Distribution functions for partons in nuclei

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We suggest that a previously conjectured relation between structure functions (SF) for nuclei and nucleons also links distribution functions (df) for partons in a nucleus and in nucleons. The above proposal ensures in principle identical results for SF F_2^A, whether computed with *effective* hadronic or partonic degrees of freedom. In practice there are differences, because of different input for F_2^n. We show that the thus-defined nuclear parton distribution functions (pdf) respect standard sum rules. We observe close agreement between moments of nuclear SF, computed in the hadronic and partonic descriptions. Despite substantial differences in the participating SF, we nevertheless find approximately the same EMC ratios in the two representations, as well as reasonable agreement with data. The apparent correlation between the above deviations is ascribed to a sum rule for F_2^A. We conclude with a discussion of alternative approaches to nuclear pdf.

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I. INTRODUCTION

A large body of data are presently available on cross sections for inclusive lepton scattering from nuclei and subsequently on extracted nuclear structure functions (SF) F_k^A. Standard descriptions of those data used hadron degrees of freedom throughout. A prototype, and possible first example of such an appraoch, is the description of inclusive scattering on a D by Atwood and West. Using the plane wave impulse approximation (PWIA), the authors proved a generalized convolution of the type [1]:

$$F_k^A = f_{N/A}\, F_k^N, \tag{1}$$

with $f_{N/A}$ related to the nucleon momentum distribution in the target A.

Much later, in one of first articles on EMC ratios, Akulinichev *et al.*, related distribution functions (df) for quarks in a nucleon, nucleons in nuclei, and quarks in nuclei through a similar convolution of seemingly different, but obviously related, content [2]:

$$f_{q/A} = f_{N/A}\, f_{q/N}. \tag{2}$$

Both above-mentioned descriptions are effective ones, using specific dynamics in terms of the chosen degrees of freedom.

A host of variations of the above basic ideas have since been proposed for the direct computation of nuclear SF as well as for an indirect analysis of EMC ratios in either the hadron or the parton representation. For instance, Frankfurt and Strikman mention nuclear parton distribution functions (pdf), but do not relate those to the pdf of a nucleon as in Eq. (1) [3]. Other approaches parametrize information on SF for Q_0^2. Borrowing perturbative quantum chromodynamics (pQCD) notions, a parametrized form for F_2^A at a given Q_0^2 is evolved to desired Q^2 values. With knowledge of F_2^D, EMC ratios can be constructed. A comparison with data ultimately determines the parameters in F_2^A for Q_0^2 [4,5].

Next we mention quark models for nuclei, which have been used in direct calculations of df [6]. Finally, there are approaches where the effect of a nuclear medium on a nucleon or a quark is replaced by mean fields [7–9].

In the present article we suggest a simple, nearly natural choice for effective nuclear pdf, which are free of adjustable parameters, satisfy sum rules either exactly or accurately, and produce the SF F_2^A, as computed in a hadronic base. The Q^2 dependence of those nuclear pdf is prescribed by the right-hand side of Eq. (2) and bears as yet no relation to pQCD and evolution from a given scale Q_0^2. In the conclusion we compare some of the above-mentioned alternative proposals with our choice. We also emphasize the need for a QCD foundation of effective descriptions, simultaneously pointing out their simplicity and accuracy of the latter.

II. A FEW ESSENTAILS

We start with the cross section per nucleon for the scattering of unpolarized electrons with energy E over an angle θ:

$$\frac{d^2\sigma^A(E;\theta,\nu)}{d\Omega\, d\nu} = \sigma_M(E;\theta,\nu)\left[\frac{2xM}{Q^2}F_2^A(x,Q^2) + \frac{2}{M}F_1^A(x,Q^2)\tan^2(\theta/2)\right], \tag{3}$$

where σ_M is the Mott cross section and $F_{1,2}^A(x,Q^2)$ are nuclear SFs per nucleon. Those depend on the squared four-momentum transfer $q^2 = -Q^2 = -(|\mathbf{q}|^2 - \nu^2)$ and on the Bjorken variable $0 \leqslant x = Q^2/2M\nu \leqslant A$ in terms of the nucleon mass M.

Next we make explicit the specific relation [Eq. (1)] between nuclear and nucleonic SF [10]

$$F_k^A(x,Q^2) = \int_x^A \frac{dz}{z^{2-k}} f^{PN,A}(z,Q^2) F_k^{\langle N\rangle}\left(\frac{x}{z},Q^2\right) \tag{4}$$

$$F_k^{\langle N\rangle} = \left(ZF_k^p + NF_k^n\right)/2A = \left(1 - \frac{\delta N}{A}\right)F_k^p + \left(1 + \frac{\delta N}{A}\right)F_k^n, \tag{5}$$

with $F_k^{\langle N\rangle}$ a nucleon SF, obtained by weighting $F_k^{p,n}$ with Z, N. $\delta N/A$ denotes the relative neutron excess. We already mentioned that Eq. (4) can be proved in the PWIA.

Our approach draws on the Gersch-Rodriguez-Smith (GRS) theory for inclusive scattering of nonrelativistic projectiles [11], in which case the linking $f^{PN,A}$ is the SF of a fictitious nucleus composed of, in principle, fully interacting pointlike nucleons. Figures 1–5 in Ref. [12] show shapes of $f^{PN,A}$ for various targets and a range of Q^2. Eq. (4) formulates a nonperturbative theory with on-mass shell-nucleon SFs F_k^N and a covariant generalization of the GRS theory for f [13]. All are in terms of hadronic degrees of freedom and their dynamics.

Equation (4), which is postulated to hold for finite $Q^2 > Q_0^2$ [10], misses contributions coming from virtual mesons [14] and (anti-)screening effects [15]. Those are negligible for $x \gtrsim 0.2$, to which range we limit our discussion. An extensive body of data in the ranges $x \gtrsim 0.2$; $Q^2 \gtrsim 2.5\,\mathrm{GeV}^2$ appears accounted for by the relation in Eq. (4) [16–20].

We shall need below the separation of the nucleon SF $F_k^N = F_k^{N,NE} + F_k^{N,NI}$ into nucleon elastic and inelastic components, which correspond to absorption processes of a virtual photon on a $N, \gamma^* + N \to N$, (NE), or $\gamma^* + N \to$ (hadrons,partons) (NI). Elastic components for a N are proportional to the standard combinations of squared electromagnetic form factors $G_{E,M}^N(Q^2)$ and vanish, unless $x = 1$. With $[\tilde{G}^{\langle N\rangle}]^2 = [Z(G^p)^2 + N(G^n)^2]/A$, one has the following:

$$F_1^{N,NE}(x, Q^2) = \frac{1}{2}\delta(1-x)\big[\tilde{G}_M^{\langle N\rangle}(Q^2)\big]^2 \tag{6}$$

$$F_2^{N,NE}(x, Q^2) = \delta(1-x) \times \frac{\big[\tilde{G}_E^{\langle N\rangle}(Q^2)\big]^2 + \eta\big[\tilde{G}_M^{\langle N\rangle}(Q^2)\big]^2}{1+\eta}. \tag{7}$$

The corresponding nuclear NE (QE) components from Eq. (4) are as follows:

$$F_1^{A,NE}(x, Q^2) = \frac{f^{PN,A}(x, Q^2)}{2}\big[\tilde{G}_M^{\langle N\rangle}(Q^2)\big]^2\big] \tag{8}$$

$$F_2^{A,NE}(x, Q^2) = x f^{PN,A}(x, Q^2) \times \frac{\big[\tilde{G}_E^{\langle N\rangle}(Q^2)\big]^2 + \eta\big[\tilde{G}_M^{\langle N\rangle}(Q^2)\big]^2}{1+\eta}. \tag{9}$$

In particular for the lightest nuclei the normalized $f^{PN,A}$ peak sharply around $x \approx 1$, and the same holds for the above QE components $F_k^{A,NE}(x, Q^2)$. The above summarizes elements of a hadronic description of nuclear SF: we now turn to a partonic representation.

III. A SIMPLE CHOICE FOR NUCLEAR PARTON DISTRIBUTION FUNCTIONS

We start with the leading-order twist contributions to the dominant NI components of nucleon SF for finite Q^2. For simplicity we occasionally omit one or both arguments x, Q^2. Decomposing quark df $q = q_v + \bar{q}$ into valence and sea quarks parts, one has upon neglect of heavy quark contributions the following:

$$\begin{aligned} F_2^p &= \frac{x}{9}(4u_v + d_v + 8\bar{u} + 2\bar{d} + 2s) \\ F_2^n &= \frac{x}{9}(u_v + 4d_v + 2\bar{u} + 8\bar{d} + 2s). \end{aligned} \tag{10}$$

Similarly for the "average" nucleon, defined as the Z, N weighted p, n we obtain:

$$F_2^{\langle N\rangle} \equiv x\sum_i a_i q_i = \frac{5x}{18}\bigg[u_v + d_v + 2\bar{u} + 2\bar{d} + \frac{4}{5}s - \frac{3\delta N}{5A}(u_v - d_v + 2\bar{u} - 2\bar{d})\bigg], \tag{11}$$

Next we turn to effective nuclear pdf, which ought to reproduce nuclear SF F_k^A, just as proton pdf do for F_k^p, Eq. (10). That demand is insufficient for a unique determination, and we exploit apparent freedom. First we choose $F_2^A(q^A)$ to be the same combination of df of partons in a nucleus, as the above $F_2^{\langle N\rangle}$, Eq. (11) for a nucleus

$$F_2^A = x\sum_i a_i q_i^A = \frac{5x}{18}\bigg[u_v^A + d_v^A + 2\bar{u}^A + 2\bar{d}^A + \frac{4}{5}s^A - \frac{3\delta N}{5A}\big(u_v^A - d_v^A + 2\bar{u}^A - 2\bar{d}^A\big)\bigg]. \tag{12}$$

Upon substitution into Eq. (4), and using Eq. (11), one finds

$$F_2^A = x\sum_i a_i\big(f^{PN,A} q_i^A\big). \tag{13}$$

Comparison with Eq. (12) does still not define a unique expressions for each individual pdf q_i^A. Guided by Eq. (4), we make the following second choice, which does not mix valence nor sea quarks

$$\begin{aligned} xq_i^A(x, Q^2) &\equiv \int_x^A dz f^{PN,A}(z, Q^2)\left(\frac{x}{z}\right) q_i\left(\frac{x}{z}, Q^2\right) \\ x\bar{q}_i^A(x, Q^2) &\equiv \int_x^A dz f^{PN,A}(z, Q^2)\left(\frac{x}{z}\right) \bar{q}_i\left(\frac{x}{z}, Q^2\right) \\ xg^A(x, Q^2) &\equiv \int_x^A dz f^{PN,A}(z, Q^2)\left(\frac{x}{z}\right) g\left(\frac{x}{z}, Q^2\right). \end{aligned} \tag{14}$$

Equations (14) in the PWIA with one f for all partons had already be suggested before Ref. [21]. We are aware that the above equations mix partonic notions with f, which is computed in a hadronic representation.

In view of the meager experimental information on nonvalence parton distributions in nuclei, we shall also investigate changes when nonvalence df are not affected by the nuclear medium (cf. [4,5]), thus

$$\bar{q}^A \equiv \bar{q}; \quad s^A = \bar{s}^A = s; \quad g^A = g^N, \tag{15}$$

Equations (12) and (14) manifestly produce the same F_2^A in the parton and the hadronic representation, provided one uses exactly the same input $f^{PN,A}$ and $F_2^{\langle N\rangle}$ in both.

This is actually not the case, in particular not for F_2^n in $F_2^{\langle N\rangle}$. In the absence of direct information, the Coordinated Theoretical-Experimental Project on QCD (CTEQ)

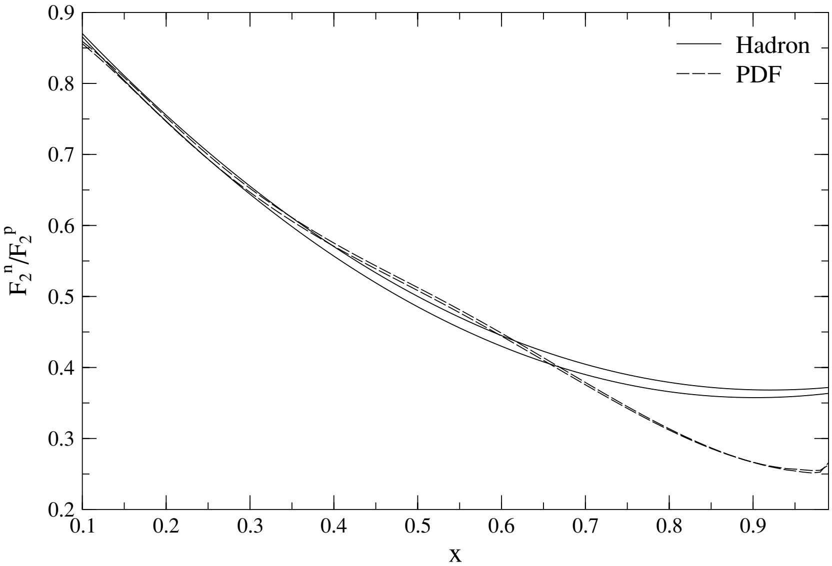

FIG. 1. The ratio $F_2^n(x, Q^2)/F_2^p(x, Q^2)$ in the hadronic (drawn lines) and the pdf representation (dashes); upper and lower curves correspond to $Q^2 = 3.5\,\mathrm{GeV}^2$ and $Q^2 = 5.0\,\mathrm{GeV}^2$.

parametrizations exploit data on F_2^D, using the "primitive" estimate $F_2^n(x) = 2F_2^D(x) - F_2^p(x)$. However, that approximation increasingly deteriorates for $x \gtrsim (0.25 - 0.30)$ and its use for larger x leads to misfits with data.

As to the options in the version CTEQ6 [22] we selected the one, with F_2^p, closest to the Arneodo parametrization of resonance-averaged data [23]. This appears possible for $x \lesssim$ 0.6–0.7. SU_3 provides F_2^n in terms of $q_i, \bar{q}_i$.

In contrast, in the hadronic approach to F_2^n, one stays as close as possible to data, which contain F_2^p and additional SFs. For the former there are available parametrizations of actual data [24]. Unfortunately, the range $Q^2 \gtrsim 3.5\,\mathrm{GeV}^2$ of our interest borders the limits of validity of the above parametrizations, and one has no choice but to use the above-mentioned Arneodo representation of F_2^p [23]. As to F_2^n, it has been obtained for *all* x by an indirect extraction method, which also requires the currently accepted FF of the p [25] (see for instance Ref. [26]). Figure 1 for $C = F_2^n/F_2^p$ in the two representations, $Q^2 = 3.5, 5.0\,\mathrm{GeV}^2$, displays the above-mentioned differences, which increase with x. Still lacking accurate experimental information on separate proton form factors causes uncertainties, whereas absence of data on G_E^n for even medium Q^2 forces one to use extrapolations of low Q^2 parametrizations [25].

A second subtle difference between the representations is the validity of Eq. (4) and (14). The former one is explicitly limited to the nucleonic part of F_2^A, i.e., roughly for $x \gtrsim$ 0.2, whereas the latter is conceivably valid out to lower x (cf. Ref. [27]).

Despite the fact that our nuclear pdf are effective ones that are as yet not related to pQCD, pdf are constrained by sum rules. We first check those, that are directly related to the normalization of $f^{PN,A}$:

(i) For any linear combination C of df for valence quarks one has the following:

$$\int_0^A dx C^A = \int_0^1 dx C^N \tag{16}$$

$$\int_0^A dx\big(u_v^A + d_v^A\big) = \int_0^1 dx(u_v + d_v) = 3 \tag{17}$$

$$\int_0^A dx \frac{2u_v^A - d_v^A}{3} = \int_0^1 dx \frac{2u_v - d_v}{3} = 1. \tag{18}$$

Equations (17) and (18) are for $C^A = u_v^A + d_v^A = u^A - \bar{u}^A + d^A - \bar{d}^A$, respectively $C^A = (2u_v^A - d_v^A)/3$. Those express the conservation (per nucleon) of the number of valence quarks in nuclei (baryon number) and of charge.

(ii) For any linear combination $C = \alpha_u u_v + \alpha_d d_v$ of nuclear pdf one has the following:

$$\begin{aligned} C^A(0, Q^2) &= C^N(0, Q^2) \\ C^A(x_0, Q^2) &\approx C^N(x_0, Q^2). \end{aligned} \tag{19}$$

By construction, Eq. (4) holds only for the contribution of partons in nucleons (i.e. not including to those in virtual mesons, etc.), and somehow the same is the case for Eq. (14). For those parts one proves the above equality for $x = 0$, whereas for $0.18 \gtrsim x_0 \gtrsim 0$, Eq. (19) is an accurate approximation [26].

(iii) For any combination $xC(x) = x\sum_i \alpha_i q_i(x,)$ or $x\sum_i \alpha_i \bar{q}_i(x)$

$$\int_0^A dx x C^A(x) = \int_0^A dz z f^{PN,A}(z) \int_0^1 dt t C^N(t) \tag{20}$$

$$\begin{aligned} &\int_0^A dx x\big(u_v^A + d_v^A + 2\bar{u}^A + 2\bar{d}^A + 2s^A + g^A\big)_x \\ &\quad = \int_0^A dz z f^{PN,A}(z) \int_0^1 dt t(u_v + d_v + 2\bar{u} \\ &\qquad + 2\bar{d} + 2s + g)_t \\ &\quad \approx \int_0^1 dt t(u_v + d_v + 2\bar{u} + 2\bar{d} + 2s + g)_t. \end{aligned} \tag{21}$$

Equation (21) is a special case, related to the momentum sum rule, which does not exactly carry over to the nuclear case. However, the peaking of the normalized $f^{PN,A}$ causes the z integral in Eq. (20) to be very close to 1. In the case of the nuclear momentum sum rule Eq. (21), the deviations are really minute. We emphasize that those are largely the result of imperfect calculations of $f^{PN,A}$ (see Ref. [28] for an entirely different way to mend the momentum sumrule violation in the PWIA).

The incompleteness of the above-mentioned Eq. (4) for $x \lesssim 0.2$ does not constitute a problem in practice. For one, the conservation of the number of valence quarks per nucleon is guaranteed by the normalization of $f^{PN,A}$, i.e., by unitarity. As the momentum sum rule Eq. (21) illustrates, more is required than unitarity, in case the fact that the widths $\Delta x(Q^2)$ of the peaked f are appreciably less than the support of x. Comparing the two sides of the expressions in Eq. (21) for four targets and $Q^2 = 3.5, 5.0\,\mathrm{GeV}^2$, we find differences of no more than $\approx 1\%$.

The above sum rules involve pdf in a nucleus and in a nucleon. Other sum rules hold for moments of nuclear SF (or, equivalently, for Mellin transforms of the latter):

$$\mathcal{M}^A(n, Q^2) = \int dx x^{n-2} F_2^A(x, Q^2),$$

and one may compare numerical results in the hadronic representation (see, for instance, Refs. [17,29,30]) and in the partonic one. For instance, Eqs. (12) and (21) hint that there exists a sum rule for $M_{-1}^A [= \mathcal{M}^A(1)] = \sum a_i q_i^A$.

The contributions of valence quarks to M_{-1}^A is 5/6, but all other parts diverge (see, for instance, Refs. [31,32]). Those divergences cancel in differences of any pair of those ratios. Thus from Eqs. (12), (15) (for simplicity we disregard $\delta N/A$ corrections) we have the following:

$$\begin{aligned} M_{-1}^A - M_{-1}^{A'} &= \int_0^A \frac{dx}{x} F_2^{A;NI} - \int_0^{A'} \frac{dx}{x} F_2^{A';NI} \\ &= \frac{5}{18}\left[\int_0^A dx\left(u_v^A + d_v^A\right)\right. \\ &\quad \left. - \int_0^{A'} dx\left(u_v^{A'} + d_v^{A'}\right)\right] \approx 0. \end{aligned} \tag{22}$$

For all $A' \neq A$ (including $A' = 1$), the upper integration limits in Eq. (22) differ. Again, for the above-mentioned reason, one may neglect the contributions to the integrals in Eq. (22) for $x \lesssim 0.20$ and $x \gtrsim 0.95$. Hence effectively there is an approximate common upper limit $x_U \approx 1 \ll A, A'$, beyond which the difference $(F_2^A - F_2^{A'})$ is negligible and the same holds for $x_L \lesssim 0.18$. For the pair D, Fe and the chosen three Q^2 values, the above difference of the integrals in the hadron and pdf representations is ≈ -0.03. Special cases are isodoublets $A = A'$, for which Eq. (22) is a generalized Gottfried sum. For a recent discussion for $A = 3$ we refer to Ref. [33].

Of particular interest is the zeroth moment of nuclear SF, which is related to the momentum sum rule. Using Eq. (4)

$$\begin{aligned} M_0^A = \mathcal{M}^A(2) &= \int_0^A dx F_2^{A;NI}(x) \\ &= \int_0^A dx \int_x^A dz f^{PN,A}(z) F_2^{\langle N\rangle}(x/z) \\ &= M_0^{\langle N\rangle} \int_0^A dz z f^{PN,A}(z) \approx M_0^{\langle N\rangle}, \end{aligned} \tag{23}$$

which result we could check numerically.

From Eqs. (12), (15), and (17) one shows that the same moment of isosinglet NI parts in the pdf representation reads as follows:

$$\begin{aligned} M_0^A &= \int_0^A dx F_2^{A;NI}(x) \\ &= \frac{5}{18}\int_0^A dx x\left[u_v^A + d_v^A + 2(\bar{u}^A + \bar{d}^A) + \frac{4}{5}s^A\right]_x \\ &\approx \frac{5}{18}\left[1 - \int_0^A dx x\left(\frac{6}{5}s^A + g^A\right)_x\right] \qquad (24) \\ &\approx \frac{5}{18}\left\{1 - \int_0^1 dx x\left[\frac{6}{5}s(x) + g(x)\right]\right\} \\ &= \int_0^1 dx F_2^{\langle N\rangle}(x). \qquad (25) \end{aligned}$$

NE parts are small for the considered Q^2. When included, the normalization of $f^{PN,A}$ guarantees $M_0^{A,NE} = M_0^{N,NE}$, i.e., NE parts of M_0^A are also A independent.

In Figs. 2(a)–2(e) we show differences of valence, sea quark, and gluon distributions functions in a nucleus and for the p. We chose five targets and display results only for $Q^2 = 5\ \mathrm{GeV}^2$, because for the Q^2 range considered there is hardly any Q^2 dependence. Differences increase with increasing mass number and change sign at roughly $x = 0.2$ and 0.8.

To the extent that nuclear sea and gluon distributions are close to the nucleonic ones, Eq. (25) shows that in the pdf representation, M_0^A is practically A independent. Emphasis is on the standard $\approx$50% reduction of the nucleon valence contributions 5/18 because of gluons (see, for instance, Ref. [31]), which result carries over to nuclear df.

Present data are for $x \gtrsim x_m \approx 0.3$ for the lowest Q^2 [for which Eq. (23) is not accurate] and $x \gtrsim x_m \approx (0.5–0.6)$ for medium Q^2. For both, the missing information for $x \lesssim 0.5$ contains the major contribution to M_0^A. It is thus not feasible to directly verify the hadronic result [Eq. (23)], which requires F_2^A to be known over the entire relevant x range.

We thus turn to an indirect method and exploit knowledge on F_k^A for all but the smallest x, and the smoothness of the same in the region $x \gtrsim x_m$ [34,35]. One then interpolates F_2^A in the intermediate region, where data are missing [26] and subsequently calculates the lowest moments M_0^A. Although the small x region contributes the major part to the integrals [Eq. (23)], the moments from the extrapolation procedure turn out to be surprisingly close to the ones computed above. Forthcoming data from JLab experiment E03–103 [36] will enable a sharpening of the above method.

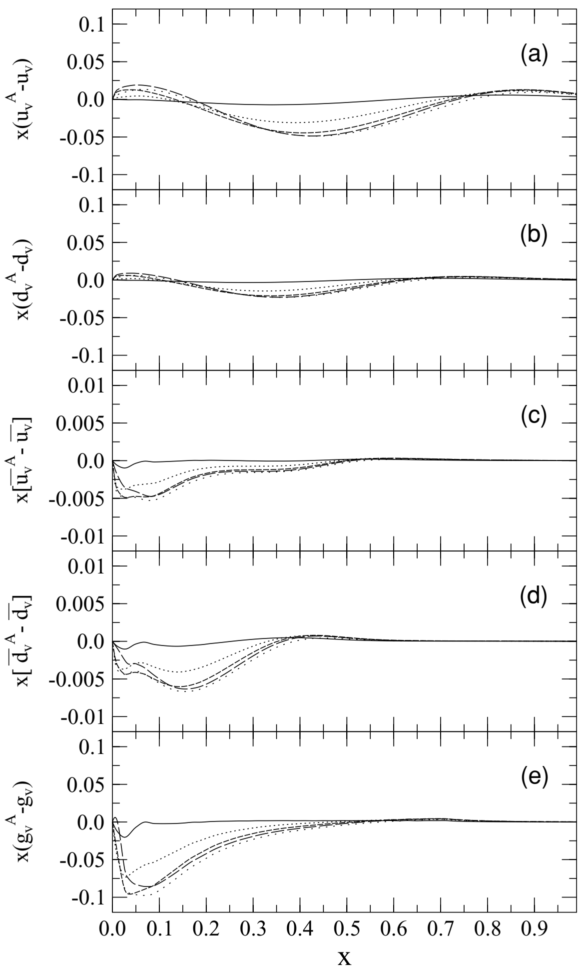

FIG. 2. Differences $x(q^A - q)$ for D (drawn line), ^{4}He (dots), C (spaced dots), Fe (short dashes), and Au (long dashes) and for a p; $Q^2 = 5\,\mathrm{GeV}^2$. (a) $q = u_v$; (b) $q = d_v$; (c) $q = \bar{u}$; (d) $q = \bar{d}$; and (e) $q = g$.

Table I displays the zeroth moments $M_0^A, A = \{D, \mathrm{He}, C, \mathrm{Fe}\}$, computed in the hadronic and the partonic representations [Eqs. (23), (24), and (25), respectively]. Comparison of columns 2 and 3 clearly shows that the shape $f^{PN,A}$ effectively cuts the upper limit of long-range integrals at $x_U \approx 1.0$.

The above moments $M_0^A(Q^2)$ have a nonsmooth, weak A dependence. Going from D to Fe, those moments are 3–4%, respectively 8% smaller than the same for the averaged nucleon $M_0^{\langle N\rangle}(Q^2)$: D and He clearly do not follow the smooth behavior of all other nuclei. One also notices that M_0^A are weakly descending functions of Q^2: The SF $f^{PN,A}$ reach their asymptotic limit only for large $Q^2 \approx 35\,\mathrm{GeV}^2$ and so will F^A and their moments.

It is of interest to compare the entries of Table I in their dependence on A and Q^2 with the QCD description of the lowest moment for a p in the Bjorken limit in terms of the number of favors N_f [31]

$$M_0^p = \frac{5N_f}{6(3N_f + 16)} = 0.1471,$$

TABLE I. Zeroth moments of F_2^A in the hadron (h) and a parton (p) representation, with upper limits A approximated by 1 and 2. The set of three columns correspond to $Q^2 = 3.5$ GeV, $Q^2 = 5.0$ GeV, and $Q^2 = 10.0\,\mathrm{GeV}^2$.

Target		$\int_0^1 dx\, F_2^A(x, Q^2)$	$\int_0^2 dx\, F_2^A(x, Q^2)$
D	h	0.1492, 0.1484, 0.1409	0.1493, 0.1484, 0.1409
	p	0.1505, 0.1470, 0.1413	0.1506, 0.1470, 0.1413
^{4}He	h	0.1455, 0.1450, 0.1378	0.1459, 0.1453, 0.1379
	p	0.1464, 0.1433, 0.1378	0.1467, 0.1435, 0.1379
C	h	0.1434, 0.1434, 0.1370	0.1440, 0.1439, 0.1372
	p	0.1430, 0.1403, 0.1353	0.1434, 0.1408, 0.1383
Fe	h	0.1402, 0.1403, 0.1338	0.1406, 0.1406, 0.1339
	p	0.1447, 0.1403, 0.1353	0.1433, 0.1405, 0.1351
$\langle N\rangle$	h	0.1554, 0.1510, 0.1438	
	p	0.1510, 0.1475, 0.1420	

where the numerical value corresponds to $N_f = 6$. The nuclear medium apparently mildly quenches the free p moment to a measure increasing with A: For the averaged N and the D the quenching seems minimal.

The approximate A independence of the above zeroth moments of any pair of SF implies the following:

$$M_0^A - M_0^{A'} = \int_0^A dx\, F_2^{A;NI} - \int_0^{A'} dx\, F_2^{A';NI} \approx \int_{x_0}^{x_U} dx\big(F_2^{A;NI} - F_2^{A';NI}\big) \approx 0. \quad (26)$$

The vanishing of the above differences is attributed to similar effective x ranges $(x_0, x_U) \approx (0.15, 0.85)$, which replace actual unequal supports [37]. Equation (26) implies that in the above common interval, the difference of two SF has to change sign at least once or in different terms: the generalized EMC ratios $\mu^{A,A'}(x, Q^2) = F_2^A(x, Q^2)/F_2^{A'}(x, Q^2)$ pass the value 1 in the above x interval, as is indeed observed for all A, Q^2 (see Ref. [13] for a discussion in an entirely different context) (Fig. 3).

The simplest cause for approximate A independence of M_0^A would be the same for F_2^A, but that appears not to be the case. In the dominant classical region $x \lesssim 0.90$, differences $F_2^A - F_2^{A'}$ in both representations grow with x beyond ≈ 0.18 and may become as large as 50–60 %, which is far larger than the spread in M_0^A (cf. Table I). We return to this point below.

IV. EMC RATIOS IN THE PARTON DISTRIBUTION FUNCTIONS REPRESENTATION

We have computed $F_2^{A,NI}$ in the pdf representations, using Eqs. (12), (14), and (15). To those we added the NE components [Eq. (9)], which are only relevant for $x \gtrsim 0.95$. The total EMC ratios F_2^A/F_2^D are then compared with recently determined counterparts in the hadron representation [13]. For the range $0.2 \lesssim x \lesssim 1.2$ and $Q^2 = 3.5\,\mathrm{GeV}^2$ and

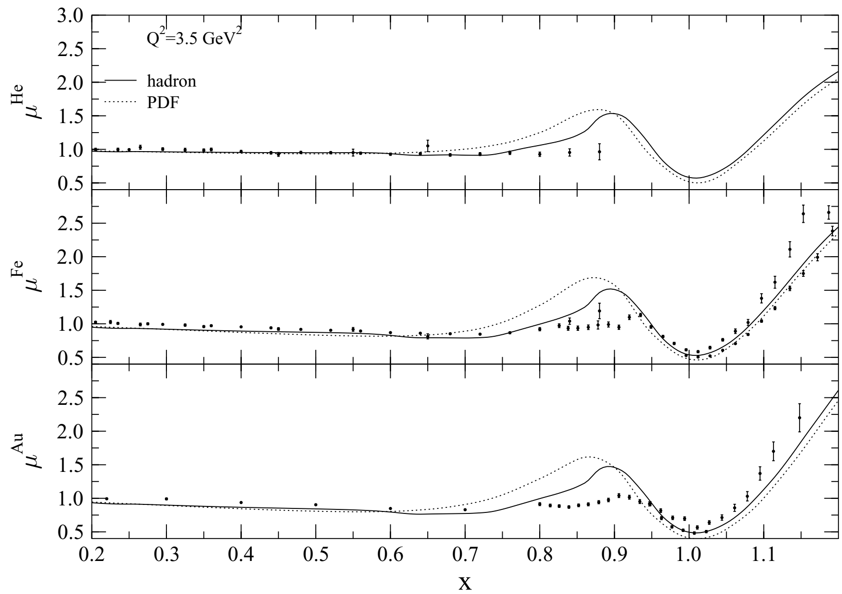

FIG. 3. Hadronic and pdf representations of EMC ratios μ^A (drawn and dotted lines), $A =$ He, C, and Fe for $0.2 \lesssim x \lesssim 1, 2$, $Q^2 = 3.5\,\mathrm{GeV}^2$. Data are from Refs. [38–40].

$Q^2 = 5.0\,\mathrm{GeV}^2$ we show in Figs. 4(a) and (b) numerical results as well as available data for He, Fe, and Au [38–40].

Up to $x \lesssim 0.65$ there is close agreement among the computed ratios. Beyond that point growing deviations set in, with pdf values in excess of the same in the hadron representation. That situation is reversed for $x \gtrsim 0.90$. Both representations overestimate the relative maxima in μ^A around $x = 0.9$, but over the entire x range, the hadronic results are closer to the data than those for the pdf. We attribute this to inferior F_2^n pdf input for larger x, which Eq. (4) shows to propagate into F_2^A. Also of interest are the slightly lower pdf results for $x \lesssim 0.65$ and the much higher ones beyond and up to $x \approx 0.9$.

The apparent insensitivity of EMC ratios μ^A to the representation, despite the large differences in the participating F_2^A, combined with points (a) and (b) in Sec. III, suggests the following: Irrespective of the cause of the dependence of nuclear SF on A and/or representation, the approximate independence of the zeroth moments [Eq. (23)] on both forces the differences in F_2^A in the regions $x \lesssim 0.18$ and $0.18 \lesssim x \lesssim 0.90$ to be nearly balanced. Consequently, if EMC ratios in one area have some order in A, that ordering must be inverted in the second one. It almost seems that deviations of EMC ratios from 1 can be generated by an integral-preserving, affine transformation with $x \approx 0.18, 0, 90$ as fixed points, having a characteristic A dependence, in particular for $A \lesssim 12$ (see also Ref. [41]).

Finally, out of sheer curiosity we followed pdf predictions down to $x = 10^{-5}$. Results for F_2^A hardly change from their A-independent values around $x = 0.15$, causing EMC ratios to

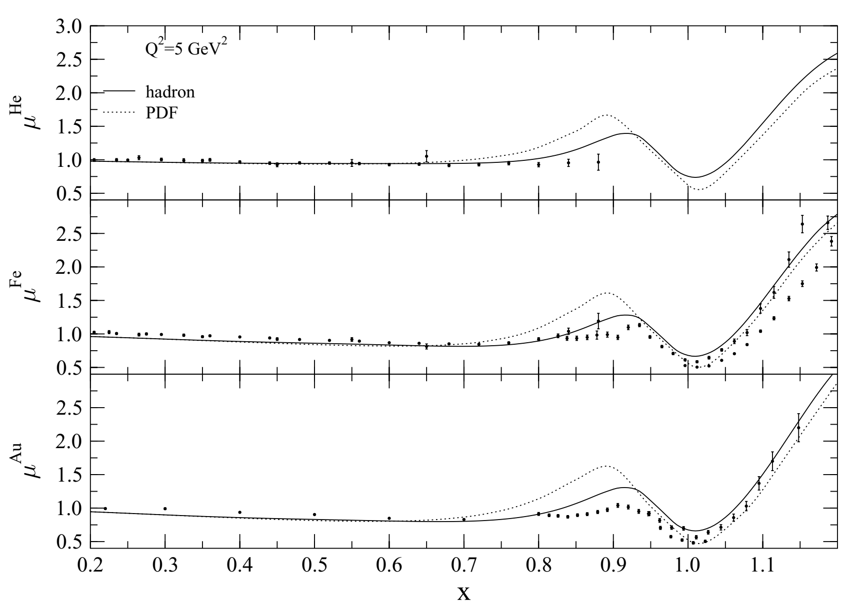

FIG. 4. Same as defined in the legend to Fig. 3 but for $Q^2 = 5\,\mathrm{GeV}^2$.

stay close to 1 for decreasing x. The above is actually observed down to $x \approx 10^{-3}$. Only for the smallest x do screening effects deplete df and cause μ^A to slowly reach values $\approx$0.6–0.7.

V. COMPARISON AND CONCLUSIONS

In this note we defined df for partons in a nucleus, suggesting a relation between nucleonic and nuclear hadron SF without adjustable parameters. We showed that, either exactly or very closely, those nuclear pdf respect basic sum rules. Observables such as F_2^A may be expressed in either representation and by construction produce in principle the same F_2^A for identical input $F_2^{p,n}$.

In practice this is not the case for F_2^n. The pdf choice rests on the "primitive" approximation $F_2^n = 2F_2^D - F_2^p$, which approximately holds for $x \lesssim 0.3$, but deteriorates with increasing x, whereas a well-founded extraction method has been used in the hadronic representation. Consequently, EMC ratios, computed in both versions, practically coincide for $x \lesssim 0.65$ but deviate for larger x. Those reflect deviations of F_2^n from the same in better founded extracted function: The hadronic representation of EMC ratios produces the better fits to the data. The above, and the fact that deviations of EMC ratios from 1 appear in distinct areas to be balanced for all A, seems linked to the lowest moment M_0^A of F_2^A: Those are for any Q^2 practically independent of A.

We have computed pdf down to the smallest x, which is the region were the above criticism does not hold. However, there Eq. (14) misses primarily (anti-)screening effects. Nevertheless, down to $x \approx 10^{-3}$ the agreement with data persists, but the pdf results cannot describe antiscreening depletion of df in μ^A for the smallest measured x.

Our almost natural choice of distribution functions of partons in a nucleus is clearly one out of many possible ones, and we mention a few suggested alternatives. For instance, Eskola *et al.*, address participating nuclear SF in EMC ratios, which are generated from parametrized input for a reference Q_0^2. Those are subsequently evolved to the desired Q^2 [4]. Parameters are constrained, for instance, by fixing the average position of minima of EMC ratios and are ultimately determined by data.

Next we mention Kumano and coworkers, who, despite different x support for F^A and F^N, assume a linear relation between df for partons in a nucleus and in a nucleon [5]. The species-dependent, relating weight functions $w_i(x, Z, A)$ contain parameters for a scale Q_0^2 and the resulting df of partons in a nucleus are again evolved to any Q^2, ultimately producing parametrized nuclear SF and EMC ratios. A large number of adjustable parameters leads to fits from the smallest x up to $x \leqslant 1$. With the connecting weight function having no meaning beyond $x = 1$, the interesting region $x \geqslant 1$ is out of reach in that approach. A serious drawback of the method may be the lack of physical meaning of the weight functions w_i and its parameters.

Finally we discuss approaches, where df of partons in a nucleus are those for a nucleon bound in scalar and vector mean fields [7,8], which couple to quarks in a nucleon [42]. Offhand, the above and our phenomenological approach seem to have little in common. This is actually not the case, and it is instructive to trace the connection. We recall that the original proposal Eq. (4) was inspired by a model where valence quarks in a nucleus cluster in bags [10]. Total interactions between quarks in two different bags have there been replaced by phenomenological *NN* forces, acting on the centers of those bags, thus replacing quark dynamics by those for hadrons. The recalled treatment of a nucleus partially reintroduces quark degrees of freedom and are in Eq. (13) seen to mix the latter with hadronic degrees of freedom through the SF $f^{PN,A}$.

In contrast, in the hybrid meson-quark coupling model the interactions of a single quark in a given nucleon with all quarks in the remaining $A - 1$ bags are replaced by mean fields. As expected from its intermediate position, it ought to be possible to sum in that model the above meson-quark interactions over the valence quarks in a bag and to construct a *NN* interaction with many-body components, mediated by the same mean boson fields. This has recently been shown to be possible [43].

The above model has been applied to nuclear matter [7,8] and it is clearly of interest to see applications of finite nuclei. A first example is a treatment of ^{3}He in the PWIA [9] and one should look forward to results for higher A.

We conclude with the observation that our derivation of nuclear pdf differs in principle from the work of, for instance, Eskola *et al.*, and Kumano *et al.* In ours the Q^2 dependence is dictated by the same of $f^{PN,A}$ and pdf of N, whereas in the other method it obtains from evolution from some scale Q_0^2. Eqs. (14) underly the same conjecture as Eq. (4) and both ultimately require a proof, based on QCD. At the same time, we foresee that effective theories will remain simple and accurate tools to describe reality. Numerous examples have proven the above in the past: Classical nuclear physics exploits effective *NN* interactions and only rarely the more fundamental boson-exchange potentials; the spectroscopy and theory of gases or fluids of diatomic molecules uses an accurate effective interaction between the centers of the atoms, forgoing one computed from electron-electron interactions, and so on.

After completion of this manuscript we found two publications, where Eqs. (14) and (4) are proved in the PWIA [44,45]. We shall elsewhere provide the generalization with inclusion of nuclear FSI, which is implied in the above equations [46].

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