Planar versus non-planar $\bar{N}N$ annihilation into mesons in the light of $\bar{q}q$ operators and the $1/N_c$ expansion

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It is argued that in the antiproton-proton annihilation into two mesons, $\bar{p}p \to m_1 m_2$, the origin of different restrictive angular momentum selection rules commonly obtained for planar annihilation diagrams $A2$ and for non-planar rearrangement diagrams $R2$ lies in the omission of momentum transfer between an annihilated antiquark-quark pair and a remaining quark or antiquark. If momentum transfer is included, there is no reason for dismissing one type of diagram in favor of another one. Some considerations in the large-N_c limit of QCD equally shed light on the planar and non-planar contributions to the total $\bar{N}N \to m_1 m_2$ annihilation amplitude.

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In antiproton-proton annihilation described by quark-line diagrams, within the context of the constituent quark model, the common wisdom is that planar diagrams $A2$ and non-planar diagrams $R2$ do not contribute equally in a given reaction, say $\bar{p}p \to \pi\pi$ or $\bar{p}p \to \pi\rho$. As depicted in Fig. 1, both the $R2$ and $A2$ diagram can lead to the same final nonstrange two-body configuration despite the different flavor-flux topology. Since the initial $\bar{p}p$ pair contains no strangeness, the annihilation $\bar{p}p \to \bar{K}K$ can proceed *directly* only via the $A2$ diagram. Indirectly, taking into account final-state interactions, the two-pion final state can couple to the $\bar{K}K$ state via the $R2$ topology.

Some authors [1] favor the $R2$ topology owing to the following observation: For $A2$ one gets the same branching ratio [assuming SU(3) symmetry is unbroken] for $\bar{p}p \to \pi\rho$ and $\bar{p}p \to \bar{K}K^*$. The experimentally observed ratio $\mathrm{Br}(\bar{K}K^*)/\mathrm{Br}(\pi\rho)$ is small and favors the $R2$ topology, which produces only $\pi\rho$ but not $\bar{K}K^*$ (where the authors ignore final-state interactions). It is therefore concluded that among the two graphs $R2$ is the dominant one. However, in the review of nucleon-antinucleon annihilation by Dover *et al.* [2], it is noted that this argument ignores a strong mechanism of SU(3) breaking, namely the suppression of $\bar{s}s$ pairs [3]. It seems thus incorrect to attribute the suppression of strange modes $\bar{K}K$, $\bar{K}K^*$, etc. to the dominance of the $R2$ over the $A2$ topology. Moreover, the authors of the review [2] reverse the argument and reason in terms of selection rules to establish instead the preponderance of $A2$ over $R2$ diagrams in $\bar{p}p$ annihilation into two mesons.

The pro $A2$ argument follows from a supposedly restrictive set of selection rules for the $R2$ diagrams, which does not allow for the experimentally observed annihilation of a $\bar{p}p$ pair into two pions (and similar restrictions also hold for the annihilation into two different mesons) with certain total angular momentum $J = l_{\bar{p}p} \pm 1 = \ell_{\pi\pi}$. In more detail, if we concentrate on $\pi\pi$ final states, the $R2$ annihilation diagram with the specific rules of Ref. [2] permits only S waves ($\ell_{\pi\pi} = 0$) when the $\bar{q}_6 q_3$ pair annihilates into an vacuum $^3P_0$ state, whereas $\bar{q}_6 q_3$ annihilation into an $^3S_1$ state with gluon quantum numbers restricts $\ell_{\pi\pi}$ to S and P waves. No final state with $\ell_{\pi\pi} = 2$ or higher is allowed in either case. The planar $A2$ diagram, in contrast, does not exhibit this restriction on the orbital angular momentum $\ell_{\pi\pi}$.

One argument speaks against these restrictions—the specific rules applied by the authors of Ref. [2] stem from the absence of momentum transfer from the annihilated $\bar{q}_6 q_3$ vertex to any of the remaining quarks or antiquarks in Fig. 1, which we claim is not realistic. If, however, momentum transfer is allowed in the non-planar $R2$ diagram [4–7], the selection rules are modified and result into $R2$ transition operators

$$\begin{aligned}\hat{T}_{R2}(^3P_0) = i\mathcal{N}\big[&A_V \boldsymbol{\sigma}\cdot\mathbf{R}' \sinh(C\,\mathbf{R}\cdot\mathbf{R}') \\ &+ B_V \boldsymbol{\sigma}\cdot\mathbf{R}\cosh(C\,\mathbf{R}\cdot\mathbf{R}') \\ &+ C_V(\boldsymbol{\sigma}\cdot\hat{\mathbf{R}}')\,R\cos\theta\cosh(C\,\mathbf{R}\cdot\mathbf{R}')\big] \\ &\times \exp\{A\mathbf{R}'^2 + B\mathbf{R}^2 + D\mathbf{R}^2\cos^2\theta\}\end{aligned} \tag{1}$$

for $\bar{q}q$ annihilation into $^3P_0$ state. The $^3S_1$ transition operator is split into a *longitudinal* component

$$\begin{aligned}\hat{T}_{R2}(^3S_1^L) = i\mathcal{N}\big[&A_L \boldsymbol{\sigma}\cdot\mathbf{R}' \sinh(C\,\mathbf{R}\cdot\mathbf{R}') \\ &+ B_L \boldsymbol{\sigma}\cdot\mathbf{R}\cosh(C\,\mathbf{R}\cdot\mathbf{R}') \\ &+ C_L(\boldsymbol{\sigma}\cdot\hat{\mathbf{R}}')\,R\cos\theta\cosh(C\,\mathbf{R}\cdot\mathbf{R}')\big] \\ &\times \exp\{A\mathbf{R}'^2 + B\mathbf{R}^2 + D\mathbf{R}^2\cos^2\theta\}\end{aligned} \tag{2}$$

and a *transversel* component

$$\begin{aligned}\hat{T}_{R2}(^3S_1^T) = \mathcal{N}\big[&A_T \boldsymbol{\sigma}\cdot\mathbf{R}' \cosh(C\,\mathbf{R}\cdot\mathbf{R}') \\ &+ B_T \boldsymbol{\sigma}\cdot\mathbf{R}\sinh(C\,\mathbf{R}\cdot\mathbf{R}') \\ &+ C_T(\boldsymbol{\sigma}\cdot\hat{\mathbf{R}}')\,R\cos\theta\sinh(C\,\mathbf{R}\cdot\mathbf{R}')\big] \\ &\times \exp\{A\mathbf{R}'^2 + B\mathbf{R}^2 + D\mathbf{R}^2\cos^2\theta\}.\end{aligned} \tag{3}$$

Here, $\mathbf{R}' = \mathbf{R}_{m_1} - \mathbf{R}_{m_2}$ and $\mathbf{R} = \mathbf{R}_{\bar{p}} - \mathbf{R}_p$ are the relative meson and antiproton-proton coordinates in the c.m. system, respectively. The c.m. angle θ is between the relative meson vector $\mathbf{R}'$ and antiproton-proton vector $\mathbf{R}$, $\boldsymbol{\sigma}$ are the usual Pauli matrices, and $\mathcal{N}$ is an overall normalization. The coefficients A_i, B_i, and C_i with $i = V, L$, and T and A, B, C, and D depend on size parameters α (proton) and β (pion) and the boost factor $\gamma = E_{\mathrm{c.m.}}/2mc^2$. They are detailed in Ref. [6].

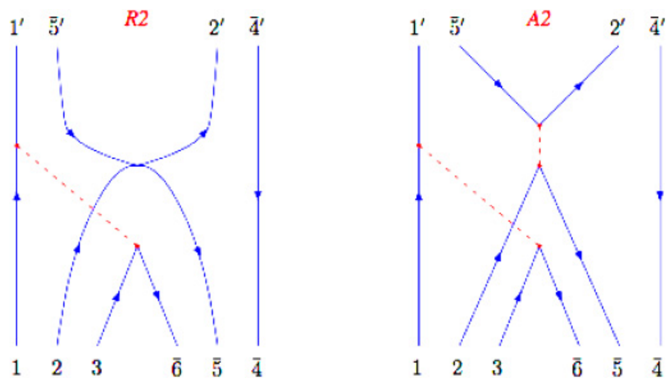

FIG. 1. (Color online) Rearrangement diagram $R2$ (left) and annihilation diagram $A2$ (right). The numbers with bars denote antiquarks; those without bars the quarks. The dashed lines represent the exchange of an effective state with either vacuum $^3P_0$ or gluon $^3S_1$ quantum numbers and with momentum transfer $\delta(\mathbf{p}_{1'} - \mathbf{p}_1 - \mathbf{p}_3 - \mathbf{p}_6)$ for $R2$ and $\delta(\mathbf{p}_{1'} - \mathbf{p}_1 - \mathbf{p}_3 - \mathbf{p}_6)\delta(\mathbf{p}_{2'} + \mathbf{p}_{5'} - \mathbf{p}_2 - \mathbf{p}_5)$ for $A2$.

Note that for $\gamma = 1$ the coefficients correspond to those in Refs. [4,5]. As was already discussed in Ref. [4] and its relativistic extension in Ref. [6], sandwiching the transition operators between the two-pion and the $\bar{N}N$ wave functions, and taking into account the parity properties of the particles, shows that $\hat{T}_{R2}(^3P_0)$ and $\hat{T}_{R2}(^3S_1^L)$ act in $\ell_{\pi\pi}$ even waves whereas $\hat{T}_{R2}(^3S_1^T)$ contributes only to $\ell_{\pi\pi}$ odd waves. Hence, one obtains $\bar{p}p \to \pi\pi$ annihilation for $\ell_{\pi\pi} = 0, 1, 2, 3, \ldots$, that is, in S, P, D, F, ... waves, provided both the $^3P_0$ and $^3S_1$ mechanisms are taken into account. A similar analysis of the $A2$ diagrams yields exactly the same selections rules.

The conclusion is that selection rules do not discriminate between the two topologies, $R2$ and $A2$, both of which describe the annihilation. So, within a constituent quark model in a nonperturbative regime, it is a more consistent approach to consider all annihilation amplitudes equally as they represent different aspects of QCD.

We mention another approach that sheds some light on the question of dominance of either $R2$ or $A2$ topology, namely the treatment of $\bar{p}p$ annihilation in the large-N_c limit of QCD. This has already been done previously [8] and in the following we summarize some salient results that underpin our previous conclusion. Again, one wants to test, if not quantitatively then at least qualitatively, whether a particular annihilation process is dominant in any reaction $\bar{N}N \longrightarrow m_1m_2$.

In his seminal paper [9], 't Hooft suggested that one can generalize QCD from three colors and an SU(3) gauge group to N_c colors and an SU(N_c) gauge group. It was shown that QCD simplifies in the limit of large N_c and g^2N_c ($g^2 = 4\pi\alpha_s$) fixed and that there exists a systematic expansion in powers of $1/N_c$. One crucial property of this limit is that at $N_c = \infty$ the meson and glue states are free, stable, and noninteracting. Other important consequences are that mesons become *pure* $\bar{q}q$ states, since the sea quarks and antiquarks vanish for large N_c. Moreover, Zweig's rule is exact in this limit. These results are restricted to color-singlet glue states (glue balls) and mesons, where a clear distinction between planar and non-planar diagrams is possible. This follows from the Euler index for each Feynman diagram, which determines the $1/N_c$ counting.

The extension to baryons was carried out by Witten, who treated the nucleon-nucleon interaction in large-N_c QCD [10]. The main result that emerged from his qualitative analysis is the N_c order of elastic baryon-baryon and baryon-meson scattering amplitudes. The former is of order N_c and the latter of order N_c^0. Since it is shown in Ref. [10] that all energy contributions to the baryon mass M are of order N_c, the (nonrelativistic) kinetic energy $Mv^2/2$ is of the same order N_c as the baryon-baryon interaction energy. Hence, the scattering cross section for baryon-baryon scattering does not vanish in the large-N_c limit. However, the baryon-meson scattering amplitude, being of order N_c^0 according to Ref. [10], is negligible compared with the baryon mass and to leading order in $1/N_c$ the baryon propagates unperturbed by mesons. The meson mass is of order N_c^0; it follows that its kinetic energy is of the same order as the baryon-meson interaction energy, which is therefore large enough to influence the motion of the meson. For the dynamics of baryon-meson systems, this means the meson is scattered off the baryon but the baryon itself remains "free" for $N_c \to \infty$.

Our aim is to verify whether these results can be applied to antinucleon-nucleon annihilation into two mesons. To begin with, it should be pointed out that for baryons the topological denominations *planar* and *non-planar* are a misnomer in the large-N_c context, as there exists no Euler index according to which the diagrams could be ordered. It is clear from Fig. 1 that it is awkward to generalize $\bar{N}N$ annihilation into two mesons for N_c quarks and N_c antiquarks, for the simple reason that mesons remain $\bar{q}q$ states for large N_c. Nevertheless, assume a naive picture in which one first chooses out of the N_c quarks and N_c antiquarks *one* quark and *one* antiquark that do not end up in the same meson. This gives a factor of N_c^2 and the color content of each of the two final mesons is fixed. Next, a quark is rearranged with an antiquark and the two-meson final state is then formed with the first quark and antiquark choice as in Fig. 1. There is no additional color factor N_c arising from this as the color neutrality of the mesons demands a unique choice in the rearrangement. A gluon exchange between the rearranged quark and antiquark does not alter the N_c counting at this stage (since the colors in the two mesons were previously fixed), however, it introduces an $1/N_c$ factor from the gluon couplings, where each vertex carries a factor $g/\sqrt{N_c}$.

One can now proceed to annihilate *all* the remaining $\bar{q}q$ pairs. In principle, the $\bar{q}q$ pairs need not form a color singlet since a gluon from the annihilation vertex can be exchanged with any other still remaining (anti)quark. Therefore, each annihilation comes along with a factor $N_c - k$, where $k = 2, 3, 4\ldots$ is the number of quarks that have been annihilated, but each also involves a factor $1/N_c$ from the gluon vertices. In our case, we start with $k = 2$ as two of the quarks (and antiquarks) are not annihilated and therefore end up in the final two mesons. An equivalent derivation is to think of the $N_c - k$ quarks and $N_c - k$ antiquarks annihilated into vacuum states, each of which can be treated as a $\bar{q}q$ state. Since any permutation of pairing a quark with an antiquark is possible, this gives a factor $(N_c - k)!$ Using this result and the factor N_c^2 derived in the previous paragraph, one obtains an N_c order for

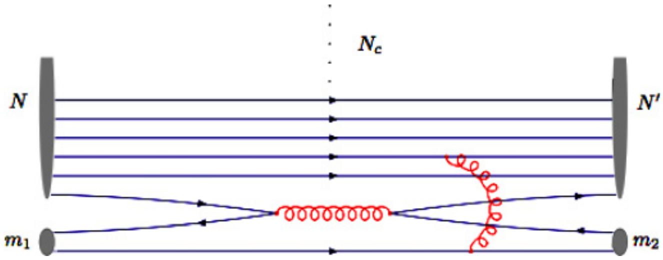

FIG. 2. (Color online) Quark-line diagram of meson-nucleon scattering in the large-N_c limit. The quark-gluon vertices carry the coupling constant $g/\sqrt{N_c}$. In principle, a gluon can be exchanged between the spectator quark in the meson and any other $N_c - 1$ quarks in the nucleon without altering the N_c order of the scattering amplitude.

$\bar{p}p$ annihilation into k mesons: $N_c^2(N_c-k)!/N_c^{N_c-k}$. It follows from this that for $k=2$, the $R2$ diagram generalization to N_c colors is of order

$$N_c^2 \times \frac{1}{N_c^{N_c-2}} \times (N_c-2)! \simeq \frac{N_c!}{N_c^{N_c}} \overset{N_c\to\infty}{\longrightarrow} e^{-N_c}, \quad (4)$$

where for large N_c one approximates $N_c - 2 \simeq N_c - 4 \simeq N_c$. In the second step of Eq. (4), Stirling's approximation $n! \simeq \sqrt{2\pi} n^{n+\frac{1}{2}} \exp(-n)$ has been employed. Repeating this procedure for the N_c generalization of the $A2$ diagram, one finds it is suppressed by a factor $1/N_c^2$ with respect to the previously discussed $R2$ case. The origin of this lies in the lack of quark-antiquark rearrangement, which contributes a nonvanishing N_c factor without a canceling effect from the gluon vertices. Instead, here a $\bar{q}q$ pair has to be created from a gluon. One eventually arrives at an expression for the $1/N_c$ order very similar to Eq. (4) except that the first factor is N_c rather than N_c^2, the denominator in Eq. (4) becomes $N_c^{N_c-1}$ since there is an additional factor from the gluon coupling to the created $\bar{q}q$ pair, and for the same reason the last factor is $(N_c-1)!$ instead of $(N_c-2)!$. Given the effect of the dominating pairwise $\bar{q}q$ annihilations in both types of diagrams, it can be concluded that either way the annihilation into two mesons is exponentially damped, which can also be deduced from the quark model calculations in Ref. [8]. We shall return to this shortly.

We allow ourselves a short digression at this point—in the cross-channel reaction $m_1 N \to m_2 N$ one also encounters a variety of processes contributing in the large-N_c limit to the scattering amplitude. An example of such scattering is illustrated in Fig. 2, where a nucleon quark is annihilated with an antiquark of the initial meson. A $\bar{q}q$ pair is produced in the final state and since color is transferred between the annihilation and the creation vertex, one gets a factor N_c from choosing a quark in the nucleon as well as a factor $1/N_c$ from the gluon couplings. The initial meson quark is still contained in the final meson and acts as a spectator. The scattering is therefore of order N_c^0. If instead of annihilation of a $\bar{q}q$ pair we resort to quark rearrangement, that is, a quark of the nucleon ends up in the final meson while the quark of the initial meson replaces this missing quark in the nucleon, we find the scattering amplitude to be of the same order N_c^0, as discussed in Ref. [10]. This, in turn, differs from the $\bar{N}N \to m_1 m_2$ annihilation, where rearrangement was shown to yield an additional factor N_c in the previous section.

Returning to $\bar{N}N$ annihilation in the large-N_c limit, we recapitulate the results derived by Pirner [8]. In this approach, overlap integrals of meson and $\bar{N}N$ wave functions generalized to N_c quarks and N_c antiquarks are worked out. Including appropriately normalized color wave functions introduces an N_c dependence from the color matrix elements. Furthermore, to obtain the annihilation cross section, one needs to calculate the N_c-dependent phase-space integral. With all these ingredients the cross section for $\bar{N}N$ annihilation into $N_c - k$ mesons and quark-antiquark rearrangement behaves in the large-N_c limit as

$$\sigma_{\text{ann.}} \overset{N_c\to\infty}{\longrightarrow} \exp\left(-4N_c\epsilon_0 r_m^2 m\right) \times \exp\{[1/2 - 3/2\ln(3/2)]N_c\}(\eta N_c)^{2k}, \quad (5)$$

where m is the mass, r_m the size, and ϵ_0 the kinetic energy of the mesons. [Equation (5) holds only for same types of mesons such as $\pi^{\pm}, \pi^0$, etc.] The nonperturbative annihilation or creation probability is $\eta^2 \simeq 0.25$. The cross section for $\bar{N}N$ annihilation into $N_c - k + l$ mesons, where l is the number of created quarks, is for large N_c

$$\sigma_{\text{ann.}} \overset{N_c\to\infty}{\longrightarrow} \exp\left(-4N_c\epsilon_0 r_m^2 m\right) \times \exp\{[1/2 - 3/2\ln(3/2)]N_c\}(\eta N_c)^{2k}\eta^{2l}. \quad (6)$$

Obviously, Eqs. (5) and (6) just differ by a factor η^{2l}. As $\eta < 1$, the pure annihilation diagram without rearrangement is slightly more damped depending on the number l of created quarks. This is in accordance with our previous remarks about the combinatorial factors in large-N_c generalizations of $R2$ and $A2$ diagrams. In the "real world," where $N_c = 3$, one can at the most annihilate $k = 3$ quarks (thus violating Zweig's rule) and create l mesons. The more mesons are produced in the final state, the more the rearrangement diagrams should dominate, however slight this difference is.

In conclusion whether using a quark model calculation or a qualitative analysis of the annihilation in the large-N_c limit of QCD, there is no evidence for dominance of either diagram, be it of the annihilation ($A2$) or rearrangement ($R2$) type. It should be mentioned that similar findings were reported in Refs. [5] and [7], where it was noted that the $R2$ diagrams suffice (although both annihilation mechanisms $^3P_0$ and $^3S_1$ are needed) to reproduce the LEAR data on $\bar{p}p \to \pi^-\pi^+$ differential cross sections and analyzing powers [11]. This contradicts the statements of Dover *et al.* [2], who based their reasoning on SU(3) symmetry breaking and selection rules. Moreover, the present discussion can be generalized to annihilation of any two baryons or hyperons into two or more mesons.

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