# $P$-wave charmed-strange mesons

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We examine charmed-strange mesons within the framework of the constituent quark model, focusing on the states with $L = 1$. We are particularly interested in the mixing of two spin states that are involved in $D_{s1}(2536)$ and the recently discovered $D_{sJ}(2460)$. We assume that these two mesons form a pair of states with $J = 1$. These spin states are mixed by a type of spin-orbit interaction that violates the total-spin conservation. Without assuming explicit forms for the interactions as functions of the interquark distance, we relate the matrix elements of all relevant spin-dependent interactions to the mixing angle and the observed masses of the $L = 1$ quartet. We find that the spin-spin interaction, among various types of spin-dependent interactions, plays a particularly interesting role in determining the spin structure of $D_{s1}(2536)$ and $D_{sJ}(2460)$.

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## I. INTRODUCTION

Recently a new charmed-strange meson, $D^*_{sJ}(2317)$, was discovered by the BaBar Collaboration [1] and confirmed by the CLEO Collaboration [2]. The CLEO reported another charmed-strange meson called $D_{sJ}(2460)$. Both these mesons were confirmed by the Belle Collaboration [3,4]. The masses and decay properties of $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ have been investigated with two types of particular structures assumed for them. One type is the ordinary $q\bar{Q}$ structure, and the other is an exotic structure such as the $KD$ molecule [5–7,9] or tetra-quark configuration [10–13]. We will work with the former structure in this paper. Then, these new entries together with $D_{s1}(2536)$ and $D_{s2}(2573)$, which were discovered earlier, are expected to form a quartet with $L = 1$ ($P$ states) of the $c\bar{s}$ (or $s\bar{c}$) system. Given this expectation, Godfrey studied various properties of $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ [14,15], following the work done prior to the discoveries of these mesons [16,17]. Also, decay modes of $D^*_{sJ}(2317)$ and $D_{sJ}(2460)$ were analyzed by Colangero and De Fazio [18], Bardeen *et al.* [19], Mehen and Springer [7], and Close and Swanson [8].

With respect to the spin structure of these mesons, there are four states, ${}^1P_1$, ${}^3P_0$, ${}^3P_1$, and ${}^3P_2$, in terms of the $JLS$ bases.[1] While $D^*_{sJ}(2317)$ and $D_{s2}(2573)$ can probably be assigned to ${}^3P_0$ and ${}^3P_2$, respectively, $D_{s1}(2536)$ and $D_{sJ}(2460)$ are probably mixtures of ${}^1P_1$ and ${}^3P_1$. The extent of the mixing can be parametrized by a mixing angle [14–17,20,21]. In addition to the masses of the mesons, the branching fractions for $B \to \bar{D}D_{sJ}$ followed by the electromagnetic (EM) decays of $D_{sJ}$ have also been measured [3]. The mixing angles are closely related to the EM decay rates of $D_{sJ}$ [15].

The purpose of this paper is to examine the spin structure of the four mesons. We use the constituent quark model with the interquark interactions that arise from the nonrelativistic expansion of the QCD-inspired Fermi-Breit interaction. We have five types of interactions in the following sense. In addition to the spin-independent interaction that consists of a confining potential and the color Coulomb interaction, we have four types of spin-dependent interactions. They are the spin-spin, tensor, and two types of spin-orbit interactions, on which we elaborate in the next paragraph. The model is the same as the one used by Godfrey *et al.* [16,17,20] except that we do not assume any explicit forms for the interactions as functions of the distance between the two quarks. We treat all spin-dependent interactions perturbatively.

By the two types of spin-orbit interactions, we mean the ones that are symmetric and antisymmetric with respect to the interchange of the two quarks. We refer to the former as SLS and the latter as ASLS interactions. The SLS interaction commutes with the total spin of the two quarks, whereas ASLS interaction does not. The ASLS interaction violates the conservation of the total spin. This is the agent that induces the mixing of ${}^1P_1$ and ${}^3P_1$. The ASLS interaction is proportional to the mass difference between the quarks. Hence, its effect can be substantial when the mass difference is large, leading to a specific amount of mixing in the heavy quark limit. This is indeed the case with the $c\bar{s}$ (or $s\bar{c}$) system as we will see. Historically, ASLS interaction effects were first examined for the $\Lambda$-$N$ interaction and hypernuclei [22–24]. Regarding the particular roles of spin-orbit interactions in $q\bar{Q}$ systems, we refer to a series of works by Schnitzer [25] and the work by Cahn and Jackson [26] in addition to those cited already [16,17,20].

As we said above, we have five types of interactions. On the other hand, there are five pieces of experimental data now

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[1] We use the ordinary spectroscopic notation ${}^{2S+1}L_J$ that is used for a two-particle system, where $S$, $L$, and $J$ are total spin, orbital angular momentum, and total angular momentum quantum numbers, respectively.

 

TABLE I. Summary of observed charmed-strange mesons.

| Label | Mass (MeV) | Assignment ($^{2S+1}L_J$) | Year of discovery |
|---|---|---|---|
| $D_s^{\pm}$ | 1968.3 $\pm$ 0.5 | $^1S_0$ | 1983 [28] |
| $D_s^{*\pm}$ | 2112.1 $\pm$ 0.7 | Probably $^3S_1$ | 1987 [29] |
| $D_{sJ}^*(2317)^{\pm}$ | 2317.4 $\pm$ 0.9 | Probably $^3P_0$ | 2003 [3] |
| $D_{sJ}(2460)^{\pm}$ | 2459.3 $\pm$ 1.3 | ? | 2003 [3] |
| $D_{s1}(2536)^{\pm}$ | 2535.35 $\pm$ 0.34 | ? | 1989 [30] |
| $D_{s2}(2573)^{\pm}$ | 2572.4 $\pm$ 1.5 | Probably $^3P_2$ | 1994 [31] |

available, which are the masses of the four mesons and the branching ratio of the EM decays. [See Eq. (29).] The matrix elements of the five interactions (within the $P$-state sector) can be determined such that the five pieces of the experimental data are reproduced. At the same time, the spin structure of the four mesons can be determined. In doing so, we do not have to know the radial dependence of the interactions. As it turns out, the spin-spin interaction, among the four types of spin-dependent interactions, plays a particularly interesting role in relation to the spin structure of $D_{s1}(2536)$ and $D_{sJ}(2460)$.

We begin Sec. II by defining a nonrelativistic model Hamiltonian that incorporates relativistic corrections as various spin-dependent interactions and proceed to determining the matrix elements of the interactions by using the mass spectra of the $L = 1$ quartet of charmed-strange mesons and the EM decay widths of $D_{sJ}(2460)$. In Sec. III, we remark on the approximations that we use. Discussions and a summary are given in the last section. In Table I we list the observed charmed-strange mesons that we consider in this paper [27].

## II. HAMILTONIAN AND MIXING ANGLE

We assume that the nonrelativistic scheme is appropriate for the system, and relativistic corrections can be treated as first-order perturbation. The nonrelativistic expansion of the Fermi-Breit interaction gives us the Hamiltonian for a charmed-strange meson in the form of

$$\begin{aligned} H = H_0 + \boldsymbol{S}_s \cdot \boldsymbol{S}_c V_S(r) + S_{12} V_T(r) \\ + \boldsymbol{L} \cdot \boldsymbol{S} V_{LS}^{(+)}(r) + \boldsymbol{L} \cdot (\boldsymbol{S}_s - \boldsymbol{S}_c) V_{LS}^{(-)}(r), \end{aligned} \tag{1}$$

where $\boldsymbol{S}_i$ is the spin operator of the strange quark when $i = s$ and of the charmed quark when $i = c$, $\boldsymbol{S} = \boldsymbol{S}_s + \boldsymbol{S}_c$, $S_{12}$ is the tensor operator, and $\boldsymbol{L}$ the orbital angular momentum operator. The lowest-order terms in the nonrelativistic expansion are all in $H_0$ which also contains a phenomenological potential to confine the quarks. More explicitly, $H_0$ reads as

$$H_0 = m_s + m_c + \frac{\boldsymbol{p}_s^2}{2m_s} + \frac{\boldsymbol{p}_c^2}{2m_c} + V_C(r) + V_{\text{conf}}(r), \tag{2}$$

where $m_i$ and $\boldsymbol{p}_i$ are the mass and momentum of quark $i$, respectively, $V_C$ is the color Coulomb interaction, and $V_{\text{conf}}$ is the confinement potential. The last two terms of Eq. (1) are the SLS and ASLS interactions, respectively. The spatial functions attached to the operators in Eq. (1) can be expressed in terms of $V_C$ and $V_{\text{conf}}$ [17,32]. However, we do not need such explicit expressions of these functions, as it will become clear shortly.

We start with the eigenstates of $H_0$ such that

$$H_0 \psi_{nJLS}(\boldsymbol{r}) = E_{nL}^{(0)} \psi_{nJLS}(\boldsymbol{r}), \tag{3}$$

where

$$\psi_{nJLS}(\boldsymbol{r}) = R_{nL}(r) \sum_{M=-J}^{J} C_M \mathcal{Y}_{JLS}^M(\theta, \phi). \tag{4}$$

Here $C_M$ are constants such that $\sum_M |C_M|^2 = 1$ and can be chosen as $(2J+1)^{-1/2}$ since there is no preferable direction. We concentrate on the $P$ states of $n = 1$ with no radial node. We denote each of the $L = 1$ states with single index $\nu$ according to

$$\nu = \begin{cases} 1 \\ 2 \\ 3 \\ 4 \end{cases} \quad \text{corresponding to} \quad \begin{cases} ^1P_1 \\ ^3P_0 \\ ^3P_1 \\ ^3P_2 \end{cases}. \tag{5}$$

Next we calculate the matrix elements of $H$ in terms of the bases defined by Eqs. (3) and (4). Nonvanishing matrix elements are

$$\begin{aligned} H_{11} &= M_0 - \tfrac{3}{4} v_S, \\ H_{22} &= M_0 + \tfrac{1}{4} v_S - 2v_{LS} - 4v_T, \\ H_{33} &= M_0 + \tfrac{1}{4} v_S - v_{LS} + 2v_T, \\ H_{44} &= M_0 + \tfrac{1}{4} v_S + v_{LS} - \tfrac{2}{5} v_T, \\ H_{13} &= H_{31} = \sqrt{2}\Delta, \end{aligned} \tag{6}$$

where

$$M_0 = \int d^3r\ \psi_{J1S}^*(\boldsymbol{r}) H_0 \psi_{J1S}(\boldsymbol{r}) = E_1^{(0)}, \tag{7}$$

$$v_S = \int_0^\infty dr r^2 V_S(r) R_1^2(r), \tag{8}$$

$$v_{LS} = \int_0^\infty dr r^2 V_{LS}^{(+)}(r) R_1^2(r), \tag{9}$$

$$v_T = \int_0^\infty dr r^2 V_T(r) R_1^2(r), \tag{10}$$

$$\Delta = \int_0^\infty dr r^2 V_{LS}^{(-)}(r) R_1^2(r). \tag{11}$$

We choose the phases of the wave functions involved in Eq. (11) such that $\Delta$ is positive. Here we have suppressed suffix $n = 1$ of the wave functions and the unperturbed $P$-state energy. We have ignored the tensor coupling of the $^3P_2$ state to the $^3F_2$ state. We will remark on this point in the next section. Note that the ASLS interaction gives rise to $\Delta \neq 0$, which causes the mixing of $^1P_1$ and $^3P_1$.

All of the matrix elements of the Hamiltonian that we need are parametrized in terms $M_0$, $v_S$, $v_{LS}$, $v_T$, and $\Delta$. These five parameters can be determined by the four observed masses and the EM decay rates of $D_{sJ}(2460)$. We have no other adjustable parameters. In this context, we do not need explicit expressions of the radial wave function nor the radial dependence of the potential functions.

The diagonalization of $H$ leads to four states whose masses are given by

$$\mathcal{M}_+ = \tfrac{1}{2}\left[2M_0 - \tfrac{1}{2}v_S - v_{LS} + 2v_T + \{(v_{LS} - 2v_T - v_S)^2 + 8\Delta^2\}^{1/2}\right], \quad (12)$$

$$\mathcal{M}_2 = M_0 + \tfrac{1}{4}v_S - 2v_{LS} - 4v_T, \quad (13)$$

$$\mathcal{M}_- = \tfrac{1}{2}\left[2M_0 - \tfrac{1}{2}v_S - v_{LS} + 2v_T - \{(v_{LS} - 2v_T - v_S)^2 + 8\Delta^2\}^{1/2}\right], \quad (14)$$

$$\mathcal{M}_4 = M_0 + \tfrac{1}{4}v_S + v_{LS} - \tfrac{2}{5}v_T. \quad (15)$$

The second and fourth states with $\mathcal{M}_2$ and $\mathcal{M}_4$ are pure $^3P_0$ and $^3P_2$ states, respectively. We identify them with $D_{sJ}^*(2317)$ and $D_{s2}(2573)$. The other two states with $\mathcal{M}_+$ and $\mathcal{M}_-$ are composed of $^1P_1$ and $^3P_1$ states. We interpret them as $D_{s1}(2536)$ and $D_{sJ}(2460)$, respectively.

Let us introduce a mixing angle $\theta$ that represents the extent of the mixing of $^1P_1$ and $^3P_1$ states in $D_{s1}(2536)$ and $D_{sJ}(2460)$. Following Godfrey and Isgur [16], we define $\theta$ by

$$\begin{aligned}\psi_+(\boldsymbol{r}) &= -\psi_{110}(\boldsymbol{r})\sin\theta + \psi_{111}(\boldsymbol{r})\cos\theta,\\ \psi_-(\boldsymbol{r}) &= \psi_{110}(\boldsymbol{r})\cos\theta + \psi_{111}(\boldsymbol{r})\sin\theta,\end{aligned} \quad (16)$$

where $\psi_+$ and $\psi_-$ are the eigenstates that correspond to $D_{s1}(2536)$ and $D_{sJ}(2460)$, respectively. The requirement that the energy eigenvalues for $\psi_\pm$ are $\mathcal{M}_\pm$ leads to

$$\tan(2\theta) = -\frac{2\sqrt{2}\Delta}{v_S - v_{LS} + 2v_T}. \quad (17)$$

It is understood that $\theta$ lies in the interval of $-\pi/2 \leqslant \theta \leqslant 0$ so that it conforms to the sign convention used in Ref. [16]. Since $-\pi/4 \leqslant \theta \leqslant 0$ (or $-\pi/2 \leqslant \theta \leqslant -\pi/4$) if $(v_S - v_{LS} + 2v_T) \geqslant 0$ (or $\leqslant 0$), we have $\theta \to 0$ (or $\to -\pi/2$) as $\Delta \to 0$ if $(v_S - v_{LS} + 2v_T) \geqslant 0$ (or $\leqslant 0$). In other words, when $(v_S - v_{LS} + 2v_T) > 0$ (or $< 0$), $D_{s1}(2536)$ develops from the $^3P_1$ (or $^1P_1$) state, while $D_{sJ}(2460)$ develops from the $^1P_1$ (or $^3P_1$) state because of the ASLS interaction.

We can express the five parameters $M_0, v_S, v_{LS}, v_T$, and $\Delta$ in terms of the four observed masses and the mixing angle such that

$$M_0 = \tfrac{1}{4}\mathcal{M}_+ + \tfrac{1}{4}\mathcal{M}_- + \tfrac{1}{12}\mathcal{M}_2 + \tfrac{5}{12}\mathcal{M}_4, \quad (18)$$

$$v_S = -\tfrac{1}{3}(1 - 2\cos(2\theta))\mathcal{M}_+ - \tfrac{1}{3}(1 + 2\cos(2\theta))\mathcal{M}_- + \tfrac{1}{9}\mathcal{M}_2 + \tfrac{5}{9}\mathcal{M}_4, \quad (19)$$

$$v_{LS} = -\tfrac{1}{8}(1 + \cos(2\theta))\mathcal{M}_+ - \tfrac{1}{8}(1 - \cos(2\theta))\mathcal{M}_- - \tfrac{1}{6}\mathcal{M}_2 + \tfrac{5}{12}\mathcal{M}_4, \quad (20)$$

$$v_T = \tfrac{5}{48}(1 + \cos(2\theta))\mathcal{M}_+ + \tfrac{5}{48}(1 - \cos(2\theta))\mathcal{M}_- - \tfrac{5}{36}\mathcal{M}_2 - \tfrac{5}{72}\mathcal{M}_4, \quad (21)$$

$$\Delta = -\tfrac{1}{2\sqrt{2}}(M_+ - M_-)\sin(2\theta). \quad (22)$$

Equation (18) states that the mass of the center of gravity of the $l = 1$ quartet is free from the spin-dependent interactions involved in Eq. (1) in the lowest-order perturbation scheme.

In order to determine the mixing angle, we consider EM decays of $D_{sJ}(2460)$ to $D_s$ and $D_s^*$. Generally the $E1$ decay width of a meson composed of quark 1 and antiquark 2 is given by

$$\Gamma(i \to f + \gamma) = \frac{4e_Q^2}{27}\, k^3(2J_f + 1)\,|\langle f\,|r|\,i\rangle|^2\, S_{if}, \quad (23)$$

where $e_Q$ is the effective charge defined by

$$e_Q = \frac{m_1 e_2 - m_2 e_1}{m_1 + m_2}, \quad (24)$$

$k$ is the momentum of the emitted photon

$$k = \frac{M_i^2 - M_f^2}{2M_i}, \quad (25)$$

and

$$S_{if} = \begin{cases} 1 \text{ for a transition between triplet states,}\\ 3 \text{ for a transition between singlet states,}\end{cases} \quad (26)$$

is a statistical factor [33]. For the decays of $D_{sJ}$, we have

$$k = \begin{cases} 322.7\text{ MeV} & \text{for the decay to } D_s^*,\\ 442.0\text{ MeV} & \text{for the decay to } D_s,\end{cases} \quad (27)$$

and $(2J_f + 1)S_{if} = 3$ for both cases. Since only the $^3P_1$ state in $D_{sJ}$ undergoes the transition to $D_s^*$ and only the $^1P_1$ state to $D_s$, the matrix element $\langle f|r|i\rangle$ is proportional to $\sin\theta$ for the decay to $D_s^*$ and to $\cos\theta$ for the decay to $D_s$ [15]. Thus we obtain

$$\frac{\Gamma(D_{sJ} \to D_s^*\gamma)}{\Gamma(D_{sJ} \to D_s\gamma)} = \left(\frac{322.7}{442.0}\right)^3 \tan^2\theta. \quad (28)$$

The Belle Collaboration made the first observation of $B \to \bar{D}D_{sJ}$ decays and reported the branching fractions for $B \to \bar{D}D_{sJ}$ followed by the EM decays of $D_{sJ}$ [3]. Colangelo *et al.* analyzed the data to extract the ratio of branching fractions for the EM decays of $D_{sJ}(2460)$ to $D_s$ and $D_s^*$ [34]. They obtained

$$R_{\text{exp}} \equiv \left[\frac{\Gamma(D_{sJ} \to D_s^*\gamma)}{\Gamma(D_{sJ} \to D_s\gamma)}\right]_{\text{exp}} = 0.40 \pm 0.28. \quad (29)$$

The experimental value has the large statistical errors which results in a large uncertainty in determining the mixing angle as can be seen in Fig. 1. The numerical value is

$$\theta = -45.4^{\circ}{}^{-7.5^\circ}_{+16.4^\circ}, \quad (30)$$

where the upper and lower increments are due to the positive and negative corrections of the statistical errors in $R_{\text{exp}}$, respectively. This may be compared with $-38^\circ$ obtained by Godfrey and Kokoski [17], and $-54.7^\circ$ that emerges from $\sin\theta = -\sqrt{2/3}$ in the heavy quark limit [15,20].

We can calculate $M_0, v_S, v_{LS}, v_T$, and $\Delta$ through Eqs. (18)–(22) by fitting the observed masses of Table I and the mixing angle of Eq. (30). Again these quantities are subject to uncertainties due to the statistical errors. Using the central values of the observed masses, we obtain

$$M_0 = 2513.6\text{ MeV}, \quad (31)$$

$$v_S = 21.0^{-13.1}_{+27.5}\text{ MeV}, \quad (32)$$

$$v_{LS} = 61.4^{+2.5}_{-5.2}\text{ MeV}, \quad (33)$$

$$v_T = 19.7^{-2.1}_{+4.3}\text{ MeV}, \quad (34)$$

$$\Delta = 26.9^{-1.0}_{-4.1}\text{ MeV}. \quad (35)$$

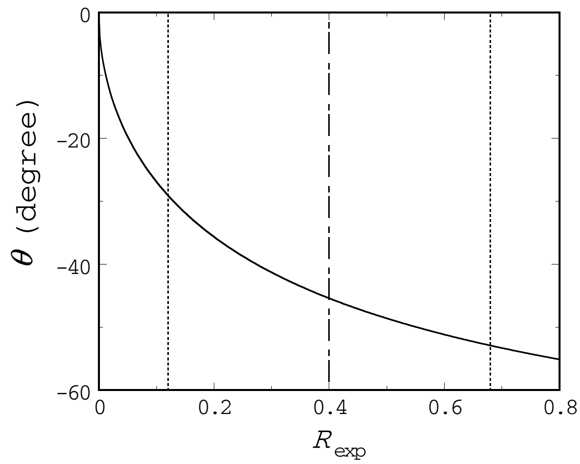


FIG. 1. Variation of the mixing angle with $R_{\rm exp}$. The dot-dashed line shows the value obtained from the central value of $R_{\rm exp}$, and the vertical dotted lines indicate the upper and lower values of $R_{\rm exp}$ allowed within the statistical errors.

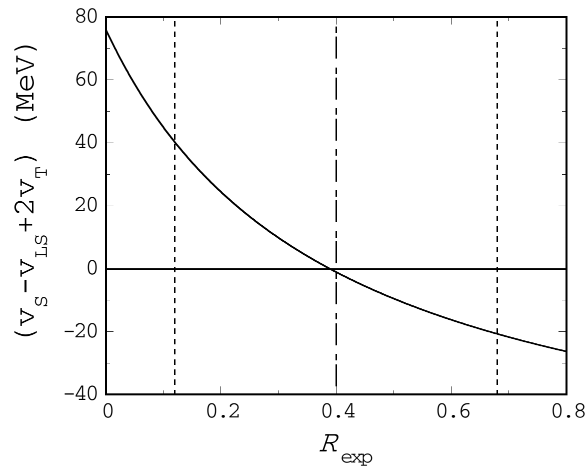


FIG. 3. Matrix element $(v_S - v_{LS} + 2v_T)$ vs $R_{\rm exp}$. The value at which the matrix element changes its sign is 0.39. The dot-dashed line shows the value obtained from the central value of $R_{\rm exp}$, and the vertical dotted lines indicate the upper and lower values of $R_{\rm exp}$ allowed within the statistical errors.

In Fig. 2 we show how the matrix elements vary when $R_{\rm exp}$ is varied within the statistical errors. Note that the matrix element of the spin-spin interaction is particularly sensitive to the variation of $R_{\rm exp}$. Since the sign of $(v_S - v_{LS} + 2v_T)$ determines the main spin states of $\mathcal{M}_\pm$, it is interesting to see the $R_{\rm exp}$ dependence of this quantity shown in Fig. 3. We see that $(v_S - v_{LS} + 2v_T)$ changes its sign from positive to negative as $R_{\rm exp}$ passes over 0.39. If $R_{\rm exp} < 0.39$ the main spin states of $D_{s1}(2536)$ and $D_{sJ}(2460)$ are, respectively, $^3P_1$ and $^1P_1$. If $R_{\rm exp}$ exceeds 0.39, these two spin states are interchanged.

In the nonrelativistic expansion of the Fermi-Breit interaction, the spin-spin interaction contains the derivative of the color Coulomb interaction. If the color Coulomb interaction is of the form of $1/r$, the spin-spin interaction behaves like the $\delta$ function near the origin. In that case, the matrix element of the spin-spin interaction will vanish in $P$ states because the $P$-state wave functions are strongly suppressed where the interaction acts. The real situation, however, is not so simple. The singular spin-dependent interactions are smeared out because of the relativistic corrections [16,17] and the asymptotic freedom. The resultant spin-spin interaction will have a well-behaved form at the origin. Consequently, the matrix element of the spin-spin interaction can become sizable. Its magnitude depends on the spatial form of the interaction which in turn depends on how one incorporates the relativistic corrections and the asymptotic freedom. Equation (32) is a constraint that the spin-spin interaction has to satisfy.

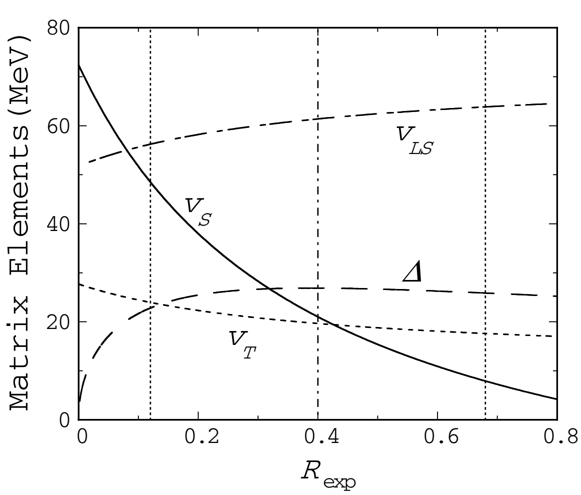


FIG. 2. Matrix elements calculated by applying the experimental value of $R_{\rm exp}$ from Eqs. (18)–(22) with Eq. (28). The dot-dashed line shows the value obtained from the central value of $R_{\rm exp}$, and the vertical dotted lines indicate the upper and lower values of $R_{\rm exp}$ allowed within the statistical errors.

Earlier we had experimental information on the effect of the spin-spin interaction on $P$ states of heavy quark systems only from the charmonia. In first-order perturbation theory, we can estimate the matrix element by calculating the difference between a weighted average of the masses of $^3P$ states and the mass of $^1P$ state. [See Eq. (6).] For the $c\bar{c}$ system, if we can regard $h_c(1P)$ as the $^1P_0$ state [27], we obtain $-0.85$ MeV for this quantity. If one assumes that the spin-spin interaction is inversely proportional to the product of the quark masses and that the wave functions of the $c\bar{c}$ system and those of the charmed-strange mesons are the same, one obtains about $-3$ MeV for the charmed-strange mesons. The value that emerged from our analysis is much larger in magnitude than this value.

Let us remark on the works of Godfrey and Isgur [16] and of Godfrey and Kokoski [17] in comparison with the present work. They used basically the same Hamiltonian that we used and diagonalized it on the basis of the harmonic oscillator eigenstates. They assumed explicit forms for the confinement potential and the color Coulomb interaction in terms of which the spatial behavior of all spin-dependent interactions can be expressed. They accomplished the relativistic corrections by

TABLE II. Matrix elements, mixing angle $\theta$, and masses of the *P*-state charmed-strange mesons in the columns with the corresponding meson symbols. They are given in MeV except for $\theta$. In the first row, the central values of the masses reported by the Particle Data Group [27] are listed and we used them to obtain the matrix elements. Mixing angle was given by Eq. (30) with the statistical errors suppressed. Values of the masses and mixing angles in the second and fourth rows are predictions by the indicated authors. Matrix elements in each row were calculated by substituting these quantities into Eqs. (18)–(22). Numbers in parentheses in the third row are the matrix elements obtained in Ref. [17].

| | $M_0$ | $v_s$ | $v_{LS}$ | $v_T$ | $\Delta$ | $\theta$ | $D_{sJ}^*$ | $D_{sJ}$ | $D_{s1}$ | $D_{s2}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| This work | 2513.6 | 21.0 | 61.4 | 19.7 | 26.9 | $-45.4^\circ$ | 2317.4 | 2459.3 | 2535.35 | 2572.4 |
| Godfrey and Kokoski [17] | 2563 | 13 | 27 | 8 | 3 | $-38^\circ$ | 2480 | 2550 | 2560 | 2590 |
| | (2564) | (15) | (27) | (7) | (3) | | | | | |
| Lucha and Schöberl [21] | 2531 | 14 | 29 | 8 | 4 | $-44.7^\circ$ | 2446 | 2515 | 2517 | 2561 |

introducing a smearing function which softens the singular behavior of the spin-dependent interactions at the origin. As a consequence, a sizable contribution from the spin-spin interaction to the matrix element for the *P* state emerged. They fixed the parameters by fitting observed meson masses known then and predicted unobserved meson masses. Although they worked beyond the perturbation theory, we thought it would be interesting to estimate the matrix elements of the spin-dependent interactions perturbatively through Eqs. (18)–(22) from the masses and the mixing angles that they obtained for charmed-strange mesons.

The results are summarized in Table II and compared with preceding works by Godfrey and Kokoski [17] and Lucha and Schöberl [21]. The last four numbers under the meson symbols in the first row are the observed masses that we used to evaluate the matrix elements in our analysis. The last four numbers in the other rows are the predicted masses. The values in parentheses in the third row are the matrix elements obtained nonperturbatively in Ref. [17]. Note that these values are very close to the corresponding ones of the second row, showing that our perturbative treatment is adequate.

The masses of $D_{sJ}^*$ predicted in Refs. [17] and [21] are much larger than the experimental value. The matrix elements of the SLS and tensor interactions come into the masses of $D_{sJ}^*$ with a negative sign as seen in Eq. (13). In Refs. [17] and [21] the magnitudes of these matrix elements are very small compared with the ones that the experiments require. This is why they had approximately 110–120 MeV larger masses for $D_{sJ}^*$ compared with the experimental value even when one corrects the overestimate of the center of gravity $M_0$ for the *P*-state masses. On the other hand, the matrix elements of the SLS and tensor interactions come with opposite signs for the mass of $D_{s2}$. This moderates the overestimate of the mass of $D_{s2}$. The mass differences between $D_{s1}$ and $D_{sJ}$ in Refs. [17] and [21] are very small as compared with 76 MeV of the experimental value. This is simply due to the feature that the values of $\Delta$ of Refs. [17] and [21] are much smaller than the one implied by the experiments.

## III. VALIDITY OF THE APPROXIMATIONS USED

Let us now discuss the approximations used in Sec. II. First, we regarded all spin-dependent interactions as perturbation and obtained their matrix elements as given in Eqs. (32)–(35). A typical mass difference $\Delta M_0$ between two consecutive principal states that emerges from $H_0$ is probably 400–500 MeV. The values of $v_S$, $v_{LS}$, $v_T$, and $\Delta$ are much smaller than $\Delta M_0$. This justifies our perturbative treatment of the spin-dependent interactions.

Secondly, we ignored the tensor coupling of the $^3P_2$ state to the $^3F_2$ state. The nonvanishing matrix element of the tensor interaction between these states gives rise to an additive correction to $H_{44}$ in Eq. (6) through the second-order perturbation. Let us estimate the second-order correction. Note that a quark in a state with $L \geqslant 1$ feels the color Coulomb interaction much less than a quark in an *S* state. This is because the wave function of the former is much suppressed near the origin as compared with the wave function of the latter. Therefore, the *P* and *F* state wave functions are not very different from those emerging from the confinement potential alone. Let us ignore the color Coulomb interaction in obtaining the *P* and *F* state wave functions and use the harmonic oscillator potential for $V_{\rm conf}$. Then the radial parts of nodeless *P* and *F* state wave functions are given by

$$R_1(r) = \sqrt{\frac{8}{3}}\left[\frac{(\mu\omega)^5}{\pi}\right]^{1/4} r\, e^{-\mu\omega r^2/2}, \tag{36}$$

$$R_3(r) = \sqrt{\frac{32}{105}}\left[\frac{(\mu\omega)^9}{\pi}\right]^{1/4} r^3\, e^{-\mu\omega r^2/2}, \tag{37}$$

where $\omega$ is an oscillator constant and $\mu = (1/m_s + 1/m_c)^{-1}$ the reduced mass. Remember that $\omega$ is related to the mass difference between two consecutive principal states and, hence, $\omega \approx \Delta M_0$. Since the tensor interaction can be expressed as

$$V_T(r) = \frac{V_C'(r) - r V_C''(r)}{12 m_s m_c r} \tag{38}$$

in terms of the color Coulomb interaction, the needed diagonal and off-diagonal matrix elements are given by

$$\langle {}^3P_2 | S_{12} V_T(r) | {}^3P_2 \rangle = -\frac{8}{45}\sqrt{\frac{(\mu\omega)^3}{\pi}}\frac{\alpha}{m_s m_c}, \tag{39}$$

$$\langle {}^3P_2 | S_{12} V_T(r) | {}^3F_2 \rangle = \frac{16}{15}\sqrt{\frac{6}{35}}\sqrt{\frac{(\mu\omega)^3}{\pi}}\frac{\alpha}{m_s m_c}, \tag{40}$$

where we used

$$V_C(r) = -\frac{4}{3}\frac{\alpha}{r} \tag{41}$$

with the strong coupling constant $\alpha$. Thus we obtain

$$\frac{\langle {}^3P_2 | S_{12} V_T(r) | {}^3F_2 \rangle}{\langle {}^3P_2 | S_{12} V_T(r) | {}^3P_2 \rangle} = -6\sqrt{\frac{6}{35}} \approx -2.5. \tag{42}$$

If we equate the denominator to the matrix element of the tensor operator times the quantity given in Eq. (34), that is,

$$\langle {}^3P_2 | S_{12} V_T(r) | {}^3P_2 \rangle \approx -8 \text{ MeV}, \tag{43}$$

an approximate magnitude of the off-diagonal element becomes

$$\langle {}^3P_2 | S_{12} V_T(r) | {}^3F_2 \rangle \approx 20 \text{ MeV}. \tag{44}$$

Since the energy difference between the ${}^3P_2$ and ${}^3F_2$ states is approximately $2\omega \approx 1$ GeV, the second-order correction will be about 0.4 MeV, that is, about 5% of the diagonal element for the ${}^3P_2$ state. Thus we conclude that the tensor coupling to the ${}^3F_2$ state will not appreciably change our result.

## IV. DISCUSSIONS AND SUMMARY

We examined the $P$-state charmed-strange mesons, focusing on the mixing of ${}^1P_1$ and ${}^3P_1$ states in $D_{s1}(2536)$ and $D_{sJ}(2460)$ that is caused by the antisymmetric spin-orbit (ASLS) interaction. We treated the spin-dependent interactions that arise from the nonrelativistic expansion of the Fermi-Breit interaction perturbatively. We did not assume any explicit forms for the interactions as functions of the interquark distance. We expressed the matrix elements of these interactions in terms of the observed masses of the $P$-state quartet and the mixing angle determined from the EM decay rates of $D_{sJ}(2460)$.

The EM decay rates have large statistical errors. If we vary the decay rates within the errors, the mixing angle varies widely. The matrix elements of the spin-dependent interactions also vary accordingly. The matrix elements of the SLS, tensor, and ASLS interactions are relatively stable with the variation of the mixing angle, varying only within 20%. On the other hand, the matrix element of the spin-spin interaction varies from 48.5 to 7.9 MeV when the mixing angle varies from one end to the other as determined from the EM decay rates with the statistical errors. Note that Godfrey and Kokoski obtained for the matrix element of the spin-spin interaction 15 MeV, which lies in this interval [17].

The matrix element of the spin-spin interaction is particularly sensitive to the mixing angle and of crucial importance in determining the dominant states of $D_{s1}(2536)$ and $D_{sJ}(2460)$. With the large variation in the mixing angle, the dominant state of $D_{s1}(2536)$ is transferred from the ${}^3P_1$ to the ${}^1P_1$ state and that of $D_{sJ}(2460)$ from the ${}^1P_1$ to the ${}^3P_1$ state. This implies that the spin-spin interaction is the most important among the spin-dependent interactions for the determination of the dominant states in $D_{s1}(2536)$ and $D_{sJ}(2460)$. It will be crucial to their assignments, provided that other mechanisms for the mixing such as the coupling to decay channels are less significant than what we have discussed [35–37].

Our analysis is based on the branching fractions that were obtained from the first observation of $B \to \bar{D}D_{sJ}$ decays by the Belle Collaboration. The analyses are accompanied by large statistical errors, and so are the mixing angles that are extracted from the branching fractions. For further discussion of the relationship between the mixing angle and the spin-dependent interactions, we need more refined data on the branching fractions from the experimental groups.

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