Minimal electromagnetic and mass difference corrections in πN scattering

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Coulomb and mass difference corrections to low-energy pion-nucleon scattering are calculated and compared with previous work including potential models, dispersion relation methods, and chiral perturbation theory calculations. Particular attention is paid to their role in testing isospin breaking.

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I. INTRODUCTION

Electromagnetic corrections are a key element in understanding low-energy pion-nucleon scattering and, in turn, the fundamental elements of chiral symmetry breaking and the pion-nucleon coupling constant. They are also crucial in the important role of low-energy pion-nucleon scattering as a testing ground for isospin breaking.

Tests of isospin breaking have been reported [1–3] that used the extracted charged pion scattering amplitudes to predict charge-exchange amplitudes. A breaking of the order of 8% in the amplitude was found. Recently, Gridnev *et al*. [4] also reported a similar breaking in the same analysis, although they found a much smaller breaking (2.4% and 1.5%) in different reactions. They concluded that the breaking is much smaller than that found in Refs. [1–3].

Piekarewicz [5] has suggested a possible explanation for the isospin breaking based on the effect of quark mass differences on the charged and neutral pion coupling constants. Breaking from ρ-ω [6] and π-η mixing are also possible [7–9]. While the ρ-ω mixing predicts about the right magnitude for the effect, it has the wrong sign.

Although a number of possible ways of breaking isospin symmetry in the πN system exist (see Ref. [10] for a classification of the types), two corrections that are isospin breaking and nonperturbative *must minimally* be included at the level of a phase-shift analysis. Any attempt to fit the three πN scattering channels at low energy without the inclusion of these effects is doomed.

The first correction is due to the coherent Coulomb amplitude. At forward angles, it provides the dominant contribution to the cross section; and at slightly larger angles, it interferes with the strong amplitude providing important phase information. A simple addition of the two amplitudes would result in a nonunitarity expression. A very common way of treating this problem is to generate a strong interaction potential and add it to the Coulomb potential. The strong interaction potential is assumed to conserve isospin.

The second effect is due to the $\pi^{\pm}$-π^0 and n-p mass differences. One result of these differences is that the charge-exchange reaction has a positive Q value and so tends to infinity at zero energy. This singularity is due to the flux factor which is present when calculating the physical amplitudes from the S matrix. Aside from this factor, there are also corrections at the S-matrix level which could be considered with a perturbative treatment.

Corrections for these effects have been reported that use a potential model for their calculation [11–13] based on pioneering work by Oades and Rasche [14]. In addition, an analysis was made [1] (hereafter referred to as GAK) in which the potential method corrections were incorporated into the fit itself. These last corrections have not been previously reported, and it is a primary purpose of this paper to report them.

For unbroken isospin, one can write the charged pion scattering amplitudes in terms of the isospin amplitudes:

$$f_{\pi^+p\to\pi^+p} = f^{\frac{3}{2}}, \quad f_{\pi^-p\to\pi^-p} = \frac{1}{3}\left(f^{\frac{3}{2}} + 2f^{\frac{1}{2}}\right),$$
$$f_{\pi^-p\to\pi^0n} = \frac{\sqrt{2}}{3}\left(f^{\frac{3}{2}} - f^{\frac{1}{2}}\right) = \frac{1}{\sqrt{2}}(f_{\pi^+p\to\pi^+p} - f_{\pi^-p\to\pi^-p}). \tag{1}$$

In the potential treatment, each of the pure isospin amplitudes is extracted from wave functions assumed to satisfy a wave equation with potentials corresponding to the isospin ($V^{\frac{3}{2}}$ and $V^{\frac{1}{2}}$). Since the π^+p amplitude consists of a single isospin amplitude, only the Coulomb correction needs to be taken into account. The formalism for this case is covered in Sec. II.

The other two amplitudes, being a mixture of isospin amplitudes, pose the additional problem of the neutron-proton and $\pi^{\pm}$-π^0 mass differences. By using a linear combination of the amplitudes, the solution is expressed as a pair of coupled equations corresponding to the charge states, that is,

$$\mathcal{T}_c\psi_c + \left(\frac{2}{3}V^{\frac{1}{2}} + \frac{1}{3}V^{\frac{3}{2}}\right)\psi_c + \frac{\sqrt{2}}{3}\left(V^{\frac{3}{2}} - V^{\frac{1}{2}}\right)\psi_0 = \epsilon_c\psi_c, \tag{2}$$

$$\mathcal{T}_0\psi_0 + \frac{\sqrt{2}}{3}\left(V^{\frac{3}{2}} - V^{\frac{1}{2}}\right)\psi_c + \left(\frac{1}{3}V^{\frac{1}{2}} + \frac{2}{3}V^{\frac{3}{2}}\right)\psi_0 = \epsilon_0\psi_0, \tag{3}$$

where ψ_c and ϵ_c are the charged wave function and kinetic energy, and ψ_0 and ϵ_0 are the corresponding neutral pion quantities. $\mathcal{T}_c$ and $\mathcal{T}_0$ are the kinetic energy operators. The corrections are then included by adding a Coulomb potential to the equation describing the π^- only. The masses entering into the kinetic energy operators $\mathcal{T}$ and the kinetic energies ϵ are also replaced by expressions with the true masses corresponding to the charged or neutral pions and nucleons. It is assumed that the potentials remain unchanged during this

process. This assumption has been questioned by Rusetsky [15].

Although the general method used in each case is very similar, each group has used different forms for the potentials, fit different data sets, used different relativistic prescriptions for the pion energies and reduced masses (energies) and, in some cases, used different wave equations. Thus, variations are to be expected among the resulting corrections.

In addition to the potential method, a dispersion approach has been used [16,17]. Sauter [18] developed the basic method for s waves only without considering mass differences. The model was extended in several works [19]. This approach has become the most common method for making the corrections, producing what are often referred to as the NORDITA [16,17] results. The corrections at low energies are made with NORDITA in the often cited work by Arndt *et al.* (see Ref. [20], for example). The advantages and disadvantages of the potential and dispersion relations method are discussed in Ref. [11].

II. π^+ PROTON SCATTERING

In this case, the corrections in a potential model are straightforward. The wave equation can be solved for each partial wave to obtain the phase shifts. While this full solution for the amplitude includes everything, the partial wave expansion diverges, as does the one for a pure Coulomb potential. Since the sum of the Coulomb series alone is known, the non-spin-flip and spin-flip amplitudes can be written as

$$f(\theta) = f_c(\theta) + \frac{1}{2ik}\sum_{\ell=0}^{\infty}[(\ell+1)(e^{2i\delta_{\ell+}} - e^{2i\sigma_\ell}) + \ell(e^{2i\delta_{\ell-}} - e^{2i\sigma_\ell})]P_\ell(\cos\theta), \quad (4)$$

$$g(\theta) = \frac{1}{2k}\sum_{\ell=0}^{\infty}[e^{2i\delta_{\ell+}} - e^{2i\delta_{\ell-}}]P_\ell^1(\cos\theta), \quad (5)$$

where σ_ℓ is the Coulomb phase shift and $f_c(\theta)$ is the Coulomb amplitude. The notation $\ell\pm$ means $j = \ell \pm \frac{1}{2}$. Here (and in GAK [1], but unlike Gashi *et al.* [11]), we neglect the spin-dependent part of the electromagnetic interaction. Equation (4) is often written as

$$f(\theta) = f_c(\theta) + \frac{1}{2ik}\sum_{\ell=0}^{\infty}e^{2i\sigma_\ell}[(\ell+1)(e^{2i\delta^n_{\ell+}} - 1) + \ell(e^{2i\delta^n_{\ell-}} - 1)]P_\ell(\cos\theta), \quad (6)$$

where the nuclear phase shift is defined as

$$\delta^n_{\ell\pm} = \delta_{\ell\pm} - \sigma_\ell. \quad (7)$$

This phase shift is then compared with the hadronic phase shift $\delta^h_{\ell\pm}$, i.e., the one that would exist if there were no Coulomb interaction. A potential model is used to calculate these two quantities in the present case. In the limit that the interactions are very weak, the hadronic and nuclear phase shifts become equal so that the difference may be expected to be small. The correction is defined as the difference between them,

$$C_{\ell\pm} = \delta^n_{\ell\pm} - \delta^h_{\ell\pm}. \quad (8)$$

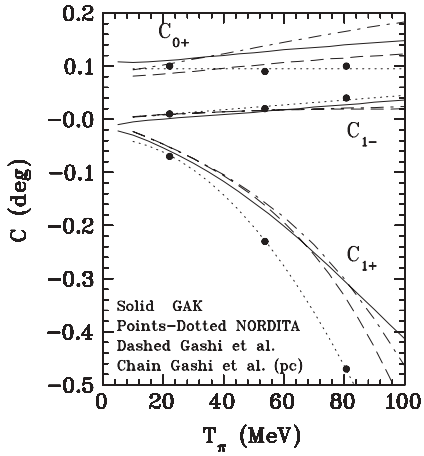

FIG. 1. Comparison among the different methods for $\pi^+ p$ elastic scattering. The difference between the point charge (pc) and full calculation of Gashi *et al.* [11] is very small except for the s wave. The dotted curve represents a fit to the NORDITA results [16] to be used later for comparisons at energies other than those given by them (solid circles) and is given in the Appendix.

Figure 1 shows a comparison of the results of the different determinations. It is seen that the potential methods are in reasonable agreement. The point Coulomb result of Gashi *et al.* [11] included no higher order electromagnetic effects, whereas the full calculation included a finite charge distribution as well. Since the corrections from GAK included a finite charge density but no higher order electromagnetic effects, there is no exact comparison possible. The dispersion relation approach is seen to give only slightly different results.

While one can judge to some extent the size and degree of agreement from Fig. 1, it is useful to see the effect on the cross section itself. Figure 2 shows such a comparison at a kinetic energy of 45 MeV. The hadronic phase shifts are taken from Ref. [1] for all of the cases. It is seen that the

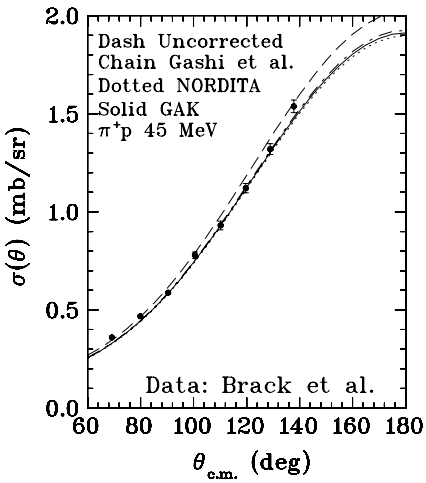

FIG. 2. Effect of the corrections for $\pi^+ p$ scattering in a typical case. The corrections are small but not negligible. The three methods compared give very nearly the same result. The data are from Brack *et al.* [21].

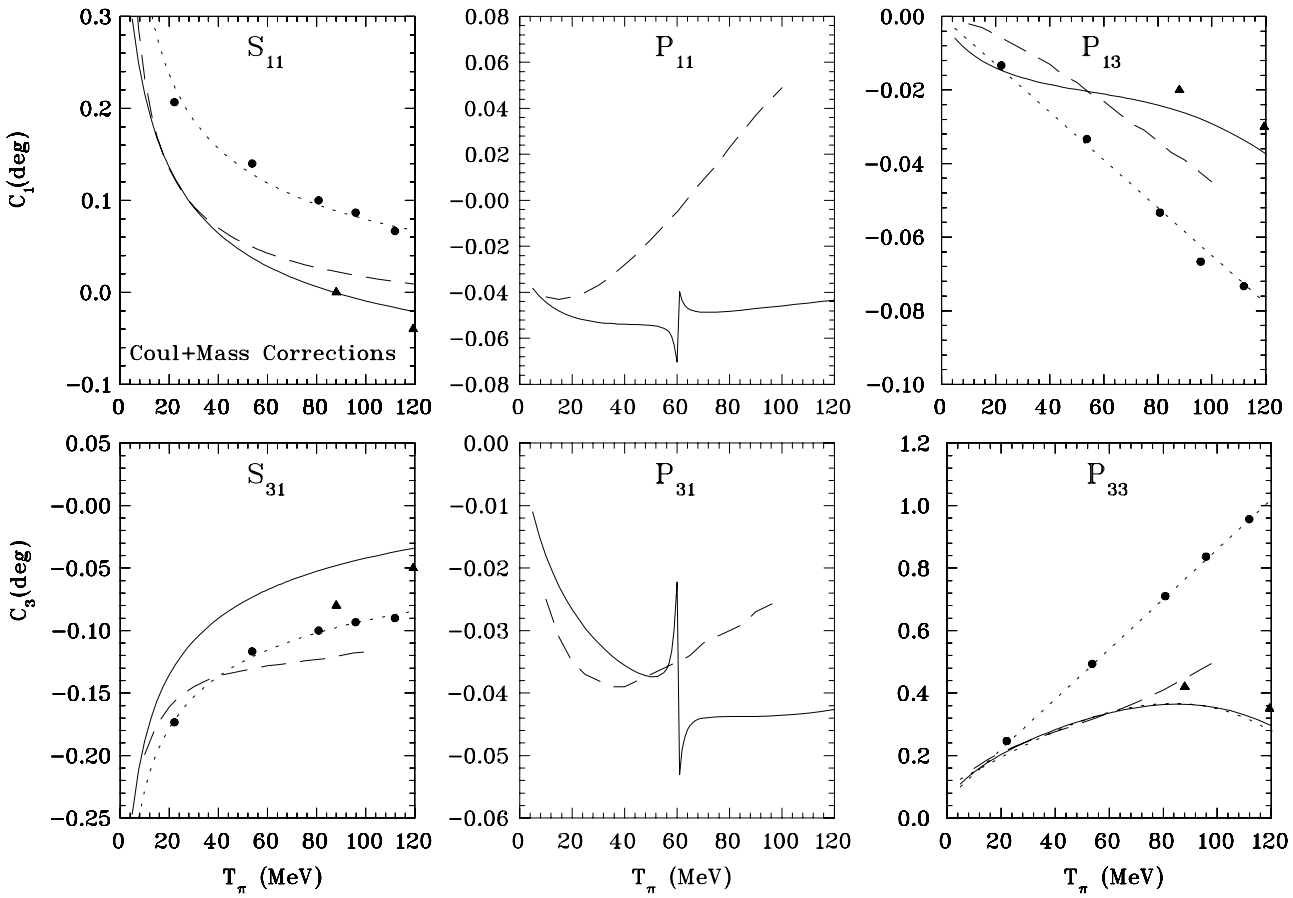

FIG. 3. Solid curve gives the result with both Coulomb and mass difference corrections from GAK. Dashed line is from Gashi *et al*. [12], solid dots are from Tromborg *et al*. (NORDITA) [16], and the solid triangles are from Zimmermann [13]. Dotted lines represent a fit to the NORDITA [16] results used later.

correction is small but not negligible, being of the order of two standard deviations of accurate data. The differences *between* the corrections would seem to be unimportant at the present level of the quality of the data.

III. π^- PROTON SCATTERING AND CHARGE EXCHANGE

In this case, the expression of the corrections is more complicated since the potential system consists of a pair of coupled differential equations. Not only is there a correction due to the finite charge in the $\pi^- p$ channel but also the neutron-proton and π^0-$\pi^\pm$ mass differences must be taken into account.

A. Method

To express the corrections in the coupled channel case, we follow the formulation of Gashi *et al*. [12]. The solution of the coupled channel system [1,22] produces the amplitude for π^- proton scattering, charge exchange, and π^0 neutron scattering. The result is expressed as a 2 by 2 symmetric S matrix with the diagonal elements representing $\pi^- p$ and $\pi^0 n$ elastic scattering and the off-diagonal elements charge exchange. The scattering matrix is expressed in terms of a 2 by 2 real K matrix,

$$S = \frac{1 + iK}{1 - iK}, \quad K = -i\frac{S-1}{S+1}. \tag{9}$$

These equations hold for each partial wave where we have suppressed their angular momentum indices. The S_{11} matrix element has been multiplied by $e^{-2i\sigma_\ell}$ to remove the pure Coulomb effect before calculation of the K matrix. Since the K matrix is real and symmetric, it can be transformed to a diagonal form with an orthogonal matrix,

$$\begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} = \begin{pmatrix} K_1 & 0 \\ 0 & K_2 \end{pmatrix} \equiv \begin{pmatrix} \tan\delta_1 & 0 \\ 0 & \tan\delta_2 \end{pmatrix}. \tag{10}$$

If isospin is unbroken, then $\cos\phi \to \sqrt{\frac{2}{3}} \equiv \cos\phi_0$ and δ_1 and δ_2 are the isospin $\frac{1}{2}$ and $\frac{3}{2}$ phase shifts. The form expressed in Eq. (10) is valid independent of isospin conservation; and the differences of the resultant phase shifts δ_1, δ_2, as well as the value of ϕ needed to diagonalize the system from the unbroken isospin values, are used to quantify the breaking.

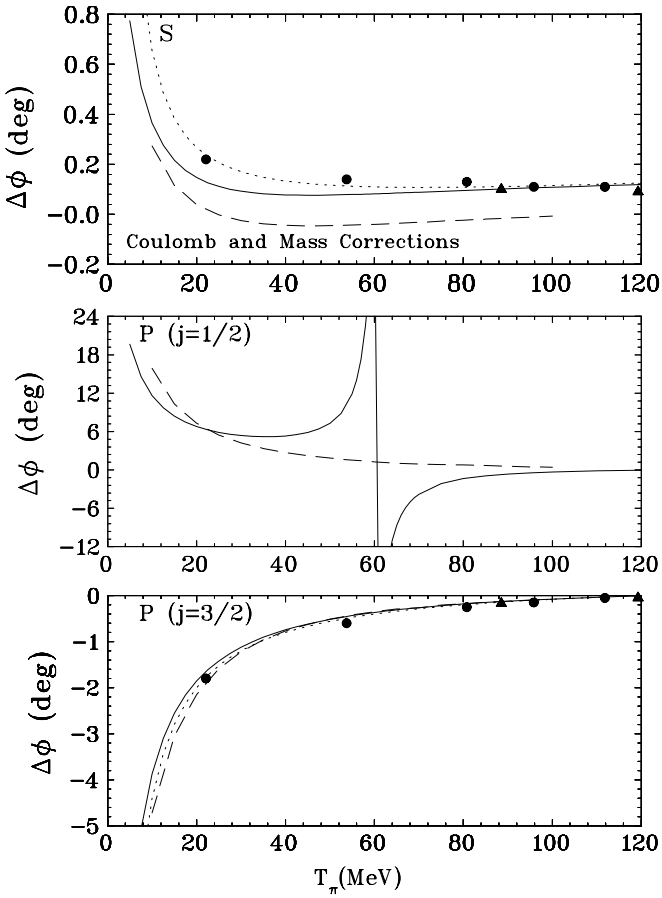

FIG. 4. Comparison with the correction for the rotation angle $\Delta\phi$. The meaning of the symbols is the same as in Fig. 3.

The corrections are expressed as

$$C_1 \equiv \bar{\delta}_{\frac{1}{2}} - \delta^h_{\frac{1}{2}}, \qquad C_3 \equiv \bar{\delta}_{\frac{3}{2}} - \delta^h_{\frac{3}{2}}, \\ \Delta\phi \equiv \bar{\phi} - \phi^h = \bar{\phi} - \phi_0, \tag{11}$$

for each partial wave, where the barred quantities are those obtained from the model by solving for δ_1, δ_2, and ϕ from the equations above with Coulomb and mass differences present. The quantities with superscript h (the hadronic values) are those obtained from the solution of the system with all pion masses equal to the charged pion mass, the neutron mass equal to the proton mass, and no Coulomb interaction.

The hope is that these corrections will be nearly model independent and that one can fit the data by choosing δ^h_1 and δ^h_2 in a fitting procedure, apply the corrections to obtain the full K and S matrices, add in the Coulomb amplitude where appropriate, and compare with data to calculate a value of χ^2. A search on the values of the isospin pure phase shifts δ^h_1 and δ^h_2 can then be made.

We now address the problem of calculating the corrections. Since K_1 and K_2 are the eigenvalues of the K matrix, we can solve directly for them with

$$\lambda_\pm = \frac{K_{11} + K_{22} \pm \sqrt{(K_{11} - K_{22})^2 + 4K_{12}^2}}{2}. \tag{12}$$

We may now identify the plus and minus signs in this expression with the isospin $\frac{1}{2}$ and $\frac{3}{2}$ states.

Multiplying Eq. (10) from the left by the transpose of the rotation matrix, we find four relations for $\tan\phi$, all of which are equivalent. Two of them are

$$\tan\phi = \frac{K_{11} - K_1}{K_{12}}, \qquad \tan\phi = \frac{K_{12}}{K_2 - K_{11}}. \tag{13}$$

B. Difficulty with the $P_{\frac{1}{2}}$ wave

In most cases, there is no difficulty in choosing the proper sign in Eq. (12) to make the correct association of δ_1 and δ_2 with the isospin phase shifts; in the case of $P_{\frac{1}{2}}$ scattering, however, a problem arises. In this case, around 60 MeV, the two hadronic phase shifts (P_{31} and P_{11}) cross. For the two eigenvalues K_1 and K_2 to be equal, the radical in Eq. (12) must vanish. This requires that the conditions

$$K_{11} = K_{22}, \qquad K_{12} = 0 \tag{14}$$

be satisfied simultaneously. For unbroken isospin, this indeed happens. In this case, isospin requires

$$K_{12} = \sqrt{2}(K_{22} - K_{11}), \tag{15}$$

so that

$$\lambda_\pm = \frac{K_{11} + K_{22} \pm 3\sqrt{(K_{11} - K_{22})^2}}{2} \\ = \frac{K_{11} + K_{22} \pm 3|K_{11} - K_{22}|}{2}. \tag{16}$$

For the eigenvalue to have a continuous derivative, the $\pm$ association must be changed at the zero of the radical. If one chooses the sign to be that of $K_{11} - K_{22}$, then

$$\lambda_+ = 2K_{11} - K_{22} = \tan\delta^{\frac{3}{2}}, \qquad \lambda_- = 2K_{22} - K_{11} = \tan\delta^{\frac{1}{2}}. \tag{17}$$

For broken isospin, in general, both conditions in Eq. (14) will NOT be satisfied at once, so the two eigenvalues can never be equal. To understand what happens in this case, it is useful to look at the direct solution of Eq. (10).

In the general case, we can expand Eq. (10) to find that the condition for the off-diagonal elements to be zero is

$$2\phi = \tan^{-1}\frac{2K_{12}}{K_{22} - K_{11}}, \tag{18}$$

so 2ϕ changes by π around the point $K_{22} = K_{11}$. Consider a simple example in which the isospin relation is slightly broken in a simple way such that

$$K_{12} = \sqrt{2}(K_{22} - K_{11}) + \epsilon, \tag{19}$$

then

$$2\phi = \tan^{-1}\left(2\sqrt{2} + \frac{2\epsilon}{K_{22} - K_{11}}\right), \tag{20}$$

and 2ϕ increases by π as the difference $K_{22} - K_{11}$ changes sign. We have assumed that $K_{22} > K_{11}$ for energies lower than the crossing point. If the opposite is true, a very similar argument gives the same effect with 2ϕ decreasing by π.

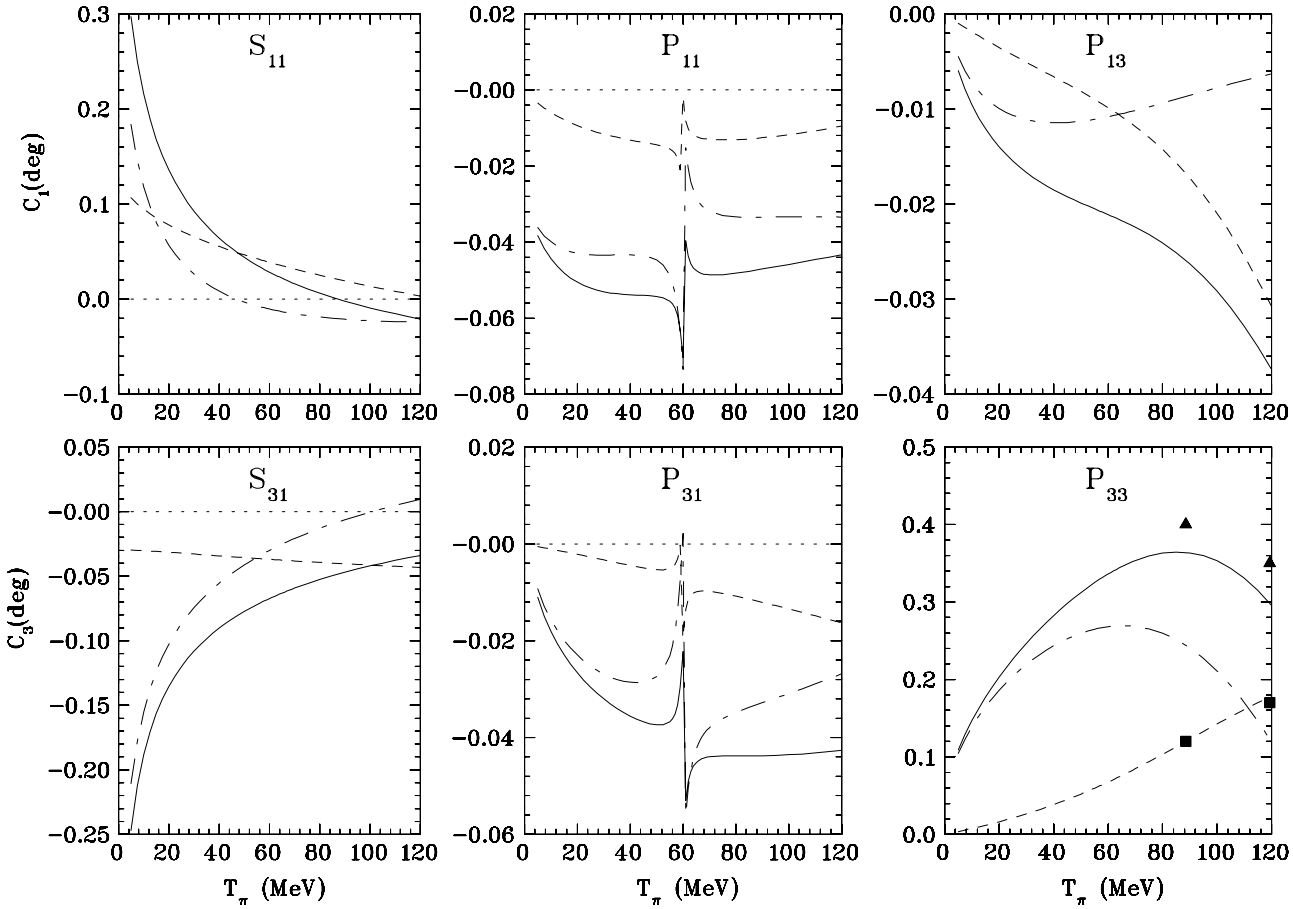

FIG. 5. Short dashed line contains the Coulomb correction only, dash-dot line is with the mass difference correction only, and solid line contains both corrections. Results of Zimmermann are given by the solid triangles (full calculation) and solid squares (Coulomb correction only).

The values of K_1 and K_2 are given by

$$K_1 = \tfrac{1}{2}K_{11}(1+\cos 2\phi) + \tfrac{1}{2}K_{22}(1-\cos 2\phi) - K_{12}\sin 2\phi \overset{\phi\to\phi_0}{\to} \tan\delta^{\frac{3}{2}}, \quad (21)$$

$$K_2 = \tfrac{1}{2}K_{11}(1-\cos 2\phi) + \tfrac{1}{2}K_{22}(1+\cos 2\phi) + K_{12}\sin 2\phi \overset{\phi\to\phi_0}{\to} \tan\delta^{\frac{1}{2}}. \quad (22)$$

Indeed, as 2ϕ increases by π, $K_1 \leftrightarrow K_2$ because the sine and cosine change sign under these conditions.

The correction is nonperturbative since the change in 2ϕ by π occurs at the point where the hadronic phase shifts cross and hence depends strongly on the hadronic phase shifts used to define the corrections. In this case, the general method fails for corrections near the crossing point.

One can choose to change the association of the plus and minus signs at the zero of K_{12} (that is what is done in the present work) or when $K_{11} = K_{22}$, but either way there will be discontinuities in the corrections extracted at the crossing point. The actual amplitudes remain continuous; it is only this representation of the breaking which shows an anomalous behavior, but this compensation only occurs for the hadronic phase shifts used in the calculation of the correction. If one uses these corrections with phase shifts other than the ones for which the corrections were derived, discontinuities in the resulting amplitudes will result. Of course, this is a small correction in a small partial wave in this case, so in a practical sense the problem may not be very serious.

The difficulty comes partly from the fact that, even for good isospin, for $K_1 = K_2$ any value of ϕ is valid for the transformation, so ϕ is undetermined at that point. For unbroken isospin there is no problem, however, since ϕ is determined arbitrarily close to the crossing point on either side.

C. Comparison with previous models

An obvious isospin breaking effect due to the mass differences leads to a positive Q value for $\pi^- p \to \pi^0 n$ and a negative Q value for $\pi^0 n \to \pi^- p$. This fact destroys the symmetry of the amplitudes derived from the symmetric S matrix, because of a factor k_f/k_i originating from the incoming and outgoing fluxes which appears in the charge-exchange cross section obtained from the procedure above. This factor is included in the calculations of charge-exchange cross sections to be shown in this section.

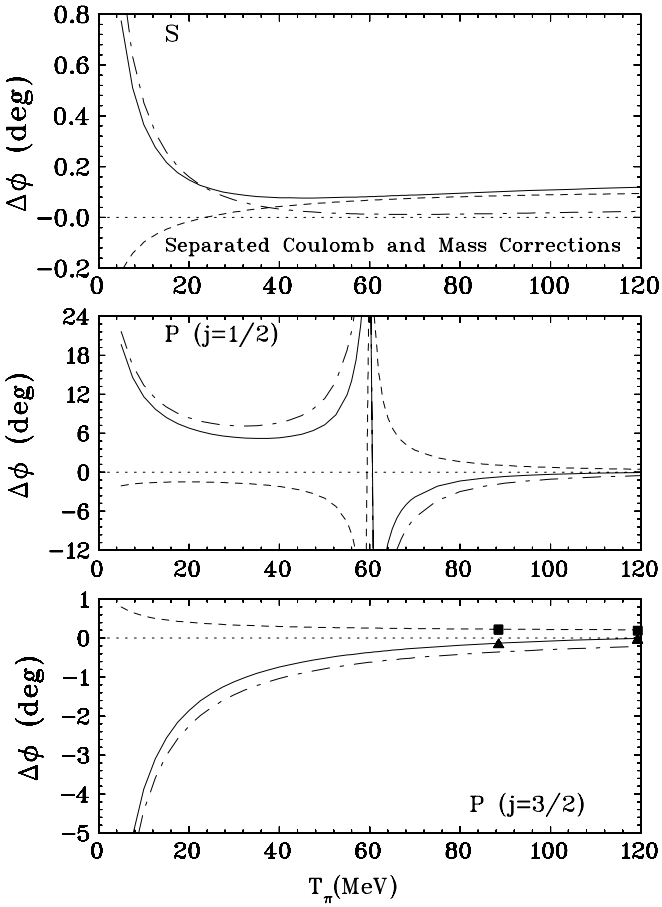

FIG. 6. Individual contributions to $\Delta\phi$. The convention for the lines and points is the same as in Fig. 5.

Figures 3 and 4 show a comparison of the corrections from GAK [1] with previous work. It is seen that there is qualitative agreement among all calculations. There is rather good agreement between GAK and Zimmermann at the higher energies where he calculated. For the S_{11} and P_{33} partial waves, there is good agreement with the work of Gashi *et al*. [12] as for the case of $\Delta\phi$ for the $P_{\frac{3}{2}}$ channel where, in fact, all calculations agree.

Figures 5 and 6 give a breakdown of the contributions to the corrections into Coulomb and mass differences for GAK. The most important contribution is from the mass differences, especially at low energies. Since Zimmermann gave a separation of the two effects for the P_{33} phase shift and the $\Delta\phi$ for the $P_{\frac{3}{2}}$ channel, a comparison can be made and the agreement is good. The NORDITA group [17] also gave a separation into the Coulomb and mass difference contributions. Their mass difference correction dominates at low energy (see Ref. [17], Figs. 2–4). Hence, while these corrections are often called electromagnetic, it is the correction arising from mass differences which is the most important.

While the expression of the correction in this form is very useful, it is difficult to appreciate the effect on the cross sections of the differences among the determinations as they may well be compensating to give similar corrections to physical observables. Hence, we consider the effects of these corrections on some charge-exchange cross sections and the $\pi^- p$ elastic amplitude. In each case, the "no correction" calculation is made from the hadronic phase shifts of GAK [1] and the corrections are made relative to them.

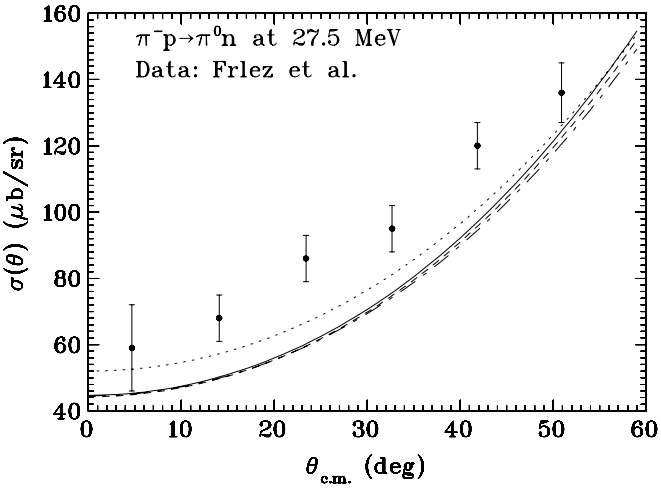

FIG. 7. Comparison of methods of corrections for charge exchange for forward angles at low energies with the data of Frlez *et al*. [23]. Dotted curve represents no correction, solid curve gives the correction from GAK [1], chain-dash curve is from Gashi *et al*. [12], and dashed curve represents the corrections from NORDITA [16].

Figure 7 shows a comparison of the correction with the charge-exchange data of Frlez *et al*. [23] at 27.5 MeV for the three groups that calculated in this energy range. The resulting corrected cross sections are all very nearly the same; the correction takes the result away from the data, whereas the uncorrected calculation (dotted line) lies closer to agreement. This last effect occurs only at forward angles.

Figure 8 gives the breakdown of the contributions. The Coulomb correction actually moves the result slightly closer to the data, while the mass difference correction takes it away.

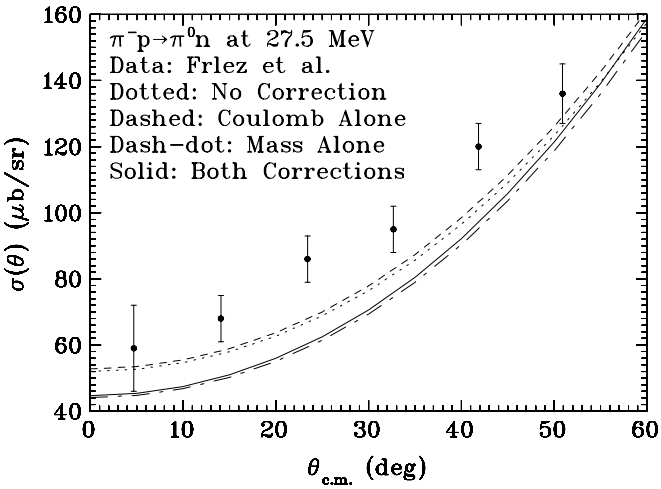

FIG. 8. Separation of the Coulomb and mass corrections compared with data of Frlez *et al*. [23].

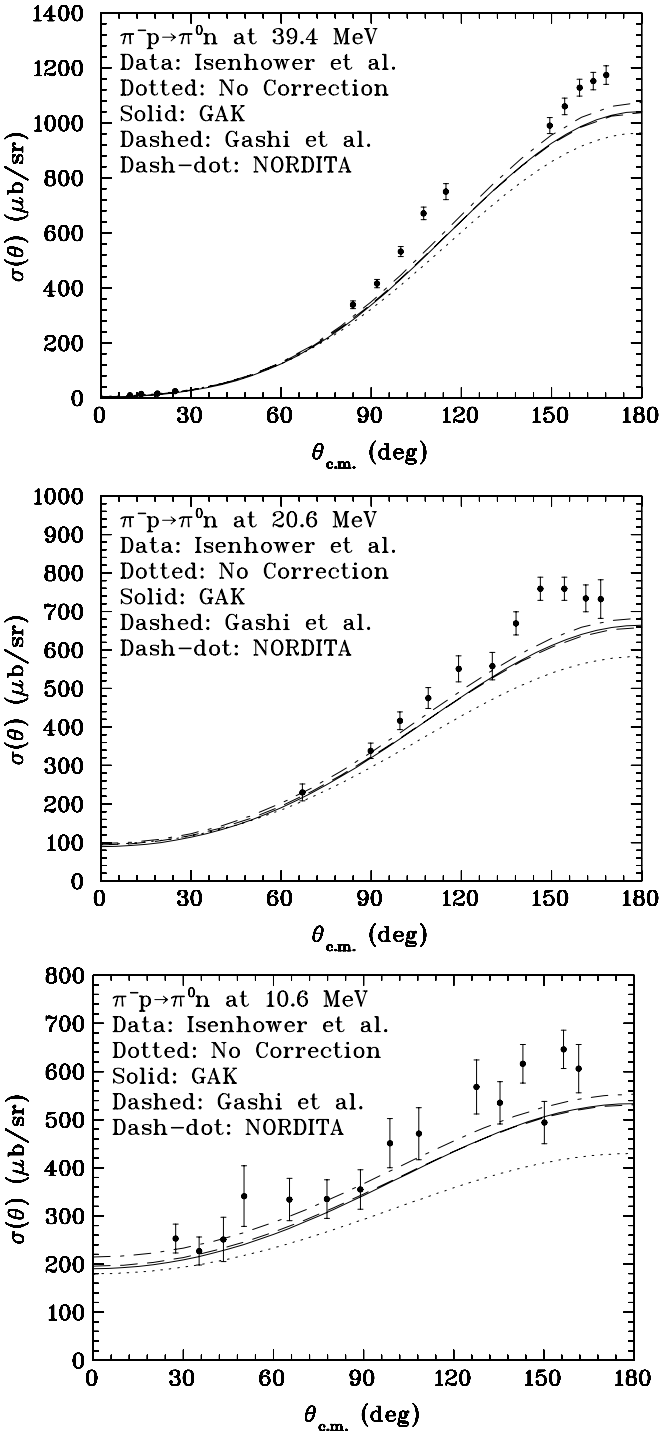

FIG. 9. Comparison of the effects of the corrections with the charge-exchange data of Isenhower *et al*. [24] at 39.4, 20.6, and 10.6 MeV. The importance of the positive Q value of the reaction is clear at the lower energies by comparing with the uncorrected case.

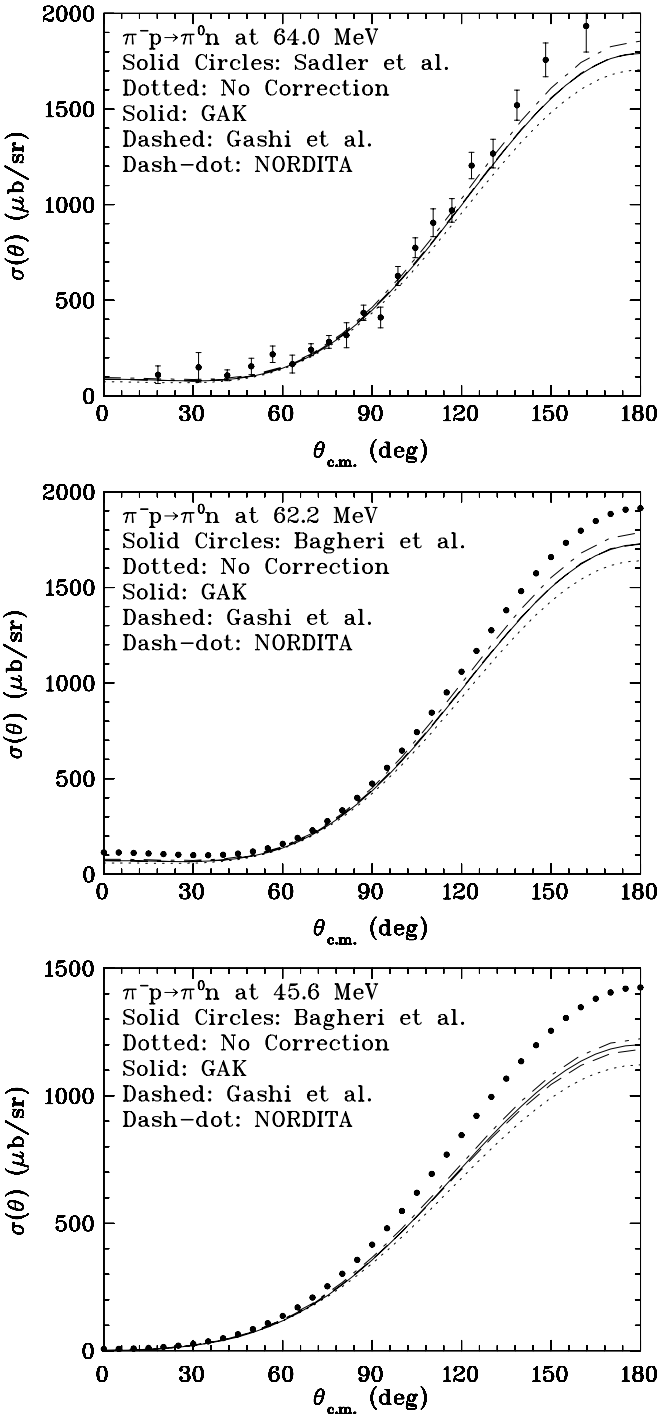

FIG. 10. Comparison of the effect of the corrections with the data of Sadler *et al*. [25] at 64 MeV (top) and Bagheri *et al*. [26] at 62.2 and 45.6 MeV (middle and bottom).

Figure 9 shows the effect over the full angular range compared with the data of Isenhower *et al*. [24]. For larger angles, the corrections move the prediction closer to the data.

At higher energies, the effect of the corrections is somewhat smaller. A comparison with the data of Sadler *et al*. [25] and Bagheri *et al*. [26] is shown in Fig. 10. For the second case, the experimental results were given as coefficients of Legendre polynomials and the dots shown were calculated from these numbers. The effect of the corrections is seen to be small, and the two potential models agree well, a difference being observable only at the lowest energy.

In general, the difference between prediction and data can be expressed as a difference in normalization of around

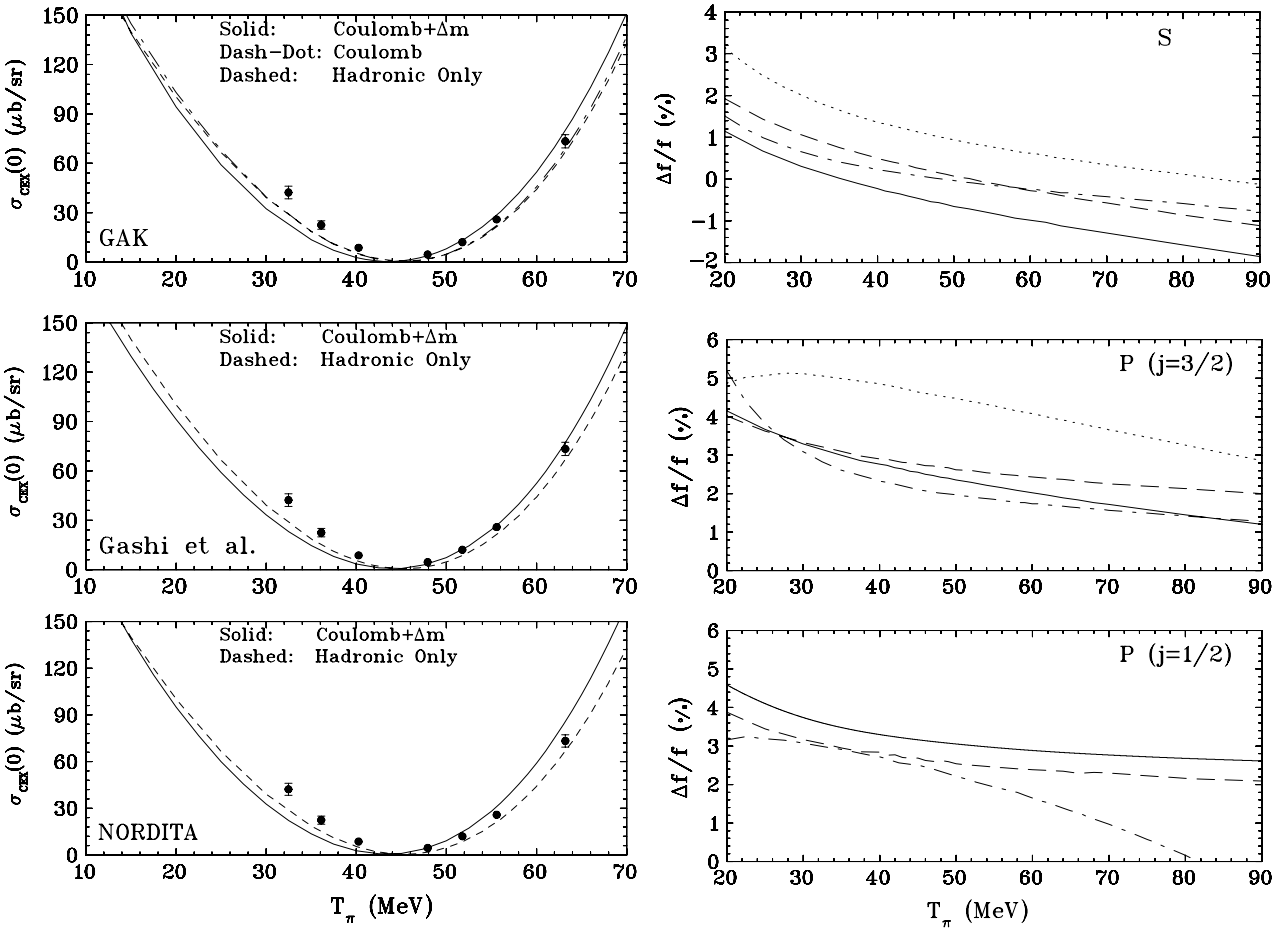

FIG. 11. Left: Zero-degree charge exchange with corrections from GAK [1], Gashi *et al.* [12], and NORDITA [16]. Data are from Fitzgerald *et al.* [27]. Right: Corrections to the real part of $f_{\pi^- p\to\pi^- p}$ for GAK [1] (dash: Coulomb only; solid: full correction), Gashi *et al.* [12] (dash-dot) and NORDITA [16] (dotted).

15% and constitutes evidence of isospin breaking. Note that the ≈8% breaking mentioned in the introduction is in the amplitude and is hence consistent with 15% in the cross section.

The forward charge-exchange cross section is a sensitive measure of isospin breaking since it shows a deep minimum near 45 MeV. While most checks on isospin depend on obtaining an absolute cross section, the position of this minimum does not; only a knowledge of the beam momentum is needed.

The left part of Fig. 11 shows the effect of the corrections of the three groups compared with the data of Fitzgerald *et al.* [27]. We see that the corrections are all very nearly the same and move the predicted cross section away from the data. While the uncorrected prediction cannot be said to give a good fit to the data, it is much better than the prediction after correction for the mass differences. The Coulomb potential has very little effect on the position of the minimum.

The right part of Fig. 11 shows the prediction of the corrections for the amplitude for $\pi^- p \to \pi^- p$. The mass correction is significantly different from the pure Coulomb effect. One can also note a large difference among the methods.

D. Comparison with chiral perturbation theory

While the results of the previous section are rather consistent, at least for the charge exchange, they are based on similar assumptions and hence could all be wrong. Thus, it is very desirable to have a comparison with an independent method calculating from an approximate QCD viewpoint.

A calculation of isospin breaking in chiral perturbation theory (ChPT) has been made by Fettes and Meißner [28]. They calculated six ratios which are measures of isospin breaking. Most of the ratios (3–6) involve quantities difficult to measure. Ratio number 1 involves isoscalar quantities that vanish at threshold, so the ratio depends sensitively on the phase-shift fit used to compute them.

Their second ratio,

$$R_2 = 2\frac{f_{\pi^+ p\to\pi^+ p} - f_{\pi^- p\to\pi^- p} - \sqrt{2} f_{\pi^- p\to\pi^0 n}}{f_{\pi^+ p\to\pi^+ p} - f_{\pi^- p\to\pi^- p} + \sqrt{2} f_{\pi^- p\to\pi^0 n}}, \qquad (23)$$

involves the same expression used to test isospin in the charge-exchange reaction, so it is very suitable for comparison. As stated by the authors of Ref. [28], a direct comparison cannot be made between their work and data, but one can calculate the ratio with the hadronic model under the same conditions.

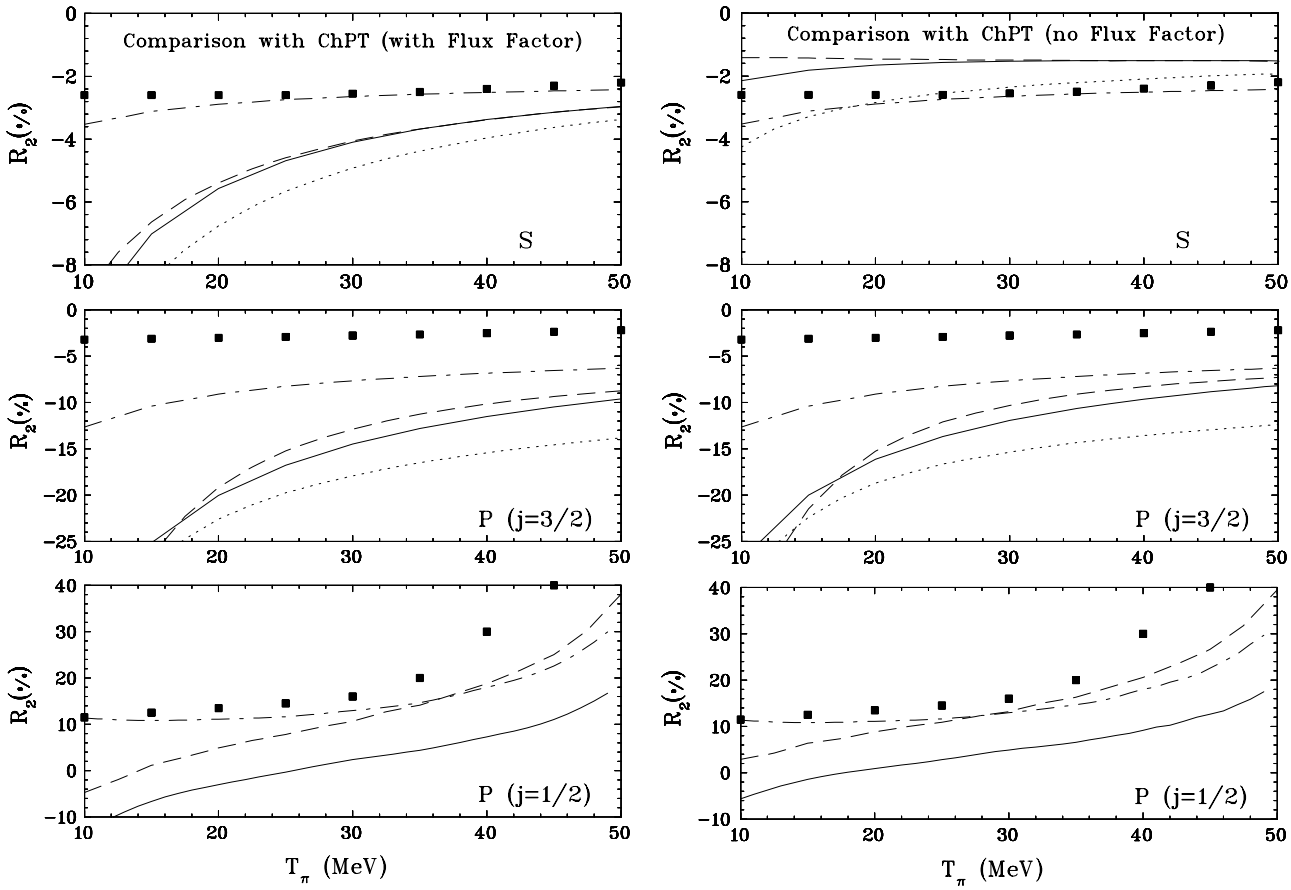

FIG. 12. Left: Ratio R_2 calculated with GAK corrections for Coulomb only (dash-dot) and Coulomb plus mass differences (solid) compared with that from Fettes and Meißner [28] (solid squares). Dashed line gives the result with the corrections of Gashi *et al.* [12]; dotted line gives that of NORDITA [16]. Flux factor $\sqrt{k_f/k_i}$ is included in the hadronic calculations. Right; Same information except that $\sqrt{k_f/k_i}$ is not included.

An attempt to compare the ChPT calculation with actual charge-exchange amplitudes obtained directly from the data would include the flux factor $\sqrt{k_f/k_i}$. Since the ChPT calculation is made at the matrix element level, this factor should not be in the comparison. We show both cases to demonstrate the importance of the factor.

Since they calculated *corrections* to the strong process, their combination

$$f_{\pi^+p\to\pi^+p} - f_{\pi^-p\to\pi^-p} \tag{24}$$

goes to zero in the limit of no strong interaction and we must remove the pure Coulomb amplitude. We might subtract it partial wave by partial wave or simply use the nuclear phase shifts. It is the latter prescription which is shown in the comparison graphs, although there is very little difference in these two ways of removing the Coulomb effect.

Figure 12 shows the comparison with and without the flux factor. For the S and $P_{\frac{1}{2}}$ waves, the pure Coulomb correction agrees rather well with the ChPT (full) calculations. The $P_{\frac{1}{2}}$ ratio is large and shows a singularity around 60 MeV because of crossing of the hadronic phase shifts leading to the vanishing of the charge-exchange amplitude in this partial wave. The position of this crossing depends on the particular fit of the hadronic phase shifts and hence cannot be expected to be the same in the different cases. For the $P_{\frac{3}{2}}$ wave, even the ratio calculated with Coulomb corrections alone is more than a factor of 2 larger than the ChPT result, perhaps because the amplitude is larger and the perturbation series has not converged.

Without the flux factor, only the $P_{\frac{3}{2}}$ partial wave shows a significant discrepancy between the models and the ChPT calculations. The two potential models and the dispersion relation approach show about the same factor of 3–5 difference in this case. Since this is the channel in which the Δ resonance occurs, one can question whether the explicit degrees of freedom of the Δ can make an important difference.

IV. CONCLUSIONS

We have seen that the hadronicly calculated corrections of GAK [1] are in rough agreement with previous determinations [11–13,16]. In most cases, the values of GAK and Gashi *et al.* [11,12] are in reasonable agreement. All of these methods give a significant (and very similar) correction to the charge-exchange cross section at low energies because of the mass differences. If one assumes that the hadronic corrections are too large, then a closer agreement can be

achieved in the case of forward angle charge exchange at low energy and the position of the forward minimum. However, this conjecture does not correct the discrepancy at larger angles. So even this rather drastic possibility does not solve the problem. Hence, the interpretation of isospin breaking based on the discrepancy between data and prediction in the charge-exchange channel (seen in Figs. 7–11) is not influenced by the variation in the corrections. We also see that the same discrepancy is present within all of the charge-exchange data sets.

Corrections calculated from ChPT are significantly smaller than those of the hadronic methods in the $P_{\frac{3}{2}}$ partial wave, especially when the mass correction is included. This may indicate that there is a general error in the hadronic corrections or that the Δ degree of freedom needs to be treated differently in the ChPT calculation. Since the mass correction in the form currently used has been questioned, it is possible that the assumption about the isospin potentials remaining unchanged is incorrect.

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APPENDIX A: FITS

Following are a set of fits to the phase shifts and corrections which are given as an aide. The form of the fits has no physical significance and should not be used outside the range 5–100 MeV. In what follows, T is the laboratory kinetic energy in MeV and δ is in degrees.

1. GAK hadronic phase shifts

The fit to the GAK hadronic phase shifts for s waves is in the form

$$\delta = \frac{a\sqrt{T}}{1 + bT + cT^2}. \tag{A1}$$

For S_1,

$$a = 1.0319, \quad b = 2.865 \times 10^{-3}, \quad c = -8.0 \times 10^{-7}. \tag{A2}$$

For S_3,

$$a = -0.5197, \quad b = -5.37 \times 10^{-3}, \quad c = 1.939 \times 10^{-5}. \tag{A3}$$

For the p waves,

$$\delta_{13} = -\frac{0.001806T^{\frac{3}{2}}(1 - 0.00414T)}{1 + 0.001T}, \tag{A4}$$

$$\delta_{33} = \frac{0.01084T^{\frac{3}{2}}(1 + 0.0085T)(1 + 0.06e^{-0.057T})}{1 - 0.000436T}.$$

$$\delta_{11} = -\frac{0.00636T^{\frac{3}{2}}(1 - 0.006190T - 0.0304T^2)}{1 + 0.0131T}, \tag{A5}$$

$$\delta_{31} = -\frac{0.00196T^{\frac{3}{2}}(1 + 0.00228T)}{1 + 0.00011T}.$$

2. $\pi^+ p$ corrections

The fits to the $\pi^+ p$ corrections by GAK are

$$C_{0+} = 0.1 + 0.000049T, \quad C_{1-} = -0.009 + 0.00045T, \quad C_{1+} = -0.0185 - 0.000397T^{1.5}. \tag{A6}$$

The fits to the $\pi^+ p$ corrections by Gashi *et al*. [11] are

$$C_{0+} = 0.075 + 0.0005T, \quad C_{1-} = 0.005 + 0.0002T, \quad C_{1+} = -0.033 - 0.0000195T^{2.2}. \tag{A7}$$

For NORDITA,

$$C_{0+} = 0.095, \qquad C_{1-} = 0.00045T, \quad C_{1+} = -0.035 - 0.000067T^2. \tag{A8}$$

3. Coupled channel corrections

The fit to the GAK corrections for the s wave is given by

$$C_1 = 0.753T^{-\frac{1}{2}}(1 - 0.0114T), \quad C_3 = -\frac{0.619T^{-\frac{1}{2}}(1 - 0.0051T)}{1. - 0.00299T}, \tag{A9}$$

$$\Delta\phi = \frac{7.9}{T^{1.3}} + 0.0011T - 0.026. \tag{A10}$$

For the $P_{\frac{1}{2}}$ partial wave,

$$C_1 = -0.035 - \frac{0.00103T}{1 + 0.0008T^2} + 0.01745\tan^{-1}\left(\frac{0.02}{T - 60.1}\right), \tag{A11}$$

$$C_3 = 0.019 - \frac{0.01137T^{0.728}}{(1 + 0.137T^{0.72} + 0.000039T^2)} - 0.01745\tan^{-1}\left(\frac{0.0155}{T - 60.1}\right), \tag{A12}$$

$$\Delta\phi = -1.2 + \frac{148}{(T + 4.3)^{0.95}} - 0.4363\tan^{-1}\left(\frac{2}{T - 60.1}\right). \tag{A13}$$

For the $P_{\frac{3}{2}}$ partial wave,

$$C_1 = -0.001 - 0.0039T^{0.4} - 0.0038\left(\frac{T}{100}\right)^4, \quad C_3 = 0.1 + 0.0048T - 0.23\left(\frac{T}{100}\right)^3, \tag{A14}$$

$$\Delta\phi = -\frac{44.5}{T} + 0.37.$$

The fit to the Gashi *et al*. [12] corrections used in this work is the following:
For s,

$$C_1 = \frac{1.15}{T^{0.6}} - 0.056, \quad C_3 = -\frac{0.3}{T^{0.3}} - 0.04, \tag{A15}$$

For $p_{\frac{3}{2}}$,

$$C_1 = -0.0005T + 0.006, \quad C_3 = 0.0036T + 0.13, \tag{A16}$$

For $p_{\frac{1}{2}}$,

$$\begin{aligned} C_1 &= -0.045 + 0.000016T^{1.9}, \\ C_3 &= -0.05 + 0.00025T + 0.046e^{-0.08T}, \end{aligned} \tag{A17}$$

$$\begin{aligned} \Delta\phi_s &= \frac{10.3}{T^{1.4}} + 0.0013T - 0.149, \\ \Delta\phi_{p_{\frac{1}{2}}} &= \frac{160}{T^{0.95}} - 1.9, \qquad \Delta\phi_{p_{\frac{3}{2}}} = -\frac{45}{T-2} + 0.4. \end{aligned} \tag{A18}$$

The fit to the NORDITA [16] corrections used in this work is as follows:
For s,

$$C_1 = \frac{1.1}{T^{0.4}} - 0.095, \quad C_3 = -\frac{0.55}{T^{0.25}} + 0.082, \tag{A19}$$

For $p_{\frac{3}{2}}$,

$$C_1 = -0.00065T, \quad C_3 = 0.008T + 0.06, \tag{A20}$$

$$\Delta\phi_s = \frac{16}{T^{1.4}} + 0.0008T + 0.01, \quad \Delta\phi_{p_{\frac{3}{2}}} = -\frac{48}{T} + 0.4. \tag{A21}$$

The NORDITA group did not give corrections for the $P_{\frac{1}{2}}$ partial wave.

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