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Orientation dependence of the heavy-ion potential between two deformed nuclei

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The deformation and orientation dependence of the real part of the interaction potential is studied for two heavy deformed nuclei using the Hamiltonian energy density approach derived from the well-known Skyrme *NN* interaction with two parameter sets SIII and SkM*. We studied the real part of the heavy ion (HI) potential for $^{238}U+^{238}U$ pair considering quadrupole and hexadecapole deformations in both nuclei and taking into consideration all the possible orientations, coplanar and noncoplanar, of the two interacting nuclei. We found strong orientation dependence for the physically significant region of the HI potential. The orientation dependence increases with adding the hexadecapole deformation. The azimuthal angle dependence is found to be strong for some orientations. This shows that taking the two system axes in one plane produces a large error in calculating the physical quantities.

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I. INTRODUCTION

The microscopic calculation of the nucleus-nucleus potential used to describe the different nuclear reactions between heavy ions (HI) is one of the main subjects that have great interest in theoretical heavy ion physics [1–3]. Different theoretical models have been proposed to describe the internuclear potential such as the folding model [1,4,5], the liquid-drop model [6], the energy density formalism [7,8], and the proximity model [9,10]. There are different degrees of freedom of the colliding nuclei that should be considered for describing well the heavy ion interaction potential. The potential between two spherically symmetric density distributions, uniform or nonuniform, is calculated to an acceptable accuracy by using the Fourier transform representation within the double folding potential which is one of the successful approaches [1,4,5,11]. The static and dynamic deformations give rise to significant changes in both Coulomb and nuclear energies. Due to the six-dimensional integration in the double folding model, it is difficult to handle the permanent static deformations of interacting nuclei. On the other hand, including some improvements like the density-dependence [12,13] of the effective nucleon-nucleon (*NN*) interaction increases the amount of calculations and its consumed time. In several studies, such as the coupled channel calculations of fusion cross section or the dynamic calculations of nuclear reactions, we needed to calculate the ion-ion potential several times, so there is a need for fast and at the same time accurate calculations for the potential. Many authors tried to calculate the interacting potential between spherical-deformed and deformed-deformed nuclear distributions [10,11,14–16] in an approximate way. Several recent studies are made to describe the potential between two deformed nuclei. One of the studies [10] generalized the "pocket formula" for the proximity potential for collisions of two equal or nonequal, axially symmetric deformed nuclei with their symmetry axes lying in one plane. In this case the azimuthal angles of the two symmetry axes are assumed to be zero, $\phi = 0°$. Authors have mentioned that their formulas may extend to be applicable to oriented nuclei in two different planes. Another study [15] provided a model to obtain approximate expressions for the Coulomb potential between two coplanar deformed nuclei with finite diffuseness and indicated a way to calculate the nuclear potential approximately, based on a zero range *NN* interaction of Migdal type [17]. In a more recent work the authors of Ref. [16] studied the deformation and orientation effects in the deriving potential of a two dinuclear system model having the symmetry axes of the two deformed nuclei lie in the same plane.

One of the methods for calculating the ion-ion potential is the Hamiltonian energy density formalism derived from the realistic density-dependent effective *NN* interaction [18,19]. This method has been applied successfully to derive the real part of the interaction potential between spherical-deformed [19,20] and coplanar deformed-deformed nuclei [21]. The Skyrme interaction is an effective *NN* force where parameters are chosen to fit the saturation properties of cold nuclear matter and to reproduce static properties such as the total binding energies and charge radii of some nuclei [22,23]. This interaction has been used frequently in nuclear structure calculations and the structure of the super heavy nuclei [24]. The simple form of the Skyrme interaction [25] has considerable advantages in nuclear structure calculations and scattering problems [18–21]. The main advantage is that its simplicity allows one to write the Hartree-Fock energy as a functional depending only on local densities and their gradients. The energy density formalism derived from the Skyrme density-dependent interaction has been used to study the orientation dependence of the real part of the ion-ion potential between two coplanar deformed nuclei

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[21,26]. This approach derived from the conventional Skyrme force [22] together with the extended Skyrme force [27] has been used to study the interaction potential between several systems [28]. The double folding model with finite range *NN* force played a fundamental role in deriving the HI potential for spherical-spherical and spherical-deformed interacting pairs. In these two cases the six-dimensional integral can be simplified using Fourier transform representation. When dealing with two deformed nuclei, it is impossible to simplify the six-dimensional integral and the problem becomes too hard to calculate with good accuracy. The energy density formalism derived from realistic *NN* interaction reduces the six-dimensional integral appearing in the double folding model to a three-dimensional one. Moreover, the parameters of its *NN* force were adjusted to reproduce the nuclear matter data and the binding energy of some nuclei. In the present work we use the energy density formalism derived from the well-known Skyrme *NN* force with parameter sets SIII and SkM* [19,23] to study the azimuthal angle dependence of the interaction potential between two deformed nuclei. In the next section we describe briefly the method of calculating the HI potential in the framework of the energy density formalism. In Sec. III we present and discuss the orientation and deformation dependence of the HI potential between two deformed nuclei calculated using Skyrme energy density.

II. THEORY

In the energy density approach, the real part of the ion-ion potential as a function of the separation distance R between the centers of the two colliding nuclei is given by [19,20]

$$V(R) = \int [H(\rho,\tau) - H_1(\rho_1,\tau_1) - H_2(\rho_2,\tau_2)]d\tau, \quad (1)$$

where ρ_i and $\tau_i, (i = 1, 2)$, are, respectively, the density distributions and the kinetic energy densities of the two separated nuclei. ρ and τ are the same quantities for the composite system. $H(\rho,\tau)$ is the energy density functional for the composite system, while $H_i(\rho_i,\tau_i)$, $(i = 1,2)$, are the energy density functions of the two separated nuclei. In the sudden approximation, ρ is given by $\rho = \rho_1 + \rho_2$. The energy density functional $H(\rho,\tau)$ for the conventional Skyrme interaction is given by [22]

$$\begin{aligned} H(\rho,\tau) = {} & \frac{\hbar^2}{2m}\tau + \frac{1}{2}t_0\left[\left(1+\frac{1}{2}x_0\right)\rho^2 - \left(x_0 + \frac{1}{2}\right)\right. \\ & \left.\times\left(\rho_n^2 + \rho_p^2\right)\right] + \frac{1}{4}(t_1 + t_2)\,\rho\tau + \frac{1}{8}(t_2 - t_1) \\ & \times(\rho_n\tau_n + \rho_p\tau_p) + \frac{1}{16}(t_2 - 3t_1)\,\rho\nabla^2\rho \\ & + \frac{1}{32}(3t_1 + t_2)\left(\rho_n\nabla^2\rho_p + \rho_p\nabla^2\rho_p\right) \\ & + \frac{1}{12}\left[\rho^2 - \frac{1}{2}\left(\rho_n^2 + \rho_p^2\right)\right]\rho^\alpha \\ & + \frac{1}{16}(t_1 - t_2)\left(J_n^2 - J_p^2\right), \end{aligned} \quad (2)$$

where n and p denote neutron and proton, respectively, t_0, x_0, t_1, t_2, and t_3 are the Skyrme parameters, J_n and J_p denote the spin-orbit density for neutrons and protons, respectively. In this work we used the Skyrme interaction with parameter sets SIII and SkM*. Their parameters are given in Refs. [19] and [23], respectively. For Skyrme energy density of SkM* and SIII, the term, in Eq. (2), resulting from the three-body interaction force has value of $\alpha = \frac{1}{6}$ and 1, respectively. For SkM* , the energy density, $H(\rho,\tau)$, differs than that given by Eq. (2), it is presented in Ref. [23].

For the volume part of the kinetic energy density of the composite system we use the Thomas-Fermi approximation [18,19]

$$\tau'_j = \tau_j^{\mathrm{TF}} = \tfrac{3}{5}(3\pi^2)^{2/3}(\rho_{1j} + \rho_{2j})^{5/3}, \quad j = n, p. \quad (3)$$

The same approximation is also used for kinetic energy density of the individual nuclei [18]

$$\tau'_i = \tau_i^{\mathrm{TF}} = \tfrac{3}{5}(3\pi^2)^{2/3}\rho_i^{5/3}, \quad i = 1, 2. \quad (4)$$

The relation between τ appearing in Eq. (2) and τ' is [19]

$$\tau = \tau^{\mathrm{TF}} + \frac{1}{3}\nabla^2\rho + \eta\frac{|\vec{\nabla}\rho|^2}{\rho}; \quad \frac{1}{4} \geqslant \eta \geqslant \frac{1}{36}. \quad (5)$$

The Thomas-Fermi term in Eq. (5) takes account of the antisymmetrization correction in the ion-ion potential. As the value of η increases, the HI potential becomes more attractive in the tail region.

The matter density distribution of the two axially symmetric deformed nuclei is assumed to be the two parameter Fermi form

$$\rho(r) = \frac{\rho_0}{1 + e^{(r - R(\theta))/a}}, \quad (6)$$

where the half density radius of this Fermi distribution is given by

$$R(\theta) = R_0[1 + \beta_2 Y_{20}(\theta,0) + \beta_4 Y_{40}(\theta,0) + \ldots]. \quad (7)$$

β_2 and β_4 are the quadruple and hexadecapole deformation parameters, respectively, and the angle θ is measured from the symmetry axis of the deformed nucleus. ρ_0 can be determined from the normalization condition

$$\int \rho(\bar{r})d\bar{r} = \text{mass number}. \quad (8)$$

We assume a system of coordinates with the z-axis in the direction of the position vector, $\bar{R}$, joining the centers of mass of the target and projectile nuclei. R describes relative motion of the two interacting nuclei (Fig. 1). Referring to Fig. 1, the angles appearing in Eq. (7) for the target and projectile are given, respectively, by

$$\begin{aligned} \cos\theta_1 &= \cos\theta\cos\theta_T + \sin\theta\sin\theta_T\cos(\phi - \phi_T) \\ \cos\theta_2 &= \frac{(\vec{r} + \vec{R}) \bullet \hat{\Omega}_P}{|\vec{r} + \vec{R}|}. \end{aligned} \quad (9)$$

$\hat{\Omega}_T(\theta_T,\phi_T)$ and $\hat{\Omega}_P(\theta_P,\phi_P)$ are unit vectors in the direction of the symmetry axes of target and projectile,

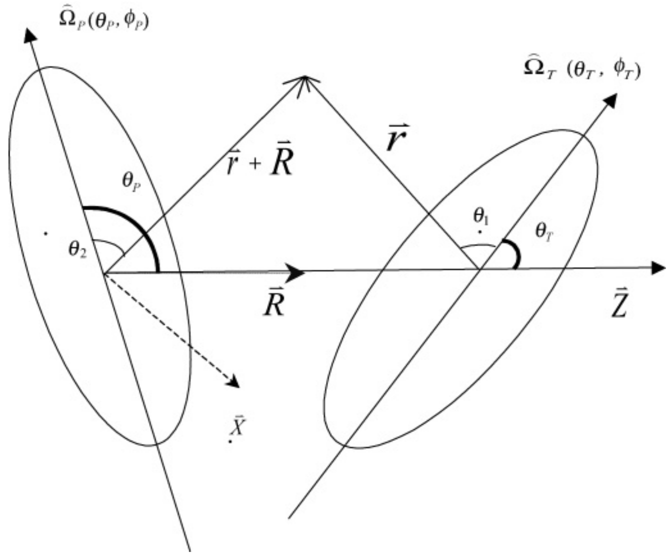

FIG. 1. Schematic representation of the two interacting axially symmetric deformed nuclei. The unit vectors in the direction of the symmetry axes of target and projectile are $\hat{\Omega}_T(\theta_T, \phi_T)$ and $\hat{\Omega}_P(\theta_P, \phi_P)$, respectively.

respectively. $\hat{\Omega}(\theta, \phi)$ is a unit vector in the direction of the vector $\bar{r}$.

In this work we propose to discuss the (θ, ϕ)-orientation dependence of the real part of the nucleus-nucleus optical potential taking into account the deformations of both colliding nuclei. We choose the $^{238}U+^{238}U$ interacting deformed-deformed pair to perform our study. The values of the static parameters of ^{238}U-nucleus used in the present work are [29], $R_0 = 6.8054$ fm, $a = 0.6049$ fm, $\beta_2 = 0.331$, and $\beta_4 = 0.087$.

III. NUMERICAL RESULTS

For two fixed directions of the two symmetry axes of the interacting nuclei, the interaction potential between them is affected by the overlap volume of the two nuclei at separation distance $R = 0.0$ fm, Fig. 2, and the value of the separation distance R. Figure 2 shows the real part of the HI potential for $^{238}U+^{238}U$ interaction pair at separation distance $R = 0.0$ fm. The results in Fig. 2 are calculated using the energy density approach for Skyrme force with parameter set SIII. The figure presents the variation of the HI potential when the two centers of mass of the two nuclei coincide, $V(R = 0.0 \text{ fm})$, with the azimuthal angle ϕ_T for the five values of the symmetry axes orientation angles $(\theta_T, \theta_P) = (30^\circ, 30^\circ), (60^\circ, 60^\circ), (90^\circ, 90^\circ), (60^\circ, 120^\circ)$, and $(60^\circ, 90^\circ)$. It is shown that complete overlap with maximum value of the potential between the two nuclei occurs when $\theta_P = \theta_T$ at $\phi_T = 0^\circ$. The overlap region becomes small at $\phi_T = 0^\circ$ for the two orientations $(30^\circ, 90^\circ)$ and $(60^\circ, 120^\circ)$ where partial overlap occurs for these orientations. The figure shows that

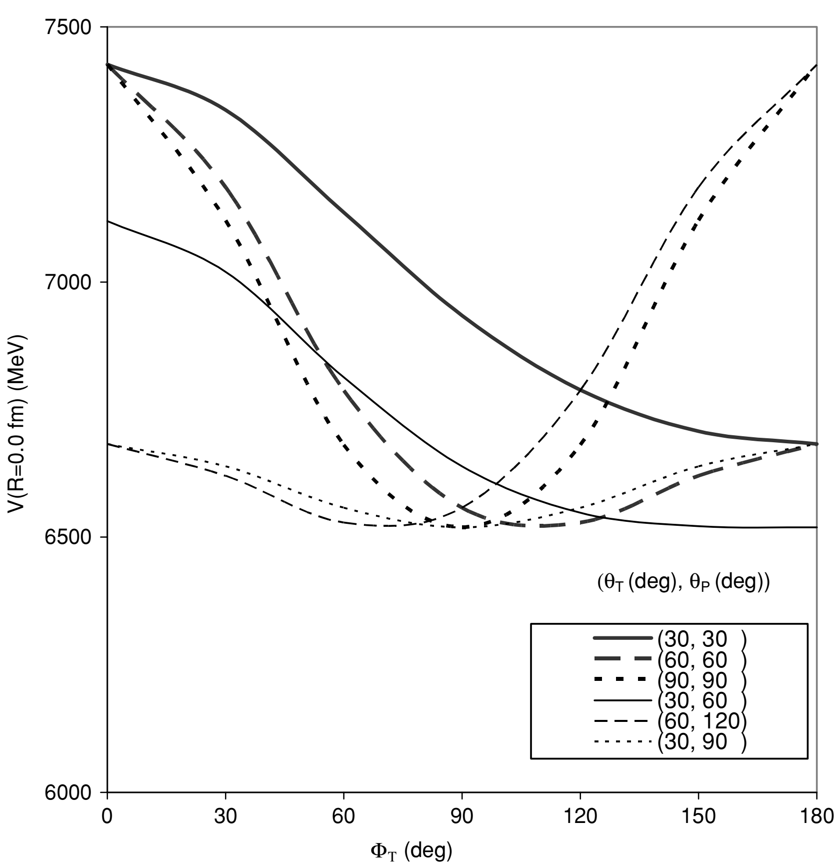

FIG. 2. The variation of the real part of the nuclear potential at $R = 0$ with the angle ϕ_T for different symmetry axes orientations for interacting pair $^{238}U + ^{238}U$.

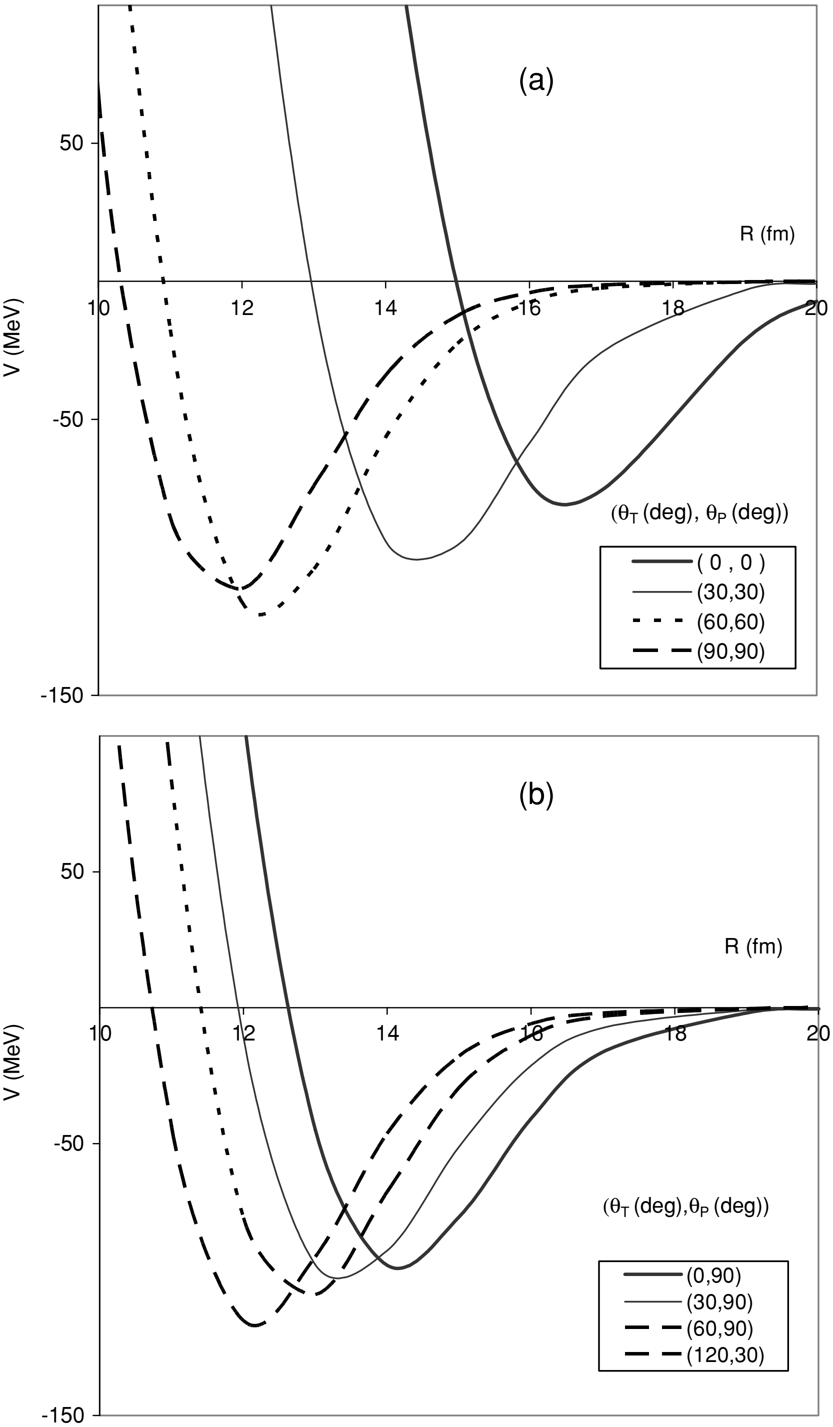

FIG. 3. (a) The real part of the nuclear potential for $^{238}U+^{238}U$ calculated using Skyrme force with parameter set SIII for various orientations with $\theta_T = \theta_P$ of the two deformed nuclei. (b) Same as (a) but for $\theta_T \neq \theta_P$.

the smallest overlap region for $R = 0.0$ fm happens when $\theta_P = \theta_T = 90^\circ$ and $\phi_T = 90^\circ$. This is expected since the two nuclei for these orientation angles cross each other at right angles. As ϕ_T increases, the overlap between the two nuclei changes, except when $\theta_T = 0^\circ$, reaching a potential value at $\phi_T = 180^\circ$ depends on the angles θ_T and θ_P. We found that the ϕ-dependence of $V(R=0$ fm) is strongest for $\theta_T = \theta_P = 90^\circ$ while it is too weak for $(30^\circ, 90^\circ)$ and vanishes at $\theta_T = 0^\circ$ or $\theta_P = 0^\circ$. For the case $\theta_T = \theta_P = 90^\circ$, the potential $V(R=0.0$ fm) is symmetric around $\phi_T = 90^\circ$ while for $\theta_T = \theta_P = 60^\circ$ and $(\theta_T, \theta_P) = (30^\circ, 60^\circ)$ the HI potential has strong ϕ_T-variation with opposite behavior in these two cases. This behavior of $V(R=0.0\,\text{fm})$ with variation of both target and projectile orientations can be understood on purely geometrical considerations.

The HI potential for two deformed nuclei was calculated assuming coplanar symmetry axes and using the folding model [11,14], energy density approach [26], and proximity method [9,10]. The behavior of $V(R)$ with the separation distance R and the orientation of the two symmetry axes when existing in the same plane was explained in Refs. [10,11,15,21,26]. Figure 3 shows our results for the $^{238}U+^{238}U$ interaction

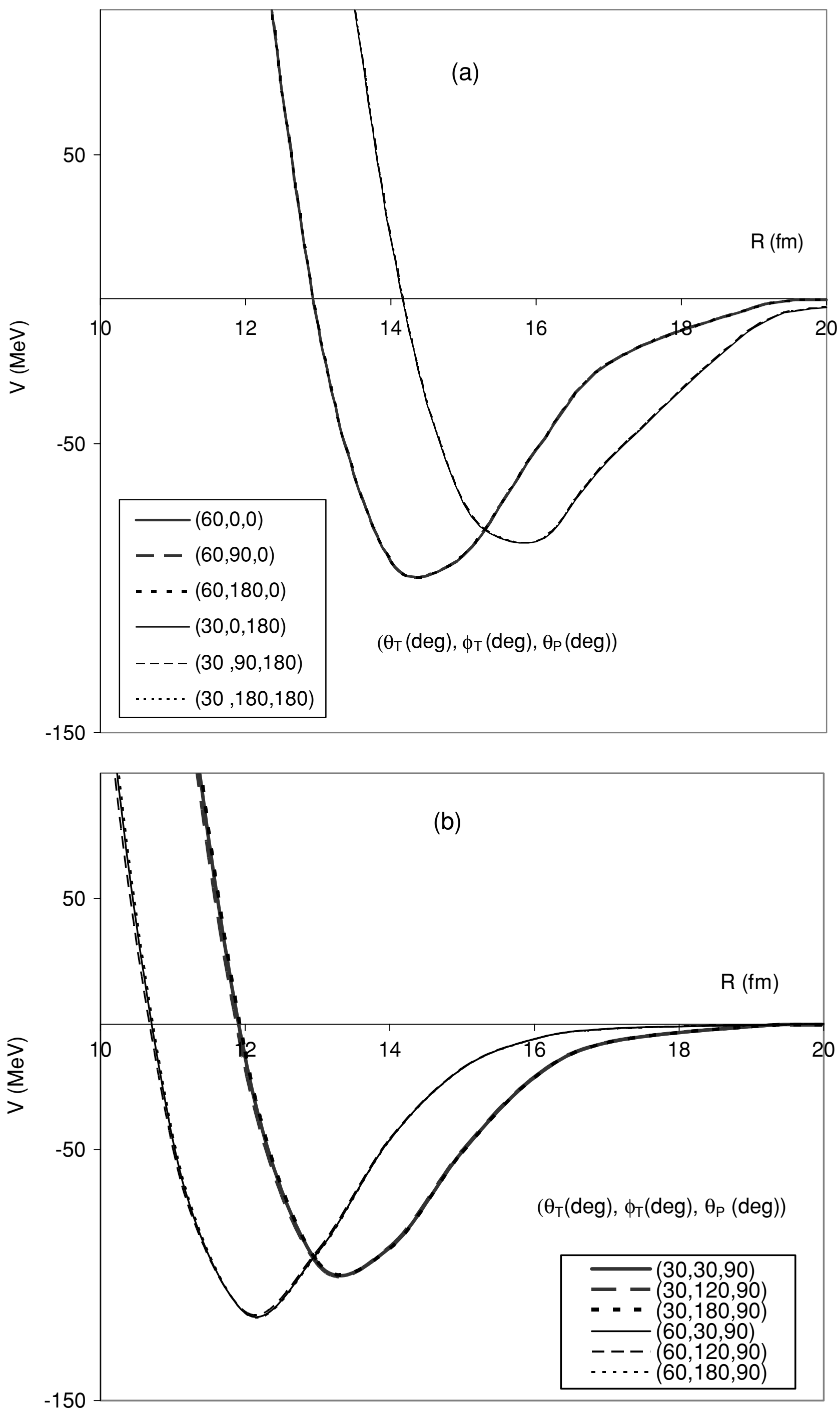

FIG. 4. (a) The ϕ-dependence of HI potential for $^{238}U+^{238}U$ calculated using SIII parameters of Skyrme force when the θ_P-orientation is 0° or 180°. (b) Same as (a) but when the θ_P-orientation is 90°.

potential assuming two coplanar symmetry axes and calculated using Skyrme force with parameter set SIII. Figure 3(a) presents the calculated potential for the orientation angles $\theta_T = \theta_P = 0^\circ, 30^\circ, 60^\circ, 90^\circ$ while Fig. 3(b) shows the calculations for the different projectile-target angles $(\theta_T, \theta_P) = (0^\circ, 90^\circ), (30^\circ, 90^\circ), (60^\circ, 90^\circ)$, and $(120^\circ, 30^\circ)$. As expected the potential curve for fixed orientation angles starts with a repulsive core followed by the attractive part. As pointed out in Refs. [18,19], the repulsive core is due to the sudden approximation, where the lack of distortion in the single particle wave functions is neglected, and the approximation used for the density and kinetic energy density of the composite system. The attractive part reaches a minimum value $V_{\min}(R_{\min})$ then it becomes less attractive and vanishes at a large distance between the two interacting nuclei. For equal values of θ_T and θ_P, the minimum occurs when the two nuclei touch each other and its value depends almost on the distances between nucleons in the surface region of one nucleus interacting with nucleons in the other nucleus surface. For $\theta_P = \theta_T = 90^\circ$ maximum area of the target surface is exposed to the projectile while for $\theta_P = \theta_T = 0^\circ$ the two surfaces exposed to each other is the smallest. In the first case we have a strong attractive potential

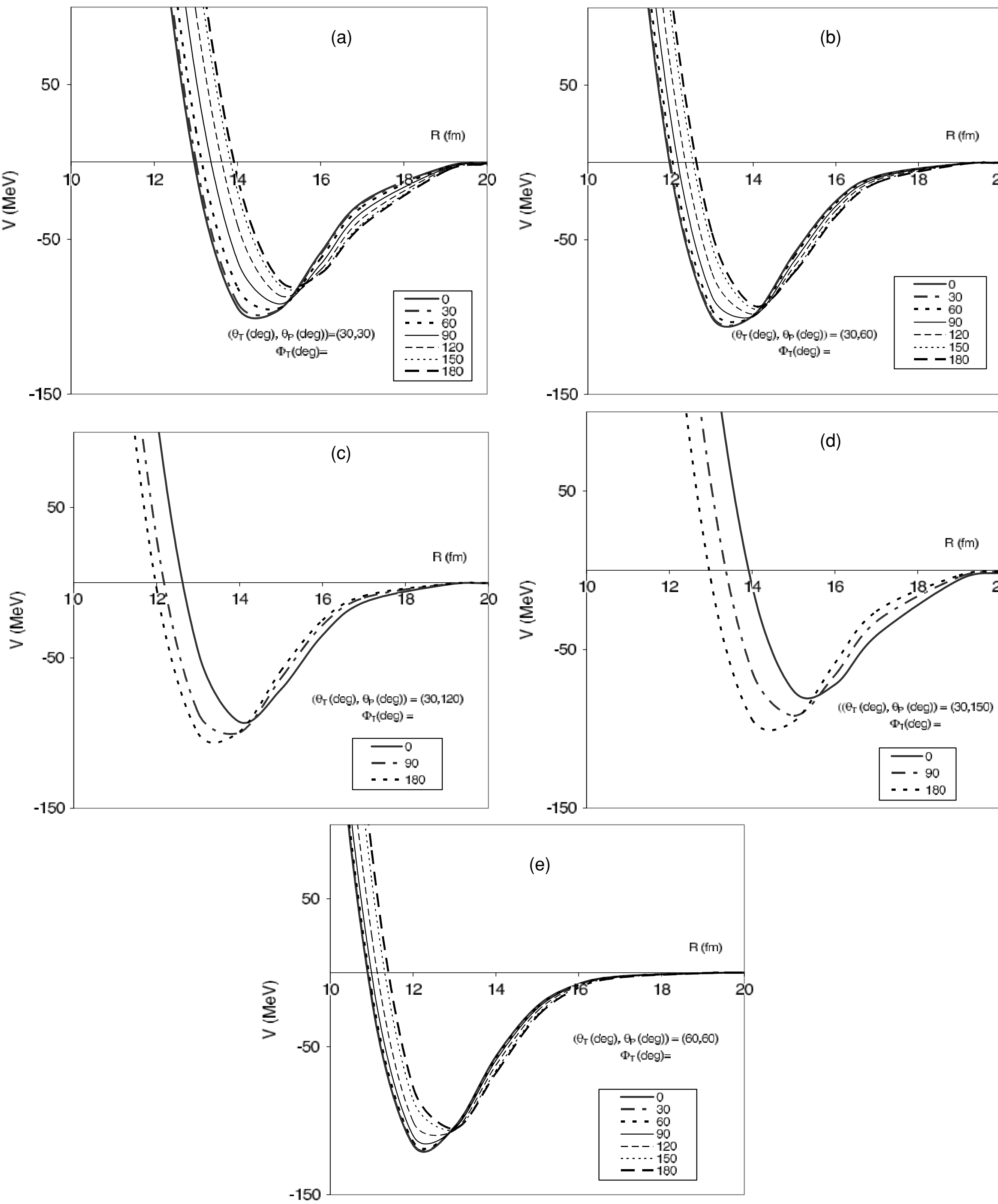

FIG. 5. (a) The ϕ-dependence of HI potential for ^{238}U+^{238}U calculated using SIII parameters of Skyrme force for the orientation angles $(\theta_T, \theta_P) = (30^\circ, 30^\circ)$. (b) For the orientation angles $(\theta_T, \theta_P) = (30^\circ, 60^\circ)$. (c) For the orientation angles $(\theta_T, \theta_P) = (30^\circ, 120^\circ)$. (d) For the orientation angles $(\theta_T, \theta_P) = (30^\circ, 150^\circ)$. (e) For the orientation angles $(\theta_T, \theta_P) = (60^\circ, 60^\circ)$.

TABLE I. The ϕ-dependence the potential minimum value, $V_{\min}$, and its radius, $R_{\min}$, for the real part of HI potential for $^{238}U+^{238}U$ interaction at the orientation angles $(\theta_T, \theta_P) = (30^\circ, 30^\circ)$. The calculations are made using SIII parameters set of Skyrme force (second and third columns) and SkM* parameters set (fourth and fifth columns) considering both the quadruple and hexadecapole deformations in projectile and target. The sixth and seventh columns present the same quantities calculated using SkM* parameters set but considering only the quadruple deformation in both projectile and target.

ϕ_T (deg)	SIII ($\beta_2 = 0.331$, $\beta_4 = 0.087$)		SkM* ($\beta_2 = 0.331$, $\beta_4 = 0.087$)		SkM* ($\beta_2 = 0.331$, $\beta_4 = 0$)	
	$R_{\min}$ (fm)	$V_{\min}$ (MeV)	$R_{\min}$ (fm)	$V_{\min}$ (MeV)	$R_{\min}$ (fm)	$V_{\min}$ (MeV)
0	14.49	−102.96	13.87	−129.40	13.90	−126.31
30	14.53	−101.57	13.91	−127.93	13.92	−125.79
60	14.67	−97.67	14.02	−123.64	13.99	−124.35
90	14.88	−92.18	14.21	−117.18	14.07	−122.36
120	15.11	−86.48	14.45	−110.13	14.16	−120.37
150	15.30	−82.19	14.64	−104.67	14.23	−118.91
180	15.38	−80.61	14.72	−102.61	14.26	−118.38

minimum while the second case produces a less attractive minimum. Since for $\theta_P = \theta_T = 90^\circ$ the separation distance R when the two surfaces touch each other is small, the minimum in this case occurs at small $R_{\min}$ value compared to the case when $\theta_P = \theta_T = 0^\circ$ where the range of the potential is the longest for all orientation angles of the two symmetry axes of the interacting nuclei.

Figure 3(b) shows that for nonequal values of θ_T and θ_P the behavior of the potential is almost the same as the case of equal values of the angles θ_T and θ_P. For $\theta_T = 0^\circ$ and $\theta_P = 90^\circ$, the number of interacting nucleons in the two surface regions is small. So, a less deep minimum is produced at larger radius. When θ_T increases to 60° this number becomes larger and the potential has deeper minimum at a smaller radius compared with the case of $\theta_T = 0^\circ$. For the cases of $\theta_T = 120^\circ$ and $\theta_P = 30^\circ$, the number of surface nucleons at $R = R_{\min}$ is less than the cases where $\theta_T = 60^\circ$ and $\theta_P = 90^\circ$.

Concerning the azimuthal angle dependence of $^{238}U+^{238}U$ HI potential calculated using the Skyrme energy density approach we found that this dependence is not important for some orientations of the two nuclei. For example, Fig. 4(a) shows that it vanishes when either θ_T or θ_P has the value 0° or 180°. This is expected since the symmetry axis, in this case, coincides with the z-axis. Also, we found that the ϕ-dependence is negligible when any of the two symmetry axes is perpendicular to the z-axis, Fig. 4(b). Strong ϕ-dependence occurs for the orientation angles $(\theta_T, \theta_P) = (30^\circ, 30^\circ), (30^\circ, 60^\circ), (30^\circ, 120^\circ), (30^\circ, 150^\circ)$, and $(60^\circ, 60^\circ)$. Figures 5(a), (b), (c), (d), and (e) show the ϕ-dependence of the $^{238}U+^{238}U$ HI potential calculated using the SIII parameters of Skyrme force for orientation angles $(\theta_T, \theta_P) = (30^\circ, 30^\circ), (30^\circ, 60^\circ), (30^\circ, 120^\circ), (30^\circ, 150^\circ)$, and $(60^\circ, 60^\circ)$, respectively. We found weak ϕ-dependence of the HI potential for other orientation angles.

The second and third columns of Tables I, II, and III, show the ϕ-dependence of the value of potential minimum, $V_{\min}$, and its radius, $R_{\min}$, for the orientation angles $(\theta_T, \theta_P) = (30^\circ, 30^\circ), (30^\circ, 60^\circ)$, and $(60^\circ, 60^\circ)$, respectively. These calculations have been performed using a Skyrme force with parameters set SIII and considering both the quadruple and hexadecapole deformations in projectile and target. For $\theta_P = \theta_T = 30^\circ$, changing the value of the azimuthal angle ϕ_T from 0° to 180° reduces the value of $V_{\min}$ by about 22% and increases its radius by 6%. It should be noted that when $\phi_T = 180^\circ$ the system of the two nuclei becomes equivalent to that with the orientation set $(\theta_T, \phi_T, \theta_P) = (150^\circ, 0^\circ, 30^\circ)$. The percentage decrease in $V_{\min}$ and its corresponding increase in $R_{\min}$ as ϕ_T changes from 0° to 180° are about 15% and 4.5%, respectively, for the orientation $\theta_T = 30^\circ$ and $\theta_P = 60^\circ$. For this orientation and at $\phi_T = 180^\circ$ the system of the two nuclei becomes equivalent to that with the orientation $(\theta_T, \phi_T, \theta_P) = (30^\circ, 0^\circ, 120^\circ)$. For $\theta_P = \theta_T = 60^\circ$, the corresponding decrease in $V_{\min}$ and increase in $R_{\min}$ are about 11% and 4%, respectively. This

TABLE II. Same as Table I but for the orientation angles $(\theta_T, \theta_P) = (30^\circ, 60^\circ)$.

ϕ_T (deg)	SIII ($\beta_2 = 0.331$, $\beta_4 = 0.087$)		SkM* ($\beta_2 = 0.331$, $\beta_4 = 0.087$)		SkM* ($\beta_2 = 0.331$, $\beta_4 = 0$)	
	$R_{\min}$ (fm)	$V_{\min}$ (MeV)	$R_{\min}$ (fm)	$V_{\min}$ (MeV)	$R_{\min}$ (fm)	$V_{\min}$ (MeV)
0	13.44	−108.60	12.89	−133.35	12.92	−132.34
30	13.46	−108.18	12.91	−132.92	12.94	−131.62
60	13.52	−106.74	12.96	−131.40	13.01	−129.55
90	13.64	−103.78	13.06	−128.22	13.10	−126.52
120	13.81	−99.25	13.21	−123.12	13.22	−123.29
150	13.98	−94.69	13.37	−117.75	13.31	−120.83
180	14.05	−92.73	13.44	−115.39	13.34	−119.91

TABLE III. Same as Table I but for the orientation angles $(\theta_T, \theta_P) = (60^\circ, 60^\circ)$.

ϕ_T (deg)	SIII ($\beta_2 = 0.331$, $\beta_4 = 0.087$)		SkM* ($\beta_2 = 0.331$, $\beta_4 = 0.087$)		SkM* ($\beta_2 = 0.331$, $\beta_4 = 0$)	
	$R_{\min}$ (fm)	$V_{\min}$ (MeV)	$R_{\min}$ (fm)	$V_{\min}$ (MeV)	$R_{\min}$ (fm)	$V_{\min}$ (MeV)
0	12.30	−120.49	11.81	−143.00	11.92	−138.49
30	12.31	−120.18	11.82	−142.78	11.94	−137.52
60	12.34	−119.28	11.84	−142.15	12.00	−134.60
90	12.40	−117.48	11.89	−140.67	12.13	−130.10
120	12.54	−114.10	12.01	−137.12	12.28	−125.09
150	12.70	−109.62	12.16	−131.79	12.42	−121.17
180	12.78	−107.33	12.24	−128.95	12.47	−119.69

indicates significant ϕ-dependence of the HI potential between two deformed nuclei.

In our calculations using Skyrme energy density we found that the θ-dependence of the HI potential between two deformed nuclei is too strong. It can change the value of the potential minimum and its radius by a maximum percentage of 50% and 40%, respectively. The value of the deepest potential minimum occurring for all possible orientations is −120.49 MeV and its radius is 12.30 fm while the corresponding less attractive potential minimum has value −80.18 MeV and it occurs at separation distance $R = 16.51$ fm. The deepest and the shallowest potential minima occur at orientation angles $(\theta_T, \phi_T, \theta_P) = (60^\circ, 0^\circ, 60^\circ)$ and $(0^\circ, 0^\circ, 0^\circ)$, respectively. The reason that the deepest minimum occurs at $\theta_P = \theta_T = 60^\circ$ and not at $\theta_P = \theta_T = 90^\circ$ as expected is the presence of hexadecapole deformation which makes the surface as diamond shape. At the orientation $(\theta_T, \phi_T, \theta_P) = (90^\circ, 0^\circ, 90^\circ)$ the potential have a minimum value −114.78 MeV at separation distance $R = 11.70$ fm.

The fourth and fifth columns in Tables I, II, and III present the azimuthal angle dependence of ^{238}U+^{238}U potential calculated using SkM* [23] parameters of Skyrme force for orientation angles $(\theta_T, \theta_P) = (30^\circ, 30^\circ), (30^\circ, 60^\circ)$, and $(30^\circ, 60^\circ)$ considering both the quadruple and hexadecapole deformations in projectile and target. Although the HI potential derived from Skyrme SkM* parameters is more attractive at $R_{\min}$ compared to that calculated using parameter set SIII, it has almost the same θ and ϕ variation. Tables I, II, and III show that the maximum ϕ-variation of the HI potential calculated using SkM* parameters, with β_2 and β_4 deformations, occurs at the relative symmetry axes orientations $\theta_T = \theta_P = 30^\circ$ of the interacting nuclei. For this orientation, the percentage variations in $V_{\min}$ and $R_{\min}$ when ϕ_T changes from 0° to 180° are about 21% and 5.8%, respectively. Also, the HI potential has a strong orientation dependence which appears in the percentage variation of $V_{\min}$ and $R_{\min}$ for the more and less deep minimum for all possible orientations considered in the present work. The deepest minimum occurs at $\theta_T = \theta_P = 60^\circ$ and $\phi_T = 0^\circ$ where it has the values $V_{\min} = -143.00$ MeV and $R_{\min} = 11.81$fm while the shallowest one occurs for the orientation $\theta_T = \theta_P = 0^\circ$ and $\phi_T = 0^\circ$ and has $V_{\min} = -103.58$ MeV and $R_{\min} = 15.82$ fm. The percentage difference between the two minima are 38.1% for $V_{\min}$ and 34.0% for $R_{\min}$.

We think that the presence of the hexadecapole deformation strongly affects the orientation dependence of the HI potential between two deformed nuclei. For this reason we calculated the ^{238}U+^{238}U interaction potential using SkM* parameters and assuming only the quadrupole deformation for both the projectile and target. The sixth and seventh columns in Tables I, II, and III show the azimuthal angle dependence of $V_{\min}$ and $R_{\min}$ calculated using SkM* parameters of Skyrme force for orientation angles $(\theta_T, \theta_P) = (30^\circ, 30^\circ), (30^\circ, 60^\circ)$, and $(30^\circ, 60^\circ)$ considering only the quadruple deformation in projectile and target ($\beta_4 = 0$). The tables show that the maximum ϕ-variation of the HI potential occurs for $\theta_T = \theta_P = 60^\circ$ where $V_{\min}$ and $R_{\min}$ vary by 13.6% and 4.4%, respectively, when ϕ_T changes from 0° to 180°. This shows less ϕ-dependence of the HI potential compared to the case when $\beta_4 = 0.087$. For the case of $\beta_4 = 0$. We found that the deepest $V_{\min}$ for all possible orientations of the deformed nuclei symmetry axes occurs at $\theta_T = \theta_P = \phi_T = 90^\circ$. Its value is $V_{\min} = -144.46$ MeV and its radius has the value 11.13 fm, it is deeper by about 24.0% compared with the smallest value of $|V_{\min}|$. Moreover, its radius varies by about 20.8% compared with the radius of the smallest $|V_{\min}|$.

In order to compare our results for orientation dependence of the HI potential derived from the realistic Skyrme *NN* potential with the same quantity derived in Ref. [10] using the proximity approach, we calculated $V_{\min}$ and $R_{\min}$ at orientation angles $(\theta_T, \theta_P) = (0^\circ, 0^\circ), (45^\circ, 45^\circ), (45^\circ, 135^\circ), (30^\circ, 135^\circ), (0^\circ, 90^\circ)$, and $(90^\circ, 90^\circ)$. Table IV shows our results using SkM* force and assuming $\beta_4 = 0$ for both

TABLE IV. The values of the potential minimum,$V_{\min}$, and its radius, $R_{\min}$, calculated using SkM* parameters of Skyrme force considering the quadruple deformation in both projectile and target for the orientations $(\theta_T, \theta_P) = (0^\circ, 0^\circ), (45^\circ, 45^\circ), (0^\circ, 90^\circ), (45^\circ, 135^\circ)$, and $(90^\circ, 90^\circ)$.

ϕ_T(deg)	ϕ_P(deg)	$R_{\min}$ (fm)	$V_{\min}$ (MeV)
45	135	13.45	−116.51
30	135	13.85	−117.83
0	0	14.95	−122.32
45	45	12.89	−131.81
0	90	13.01	−133.66
90	90	11.13	−144.46

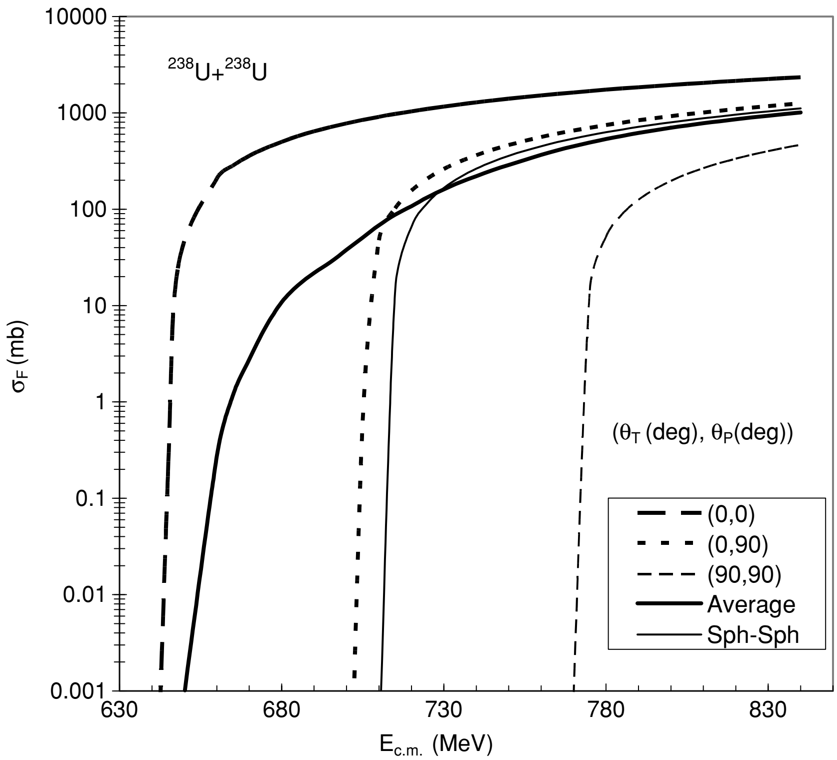

FIG. 6. Fusion cross sections for ^{238}U+^{238}U calculated at the three orientations $(\theta_T, \theta_P) = (0^\circ, 0^\circ)$, $(90^\circ, 90^\circ)$, and $(0^\circ, 90^\circ)$ of the symmetry axes of the two interacting nuclei. The quadrupole deformations of both target and projectile are considered with the value $\beta_2 = 0.331$. The thick solid curve shows the average fusion cross section over all possible orientations. The thin solid curve is the fusion cross section calculated assuming two spherical nuclei.

interacting nuclei. Our results show strong orientation dependence compared to the results in Ref. [10]. For example, Table IV shows large differences in $V_{\min}$ for the orientations $(\theta_T, \theta_P) = (0^\circ, 0^\circ)$, $(45^\circ, 45^\circ)$, $(45^\circ, 135^\circ)$, $(30^\circ, 135^\circ)$, and $(0^\circ, 90^\circ)$ while in Ref. [10] $V_{\min}$ is almost the same for all these orientations.

Many authors studied the effect of deformation of a deformed target nucleus on some physical quantities such as fusion, fission, and reaction cross sections. Most of these studies consider deformation of one nucleus while the other is assumed spherical. This is because the potential between two deformed nuclei which is input for these studies depends on many different possible orientations besides that its microscopic derivation needs the numerical calculation of six-dimensional integral [10,15]. Since the present paper deals with the nuclear heavy ion potential between two deformed nuclei, it is interesting to show the effect of deformation of both nuclei on the fusion cross section. For this purpose we consider the fusion cross section, σ_F, for the reaction ^{238}U+^{238}U. Beside the nuclear part of the HI potential between two deformed nuclei we need to calculate the Coulomb potential for this system. The latter is needed to determine the fusion barrier parameters which are input to calculate σ_F. To simplify the calculations we calculate σ_F using Wong's formula [30] and we assume coplanar symmetry axes for the nuclei. This assumption reduces the amount of calculations needed to calculate the Coulomb potential. The Coulomb interaction between two deformed nuclei is calculated by the method of Ref. [11] which is based on multipole expansion of two deformed density distributions. This method of calculating the Coulomb potential is more accurate than other methods based on expanding the potential in terms of deformation parameters [31,32]. Figure 6 shows the calculations of the ^{238}U+^{238}U fusion cross section for the three orientation angles $(\theta_T, \theta_P) = (0^\circ, 0^\circ)$, $(90^\circ, 90^\circ)$, and $(0^\circ, 90^\circ)$ of the symmetry axes of the two interacting nuclei. We considered the quadrupole deformations of both target and projectile with the value $\beta_2 = 0.331$. The figure also indicates the fusion cross section averaged over all possible orientations, σ^{av}, and the fusion cross section calculated assuming two spherical nuclei, σ^{sph}, with the same root mean square, matter radii, r_{rms}, as the interacting deformed nuclei. The figure shows strong orientation dependence of the fusion cross section at sub-barrier energies. This is clear from the difference between $\sigma_F(0^\circ, 0^\circ)$ and $\sigma_F(90^\circ, 90^\circ)$. For the orientations $(\theta_T, \theta_P) = (0^\circ, 0^\circ)$ and $(\theta_T, \theta_P) = (90^\circ, 90^\circ)$ the Coulomb barrier heights are 646.46 MeV and 773.08 MeV, respectively, while the fusion radii occur at $R_B = 17.99$ fm and 13.65 fm, respectively. These large differences between lowest and largest Coulomb barriers produce a too strong orientation dependence. In Ref. [33], the orientation dependence of σ_F from $\theta_T = 0^\circ$ to $\theta_T = 90^\circ$ for the spherical-deformed interacting pair ^{16}O+^{154}Sm at $E_{\text{c.m.}} = 54.3$ MeV ($\approx$ the spherical-spherical barrier height of ^{16}O+^{154}Sm) is a factor about 10^3. In our case the orientation dependence of σ_F from $(\theta_T, \theta_P) = (0^\circ, 0^\circ)$ to $(\theta_T, \theta_P) = (90^\circ, 90^\circ)$ at $E_{\text{c.m.}} = 770$ MeV [$\approx$ the maximum barrier height of ^{238}U+^{238}U at $(\theta_T, \theta_P) = (90^\circ, 90^\circ)$] is a factor of about 10^7. Also, in Ref. [33] the deformation dependence at $E_{\text{c.m.}} = 54.3$ MeV, $\sigma_F(\beta_2)/\sigma_F(0) \approx 18$. In our study the deformation dependence at $E_{\text{c.m.}} = 713.6$ MeV ($\approx$ the spherical-spherical barrier height of ^{238}U+^{238}U), $\sigma_F^{av}(\beta_2)/\sigma_F(0) \approx 36$.

IV. CONCLUSION AND OUTLOOK

In most calculations of the nuclear physical quantities such as fusion cross section, reaction cross section, and angular distribution the authors neglect the deformation of one of the interacting nuclei to avoid heavy numerical calculations. In almost all the cases it is assumed that the two interacting nuclei have coplanar symmetry axes [10,11,14,16,21] and even $\theta_P = \theta_T$. In the present work we used the Hamiltonian energy density approach derived from the well-known Skyrme interaction to study the orientation dependence of the HI potential between two deformed nuclei. We found strong θ-dependence of the HI potential beside non-negligible ϕ-dependence. In most calculations, the physical quantity is calculated first at fixed orientation angles then it is averaged over all possible orientations. In the averaging process, the quantity at a given value of θ is multiplied by $\sin\theta$ while for ϕ the average process takes equal contributions from all ϕ values. Since we found in the present calculations that large ϕ-dependence occurs at $\theta = 30^\circ$ or $\theta = 60^\circ$ whose sine values equal $1/2$ and $\sqrt{3}/2$, respectively, and no ϕ-dependence at $\theta = 0^\circ$, 180° and too small ϕ-dependence at $\theta = 90^\circ$ we conclude that the present work is important to show both θ and ϕ dependences of the HI potential.

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