Unified analysis of spin isospin responses of nuclei

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We investigate the Gamow-Teller (GT) response functions at a momentum transfer of $q = 0\,\mathrm{fm}^{-1}$ and the pionic response functions for quasielastic scattering (QES) at $q \approx 1.7\,\mathrm{fm}^{-1}$ using the continuum random phase approximation with the $\pi + \rho + g'$ model interaction. The Landau-Migdal (LM) parameters, g'_{NN} and $g'_{N\Delta}$, are estimated by comparing the calculations with recent experimental data. The peak of the GT resonance and the pionic response functions below the QES peak constrain g'_{NN}, whereas the quenching of the GT total strength and the enhanced pionic strength around the QES peak provide information about $g'_{N\Delta}$. We obtained $g'_{NN} = 0.6 \pm 0.1$ and $g'_{N\Delta} = 0.35 \pm 0.16$ at $q = 0\,\mathrm{fm}^{-1}$ and $g'_{NN} = 0.7 \pm 0.1$ and $g'_{N\Delta} = 0.3 \pm 0.1$ at $q \approx 1.7\,\mathrm{fm}^{-1}$. These results indicate that the q dependence of the LM parameters is weak.

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Recent (p, n) and (n, p) experiments at intermediate energies have yielded reliable information on nuclear spin-isospin responses [1]. Two contrasting issues have arisen that are especially interesting. One is quenching of the total strength of the Gamow-Teller (GT) transitions [2] from the sum rule value $3(N - Z)$ [3] and the other is enhancement of the pionic (isovector spin-longitudinal) response functions in the quasielastic scattering (QES) region [4–6] as a precursor of pion condensation [7]. A common key concept in understanding these contrasting phenomena is the Landau-Migdal (LM) parameters, g'_{NN}, $g'_{N\Delta}$, and $g'_{\Delta\Delta}$, which specify the LM interactions V_{LM}, namely the zero-range interactions between particle-hole (ph) and delta-hole (Δh) states.

In this Brief Report we present a unified analysis of the GT strength distribution and quenching factor observed at 295 MeV at the Research Center for Nuclear Physics (RCNP) and the spin-longitudinal cross section ID_q of $(\vec{p}, \vec{n})$ at 346 MeV at RCNP and at 494 MeV at the Los Alamos Meson Physics Facility (LAMPF), which represents the pionic response function R_L. To these measurements we apply the continuum random phase approximation (RPA) with the $\pi + \rho + g'$ model interaction, which properly treats finite geometry and continuum single-particle spectra. We then determine the LM parameters that best reproduce the experimental data.

Estimations of g'_{NN} from GT giant resonances (GTGR) have been carried out by several researchers [8]. For instance, Suzuki [9] used the energy-weighted-sum technique and Bertsch, Cha, and Toki [10] used the continuum RPA. By fitting the peak position of the GTGR, these two groups obtained similar values of $g'_{NN} \approx 0.6$ for ^{90}Zr. However, their analysis used only the LM interaction for nucleons. Most later works with Δ [11,12] used the universality ansatz $g'_{NN} = g'_{N\Delta} = g'_{\Delta\Delta}$. We re-investigate the GTGR spectrum using the $\pi + \rho + g'$ model interaction without the universality ansatz.

From the GT quenching factor, Suzuki and Sakai [13] estimated $g'_{N\Delta} \approx 0.2$ for ^{90}Zr, using the Fermi gas model with only V_{LM} and treating the finite-size effect crudely. Using the first-order perturbation on the $N\Delta$ transition part of the $\pi + \rho + g'$ model interaction, Arima *et al.* [14] obtained $g'_{N\Delta} \approx 0.3$. This increase of 0.1 from the Suzuki-Sakai result arises from the π and ρ exchange interactions resulting from the nuclear finite size. In this Brief Report we present an integrated RPA analysis.

It has been shown [15] that for pionic responses in the QES region, a conventional eikonal approximation for the nuclear distortion is not quantitatively reliable for extracting the pionic response function R_L from ID_q. Thus, in Ref. [6] we calculated ID_q for the RCNP data by the distorted wave impulse approximation (DWIA), incorporating continuum RPA response functions, and compared the theoretical and experimental results of ID_q directly. We also found that two-step processes contribute appreciably to the background. Here we extend the same DWIA + two-step analysis to the LAMPF data and attempt to find suitable values for g'.

We write the $\beta^{\pm}(\mathrm{GT}^{\pm})$ transition operators with N and Δ in the unit of g_A as

$$O^{\pm}_{\mathrm{GT}} = \mp\frac{1}{\sqrt{2}}\sum_{k=1}^{A}\left[\tau_{k,\pm1}\boldsymbol{\sigma}_k + \frac{g_A^{N\Delta}}{g_A}\left(T_{k,\pm1}\boldsymbol{S}_k + T^{\dagger}_{k,\pm1}\boldsymbol{S}^{\dagger}_k\right)\right], \tag{1}$$

with $\tau_{\pm1} = \mp\frac{1}{\sqrt{2}}(\tau_x \pm i\tau_y)$ and $T_{\pm1} = \mp\frac{1}{\sqrt{2}}(T_x \pm iT_y)$, where g_A and $g_A^{N\Delta}$ are the axial-vector weak coupling constants for the *NN* and $N\Delta$ transitions, $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ are the nucleon Pauli spin and isospin matrixes, and $\boldsymbol{S}$ and $\boldsymbol{T}$ are the spin and isospin transition operators from N to Δ. Similarly, we write the isovector spin-longitudinal transition operators with momentum transfer $\boldsymbol{q}$ as

$$O^{\lambda}_{L}(\boldsymbol{q}) = \sum_{k=1}^{A}\left[\tau_{k,\lambda}\boldsymbol{\sigma}_k\cdot\hat{\boldsymbol{q}} + \frac{f_{\pi N\Delta}}{f_{\pi NN}}\left(T_{k,\lambda}\boldsymbol{S}_k\cdot\hat{\boldsymbol{q}} + T^{\dagger}_{k,\lambda}\boldsymbol{S}^{\dagger}_k\cdot\hat{\boldsymbol{q}}\right)\right]e^{i\boldsymbol{q}\cdot\boldsymbol{r}_k}, \tag{2}$$

where $\lambda = 0, \pm1$ and $f_{\pi NN}$ and $f_{\pi N\Delta}$ are the πNN and $\pi N\Delta$ coupling constants. We have neglected the transitions from Δ to Δ in both $O^{\pm}_{\mathrm{GT}}$ and $O^{\lambda}_{L}(\boldsymbol{q})$ and we have used the quark model relation $f_{\pi N\Delta}/f_{\pi NN} = g_A^{N\Delta}/g_A = \sqrt{72/25} \simeq 1.70$. Having defined these operators, we are interested in how the nuclei respond to them.

Since neither momentum $\boldsymbol{q}$ nor spin direction are conserved in finite nuclei, we introduce the spin-isospin transition densities

$$O^N_{\lambda,a}(\boldsymbol{r}) = \sum_{k=1}^{A} \tau_{k,\lambda}\sigma_{k,a}\delta(\boldsymbol{r}-\boldsymbol{r}_k),$$
$$O^\Delta_{\lambda,a}(\boldsymbol{r}) = \sum_{k=1}^{A} T_{k,\lambda}S_{k,a}\delta(\boldsymbol{r}-\boldsymbol{r}_k), \tag{3}$$

with $a = x, y, z$, and calculate the spin-isospin response functions

$$R^{\alpha\beta}_{\lambda,ba}(\boldsymbol{r}',\boldsymbol{r},\omega) = \sum_{n\neq 0}\langle\Psi_0|O^{\alpha\dagger}_{\lambda,b}(\boldsymbol{r}')|\Psi_n\rangle\langle\Psi_n|O^{\beta}_{\lambda,a}(\boldsymbol{r})|\Psi_0\rangle \times \delta[\omega-(\mathcal{E}_n-\mathcal{E}_0)], \tag{4}$$

using the continuum RPA with the orthogonality condition in coordinate space [16].

The $\pi + \rho + g'$ model interaction is written as

$$V^{\rm eff}(\boldsymbol{q},\omega) = V_{\rm LM} + V_\pi(\boldsymbol{q},\omega) + V_\rho(\boldsymbol{q},\omega), \tag{5}$$

where V_π and V_ρ are the one-pion and the one-rho-meson exchange interactions, respectively. The LM interaction $V_{\rm LM}$ is written by the LM parameters as

$$V_{\rm LM} = \Bigg[\frac{f^2_{\pi NN}}{m^2_\pi} g'_{NN}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2) + \frac{f_{\pi NN}f_{\pi N\Delta}}{m^2_\pi} \times g'_{N\Delta}\{[(\boldsymbol{\tau}_1\cdot\boldsymbol{T}_2)(\boldsymbol{\sigma}_1\cdot\boldsymbol{S}_2) + (\boldsymbol{\tau}_1\cdot\boldsymbol{T}^\dagger_2)(\boldsymbol{\sigma}_1\cdot\boldsymbol{S}^\dagger_2)] + (1\leftrightarrow 2)\} + \frac{f^2_{\pi N\Delta}}{m^2_\pi} g'_{\Delta\Delta}\{[(\boldsymbol{T}_1\cdot\boldsymbol{T}_2)(\boldsymbol{S}_1\cdot\boldsymbol{S}_2) + (\boldsymbol{T}_1\cdot\boldsymbol{T}^\dagger_2)(\boldsymbol{S}_1\cdot\boldsymbol{S}^\dagger_2)] + \text{h.c.}\}\Bigg]\delta(\boldsymbol{r}_1-\boldsymbol{r}_2). \tag{6}$$

We fixed $g'_{\Delta\Delta} = 0.5$ [17], since the calculated results depend on it very weakly. Nonlocality of mean fields is taken into account by a local effective mass approximation in the form

$$m^*(r) = m_N - \frac{f_{\rm WS}(r)}{f_{\rm WS}(0)}[m_N - m^*(0)], \tag{7}$$

with the free nucleon mass m_N and the Woods-Saxon radial form $f_{\rm WS}(r)$.

We first discuss the strength distributions of the GT$^-$ transitions, which are expressed by the GT$^\pm$ response functions for the ground state $|\Psi_0\rangle$ as

$$R^\pm_{\rm GT}(\omega) = \sum_{n\neq 0}|\langle\Psi_n|O^\pm_{\rm GT}|\Psi_0\rangle|^2\delta[\omega-(\mathcal{E}_n-\mathcal{E}_0)], \tag{8}$$

where $|\Psi_n\rangle$ and $\mathcal{E}_n$ denote the n-th nuclear state and its energy. The response functions are experimentally extracted from the $\Delta J^\pi = 1^+$ cross sections $d^2\sigma_{1^+}(q,\omega)/d\Omega d\omega$ deduced by multipole decomposition analysis (MDA) as [18]

$$\frac{d^2\sigma_{1^+}(q,\omega)}{d\Omega d\omega} = \hat{\sigma}_{\rm GT}F(q,\omega)R^\pm_{\rm GT}(\omega), \tag{9}$$

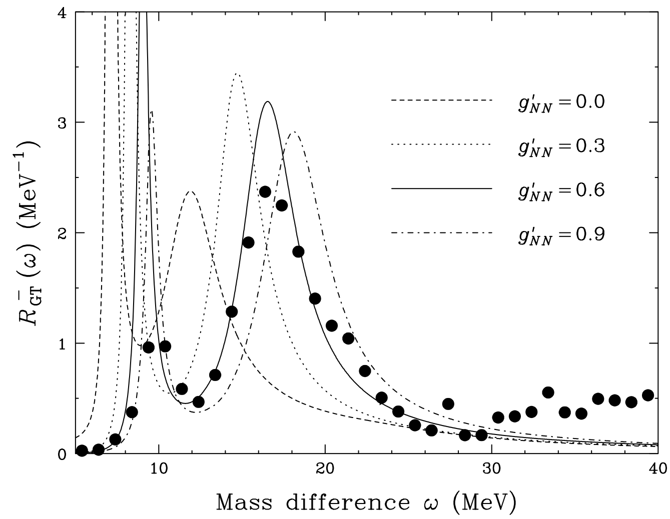

FIG. 1. g'_{NN} dependence of GT$^-$ strength distributions from ^{90}Zr to ^{90}Nb, where $g'_{N\Delta}$ and $m^*(0)/m_N$ are set to 0.3 and 0.7, respectively. The filled circles are experimental data taken from Ref. [18].

with the GT unit cross section $\hat{\sigma}_{\rm GT}$ and the (q,ω) dependence factor $F(q,\omega)$.

Converting the calculated response functions $R^{\alpha\beta}(\boldsymbol{r}',\boldsymbol{r},\omega)$ into the momentum representation, $R^{\alpha\beta}(\boldsymbol{q}',\boldsymbol{q},\omega)$ gives the GT response functions $R^\pm_{\rm GT}(\omega)$ of Eq. (8):

$$R^\pm_{\rm GT}(\omega) = \frac{1}{2}\sum_a\Bigg[R^{NN}_{\pm 1,aa}(\omega) + 2\frac{g^{N\Delta}_A}{g_A}R^{N\Delta}_{\pm 1,aa}(\omega) + \left(\frac{g^{N\Delta}_A}{g_A}\right)^2 R^{\Delta\Delta}_{\pm 1,aa}(\omega)\Bigg], \tag{10}$$

where $R^{\alpha\beta}(\omega) = R^{\alpha\beta}(\boldsymbol{q}'=0,\boldsymbol{q}=0,\omega)$. The strength distribution $R^-_{\rm GT}(\omega)$ from ^{90}Zr to ^{90}Nb was obtained by MDA of (p,n) data [18,19], which cover not only the GTGR region but also excitation energies up to 50 MeV.

Figure 1 shows the g'_{NN} dependence of the GTGR peak position. The curves correspond to the results for $g'_{NN} =$ 0.0–0.9 in 0.3 steps, with $g'_{N\Delta} = 0.3$ and $m^*(0)/m_N = 0.7$. The result for $g'_{NN} = 0.6$ reproduces the peak position well and is very close to previous results [9,10]. The excess of the theoretical values around the peaks can be redistributed by mixing $2p2h$ and other excitations [20], interpreted as being significant experimental strength beyond the GTGR. This is a quenching mechanism that should be distinguished from that resulting from Δh mixing discussed in the following.

Figure 2 shows the $g'_{N\Delta}$ and m^* dependences of the GTGR spectrum. In the left panel, the curves denote the results for $g'_{N\Delta} = 0.0$–0.9 in 0.3 steps. The peak position barely depends on $g'_{N\Delta}$, though the peak height strongly does. Since $g'_{N\Delta}$ governs the coupling between ph and Δh, it controls the amount by which the GT$^-$ strength in the GTGR region moves into the Δh region. The m^* dependence is shown in the right panel, where the curves represent the results for $m^*(0)/m_N = 1.0$–0.6 in 0.2 steps. It is hard to make a conclusion about the effect of m^* since m^* affects the GTGR so weakly. From this analysis, we determined $g'_{NN} = 0.6 \pm 0.1$

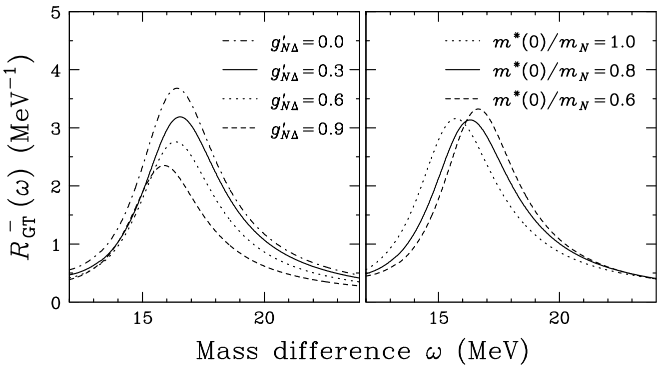

FIG. 2. $g'_{N\Delta}$ (left panel) and $m^*(0)/m_N$ (right panel) dependences of the RPA calculations. In the left panel g'_{NN} and $m^*(0)/m_N$ are set to 0.6 and 0.7, respectively. In the right panel g'_{NN} and $g'_{N\Delta}$ are set to 0.6 and 0.3, respectively.

as an appropriate value, accounting for the small $g'_{N\Delta}$ and m^* dependences.

We next discuss the GT quenching factor Q, which is defined as

$$Q = \frac{S^-_{\rm GT}(\omega^-_{\rm top}) - S^+_{\rm GT}(\omega^+_{\rm top})}{3(N-Z)}, \tag{11}$$

with

$$S^{\pm}_{\rm GT}(\omega^{\pm}_{\rm top}) = \int^{\omega^{\pm}_{\rm top}} R^{\pm}_{\rm GT}(\omega) d\omega. \tag{12}$$

Recently Yako *et al.* [21] applied MDA to ^{90}Zr(p,n) and ^{90}Zr(n,p) data and obtained $Q = 0.86 \pm 0.07$ using an end energy of $\omega^-_{\rm top} = 57$ MeV and selected a corresponding $\omega^+_{\rm top}$ accounting for the Coulomb energy shift and the nuclear mass difference.

Since Q is almost exclusively determined by $g'_{N\Delta}$ in the calculations, we display the $g'_{N\Delta}$ dependence in Fig. 3 with

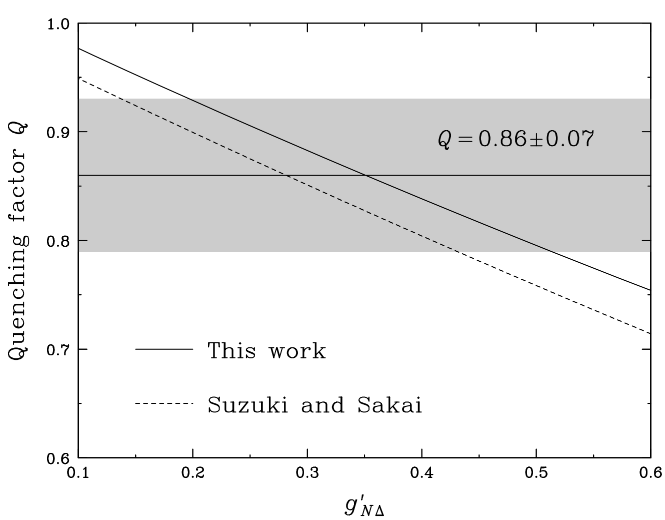

FIG. 3. GT quenching factor Q as a function of $g'_{N\Delta}$. The experimental result for $Q = 0.86 \pm 0.07$ [21] is shown by the horizontal solid line and band. The dashed curve is the theoretical prediction of Suzuki and Sakai [13].

a fixed $g'_{NN} = 0.6$. The solid line shows the results of the continuum RPA and the dashed line shows those of the Suzuki-Sakai formulas [13]. The experimental Q and its uncertainty are shown by the horizontal solid line and the horizontal band, respectively. From this comparison we obtained $g'_{N\Delta} = 0.35 \pm 0.16$. The difference between the present calculation and the Suzuki-Sakai line can be understood by the mechanism of Arima *et al.* [14].

We next investigate the enhancement of the pionic modes in the QES region. The relevant spin-longitudinal cross sections ID_q were measured for ^{12}C and ^{40}Ca at $T_p = 346$ MeV [5,6] and 494 MeV [4], taken at RCNP and LAMPF, respectively. We performed DWIA calculations using the response functions $R^{\alpha\beta}(\boldsymbol{r}', \boldsymbol{r}, \omega)$ and estimated the two-step contributions in the same manner as in Ref. [6]. Since the obtained characteristics are very similar for both ^{12}C and ^{40}Ca, in Fig. 4 we compare the calculations with the experimental ID_q only for ^{12}C taken at RCNP and LAMPF in the left and right panels, respectively.

The top panels show the g'_{NN} dependence for $g'_{NN} = 0.0$–0.9 in 0.3 steps with fixed $g'_{N\Delta} = 0.3$ and $m^*(0)/m_N = 0.7$. The calculations are sensitive to g'_{NN} near and below the QES peak. The experimental data are reasonably reproduced for $g'_{NN} = 0.7$, with an uncertainty of about 0.1. This result is close to the value of $g'_{NN} = 0.6 \pm 0.1$ evaluated from the GTGR spectrum.

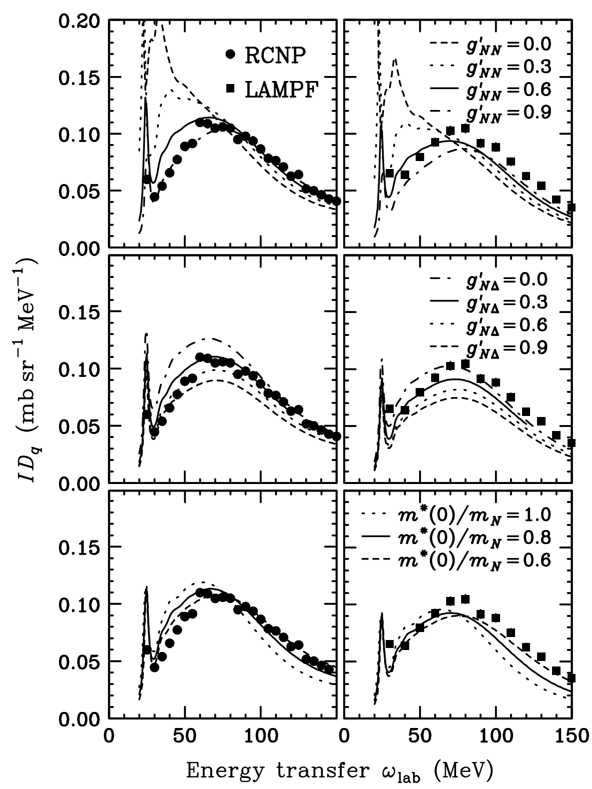

FIG. 4. Spin-longitudinal polarized cross section ID_q for the ^{12}C reaction at $T_p = 346$ MeV [5,6] (left panels) and $T_p = 494$ MeV [4] (right panels). The top, middle, and bottom panels show the g'_{NN}, $g'_{N\Delta}$, and $m^*(0)/m_N$ dependences of the calculations. The notation for the curves is the same as in Figs. 1 and 2 except that $g'_{NN} = 0.7$ for the middle and bottom panels.

The middle panels denote the $g'_{N\Delta}$ dependence for $g'_{N\Delta} = 0.0\text{--}0.9$ in 0.3 steps with fixed $g'_{NN} = 0.7$ and $m^*(0)/m_N = 0.7$. The dependence is seen around the QES peak. The most likely choices of $g'_{N\Delta}$ are about 0.4 and 0.2 for the RCNP and LAMPF data, respectively. The systematic uncertainties of the data are in the range 6–8% [4–6], which corresponds to the $\approx \pm 0.1$ uncertainty in $g'_{N\Delta}$. Thus the difference between $g'_{N\Delta} = 0.2$ and 0.4 seems acceptable in the present analysis. Note that this difference gives rise to the difference of the full spin-longitudinal interaction strength at $q \approx 1.7\,\text{fm}^{-1}$ including a one-pion exchange contribution by about 30%. From these results, we estimate $g'_{N\Delta} = 0.3 \pm 0.1$.

The bottom panels display the m^* dependence for $m^*(0)/m_N = 1.0\text{--}0.6$ in 0.2 steps with fixed $g'_{NN} = 0.7$ and $g'_{N\Delta} = 0.3$. The theoretical estimate [22,23] of $m^*(0)/m_N \approx 0.7$ is consistent with the data.

In summary, we have reported the theoretical analysis of two contrasting phenomena, the quenching of the GT transition at $q = 0\,\text{fm}^{-1}$ and the enhancement of the pionic response for QES at $q \approx 1.7\,\text{fm}^{-1}$. The GT strength distribution and the latest value for the quenching factor were calculated using the continuum RPA with $\pi + \rho + g'$ interactions including Δ degrees of freedom. In the same structure calculation framework, incorporating the DWIA and two-step calculations, we also calculated the spin-longitudinal cross sections ID_q at different incident energies. By these elaborated and comprehensive calculations we obtained $g'_{NN} = 0.6 \pm 0.1$ and $g'_{N\Delta} = 0.35 \pm 0.16$ at $q = 0\,\text{fm}^{-1}$ and $g'_{NN} = 0.7 \pm 0.1$ and $g'_{N\Delta} = 0.3 \pm 0.1$ at $q \approx 1.7\,\text{fm}^{-1}$. Comparing these results, we conclude that the q dependence of the LM parameters is weak.

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