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Gauge invariance in two-particle scattering

B. Borasoy,[1,*] P. C. Bruns,[1,†] U.-G. Meißner,[1,2,‡] and R. Nißler[1,§]
[1]*Helmholtz-Institut für Strahlen- und Kernphysik (Theorie), Universität Bonn, Nußallee 14-16, D-53115 Bonn, Germany*
[2]*Institut für Kernphysik (Theorie), Forschungszentrum Jülich, D-52425 Jülich, Germany*
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It is shown how gauge invariance is obtained for the coupling of a photon to a two-body state described by the solution of the Bethe-Salpeter equation. This is illustrated both for a complex scalar field theory and for interaction kernels derived from chiral effective Lagrangians.

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I. INTRODUCTION

Chiral perturbation theory provides an appropriate framework for studying hadronic processes at low energies [1]. In chiral perturbation theory (ChPT) the most general effective Lagrangian incorporating the same symmetries and symmetry breaking patterns as the underlying theory, QCD, is formulated in terms of the relevant degrees of freedom, i.e., mesons and baryons. Moreover, external fields representing, e.g., external photons are included in a gauge-invariant fashion. The Green's functions are then expanded perturbatively in powers of Goldstone boson masses and small three-momenta. By employing a regularization scheme that respects chiral symmetry, gauge invariance is maintained at every order in the loop expansion of ChPT.

However, the systematic loop expansion involves a characteristic scale $\Lambda_\chi \simeq 4\pi F_\pi \approx 1.2$ GeV at which the chiral series is expected to break down and the limitation to very low-energy processes is even enhanced in the vicinity of resonances. The appearance of resonances in certain channels constitutes a major problem for the loop expansion of ChPT, because a resonance cannot be reproduced at any given order of the chiral series. Nevertheless, at low energies the contribution from such resonances is encoded in the numerical values of certain low-energy constants (frequently called resonance saturation).

Recently, considerable effort has been undertaken to combine the effective chiral Lagrangian approach with nonperturbative methods, both in the meson-baryon sector [2–4] and in the purely mesonic sector [5]. The combination with nonperturbative schemes has made it possible to go to energies beyond Λ_χ and to generate resonances dynamically (giving up, however, certain aspects of the rigorous framework constituted by ChPT). Two prominent examples in the baryonic sector are the $\Lambda(1405)$ and the $S_{11}(1535)$. The first one is an s-wave resonance just below the $K^- p$ threshold and dominates the interaction of the $\bar{K}N$ system, whereas the $S_{11}(1535)$ was identified in Ref. [2] as a quasibound $K\Lambda$-$K\Sigma$ state.

Such chiral unitary approaches have been extended to photo- and electroproduction processes of mesons on baryons, see, e.g. [6–8]. In these coupled-channel models the initial photon scatters with the incoming baryon into a meson-baryon pair that in turn rescatters (elastically or inelastically) an arbitrary number of times. The two-body final state interactions are taken into account in a coupled-channels Bethe-Salpeter equation (BSE) or—in the nonrelativistic framework—Lippmann-Schwinger equation. The coupling of the incoming photon to other possible vertices is omitted, and although these approaches appear to describe the available data well, the issue of gauge invariance is not discussed in these chiral unitary approaches. Conversely, a method to obtain conservation of the electromagnetic current of a two-nucleon system is presented in Ref. [9] and extended to a resonance model for pion photoproduction in Ref. [10]. Alternatively, the so-called gauging of equations method has been developed in Refs. [11,12] to incorporate an external electromagnetic field in the integral equation of a few-body system in a gauge-invariant fashion. Further investigations of gauge invariance in pion photoproduction within πN models can be found in Refs. [13,14]. Gauge invariance is also of interest in coupled-channels approaches in the mesonic sector, e.g., in radiative ϕ, ρ decays [15,16] and in the anomalous decays $\eta, \eta' \to \gamma\gamma, \pi^+\pi^-\gamma$ [17,18]. For related recent work, see also Ref. [19].

The purpose of the present work is to illustrate how gauge invariance can be maintained when an external photon couples to a two-particle state described by the BSE within the chiral effective framework. We start in the next section by first outlining the procedure for a simple scalar field theory. With the insights gained from this example we can then address gauge invariance in meson-baryon scattering processes with chiral effective Lagrangians. In Sec. III we discuss the case with the interaction kernel of the BSE derived from the leading order Lagrangian—the Weinberg-Tomozawa term. The extension to driving terms from higher chiral orders is presented in Sec. IV. In Sec. V it is shown that the corresponding amplitudes also satisfy unitarity constraints. Our conclusions and outlook are presented in Sec. VI.

II. COMPLEX SCALAR FIELDS

In this section, we demonstrate in a simple field theory with complex scalar fields that the coupling of an external photon to a two-body state, which corresponds to the solution of the BSE,

*Electronic address: borasoy@itkp.uni-bonn.de
†Electronic address: bruns@itkp.uni-bonn.de
‡Electronic address: meissner@itkp.uni-bonn.de
§Electronic address: rnissler@itkp.uni-bonn.de

is gauge invariant. To this end, consider the (normal-ordered) Lagrangian for complex fields ϕ and ψ

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - m^2 \phi^* \phi + \partial_\mu \psi^* \partial^\mu \psi - M^2 \psi^* \psi - g(\phi^*\phi)(\psi^*\psi), \tag{1}$$

with masses m and M, respectively. For small values of the coupling constant g, the scattering process $\phi(p_1)\psi(p_2) \to \phi(p_3)\psi(p_4)$ may be calculated perturbatively. For general values of g, however, and if one is interested in bound states, one must resort to nonperturbative techniques such as the BSE. The Bethe-Salpeter equation for the scattering matrix T of the two-particle scattering process can be written as follows:

$$T(s) = g + gG(s)T(s), \tag{2}$$

where $s = p^2 = (p_1 + p_2)^2 = (p_3 + p_4)^2$ and G is the scalar loop integral

$$G(p^2) = i \int_l \Delta_\phi(l)\Delta_\psi(p + l) \tag{3}$$

utilizing the short-hand notation

$$\int_l = \int \frac{d^4 l}{(2\pi)^4} \tag{4}$$

and the propagators

$$i\Delta_\phi(l) = \frac{i}{l^2 - m^2}, \qquad i\Delta_\psi(l) = \frac{i}{l^2 - M^2}. \tag{5}$$

The solution of the BSE is given by the following:

$$T(s) = \frac{g}{1 - gG(s)} = g + gG(s)g + gG(s)gG(s)g + \ldots, \tag{6}$$

which amounts to the summation of an infinite series of s-channel loops, the so-called bubble chain (or bubble sum). From inversion of Eq. (6) it immediately follows that

$$\mathrm{Im}\, T^{-1} = -\mathrm{Im}\, G = \frac{|\boldsymbol{q}_{\mathrm{cm}}|}{8\pi\sqrt{s}} \theta[s - (m + M)^2], \tag{7}$$

where the second relation is deduced from unitarity of the amplitude T and q_{cm} is the three-momentum in the center-of-mass frame. The explicit calculation of the scalar loop integral G, e.g., in dimensional regularization, indeed confirms the above relation.

Let us now turn to the coupling of an external photon field to the solution of the BSE. By minimal substitution

$$\begin{aligned} \partial_\mu &\to \nabla_\mu = \partial_\mu + ie_\phi \mathcal{A}_\mu \quad \text{for } \phi, \\ \partial_\mu &\to \nabla_\mu = \partial_\mu + ie_\psi \mathcal{A}_\mu \quad \text{for } \psi, \end{aligned} \tag{8}$$

where $\mathcal{A}_\mu$ is the photon field and $e_\phi(e_\psi)$ the charge of the meson $\phi(\psi)$, one obtains a locally gauge-invariant Lagrangian. The coupling of the photon with incoming momentum k to the four external legs of a bubble chain leads to the following amplitudes:

$$\begin{aligned} T_1^\mu &= e_\phi T(s')\Delta(p_1 + k)(2p_1 + k)^\mu, \\ T_2^\mu &= e_\psi T(s')\Delta(p_2 + k)(2p_2 + k)^\mu, \\ T_3^\mu &= e_\phi (2p_3 - k)^\mu \Delta(p_3 - k) T(s), \\ T_4^\mu &= e_\psi (2p_4 - k)^\mu \Delta(p_4 - k) T(s), \end{aligned} \tag{9}$$

where $s' = (p + k)^2$. Multiplying these contributions with the four-momentum of the photon, k_μ, and setting the external legs on-shell yields the following:

$$(e_\phi + e_\psi)[T(s') - T(s)], \tag{10}$$

which in general does not vanish. This underlines that, to achieve gauge invariance, it is not sufficient to couple the photon only to external legs. One rather has to include contributions that arise because of the coupling of the photon to intermediate states within the bubble chain, leading to the additional contributions

$$\begin{aligned} T_5^\mu &= ie_\phi T(s') \int_l \Delta_\phi(l + k)(2l + k)^\mu \Delta_\phi(l)\, \Delta_\psi(p - l) T(s), \\ T_6^\mu &= ie_\psi T(s') \int_l \Delta_\psi(l + k)(2l + k)^\mu \Delta_\psi(l)\, \Delta_\phi(p - l) T(s). \end{aligned} \tag{11}$$

By employing the Ward-Takahashi identities

$$k_\mu (2l + k)^\mu = \Delta^{-1}_{\phi/\psi}(l + k) - \Delta^{-1}_{\phi/\psi}(l) \tag{12}$$

it is straightforward to show that

$$k_\mu \left(T_5^\mu + T_6^\mu\right) = (e_\phi + e_\psi)T(s')[G(s) - G(s')]T(s). \tag{13}$$

The last expression can be rewritten by making use of the BSE

$$\begin{aligned} T(s')&[G(s) - G(s')]T(s) \\ &= T(s')[g^{-1}T(s) - 1] - [T(s')g^{-1} - 1]T(s) \\ &= T(s) - T(s'). \end{aligned} \tag{14}$$

Adding up all contributions we arrive at

$$k_\mu \sum_{i=1}^{6} T_i^\mu = 0, \tag{15}$$

which confirms the gauge invariance of the photon coupling to the bubble chain.

At the same time, insertion of the photon coupling at all possible places in the bubble chain guarantees unitarity of the scattering matrix up to radiative corrections of order $\mathcal{O}(e^3)$. To this end, we remark that in the transition $\phi\psi\gamma \to \phi\psi$ we can restrict ourselves to the states $|\phi\psi\rangle$ and $|\phi\psi\gamma\rangle$. Unitarity of the $\mathcal{S}$ matrix, $\mathcal{S} = 1 - i\mathcal{T}$, implies then

$$\begin{aligned} \langle\phi\psi|\mathcal{T} - \mathcal{T}^\dagger|\phi\psi\gamma\rangle = -i \int_{\mathrm{PS}} \{&\langle\phi\psi|\mathcal{T}^\dagger|\phi'\psi'\rangle\langle\phi'\psi'|\mathcal{T}|\phi\psi\gamma\rangle \\ &+ \langle\phi\psi|\mathcal{T}^\dagger|\phi'\psi'\gamma'\rangle\langle\phi'\psi'\gamma'|\mathcal{T}|\phi\psi\gamma\rangle\}, \end{aligned} \tag{16}$$

where $\int_{\mathrm{PS}}$ denotes the phase-space integral for the set of intermediate states $|\phi'\psi'\rangle$ and $|\phi'\psi'\gamma'\rangle$. We have introduced a superscript for the intermediate particles with running three-momenta, ϕ', ψ', and γ', to distinguish them from the external particles ϕ, ψ, and γ with fixed momenta. Note that the last matrix element $\langle\phi'\psi'\gamma'|\mathcal{T}|\phi\psi\gamma\rangle$ contains a disconnected piece of order $\mathcal{O}(e^0)$ in which the photon does not couple to the bubble chain and thus appears in the $\mathcal{O}(e)$ part of the unitarity relation. The remaining connected diagrams of this matrix element that contain the coupling of the photon to the bubble chain are of order $\mathcal{O}(e^2)$ and are neglected in the following. By making use of the symmetry $\langle i|\mathcal{T}|j\rangle = \langle j|\mathcal{T}|i\rangle$ because of time reversal invariance, Eq. (16) can be rewritten as follows:

$$-2\,\mathrm{Im}\langle\phi\psi|\mathcal{T}|\phi\psi\gamma\rangle = \int_{\mathrm{PS}}\{\langle\phi\psi|\mathcal{T}|\phi'\psi'\rangle^*\langle\phi'\psi'|\mathcal{T}|\phi\psi\gamma\rangle + \langle\phi\psi|\mathcal{T}|\phi'\psi'\gamma\rangle^*\langle\phi'\psi'\gamma|\mathcal{T}|\phi\psi\gamma\rangle\}, \tag{17}$$

where now the phase-space integral applies only to the particles ϕ' and ψ'. The last equation represents the Cutkosky cutting rules [20]. At the diagrammatic level this amounts to cutting a pair of ϕ and ψ propagators at all possible places in the bubble chain (keeping in mind that the photon is merely an external particle). The two terms on the right-hand side represent the two possibilities for the photon to couple to the bubble chain before or after the cut. Because these are the only possible cuts in the bubble chain leading to imaginary pieces in the relevant kinematic region, one verifies Eq. (17) and hence unitarity of the $\mathcal{S}$ matrix up to radiative corrections. However, if the photon couples to a propagator, there is also the possibility to cut in the corresponding diagram both propagators that are directly connected to the photon. For a photon with $k^2 < 4\min(m^2, M^2)$, which is the case both for physical photons and for virtual photons from electron scattering, these cuts do not yield imaginary values and can be safely omitted here.

Having convinced ourselves that it is possible to obtain gauge invariant and (up to radiative corrections) unitary amplitudes by taking into account the coupling of the photon to scalar fields in all possible ways in the bubble chain, we can now continue by applying this procedure to the slightly more complicated case of the chiral effective meson-baryon Lagrangian.

III. WEINBERG-TOMOZAWA TERM

The chiral effective Lagrangian describing the interactions between the octet of Goldstone bosons (π, K, η) and the ground-state baryon octet $(N, \Lambda, \Sigma, \Xi)$ is given at leading chiral order by

$$\mathcal{L}^{(1)}_{\phi B} = i\langle\bar{B}\gamma_\mu[D^\mu, B]\rangle - m_0\langle\bar{B}B\rangle + \cdots, \tag{18}$$

where $\langle\ldots\rangle$ denotes the trace in flavor space. The 3×3 matrix B collects the ground-state baryon octet and m_0 is the common baryon octet mass in the chiral limit. We have displayed only the terms relevant for the present investigation and omitted two operators that contain the axial vector couplings of the mesons to the baryons. In the present investigation, we restrict ourselves to interaction kernels of the BSE given by contact interactions. The axial vector couplings of the mesons to the baryons could in principle contribute via direct and crossed Born terms, but the crossed Born term corresponds to three-body intermediate states which are beyond the scope of this work and we neglect the Born terms throughout. (The inclusion of Born terms in the interaction kernel is deferred to future work [21].) In fact, many coupled-channels approaches only take into account the contact interaction originating from the Lagrangian in Eq. (18), see, e.g. [22] and references therein.

The covariant derivative of the baryon field is given by

$$[D_\mu, B] = \partial_\mu B + [\Gamma_\mu, B] \tag{19}$$

with the chiral connection

$$\Gamma_\mu = \frac{1}{2}[u^\dagger, \partial_\mu u] - \frac{i}{2}(u^\dagger v_\mu u + u v_\mu u^\dagger) \tag{20}$$

and $v_\mu = -e\mathcal{A}_\mu Q$, where $Q = \frac{1}{3}\mathrm{diag}(2, -1, -1)$ is the quark charge matrix. The pseudoscalar meson octet ϕ is arranged in a matrix valued field

$$U(\phi) = u^2(\phi) = \exp\left(\sqrt{2}i\frac{\phi}{F}\right), \tag{21}$$

with F the pseudoscalar decay constant in the chiral limit. Expansion of the chiral connection in the meson fields ϕ yields at leading order a $\phi^2\bar{B}B$ contact interaction, the Weinberg-Tomozawa term, which we choose to be the driving term for the BSE in this section.

The mesonic piece of the Lagrangian at leading chiral order is given by [1]

$$\mathcal{L}^{(2)}_\phi = \frac{F^2}{4}\langle\nabla_\mu U^\dagger\nabla^\mu U\rangle + \frac{F^2}{4}\langle\chi_+\rangle, \tag{22}$$

where $\chi_+ = 2B_0(u^\dagger\mathcal{M}u^\dagger + u\mathcal{M}u)$ describes explicit chiral symmetry breaking via the quark mass matrix $\mathcal{M} = \mathrm{diag}(m_u, m_d, m_s)$ and $B_0 = -\langle 0|\bar{q}q|0\rangle/F^2$ represents the order parameter of spontaneously broken chiral symmetry. The covariant derivative of the meson fields is given by (neglecting external axial-vector fields)

$$\nabla_\mu U = \partial_\mu U - i v_\mu U + i U v_\mu. \tag{23}$$

In the Bethe-Salpeter formalism we choose to work with the propagators

$$\Delta_i(p) = \frac{1}{p^2 - M_i^2}, \qquad S_a(p) = \frac{1}{\not{p} - m_a}, \tag{24}$$

with flavor indices i, a and physical meson and baryon masses M_i and m_a, respectively. However, the following calculations are valid for all propagators satisfying the Ward-Takahashi identities

$$k^\mu V^\phi_\mu(p+k, k) = \Delta^{-1}(p+k) - \Delta^{-1}(p), \qquad k^\mu V^B_\mu(p+k, k) = S^{-1}(p+k) - S^{-1}(p), \tag{25}$$

where $V^\phi_\mu(V^B_\mu)$ are the corresponding $\gamma\phi^2(\gamma\bar{B}B)$ three-point functions.

In the presence of a general interaction kernel A and coupled channels consisting of a meson-baryon pair the BSE for the

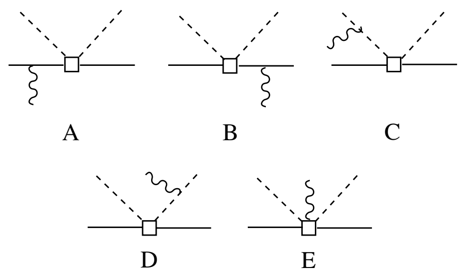

FIG. 1. Tree diagrams for the process $\gamma\phi B \to \phi B$. Solid, dashed, and wavy lines correspond to baryons, mesons, and photons, respectively. The square denotes the vertex from the leading order Lagrangian.

process $\phi(q_i)B(p_i) \to \phi(q_f)B(p_f)$ generalizes to

$$\begin{aligned} T_{fi}(p;q_f,q_i) &= A_{fi}(p;q_f,q_i) + i\sum_l \int_k T_{fl}(p;q_f,k) \\ &\quad \times \Delta_j(k)S_a(p+k)A_{li}(p;k,q_i) \\ &= A_{fi}(p;q_f,q_i) + i\sum_l \int_k A_{fl}(p;q_f,k) \\ &\quad \times \Delta_j(k)S_a(p+k)T_{li}(p;k,q_i), \end{aligned} \tag{26}$$

with $p = p_i + q_i = p_f + q_f$ and $l = \{\phi_j, B_a\}$ the channels that couple both to the initial and final state, i and f. Note that we have replaced the common mass of the ground-state baryon octet, m_0, by the physical baryon masses m_a. This is consistent with the chiral order of the interaction kernel derived from the Weinberg-Tomozawa term and, in particular, produces the unitarity cuts at the physical thresholds.

After setting up the formalism we first calculate the tree-level contributions for the coupling of a photon to meson-baryon scattering. The pertinent Feynman diagrams for the process $\gamma(k)\phi_i(q_i)B_a(p_i) \to \phi_j(q_f)B_b(p_f)$ are depicted in Fig. 1. In addition to the coupling of the photon to the propagators the chiral connection in Eq. (18) gives rise to a $\gamma\phi^2\bar{B}B$ vertex, Fig. 1(E).

The tree contributions to the transition amplitude read as follows:

$$\begin{aligned} T_\mu^{(\text{tree})bj,ai} &= -\frac{e}{4F^2}\{(\not{q}_i + \not{q}_f)S_a(p_i+k)\gamma_\mu \hat{Q}^a \\ &\quad + \gamma_\mu \hat{Q}^b S_b(p_f - k)(\not{q}_i + \not{q}_f) \\ &\quad + (\not{q}_i + \not{q}_f + \not{k})\Delta_i(q_i+k)[2q_i + k]_\mu \hat{Q}^i \\ &\quad + (\not{q}_i + \not{q}_f - \not{k})\Delta_j(q_f-k)[2q_f - k]_\mu \hat{Q}^j \\ &\quad - \gamma_\mu(\hat{Q}^j + \hat{Q}^i)\}\, \langle \lambda^{b\dagger}[[\lambda^{j\dagger}, \lambda^i], \lambda^a]\rangle, \end{aligned} \tag{27}$$

where $\hat{Q}^a\lambda^a = [Q, \lambda^a]$ (no summation over a) is the charge of the particle a in units of e and the λ^i are the generators of the SU(3) Lie-Algebra in the physical basis. By multiplying the tree contributions in Eq. (27) with k_μ gauge invariance is easily verified, if the momenta of the particles are put on-shell.

To prove gauge invariance for the coupling of the photon to the bubble chain, it is convenient to consider first the diagrams presented in Fig. 2 with the pertinent contributions given by the following (a, b, c, d denote baryon flavor indices, whereas i, j, m, n represent meson flavors):

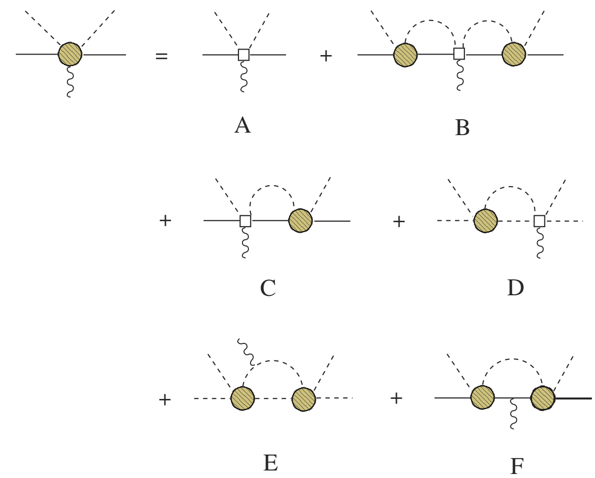

FIG. 2. (Color online) Bubble-chain diagrams for the process $\gamma\phi B \to \phi B$. Solid, dashed, and wavy lines correspond to baryons, mesons, and photons, respectively. The square denotes the vertex from the leading order Lagrangian, the filled circle represents the bubble chain derived from the BSE.

Fig. 2(A):

$$A_\mu^{bj,ai} = \frac{e}{4F^2}\gamma_\mu(\hat{Q}^j + \hat{Q}^i)\langle\lambda^{b\dagger}[[\lambda^{j\dagger}, \lambda^i], \lambda^a]\rangle, \tag{28}$$

Fig. 2(B):

$$\begin{aligned} i^2 \int_l \int_q & T^{bj,dn}(p'; q_f, q) S_d(p'-q)\Delta_n(q) A_\mu^{dn,cm} \\ & \times S_c(p-l)\Delta_m(l)T^{cm,ai}(p;l,q_i), \end{aligned} \tag{29}$$

Fig. 2(C):

$$i\int_l T^{bj,cm}(p';q_f,l)S_c(p'-l)\Delta_m(l)A_\mu^{cm,ai}, \tag{30}$$

Fig. 2(D):

$$i\int_l A_\mu^{bj,cm} S_c(p-l)\Delta_m(l)T^{cm,ai}(p;l,q_i), \tag{31}$$

Fig. 2(E):

$$\begin{aligned} i^2 \int_l & T^{bj,cm}(p';q_f,l+k)S_c(p-l)\Delta_m(l+k) \\ & \times (-ie\hat{Q}^m(2l+k)_\mu)\Delta_m(l)T^{cm,ai}(p;l,q_i), \end{aligned} \tag{32}$$

Fig. 2(F):

$$\begin{aligned} i^2 \int_l & T^{bj,cm}(p';q_f,l)S_c(p'-l)(-ie\hat{Q}^c\gamma_\mu)S_c(p-l) \\ & \times \Delta_m(l)T^{cm,ai}(p;l,q_i), \end{aligned} \tag{33}$$

with $p' = p + k = p_i + q_i + k = p_f + q_f$.

For general momenta p, q (i.e., not necessarily on-shell), the quantity $A_\mu^{bj,ai}$ satisfies the following relation:

$$\begin{aligned} k^\mu A_\mu^{bj,ai} &= e\{(\hat{Q}^b - \hat{Q}^a)A^{bj,ai}(q,p) - \hat{Q}^i \\ &\quad \times A^{bj,ai}(q, p+k) + \hat{Q}^j A^{bj,ai}(q-k, p)\}, \end{aligned} \tag{34}$$

where A is the amplitude deduced from the Weinberg-Tomozawa term as follows:

$$A^{bj,ai}(q,p) = -\frac{1}{4F^2}(\not{q}+\not{p})\langle\lambda^{b\dagger}[[\lambda^{j\dagger},\lambda^i],\lambda^a]\rangle. \quad (35)$$

Making extensively use of Eq. (34) and the BSE it is straightforward to show that the contributions of Eqs. (28)–(33) multiplied by k_μ yield in total

$$e[-\hat{Q}^a T^{bj,ai}(p';q_f,q_i) + \hat{Q}^b T^{bj,ai}(p;q_f,q_i) - \hat{Q}^i \times T^{bj,ai}(p';q_f,q_i+k) + \hat{Q}^j T^{bj,ai}(p;q_f-k,q_i)]. \quad (36)$$

This compensates exactly the contributions from the remaining four diagrams where the photon couples to the external on-shell legs of the bubble chain. We have thus confirmed that gauge invariance is achieved if all possible diagrams of a photon coupling to a bubble chain are taken into account. In particular, it is not sufficient to consider only the coupling of the photon to external legs, because this will lead to a gauge-dependent amplitude. Conversely, within the field theoretical framework applied here it is also not sufficient to take into account only the coupling of the photon to the interaction kernel. Note that in the proof given here we do not assume the so-called on-shell approximation for the interaction kernel.

It is also important to stress that an explicit evaluation of the BSE was not necessary and thus we do not need to specify the regularization scheme to render the loop integral in the BSE finite. Any regularization procedure that satisfies the Ward identities for both the propagators, Eq. (23), and the interaction kernel, Eq. (34), will maintain gauge invariance in the BSE as outlined in the proof.

Finally, the unitarity constraint for the full amplitude T_μ can be proven in analogy to the treatment in Sec. II. The detailed derivation of unitarity is deferred to Sec. V. Thus, also to obtain a unitarized amplitude (up to radiative corrections) one must take into account the coupling of the photon at all possible places in the bubble chain.

IV. HIGHER ORDER INTERACTION KERNELS

In this section, we consider more complicated structures for the interaction kernel as they arise at higher chiral orders in the effective Lagrangian. Because we restrict ourselves to $\phi^2\bar{B}B$ contact interactions, the most general form of a term without the chiral invariant field strength tensor $f^+_{\mu\nu}$ is given by the following:

$$\mathcal{L}_{\text{int}} = \bar{B}C_{\mu_1\cdots\mu_l\mu_{l+1}\cdots\mu_m\mu_{m+1}\cdots\mu_n}(D^{\mu_1}\ldots D^{\mu_l}\tilde{\phi}) \times (D^{\mu_{l+1}}\ldots D^{\mu_m}\phi^\dagger)(D^{\mu_{m+1}}\ldots D^{\mu_n}\tilde{B}), \quad (37)$$

where we have suppressed flavor indices for brevity (we merely kept the $\sim$ symbol as a reminder of the flavor structure indicating that the in- and outgoing baryons and mesons can be different) and introduced the notation $D_\mu = \partial_\mu + i\hat{e}\mathcal{A}_\mu$ with $\hat{e}$ the charge of the particle D_μ is acting on. As the contact interactions originate from a gauge-invariant Lagrangian, charge conservation is guaranteed at each vertex. The introduction of explicit flavor indices does not change any of the following conclusions. Note that the constant C in Eq. (37) may also contain elements of the Clifford algebra. From this Lagrangian one derives both a $\tilde{\phi}(\tilde{q})\tilde{B}(\tilde{p}) \to \phi(q)B(p)$ contact interaction $A(\tilde{p},\tilde{q},q)$ and a $\gamma(k)\tilde{\phi}(\tilde{q})\tilde{B}(\tilde{p}) \to \phi(q)B(p)$ vertex $\epsilon_\mu A^\mu(\tilde{p},\tilde{q},q,k)$, where ϵ_μ is the polarization vector of the photon. Because of the form of the contact term (37), which can always be obtained by partial integration, the vertices do not depend explicitly on the momentum p. In the appendix it is shown that they satisfy the relation

$$\begin{aligned} k_\mu A^\mu(p_i,q_i,q_f,k) &= \hat{e}_\phi A(p_i,q_i,q_f-k) - \hat{e}_{\tilde{\phi}} A(p_i,q_i+k,q_f) \\ &\quad + \hat{e}_B A(p_i,q_i,q_f) - \hat{e}_{\tilde{B}} A(p_i+k,q_i,q_f) \end{aligned} \quad (38)$$

for general momenta of the particles. This equation is the analog of Eq. (34) for the Weinberg-Tomozawa term. It follows then immediately by applying the same arguments as in the previous section that the coupling of the photon to the two-particle state of the BSE yields a gauge-invariant amplitude also in the presence of more complicated contact interactions of the type Eq. (37).

Note that throughout we have not considered the dimension-two magnetic-moment coupling $\sim\sigma^{\mu\nu}f^+_{\mu\nu}$ and higher order operators involving the chiral covariant field-strength tensor $f^+_{\mu\nu}$. Such terms are of course present in the effective Lagrangian and must be considered at the appropriate order in the chiral expansion of the interaction kernel. However, these are of the form $\partial_\nu v_\mu(\mathcal{O}^{\nu\mu} - \mathcal{O}^{\mu\nu})$ with some operator $\mathcal{O}^{\nu\mu}$ and the pertinent vertex in momentum space vanishes upon contraction with the photon momentum k_μ.

As already mentioned in the previous section, the proof of gauge invariance does not depend on the specific choice of the meson and baryon propagators but rather is valid for all propagators satisfying the Ward-Takahashi identities with the corresponding $\gamma\phi^2$ and $\gamma\bar{B}B$ three-point functions. For example, one can define the BSE by employing propagators with the physical masses for the intermediate states and derive the interaction kernel from the effective Lagrangian to a given chiral order. This automatically produces the correct physical thresholds of the unitarity cuts and we have followed this path in the present investigation.

Alternatively, one may prefer to deduce the propagators from the effective Lagrangian as well. To leading chiral order this implies a common baryon octet mass shifting the threshold of the unitarity cuts to unphysical values. At higher chiral orders the inclusion of self-energy diagrams for the meson and baryon propagators will cure the situation by restoring the physical thresholds. The self-energy diagrams will modify the simple form of the propagators given in Eq. (26); e.g., the meson propagators will acquire the following form:

$$\Delta(p) = \frac{Z}{p^2 - M^2 - \Sigma_R(p)}, \quad (39)$$

with M being the physical meson mass, Z the appropriate wave function renormalization constant, and $\Sigma_R(p)$ the renormalized self-energy.

To prove gauge invariance in the latter approach, one must also take into account the coupling of the photon to the self-energy corrections of the propagators. Because the effective Lagrangian is gauge invariant, the corresponding propagators and three-point functions satisfy the pertinent Ward-Takahashi identities and the proof is equivalent to the one given in the previous section.

We also compare the present investigation with the work of Refs. [11,12]. Although similar in spirit, the authors study therein an integral equation for the two-body Green's function, whereas we prefer to work with an equation for the scattering amplitude. "Gauging" the integral equation for the Green's function as outlined in Refs. [11,12], i.e., adding a vector index μ to all the terms of the equation such that a linear equation in μ-labeled quantities results, amounts to attaching an external photon everywhere, including the external legs. In our framework, the straightforward application of the gauging method to the integral equation for the scattering amplitude fails, as in this case the procedure does not yield the contributions where the photon couples to the external legs. (Of course, one could correct this by adding the missing contributions by hand.) Conversely, our approach is more convenient within the chiral effective framework, as it allows a direct comparison with the scattering amplitude derived in the perturbative scheme of ChPT. Moreover, the authors of Refs. [11,12] restrict themselves merely to one two-body channel, whereas in the present investigation this is generalized to several coupled channels. In contrast to Refs. [11,12] we explicitly specify the interaction kernel by deriving the vertices from the chiral effective Lagrangian and utilizing them as interaction kernels in the BSE.

V. UNITARITY

After having constructed a gauge-invariant amplitude for $B\phi\gamma \to B\phi$ with Weinberg-Tomozawa or more general contact interaction kernels, we investigate unitarity of the obtained amplitude. The calculation presented in this section generalizes the findings for the scalar field theory presented at the end of Sec. II, as one must take care of the noncommutative nature of the matrix amplitudes because of the Clifford algebra and coupled channels. Moreover, we do not assume symmetry of the transition amplitudes under exchange of initial and final states.

In operator form the statement of a unitary scattering matrix amounts to

$$\mathcal{T} - \mathcal{T}^\dagger = -i\mathcal{T}^\dagger\mathcal{T}. \tag{40}$$

For brevity we introduce a short-hand notation for the BSE, Eq. (26),

$$T(p) = A + \int T(p)G(p)A = A + \int AG(p)T(p), \tag{41}$$

where p is the external momentum and $G = iS\Delta$. The BSE for the meson-baryon scattering amplitude T is easily transformed into the unitarity relation

$$T - \bar{T} = \int \bar{T}(G - \bar{G})T, \tag{42}$$

with $\bar{O} \equiv \gamma_0 O^\dagger \gamma_0$, as both T and G are elements of the Clifford algebra. Note that the adjoint $O^\dagger$ also implies taking the transposed matrix in channel space. We see that the quantity $G - \bar{G}$ is equal to setting the intermediate meson-baryon pairs on-shell in Eq. (42). For invariant energies below the lowest three-particle threshold Eq. (42) is thus equivalent to the unitarity constraint (40).

If T is an analytic function, one can apply the residue theorem and rewrite the difference $G - \bar{G}$ as

$$\begin{aligned} &i[S(p-l)\Delta(l) + \bar{S}(p-l)\bar{\Delta}(l)] \\ &\quad \to i(-2\pi i)^2\delta_+(l^2 - M^2)\delta_+[(p-l)^2 - m^2][\not{p} - \not{l} + m], \end{aligned} \tag{43}$$

where l and p are the loop and external momentum, respectively, and $\delta_+(k^2 - \mu^2) = \delta(k^2 - \mu^2)\theta(k_0)$. For a detailed derivation of this replacement, see, e.g., Ref. [23]. The last equation can even be generalized to nonanalytic T that does not have coinciding poles with G. In particular, the above replacement is valid if T describes the solution of the BSE with polynomial interaction kernels, as can be seen by insertion of its defining Eq. (26) into Eq. (42).

To prove unitarity for the transition $B\phi\gamma \to B\phi$, it is convenient to introduce the amplitude

$$M^\mu_{\phi\gamma} = V^\mu_{\rm disc} + T^\mu \tag{44}$$

with the disconnected piece

$$\begin{aligned} V^\mu_{\rm disc} = {} & 2E_{q_i}(2\pi)^3\delta^{(3)}(\boldsymbol{q}_i - \boldsymbol{q}_f)V^\mu_B \\ & + 2E_{p_i}(2\pi)^3\delta^{(3)}(\boldsymbol{p}_i - \boldsymbol{p}_f)V^\mu_\phi \end{aligned} \tag{45}$$

and T^μ the transition amplitude calculated in Secs. III and IV. The energies of the particles are given by $E_{q_i} = \sqrt{q_i^2 + M_\phi^2}$ and $E_{p_i} = \sqrt{p_i^2 + m_B^2}$, respectively. The piece $V^\mu_{\rm disc}$ is represented by the two disconnected diagrams in which the photon couples either to the baryon or the meson, whereas the other particle does not interact at all. Although $V^\mu_{\rm disc}$ does not contribute to on-shell matrix elements and could in principle be omitted in the unitarity relation, its introduction generalizes unitarity beyond the physical region. From the amplitude $M^\mu_{\phi\gamma}$ one constructs the reversed amplitude $M^\mu_{\gamma\phi}$ for the process $B(p_f)\phi(q_f) \to B(p_i)\phi(q_i)\gamma(k)$. Neglecting radiative corrections and below the lowest three-particle (i.e., baryon two-meson) threshold unitarity implies

$$\begin{aligned} M^\mu_{\phi\gamma} - \bar{M}^\mu_{\gamma\phi} = {} & \int \bar{T}(p')[G(p') - \bar{G}(p')]M^\mu_{\phi\gamma} \\ & + \int \bar{M}^\mu_{\gamma\phi}[G(p) - \bar{G}(p)]T(p), \end{aligned} \tag{46}$$

where we have replaced again the two-body phase-space integration by the four-dimensional integral over $G - \bar{G}$. Note that in the physical region $M^\mu_{\phi\gamma}$ reduces to T^μ. By inserting the amplitudes T^μ and T from the BSE and making use of the unitarity statement for T, Eq. (42), as well as $\bar{V}^\mu_{\phi/B} = V^\mu_{\phi/B}$ and $\bar{A}^\mu = A^\mu$ from Eq. (28), one can indeed confirm the unitarity constraint for $M^\mu_{\phi\gamma}$ and thus for T^μ for on-shell matrix elements.

We refrain from presenting the entire and tedious calculation here but would like to comment on two points. First, the contribution of the disconnected graphs drops out on the l.h.s. of the unitarity statement (46) (because of the symmetry of these graphs under interchange of incoming and outgoing particles and $\bar{V}^{\mu}_{\phi/B} = V^{\mu}_{\phi/B}$). On the r.h.s., they produce terms of the following type:

$$\begin{aligned}
&\int_l \bar{T}(p')[G(p') - \bar{G}(p')]2E_{q_i}(2\pi)^3\delta^{(3)}(\boldsymbol{q}_i - \boldsymbol{l})V^{\mu}_B \\
&= \int_l \bar{T}(p')i(-2\pi i)^2\delta_+[(p'-l)^2 - m^2] \\
&\quad \times \delta_+(l^2 - M^2)[\not{p}' - \not{l} + m]2E_{q_i}(2\pi)^3\delta^{(3)}(\boldsymbol{q}_i - \boldsymbol{l})V^{\mu}_B \\
&= \int d^3l\bar{T}(p')(-2\pi i)\delta_+((p'-l)^2 - m^2) \\
&\quad \times [\not{p}' - \not{l} + m]\delta^{(3)}(\boldsymbol{q}_i - \boldsymbol{l})V^{\mu}_B \\
&= \bar{T}(p')(-2\pi i)\delta_+((p'-q_i)^2 - m^2)[\not{p}' - \not{q}_i + m]V^{\mu}_B \\
&= \bar{T}(p')[S(p_i + k) - \bar{S}(p_i + k)]V^{\mu}_B.
\end{aligned} \tag{47}$$

For on-shell matrix elements (and $k \neq 0$) this vanishes as $\epsilon \to 0$ in the propagators, but it happens that all terms of this type cancel each other on the r.h.s. of Eq. (46), even for general external momenta. Our second comment concerns some contributions from diagrams 2(E) and (F). On the l.h.s. of Eq. (46) one obtains, e.g., the following combination:

$$\begin{aligned}
&i\int_l \bar{T}(p')S(p'-l)\Delta(l)V^{\mu}_B S(p-l)T(p) \\
&\quad + i\int_l \bar{T}(p')\bar{S}(p'-l)\bar{\Delta}(l)V^{\mu}_B \bar{S}(p-l)T(p),
\end{aligned} \tag{48}$$

which represents the discontinuity of the transition amplitude stemming from Fig. 2(F) (the two integrals only differ in the sign of the "$i\epsilon$" terms in the propagators). According to the Cutkosky cutting rules and for momenta $k^2 < 4m^2$ this discontinuity is given by the following:

$$\begin{aligned}
&\int_l \bar{T}(p')[G(p') - \bar{G}(p')]V^{\mu}_B S(p-l)T(p) \\
&\quad + \int_l \bar{T}(p')\bar{S}(p'-l)V^{\mu}_B[G(p) - \bar{G}(p)]T(p).
\end{aligned} \tag{49}$$

In the kinematical region that is of relevance here the cut through the two propagators connecting to the photon does not contribute to the discontinuity and can be safely neglected. We conclude by emphasizing that the unitarity constraint [Eq. (46)] is fulfilled only if the photon couples to all possible places in the bubble chain.

VI. CONCLUSIONS

In the present work, we have studied how gauge invariance is obtained for a photon coupling to a two-body state described by the solution of the Bethe-Salpeter equation. We have discussed the procedure both for a simple complex scalar field theory and for interaction kernels derived from chiral effective Lagrangians in the meson-baryon sector. In the latter case, we have first considered the Weinberg-Tomozawa term and afterwards the most general contact interaction consisting of two mesons and two baryons that can arise in the chiral effective framework. Our study underlines that it is *not* sufficient to take into account only the coupling of the photon to the external legs, as has been done in many calculations based on chiral unitary approaches, but rather one has to include all possible contributions of the photon coupling to the vertices and intermediate states. Neither is it sufficient to consider only the coupling of the photon to the interaction kernel. At the same time, coupling of the photon to the bubble chain at all possible places is necessary to guarantee a unitary scattering matrix up to radiative corrections.

For the interaction kernels discussed in the present work we have shown explicitly that gauge invariance is maintained in this manner. It is accomplished without assuming the on-shell approximation for the interaction kernel. Moreover, the explicit evaluation of the loop integral in the Bethe-Salpeter equation is not necessary and hence the proof does not depend on the chosen regularization scheme. But the regularization procedure is required to satisfy the Ward identities both for the propagators and the interaction kernels. This study will also be of importance for photo- and electroproduction processes of mesons on nucleons and for radiative decays of baryons and mesons that must be treated in a similar way to achieve gauge-invariant and unitarized amplitudes. Work along these lines is in progress.

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APPENDIX

Higher chiral orders

In this appendix we derive Eq. (38), which follows from a contact interaction of the type

$$\begin{aligned}
\mathcal{L}_{\rm int} &= \bar{B}C_{\mu_1\cdots\mu_l\mu_{l+1}\cdots\mu_m\mu_{m+1}\cdots\mu_n}(D^{\mu_1}\ldots D^{\mu_l}\tilde{\phi}) \\
&\quad \times (D^{\mu_{l+1}}\ldots D^{\mu_m}\phi^{\dagger})(D^{\mu_{m+1}}\ldots D^{\mu_n}\tilde{B}),
\end{aligned} \tag{A1}$$

with $D_\mu = \partial_\mu + i\hat{e}\mathcal{A}_\mu$. The $\gamma(k)\tilde{\phi}(q_i)\tilde{B}(p_i) \to \phi(q_f)B(p_f)$ vertex is obtained from this Lagrangian by extracting the part linear in $\mathcal{A}_\mu$. A partial derivative acting, e.g., on an incoming meson $\phi(q)$ yields a factor $-iq$ in momentum space, whereas a partial derivative on $\phi(q)\mathcal{A}(k)$ (both momenta incoming) leads to $-i(q+k)$. Therefore, one obtains the following vertex:

$$\begin{aligned}
A_\mu = i\sum_{s=1}^{l} C_{\mu_1\cdots\mu_n}\Big|_{\mu_s=\mu} (-i)^{(2l-2m+n)}\left[\prod_{t=1}^{s-1}(q_i+k)^{\mu_t}\right] \\
\times(-\hat{e}_{\tilde{\phi}})\left[\prod_{t=s+1}^{l} q_i^{\mu_t}\right]\left[\prod_{t=l+1}^{m} q_f^{\mu_t}\right]\left[\prod_{t=m+1}^{n} p_i^{\mu_t}\right]
\end{aligned}$$

$$+ i \sum_{s=l+1}^{m} C_{\mu_1\cdots\mu_n}\Big|_{\mu_s=\mu} (-i)^{(2l-2m+n)} \left[\prod_{t=1}^{l} q_i^{\mu_t}\right] \times \left[\prod_{t=l+1}^{s-1} (q_f - k)^{\mu_t}\right](-\hat{e}_\phi)\left[\prod_{t=s+1}^{m} q_f^{\mu_t}\right] \times \left[\prod_{t=m+1}^{n} p_i^{\mu_t}\right] + i \sum_{s=m+1}^{n} C_{\mu_1\cdots\mu_n}\Big|_{\mu_s=\mu} (-i)^{(2l-2m+n)} \times \left[\prod_{t=1}^{l} q_i^{\mu_t}\right]\left[\prod_{t=l+1}^{m} q_f^{\mu_t}\right]\left[\prod_{t=m+1}^{s-1} (p_i + k)^{\mu_t}\right] \times (-\hat{e}_{\tilde{B}})\left[\prod_{t=s+1}^{n} p_i^{\mu_t}\right]. \quad \text{(A2)}$$

Multiplying this equation with k_μ and making use of charge conservation at the vertex that follows from gauge invariance of the Lagrangian one arrives at the following:

$$\begin{aligned} k^\mu A_\mu = {} & i C_{\mu_1\cdots\mu_n}(-i)^{(2l-2m+n)}(-\hat{e}_{\tilde{\phi}})(q_i + k)^{\mu_1}\cdots(q_i+k)^{\mu_l} q_f^{\mu_{l+1}}\cdots q_f^{\mu_m} p_i^{\mu_{m+1}}\cdots p_i^{\mu_n} \\ & - i C_{\mu_1\cdots\mu_n}(-i)^{(2l-2m+n)}(-\hat{e}_{\phi}) q_i^{\mu_1}\cdots q_i^{\mu_l}(q_f - k)^{\mu_{l+1}}\cdots(q_f-k)^{\mu_m} p_i^{\mu_{m+1}}\cdots p_i^{\mu_n} \\ & + i C_{\mu_1\cdots\mu_n}(-i)^{(2l-2m+n)}(-\hat{e}_{\tilde{B}}) q_i^{\mu_1}\cdots q_i^{\mu_l} q_f^{\mu_{l+1}}\cdots q_f^{\mu_m}(p_i + k)^{\mu_{m+1}}\cdots(p_i+k)^{\mu_n} \\ & - i C_{\mu_1\cdots\mu_n}(-i)^{(2l-2m+n)}(-\hat{e}_{B}) q_i^{\mu_1}\cdots q_i^{\mu_l} q_f^{\mu_{l+1}}\cdots q_f^{\mu_m} p_i^{\mu_{m+1}}\cdots p_i^{\mu_n}. \end{aligned} \quad \text{(A3)}$$

The vertex $\tilde{\phi}(q_i)\tilde{B}(p_i) \to \phi(q_f)B(p_f)$ from the Lagrangian (A1) is given by the piece without the photon field $\mathcal{A}_\mu$ and reads as follows:

$$A(p_i, q_i, q_f) = i C_{\mu_1\cdots\mu_n}(-i)^{(2l-2m+n)} q_i^{\mu_1}\cdots \times q_i^{\mu_l} q_f^{\mu_{l+1}}\cdots q_f^{\mu_m} p_i^{\mu_{m+1}}\cdots p_i^{\mu_n}. \quad \text{(A4)}$$

This proves the Ward identity [Eq. (38)]

$$\begin{aligned} k^\mu A_\mu(p_i, q_i, q_f, k) &= \hat{e}_\phi A(p_i, q_i, q_f - k) - \hat{e}_{\tilde{\phi}} A(p_i, q_i + k, q_f) \\ &\quad + \hat{e}_B A(p_i, q_i, q_f) - \hat{e}_{\tilde{B}} A(p_i + k, q_i, q_f). \end{aligned} \quad \text{(A5)}$$

[1] J. Gasser and H. Leutwyler, Nucl. Phys. **B250**, 465 (1985).

[2] N. Kaiser, P. B. Siegel, and W. Weise, Phys. Lett. **B362**, 23 (1995); N. Kaiser, P. B. Siegel, and W. Weise, Nucl. Phys. **A594**, 325 (1995).

[3] J. A. Oller and U.-G. Meißner, Phys. Lett. **B500**, 263 (2001); U.-G. Meißner and J. A. Oller, Nucl. Phys. **A673**, 311 (2000).

[4] E. E. Kolomeitsev and M. F. M. Lutz, Phys. Lett. **B585**, 243 (2004).

[5] J. A. Oller and E. Oset, Nucl. Phys. **A620**, 438 (1997); [**A652**, 407(E) (1999)]; J. A. Oller, E. Oset, and J. R. Pelaez, Phys. Rev. D **59**, 074001 (1999); [**60**, 099906(E) (1999)].

[6] N. Kaiser, T. Waas, and W. Weise, Nucl. Phys. **A612**, 297 (1997); J. Caro Ramon, N. Kaiser, S. Wetzel, and W. Weise, *ibid.* **A672**, 249 (2000).

[7] S. D. Bass, S. Wetzel, and W. Weise, Nucl. Phys. **A686**, 429 (2001).

[8] B. Borasoy, E. Marco, and S. Wetzel, Phys. Rev. C **66**, 055208 (2002).

[9] F. Gross and D. O. Riska, Phys. Rev. C **36**, 1928 (1987).

[10] Y. Surya and F. Gross, Phys. Rev. C **53**, 2422 (1996).

[11] A. N. Kvinikhidze and B. Blankleider, Phys. Rev. C **60**, 044003 (1999).

[12] A. N. Kvinikhidze and B. Blankleider, Phys. Rev. C **60**, 044004 (1999).

[13] C. H. M. van Antwerpen and I. R. Afnan, Phys. Rev. C **52**, 554 (1995).

[14] H. Haberzettl, Phys. Rev. C **56**, 2041 (1997).

[15] J. A. Oller, Phys. Lett. **B426**, 7 (1998).

[16] E. Marco, S. Hirenzaki, E. Oset, and H. Toki, Phys. Lett. **B470**, 20 (1999).

[17] B. Borasoy and R. Nißler, Eur. Phys. J. A **19**, 367 (2004).

[18] B. Borasoy and R. Nißler, Nucl. Phys. **A740**, 362 (2004).

[19] H. Haberzettl and K. Nakayama, *in preparation*.

[20] R. E. Cutkosky, Phys. Rev. **96**, 1135 (1954).

[21] B. Borasoy, P. Bruns, U.-G. Meißner, and R. Nißler, *in preparation*.

[22] J. A. Oller, E. Oset, and A. Ramos, Prog. Part. Nucl. Phys. **45**, 157 (2000).

[23] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory* (Westview Press, Boulder, Co, 1995).