	0.18(3)
$V_{\Lambda ij}^S$	−0.001(0)	−0.015(1)	0.014(1)	−0.006(0)	−0.015(2)	0.009(2)	−0.010(0)	−0.015(2)	0.005(2)
$V_{\Lambda ij}^{2\pi}=V_{\Lambda ij}^P+V_{\Lambda ij}^S$	−2.88(2)	−2.73(2)	−0.15(3)	−2.69(2)	−2.73(2)	0.04(3)	−2.54(2)	−2.73(2)	0.19(3)
$V_{\Lambda ij}=V_{\Lambda ij}^D+V_{\Lambda ij}^{2\pi}$	−0.58(2)	−0.30(1)	−0.28(2)	−0.44(2)	−0.30(1)	−0.14(2)	−0.39(2)	−0.30(2)	−0.09(3)
$V_\Lambda=v_{\Lambda i}+V_{\Lambda ij}$	−18.28(6)	−18.47(6)	0.19(8)	−17.37(6)	−18.24(6)	0.87(8)	−16.32(6)	−18.01(6)	1.69(8)
$E_\Lambda=T_\Lambda+V_\Lambda$	−9.51(4)	−9.46(4)	−0.05(6)	−8.88(4)	−9.23(4)	0.35(6)	−8.20(4)	−9.01(4)	0.81(6)
$T_{\rm NC}$	118.00(15)	118.12(15)	−0.12(21)	117.52(15)	118.12(15)	−0.60(21)	117.47(15)	118.12(15)	−0.65(21)
$V_{\rm NC}=v_{ij}+V_{ijk}$	−140.52(15)	−139.98(15)	−0.54(21)	−140.31(15)	−139.98(15)	−0.33(21)	−140.63(15)	−140.11(15)	−0.52(21)
$E_{\rm NC}=T_{\rm NC}+V_{\rm NC}$	−22.51(4)	−21.86(4)	−0.65(6)	−22.79(4)	−21.86(4)	−0.93(6)	−23.16(4)	−21.86(4)	−1.30(6)
$E=E_\Lambda+E_{\rm NC}$	−32.02(2)	−31.32(2)	−0.70(3)	−31.68(2)	−31.09(2)	−0.58(3)	−31.36(2)	−30.86(2)	−0.50(3)
B_Λ	4.29(2)	3.59(2)	0.70(3)	3.95(2)	3.36(2)	0.58(3)	3.63(2)	3.13(2)	0.50(3)
NCP	3.59(4)	4.31(3)	−0.72(5)	3.16(4)	4.31(3)	−1.15(5)	2.70(4)	4.31(3)	−1.61(5)
				$\bar{v}2$($\bar{v}=6.10$ and $v_\sigma=0.24$)					
T_Λ	8.25(3)	8.14(3)	0.11(4)	8.04(3)	8.14(3)	−0.10(4)	7.77(3)	8.14(4)	−0.37(5)
$v_0(r)(1-\varepsilon)$	−14.68(5)	−14.67(5)	−0.01(7)	−12.74(5)	−13.04(5)	0.30(7)	−10.71(5)	−11.41(4)	0.70(6)
$v_0(r)\varepsilon P_x$	−1.41(1)	−1.41(1)	−0.00(1)	−2.75(1)	−2.82(1)	0.07(2)	−3.93(2)	−4.24(2)	0.31(3)
$(\frac{1}{4})v_\sigma T_\pi^2(r)\boldsymbol{\sigma}_\Lambda\cdot\boldsymbol{\sigma}_i$	0.007(0)	0.058(0)	−0.051(0)	0.007(0)	0.058(0)	−0.051(0)	0.005(0)	0.058(0)	−0.053(0)
$v_{\Lambda i}$	−16.08(6)	−16.02(6)	−0.06(8)	−15.48(6)	−15.81(6)	0.33(8)	−14.64(6)	−15.59(6)	0.95(8)
$V_{\Lambda ij}^D$	2.05(2)	2.08(1)	−0.03(2)	2.02(2)	2.08(1)	−0.06(2)	1.96(1)	2.08(2)	−0.12(2)
$V_{\Lambda ij}^P$	−2.63(2)	−2.22(2)	−0.41(3)	−2.43(2)	−2.22(2)	−0.21(3)	−2.38(2)	−2.22(2)	−0.16(3)
$V_{\Lambda ij}^S$	−0.004(2)	−0.032(2)	0.028(3)	−0.013(2)	−0.032(2)	0.019(3)	−0.013(2)	−0.032(2)	0.019(3)
$V_{\Lambda ij}^{2\pi}=V_{\Lambda ij}^P+V_{\Lambda ij}^S$	−2.64(2)	−2.25(2)	−0.39(3)	−2.44(2)	−2.25(2)	−0.19(3)	−2.40(2)	−2.25(2)	−0.12(3)
$V_{\Lambda ij}=V_{\Lambda ij}^D+V_{\Lambda ij}^{2\pi}$	−0.59(2)	−0.17(1)	−0.42(2)	−0.43(2)	−0.17(1)	−0.26(2)	−0.44(2)	−0.17(1)	−0.27(2)
$V_\Lambda=v_{\Lambda i}+V_{\Lambda ij}$	−16.67(6)	−16.20(6)	−0.47(8)	−15.91(6)	−15.98(6)	0.07(8)	−15.08(6)	−15.76(6)	0.68(8)
$E_\Lambda=T_\Lambda+V_\Lambda$	−8.42(4)	−8.06(3)	−0.36(5)	−7.86(4)	−7.84(3)	−0.02(5)	−7.31(4)	−7.62(3)	0.31(5)
$T_{\rm NC}$	117.02(15)	116.43(15)	0.59(21)	116.78(15)	116.43(15)	0.35(21)	116.99(15)	116.43(15)	0.56(21)
$V_{\rm NC}=v_{ij}+V_{ijk}$	−139.90(14)	−138.86(14)	−1.04(20)	−139.88(14)	−138.86(14)	−1.02(20)	−140.30(14)	−138.86(14)	−1.44(20)
$E_{\rm NC}=T_{\rm NC}+V_{\rm NC}$	−22.87(4)	−22.44(4)	−0.43(6)	−23.10(4)	−22.44(4)	−0.66(6)	−23.31(4)	−22.44(4)	−0.87(6)
$E=E_\Lambda+E_{\rm NC}$	−31.29(2)	−30.50(2)	−0.79(3)	−30.96(2)	−30.28(2)	−0.68(3)	−30.62(2)	−30.06(2)	−0.56(3)
B_Λ	3.56(2)	2.77(2)	0.78(3)	3.23(2)	2.55(2)	0.68(3)	2.89(2)	2.33(2)	0.56(3)
NCP	2.93(4)	3.22(4)	−0.29(6)	2.61(4)	3.22(4)	−0.71(6)	2.39(4)	3.22(4)	−0.83(6)
				$\bar{v}3$ ($\bar{v}=6.05$ and $v_\sigma=0.24$)					
T_Λ	7.75(3)	7.54(3)	0.21(4)	7.51(3)	7.54(3)	−0.03(4)	7.25(3)	7.54(3)	−0.29(4)
$v_0(r)(1-\varepsilon)$	−13.27(5)	−13.18(5)	−0.09(7)	−11.42(5)	−11.72(4)	0.30(6)	−9.63(4)	−10.25(4)	0.62(6)
$v_0(r)\varepsilon P_x$	−1.27(1)	−1.26(1)	−0.01(1)	−2.44(2)	−2.52(1)	0.08(2)	−3.51(2)	−3.78(2)	0.27(3)
$(\frac{1}{4})v_\sigma T_\pi^2(r)\boldsymbol{\sigma}_\Lambda\cdot\boldsymbol{\sigma}_i$	−0.003(1)	0.034(0)	−0.037(1)	0.004(1)	0.034(0)	−0.030(1)	0.002(1)	0.034(0)	−0.032(1)
$v_{\Lambda i}$	−14.55(6)	−14.41(6)	−0.14(8)	−13.85(6)	−14.21(6)	0.36(8)	−13.14(6)	−14.25(6)	1.11(8)
$V_{\Lambda ij}^D$	1.83(2)	1.83(1)	0.00(2)	1.80(2)	1.83(1)	−0.03(2)	1.74(2)	1.83(1)	−0.09(2)
$V_{\Lambda ij}^P$	−2.40(2)	−1.92(1)	−0.48(2)	−2.30(2)	−1.92(1)	−0.38(2)	−2.16(2)	−1.92(1)	−0.24(2)
$V_{\Lambda ij}^S$	−0.012(1)	−0.034(0)	0.022(1)	−0.018(1)	−0.037(0)	0.019(1)	−0.017(1)	−0.034(0)	0.017(1)
$V_{\Lambda ij}^{2\pi}=V_{\Lambda ij}^P+V_{\Lambda ij}^S$	−2.41(2)	−1.96(2)	−0.45(3)	−2.32(2)	−1.96(2)	−0.36(3)	−2.17(2)	−1.96(2)	−0.21(3)
$V_{\Lambda ij}=V_{\Lambda ij}^D+V_{\Lambda ij}^{2\pi}$	−0.58(2)	−0.12(1)	−0.46(2)	−0.52(2)	−0.12(1)	−0.40(2)	−0.43(2)	−0.12(1)	−0.31(2)
$V_\Lambda=v_{\Lambda i}+V_{\Lambda ij}$	−15.12(6)	−14.54(6)	−0.58(9)	−14.37(6)	−14.33(6)	−0.04(9)	−13.57(6)	−14.13(6)	0.56(9)
$E_\Lambda=T_\Lambda+V_\Lambda$	−7.38(4)	−6.99(3)	−0.39(5)	−6.86(4)	−6.79(3)	0.07(5)	−6.32(4)	−6.59(3)	0.27(5)
$T_{\rm NC}$	115.90(15)	115.24(15)	0.66(21)	115.78(15)	115.14(15)	0.64(21)	116.12(15)	115.14(15)	0.98(21)
$V_{\rm NC}=v_{ij}+V_{ijk}$	−139.19(14)	−138.03(14)	−1.16(20)	−139.28(14)	−138.03(14)	−1.25(20)	−139.85(14)	−138.03(14)	−1.82(20)
$E_{\rm NC}=T_{\rm NC}+V_{\rm NC}$	−23.28(4)	−22.89(4)	−0.39(6)	−23.49(4)	−22.89(4)	−0.60(6)	−23.73(4)	−22.89(4)	−0.84(6)
$E=E_\Lambda+E_{\rm NC}$	−30.66(2)	−29.88(2)	−0.78(3)	−30.35(2)	−29.68(2)	−0.67(3)	−30.05(2)	−29.48(2)	−0.57(3)
B_Λ	2.93(2)	1.15(2)	1.78(3)	2.62(2)	1.95(2)	0.67(3)	2.32(2)	1.74(2)	0.58(3)
NCP	2.19(4)	2.39(4)	−0.20(6)	1.91(4)	2.39(4)	−0.48(6)	1.69(4)	2.39(4)	−0.70(6)

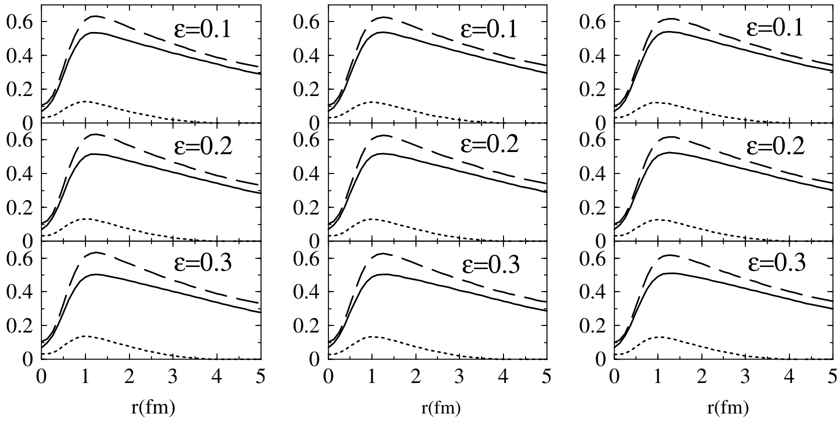

FIG. 1. The ΛN correlation function $f^c_{\Lambda N}(r)$ with and without SEC, represented by solid and dashed lines, respectively. The dotted line shows the $u^x_{\Lambda N}(r)$ function. The left, middle, and right columns represent $\bar{v}1$, $\bar{v}2$, and $\bar{v}3$, respectively.

The repulsive strength W^D is varied to reproduce the experimental $B_\Lambda^{\exp} = 3.12(2)$ MeV, which is found to be 0.013 MeV. Because of the constant nature of the WF with no SEC, among the entire energy breakdown only the central and space-exchange parts of $v_{\Lambda i}$ [Eq. (1)] are affected with the variation of ε. Hence the strange energy $E_\Lambda = T_\Lambda + v_{\Lambda i} + V_{\Lambda ij}$ and the total energy $E = E_\Lambda + E_{\rm NC}$. E_Λ includes the nucleon kinetic energy and $E_{\rm NC}$ (nuclear core energy) includes the Λ kinetic energy through ΛN correlations. They exhibit a linear behavior: $\partial v_{\Lambda i}/\partial\varepsilon = \partial E_\Lambda/\partial\varepsilon = \partial E/\partial\varepsilon = -\partial B_\Lambda/\partial\varepsilon \approx 2.3$ MeV for every $\bar{v}$. However, the SEC modifies the WF through $f^c_{\Lambda N}(r)$ and $u^x_{\Lambda N}(r)$; hence it affects the entire energy breakdown, giving additional binding. The effect is more evident with increasing ε. Interestingly, for the fine-tuned WFs, E_Λ obeys a linear behavior: $\partial E_\Lambda/\partial\varepsilon \approx$ constant. Because this is true for every $\bar{v}$, it cannot be an accident. With increasing ε, both δ^P and δ^S of $U_{\Lambda ij}$ set to lower values. Being sensitive to its own correlation, the attraction from $\langle V^{2\pi}_{\Lambda ij}\rangle$ is reduced with the reduction in $U_{\Lambda ij}$. This offsets other energy pieces of E_Λ. Therefore, any change in C^P leads to a quadratic dependence of $\langle V^{2\pi}_{\Lambda ij}\rangle$. This may be understood as follows: The $\{X_{i\Lambda}, X_{\Lambda j}\}$ may be expressed in terms of operators $(\boldsymbol{\sigma}_i\cdot\mathbf{r}_{\Lambda i})(\boldsymbol{\sigma}_j\cdot\mathbf{r}_{\Lambda j})$, $(\boldsymbol{\sigma}_i\cdot\mathbf{r}_{\Lambda i})(\boldsymbol{\sigma}_j\cdot\mathbf{r}_{\Lambda i})$, $(\boldsymbol{\sigma}_i\cdot\mathbf{r}_{\Lambda j})(\boldsymbol{\sigma}_j\cdot\mathbf{r}_{\Lambda j})$, and $(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)$ followed by $(\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)$ and hence is a generalization of the $S_{ij}\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$ operator. The expectation value of S_{ij} in a Jastrow WF for a closed-shell nucleus is zero, whereas the expectation value of S^2_{ij} is nonzero. Therefore, $V^{2\pi}_{\Lambda ij}$ is sensitive to both $U_{\Lambda ij}$ and S_{ij} [2,12,13]. As no other operatorial correlation is affected by the variation of ε, the change in $U_{\Lambda ij}$ is basically due to the change in baryon densities (discussed later), which affects $\{X_{i\Lambda}, X_{\Lambda j}\}$ through $T_\pi(r)$ and $Y_\pi(r)$ functions. A little variation in U^{TNI}_{ijk} is also noticed. The difference in the ΛN "central" potential $[v_0(r)(1-\varepsilon)]$ with and without SEC too is due to the change in density profiles. With the observations that $\partial B_\Lambda/\partial\varepsilon = c_1$ and $\partial V^D_{\Lambda ij}/\partial W^D = \partial E/\partial W^D = -\partial B_\Lambda/\partial W^D = c_2$, the $B_\Lambda^{\exp}$ for any value of ε may be reproduced through variation in W^D as $\partial W^D/\partial\varepsilon \sim -0.016$ MeV. Here c_1 and c_2 are positive numbers. The behavior of B_Λ with $\bar{v}$ is not linear. We can now examine the behavior of $B_\Lambda(\bar{v}, v_\sigma, \varepsilon, C^p, C^S, W^D)$ compared with strengths in detail.

The average $\langle P_x\rangle = \langle v_0(r)\varepsilon P_x\rangle/\langle\varepsilon v_0(r)\rangle$ values as extracted from Table II for the no-SEC case are 0.88(1), 0.87(1), and 0.86(1) for $\bar{v}1$, $\bar{v}2$, and $\bar{v}3$, respectively. But with SEC, the WF involves another P_x operator. For a properly weighted independent calculation using Eq. (10) for the configurations with no P_x operation, $v_{\Lambda N}$ [Eq. (1)] would be independent of ε. Hence ε may be treated as a variational parameter, which implicitly appears in the WF through $u^x_{\Lambda N}$. The best energy is found about $\varepsilon = 0.2$. This is compared with the full calculation involving P_x [Eq. (11)] in Table III. The difference in every energy piece is significant. The WF with the SEC term [Eq. (8)] even with no P_x operation gives better results than the no-SEC case.

The difference of the internal energy ($E^{\rm int}_{\rm NC} = T^{\rm int}_{\rm NC} + V_{\rm NC}$) of the $(A-l)$ subsystem and the energy of an identical isolated bound nucleus is defined as the NCP. Here,

$$T^{\rm int}_{\rm NC} = \sum_{i=1}^{A-l}\frac{p_i^2}{2m_N} - \frac{\left(\sum_{i=1}^{A-l}p_i\right)^2}{2(A-l)m_N} \equiv T_{\rm NC} - T^{\rm c.m.}_{\rm NC}, \qquad (19)$$

where $T^{\rm c.m.}_{\rm NC}$ is the kinetic energy resulting from the c.m. motion of the subsystem around the c.m. of the hypernucleus. As reported in Table II, at a fixed ε, NCP increases with increasing $\bar{v}$ with SEC as well as with no SEC. However, at a fixed $\bar{v}$ it decreases with increasing ε with SEC but remains constant with no SEC as WF does not change. A similar dependence is found for the point proton radius (Table IV), which establishes a direct correlation between the two. The obvious reason is the significant reduction in the repulsive $f^c_{\Lambda N}$ correlation at moderate and large r, if the SEC is invoked (Fig. 1). As a result, all the baryons receive an inward pull, leading to a reduction

TABLE III. Independently minimum energy breakdown with P_x operation on the configurations (A) and without it (B). All quantities are in units of MeV.

	A	B	A − B
T_Λ	8.49(3)	9.01(3)	−0.52(4)
$v_0(r)(1-\varepsilon)$	−13.92(5)	−14.55(5)	0.63(7)
$v_0(r)\varepsilon P_x$	−3.02(2)	−3.70(2)	0.68(3)
$(1/4)v_\sigma T^2_\pi(r)\boldsymbol{\sigma}_\Lambda\cdot\boldsymbol{\sigma}_i$	0.012(0)	0.010(0)	0.002(0)
$V^D_{\Lambda ij}$	2.25(2)	2.31(1)	−0.06(2)
$V^P_{\Lambda ij}$	−2.68(2)	−2.80(1)	0.12(2)
$V^S_{\Lambda ij}$	−0.006(2)	−0.024(1)	0.018(2)
$T_{\rm NC}$	117.52(15)	117.94(15)	−0.42(21)
$V_{\rm NC}$	−140.31(15)	−139.66(15)	−0.65(21)
E	−31.68(2)	−31.46(2)	−0.22(3)

TABLE IV. Point proton radius of the NC in units of femtometers.

	$\varepsilon = 0.1$		$\varepsilon = 0.2$		$\varepsilon = 0.3$	
	SEC	No SEC	SEC	No SEC	SEC	No SEC
$\bar{v}1$	1.588(1)	1.619(1)	1.585(1)	1.619(1)	1.586(1)	1.619(1)
$\bar{v}2$	1.605(1)	1.647(1)	1.602(1)	1.647(1)	1.600(1)	1.647(1)
$\bar{v}3$	1.624(1)	1.676(1)	1.621(1)	1.676(1)	1.620(1)	1.676(1)

in p and Λ densities at the periphery (right column of Fig. 2). The change in the interior is weak enough to be noticed (left column of Fig. 2). Compared to the no-SEC case, $^{5}_{\Lambda}$He and its NC are more compact with SEC owing to "space-exchange pressure." The NC is found to be quite spherical. The Λ skin is also seen. The features are similar for $\bar{v}2$ and $\bar{v}3$.

We conclude that the SEC is an important correlation. It significantly affects energy breakdown, Λ-separation energy, nuclear core polarization, point proton radius, and density profiles. Findings suggest that a study without SEC would be misleading. Hence, any such effort to reslove the outstanding $A = 5$ anomaly [11,15,25] or to pin down the strengths of ΛN and ΛNN potentials would be deficient.

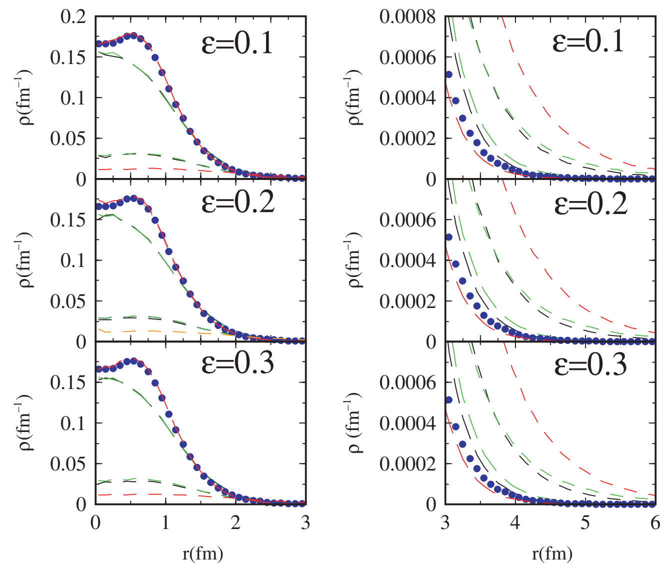

FIG. 2. (Color online) The dashed and long dashed lines represent Λ and p densities in $^{5}_{\Lambda}$He with black and green colors for the SEC and the no-SEC cases, respectively. The red color shows the NC. Blue circles show p in ^{4}He.

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