

Extraction of structure functions from quasielastic electron scattering (e, e') off medium and heavy nuclei

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Using a relativistic mean-field single-particle knockout model for (e, e') reactions on nuclei, we investigate approximate treatments of Coulomb distortion effects and the extraction of longitudinal and transverse structure functions. We show that an effective momentum approximation when coupled with a focusing factor provides a good description of the transverse contributions to the (e, e') cross sections for electron energies above 300 MeV on ^{208}Pb . This approximation is not as good for the longitudinal contributions even for incident electron energies above 1 GeV, and if one requires very precise extraction of longitudinal and transverse structure functions in the quasielastic region, one must utilize distortion factors based on a nuclear model and a more accurate inclusion of Coulomb distortion effects.

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I. INTRODUCTION

There continues to be considerable theoretical and experimental interest in extracting longitudinal and transverse structure functions as a function of energy loss for fixed three-momentum transfer for a range of nuclei [1]. For low to medium electron energies ($200 < E < 600$ MeV), the (e, e') cross section is significantly affected by the static Coulomb field of the target nucleus for $Z > 20$, and it is necessary to use some method of removing the so-called Coulomb distortion effects in order to investigate the underlying physical processes in quasielastic scattering. For a precise extraction of the longitudinal response even at incident electron energies of 2 GeV, some correction for Coulomb distortion effects is needed. It would be very appealing if Coulomb effects could be removed by shifting the value of the experimental energy or scattering angle so as to compensate for the Coulomb distortion [2]. While it is quite clear that this cannot be done exactly, the question is whether one can find some approximate treatment of Coulomb distortion effects that improves with increasing electron energy.

In this paper, we investigate this question and report very good success in handling the Coulomb distortion effects in the transverse parts of the quasielastic cross section, but with less success in the longitudinal portion of the cross section unless we utilize a nuclear model of the transition current arising from the quasielastic knockout of nucleons.

In order to frame the discussion, it is useful to note that in the plane wave Born approximation (PWBA), the inclusive (e, e') cross section for electrons or positrons (assuming nuclear

current conservation) is given by

$$\frac{d^2\sigma}{d\Omega_e d\omega} = \sigma_M \left\{ \frac{Q^4}{q^4} S_L(q, \omega) + \left[\tan^2 \frac{\theta_e}{2} + \frac{Q^2}{2q^2} \right] S_T(q, \omega) \right\}, \quad (1)$$

where $q_\mu^2 = \omega^2 - \mathbf{q}^2 = -Q^2$ is the four-momentum transfer, σ_M is the Mott cross section given by $\sigma_M = \left(\frac{\alpha}{2E_i} \right)^2 \frac{\cos^2 \frac{\theta_e}{2}}{\sin^4 \frac{\theta_e}{2}}$, and S_L and S_T are the longitudinal and transverse structure functions which depend only on the momentum transfer q and the energy transfer ω . As is well known, by keeping the momentum and energy transfers fixed while varying the electron energy E_i and scattering angle θ_e , it is possible to extract the two structure functions with two measurements. The longitudinal and transverse structure functions in Eq. (1) are squares of the Fourier transform of the components of the nuclear transition current density integrated over outgoing nucleon angles. Explicitly, the structure functions for knocking out nucleons from a nucleus are given by

$$S_L(q, \omega) = \sum_{\alpha_b, s_p} S_{\alpha_b} \rho_p \int |N_0|^2 d\Omega_p, \quad (2)$$

$$S_T(q, \omega) = \sum_{\alpha_b, s_p} S_{\alpha_b} \rho_p \int (|N_x|^2 + |N_y|^2) d\Omega_p, \quad (3)$$

where the nucleon density of states $\rho_p = \frac{pE_p}{(2\pi)^2}$, s_p is the spin projection of the continuum nucleon, the z axis is taken to be along \mathbf{q} , and S_{α_b} contains the spectroscopic and averaging factors for each bound orbital. The Fourier transform of the nuclear current $J^\mu(\mathbf{r})$ is simply

$$N^\mu = \int J^\mu(\mathbf{r}) e^{i\mathbf{q}\cdot\mathbf{r}} d^3r. \quad (4)$$

In Eqs. (2) and (3), the continuity equation has been used to eliminate the z component N_z via the equation $N_z = -\frac{\omega}{q} N_0$.

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As noted above, the PWBA calculation breaks down for cases when the static Coulomb potential at the surface of the nucleus is not negligible when compared to the incident and outgoing electron energy. To set the scale, note that the Coulomb potential at the surface of ^{208}Pb is about 20 MeV.

Based on our previous investigations of inclusive quasielastic scattering and a new appreciation of a focusing factor [3–8], we investigate an improved effective momentum approximation with focusing (EMA-f) in this paper. Our goal is to seek a procedure for extracting the longitudinal and transverse structure functions from experimental data with minimal use of nuclear models.

II. COULOMB DISTORTION EFFECTS IN INCLUSIVE QUASIELASTIC SCATTERING

As discussed in a previous paper [7], we found a treatment of Coulomb distortion, labeled approximate DW, for (e, e') from medium and heavy nuclei that agrees to within a few percent to a full DWBA partial wave analysis which includes the static Coulomb field of the target exactly in the electron (or positron) wave functions. In this approximation, we define an r -dependent momentum for the incoming and outgoing electron wave functions, $\mathbf{p}'_{i,f}(r) = (p_{i,f} - \frac{1}{r} \int_0^r V_c(r) dr) \hat{\mathbf{p}}_{i,f}$, where $V_c(r)$ is the potential energy of the lepton moving in the static Coulomb field of the target nucleus. This implies an r -dependent momentum transfer $\mathbf{q}'(r) = \mathbf{p}'_i(r) - \mathbf{p}'_f(r)$. The choice of an r -dependent momentum allows the distorted partial wave solutions of the Dirac equation with the static Coulomb potential present to be approximated by spherical Bessel functions with argument $x = p'(r)r$ quite well from the origin out to more than 3 times the nuclear radius R . An additional requirement of approximating Coulomb distorted waves is to incorporate the Coulomb scattering phase shifts into the problem. As discussed in previous papers [5,7], we were able to achieve this by fitting the phase shifts as a function of the square of the Dirac quantum number κ for the incoming and outgoing electron energies for the nucleus under investigation and then to replace κ^2 by the classical angular momentum squared $(\mathbf{r} \times \mathbf{p})^2$ so that “plane-wave-like” lepton wave functions can be constructed that contain the effects of Coulomb distortion in the local momentum and the parametrized phase shifts which lead to what other authors refer to as focusing. Using this plane-wave-like wave function, we obtained an approximate “Møller-like” potential given by,

$$A_{\mu}^{\text{approx. DW}}(\mathbf{r}) = \frac{4\pi e}{q^2 - \omega^2} e^{i[\delta_i + \delta_f]} e^{i(\Delta_i - \Delta_f)} e^{i\mathbf{q}'(r) \cdot \mathbf{r}} \bar{u}_f \gamma_{\mu} u_i, \quad (5)$$

where the phase-shift parametrization is given by

$$\delta(\kappa^2) = \left[a_0 + a_2 \frac{\kappa^2}{(pR)^2} \right] e^{-\frac{1.4\kappa^2}{(pR)^2}} - \frac{\alpha Z}{2} \left(1 - e^{-\frac{\kappa^2}{(pR)^2}} \right) \ln(1 + \kappa^2), \quad (6)$$

where $\kappa^2 = (\mathbf{r} \times \mathbf{p})^2$, p is the electron momentum (for the initial and final kinematics), and the nuclear radius is $R = 1.12A^{1/3} - 0.86A^{-1/3}$. The two constants a_0 and a_2 are fitted

to two of the elastic scattering phase shifts [$\kappa = 1$ and $\kappa = \text{Int}(pR) + 5$] for the incident and final electron energy. The parameter $\Delta = a[\hat{\mathbf{p}} \cdot \hat{\mathbf{r}}] \mathbf{L}^2$ denotes a small higher order correction to the electron wave number, which we have written in terms of the parameter $a = -\alpha Z (\frac{16 \text{ MeV}/c}{p})^2$. The potential in Eq. (5) cannot be easily decomposed into a multipole expansion because of the angular dependence of the vector \mathbf{r} in the expression for the phase shifts. However, when combined with the nuclear transition current density $J^{\mu}(\mathbf{r})$ as in Eq. (4), the modified Fourier transform can be obtained by three-dimensional integration since the volume is limited to a sphere with radius of 3–4 times the nuclear radius R . We confirmed that use of the potential given in Eq. (5) reproduces the full DWBA results for the cross section very well for electron energies above 300 MeV and for momentum transfer greater than about 250 MeV. With this approximate DW four potential A_{μ} , it is straightforward to calculate the exclusive $(e, e'p)$ cross sections and modified structure functions. We showed [6] that using this new phase shift we can reproduce the full DWBA cross sections for $(e, e'p)$ from medium and heavy nuclei very well.

Again as noted in our previous paper [7], this approximate DW potential which includes the local value of the potential (i.e, a function of r) and the focusing effect due to the phase shifts is very time consuming for the inclusive (e, e') reaction since we need to integrate over the solid angle of the outgoing nucleons. Thus, we proposed making further approximations for the inclusive process. In order to allow a straightforward multipole expansion, we must remove any angular dependence from the phase shifts in Eq. (5). We investigated various methods of achieving this and found that neglecting the phase shifts entirely but including a focusing factor was sufficient for the transverse contribution. However, this was not sufficient for the longitudinal term, so we chose to include some of the effects of the phase shifts by averaging over the angles of the vector \mathbf{r} in the phase-shift parametrization. With these further approximations, we were able to write the (e, e') cross section [7] as

$$\left(\frac{d^2\sigma}{d\Omega_e d\omega} \right)^{\text{ad hoc}} = \sigma_M \left\{ \frac{Q^4}{q^4} S'_L(q', \omega) + \left[\tan^2 \frac{\theta_e}{2} + \frac{Q^2}{2q'^2} \right] S'_T(q', \omega) \right\}, \quad (7)$$

where the Fourier transforms of the transition current in S'_L and S'_T are replaced by

$$N_0^{\text{ad hoc}} = \int \left(\frac{q'_{\mu}(r)}{Q} \right)^2 \left(\frac{q}{q'(r)} \right)^2 e^{i\langle \delta_i + \delta_f \rangle} e^{i\mathbf{q}'(r) \cdot \mathbf{r}} J_0(\mathbf{r}) d^3r, \quad (8)$$

$$\mathbf{N}_T^{\text{ad hoc}} = \left(\frac{p'_i(0)}{p_i} \right) \int e^{i\mathbf{q}'(r) \cdot \mathbf{r}} \mathbf{J}_T(\mathbf{r}) d^3r.$$

Here $\langle \delta_{i,f} \rangle$ denotes an average of $\delta(\kappa^2)$ over the angles of the vector \mathbf{r} . That is, the argument of the parametrization for the phase shifts in Eq. (6) is given by $\langle \kappa_{i,f}^2 \rangle = \langle (\mathbf{r} \times \mathbf{p}_{i,f})^2 \rangle = r^2 p_{i,f}^2 (3 - \cos^2 \theta_{p_{i,f}}) / 4$. In addition, we fix the direction of \mathbf{q}' to be equal to the asymptotic momentum transfer, but we use $q^2(r) = p_i^2(r) + p_f^2(r) - 2p'_i(r)p'_f(r) \cos \theta_e$ for the

magnitude. We confirmed that for the kinematics under consideration this change is negligible. Using a toy model and the full three-dimensional integration for the longitudinal terms and by comparing to the full DWBA for the cross section, we found that this so-called *ad hoc* model works very well for (e, e') on medium and heavy nuclei.

However, use of an r -dependent momentum transfer requires a nuclear model to extract longitudinal and transverse structure functions. The challenge is to find some approximation with a constant shift of momentum transfer which approximates Coulomb distortion. One such approximation, referred to as the effective momentum approximation (EMA) replaces the r -dependent momentum for the incoming and outgoing lepton wave functions with a fixed value given by $p' = p - V_c(R_c)$, where R_c is usually taken to be equal to 0 or the nuclear radius R . Then one calculates the effective momentum transfer in terms of these momenta and the electron scattering angle. It is convenient to calculate $V_c(R_c)$ for a uniform charge distribution of radius $R = 1.12A^{2/3} - 0.86A^{-1/3}$ containing a charge of Ze . The result for $R_c < R$ is simply, $V_c(R_c) = -\frac{Ze}{2R}(3 - R_c^2/R^2)$ for electrons. However, this approximation does not include the effect of the phase shifts which result in what other authors refer to as focusing. As discussed in some detail by the Basel group [8], the effects of focusing can be included by multiplying the potential arising from the EMA wave function by the factor $p'_i(0)/p_i$. We will refer to this approximation as EMA-f and we note that the (e, e') cross section for EMA-f is simply this factor squared times the EMA cross section. Thus, the EMA-f cross section is given by

$$\left(\frac{d^2\sigma}{d\Omega_e d\omega}\right)^{\text{EMA-f}} = \left(\frac{p'_i(0)}{p_i}\right)^2 \sigma_M \left\{ \frac{Q'^4}{q'^4} S_L(q', \omega) + \left[\tan^2 \frac{\theta_e}{2} + \frac{Q'^2}{2q'^2} \right] S_T(q', \omega) \right\}. \quad (9)$$

The structure functions depend on the standard Fourier transforms of the nuclear transition current except that they are calculated as a function of q' rather than q . Note that $Q'^2 = q'^2 - \omega^2$. Since the sign of V_c changes for positron-induced reactions, clearly one can shift E_i for positron-induced reactions as compared to electron-induced reactions such that $q'_{e-} = q'_{e+}$, and therefore the quantity $(\frac{d^2\sigma}{d\Omega_e d\omega})^{\text{EMA-f}} / [\sigma_M (\frac{p'_i(0)}{p_i})^2]$ is the same for electrons and positrons in this approximation. That is, the focusing factor must be removed before the overall structure functions are equal for electrons and positrons.

We have investigated the validity of the EMA-f approximation for different choices of R_c , the argument of V_c , by calculating the longitudinal and transverse contributions to the cross section using a relativistic nuclear model that we have successfully used to describe a great deal of (e, e') data [2,7,9]. In Fig. 1, we compare the cross section for the inclusive reaction (e, e') on ^{208}Pb for an incident electron energy of 310 MeV at a backward angle ($\theta = 143^\circ$). We show three curves in addition to black circles representing the full DWBA calculation. As we have noted before, even at these low energies, our *ad hoc* result agrees quite well with the full

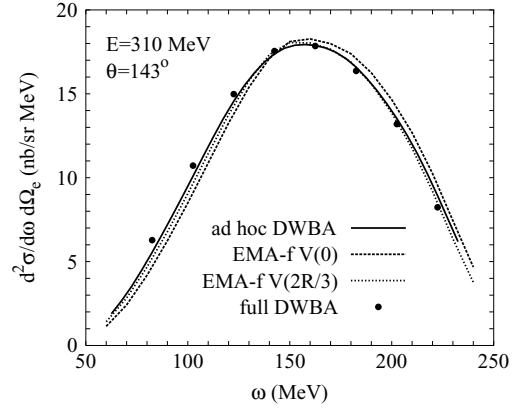


FIG. 1. Theoretical quasielastic scattering cross sections for ^{208}Pb with incident electron energy $E_i = 310$ MeV and electron scattering angle $\theta = 143^\circ$ as a function of energy transfer. Black circles are the results of our full DWBA partial wave calculation.

DWBA results. We also show two EMA-f results. In one case, we evaluated the Coulomb potential energy at the origin; and in the other, at two thirds of the nuclear radius, which is in better agreement with the full DWBA results. Note that the cross section at such a large angle is dominated by the transverse contributions. In Fig. 2, we compare our *ad hoc* results to

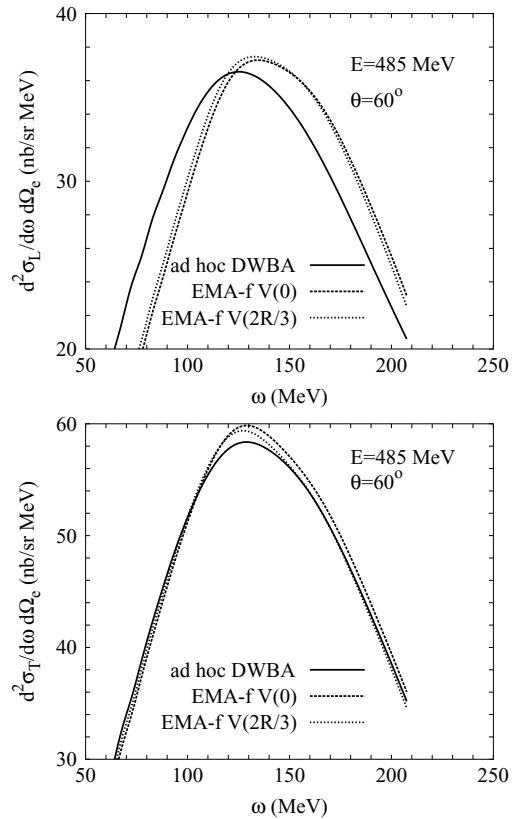


FIG. 2. Theoretical quasielastic scattering partial cross sections for ^{208}Pb with incident electron energy $E_i = 485$ MeV and electron scattering angle $\theta = 60^\circ$ as a function of energy transfer. Upper panel shows the longitudinal contribution to the cross section; lower panel shows the transverse contribution.

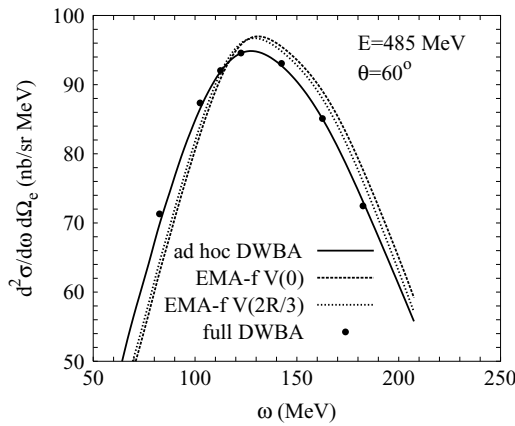


FIG. 3. Combined longitudinal and transverse contributions for the kinematics described in Fig. 1.

EMA-f for the same two choices of the Coulomb potential for the longitudinal (upper panel) and transverse (lower panel) contributions to the cross section for 485 MeV electron on ²⁰⁸Pb at a scattering angle of 60°. Note that the longitudinal and transverse contributions are of similar magnitude, but the

transverse cross section is much better described by the EMA-f than is the longitudinal cross section. Furthermore, using the Coulomb potential at 2/3R to calculate q' is a somewhat better approximation than using the origin value. In Fig. 3, we show the full cross section for this case as compared to the full DWBA calculation (where we cannot separate out the longitudinal and transverse terms) and we note that the *ad hoc* result is very good and the EMA-f does not look so bad since over half the cross section comes from the transverse term, which is reasonably well described by EMA-f. Of course, if you were to use EMA-f to make a Rosenbluth separation, the differential quality of the description of the longitudinal and transverse terms would lead to large errors.

In Fig. 4, we examine the two separated contributions as in Fig. 2, except that we have increased the electron energy to 800 MeV. Clearly the EMA-f approximation is much better at this higher energy, although the discrepancy in the longitudinal case is still almost 10% at the quasielastic peak. In Fig. 5, we increased the electron energy up to 2 GeV while reducing the scattering angle to 20°. In all cases, we used ²⁰⁸Pb as the target. The EMA-f is almost exact for the transverse contribution to the cross section (lower panel), while the longitudinal contribution (upper panel) continues to have problems.

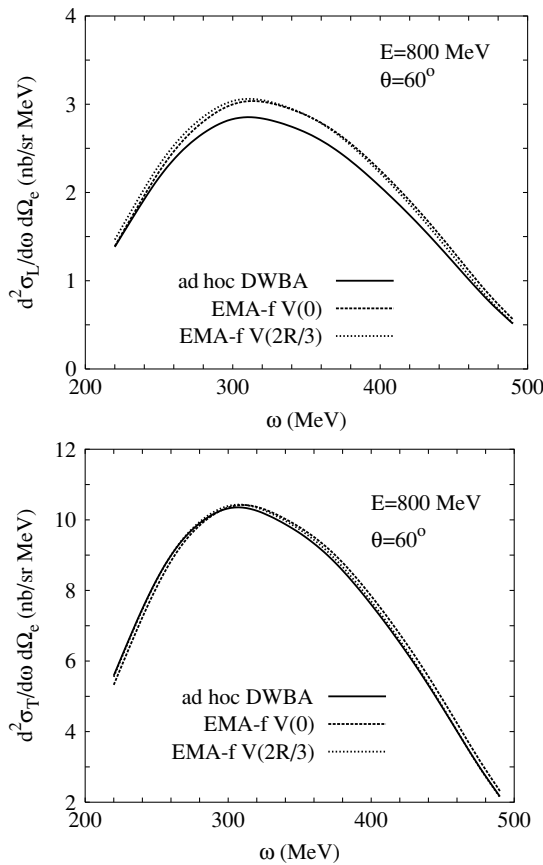


FIG. 4. Quasielastic longitudinal and transverse contributions to the cross section for ²⁰⁸Pb with incident electron energy of $E_i = 800$ MeV and electron scattering angle $\theta = 60^\circ$ calculated with the *ad hoc* DWBA as compared to the EMA-f with two different values of the Coulomb potential energy.

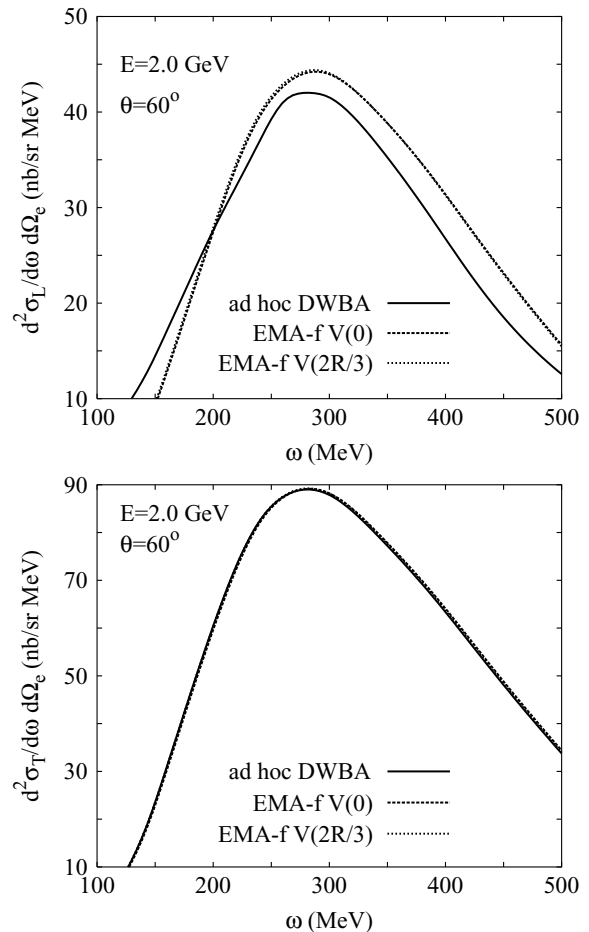


FIG. 5. Same as Fig. 4, except that $E_i = 2.0$ GeV and the electron scattering angle $\theta = 20^\circ$.

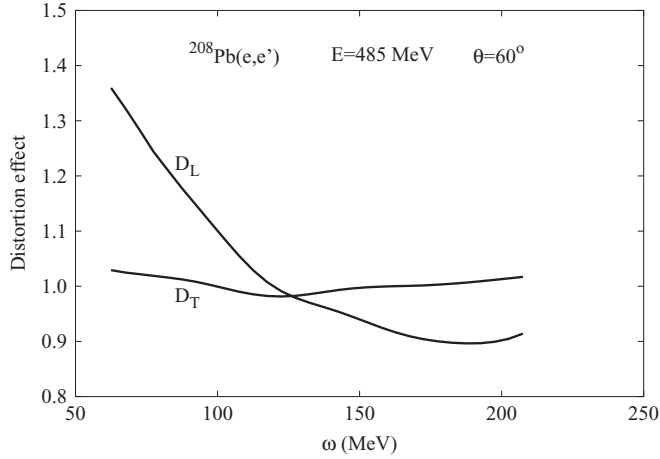


FIG. 6. Ratio of the longitudinal and transverse (D_L and D_T) contributions to the quasielastic cross section calculated using the *ad hoc* DWBA method of including Coulomb effects and the EMA-f method of approximating Coulomb distortion for 485 MeV electrons on ^{208}Pb at a scattering angle of $\theta = 60^\circ$.

III. PROPOSED SOLUTION AND CONCLUSIONS

Our results show that even at rather high electron energies, the longitudinal contributions to the quasielastic cross section are not well described by the EMA approximation even with a focusing factor. Based on these results, we do not believe that a model-independent EMA-like approach can be used to extract the longitudinal structure function in quasielastic scattering. However, as the electron energy increases, the EMA-f approach does get better so perhaps it can be used as the basic analysis tool, but with some model-dependent corrections to the Coulomb distortion effects [10]. In Fig. 6, we show the ratio of the contributions to the quasielastic cross section for 485 MeV electrons on ^{208}Pb at a scattering angle of 60° calculated using our *ad hoc* model over the EMA-f calculation using the Coulomb potential at $\frac{2}{3}R$ for the longitudinal and transverse contributions. As was clear in Fig. 2, the distortion factor D_T for the transverse contributions differs from 1 by only a few percent over the quasielastic peak. However, the distortion factor D_L varies considerably from 1 at this energy. In Fig. 7, we repeat this calculation for incident electrons of 800 MeV. At this higher energy, D_T differs from 1 by less than 3% across the quasielastic peak, while D_L deviates from 1 by up to about 7%. Based on these results and our results at higher energies, we propose that the quasielastic scattering cross section for (e, e') be written as

$$\left(\frac{d^2\sigma}{d\Omega_e d\omega} \right) = \sigma_M \left\{ \frac{Q'^4}{q'^4} D_L S_L(q', \omega) + \left[\tan^2 \frac{\theta_e}{2} + \frac{Q'^2}{2q'^2} \right] D_T S_T(q', \omega) \right\}, \quad (10)$$

where the distortion factors D_L and D_T are given by

$$D_L = \left(\frac{Q}{Q'} \right)^4 \left(\frac{q'}{q} \right)^4 \frac{S'_L(q', \omega)}{S_L(q', \omega)}, \quad (11)$$

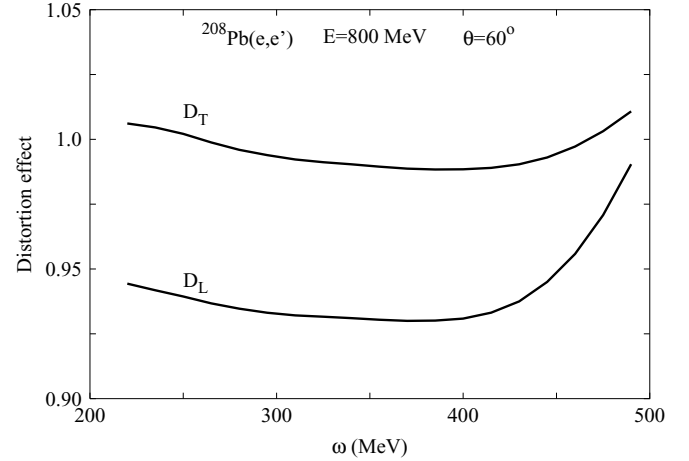


FIG. 7. Ratio of the longitudinal and transverse D_T contributions to the quasielastic cross section calculated using the *ad hoc* DWBA method of including Coulomb effects and the EMA-f method of approximating Coulomb distortion for 800 MeV electrons on ^{208}Pb at a scattering angle of $\theta = 60^\circ$.

$$D_T = \left(\frac{p'_i(0)}{p_i} \right)^2 \frac{S'_T(q', \omega)}{S_T(q', \omega)}. \quad (12)$$

Note that in the factor before the transverse structure function in Eq. (10), we are using the factor $Q^2/2q^2$ rather than primed values so as to more easily define the distortion factor D_T . Clearly, by using distortion factors D_L and D_T calculated from any reasonable nuclear model, Eq. (10) can be used in a Rosenbluth mode to extract the longitudinal and transverse functions as a function of q' and ω from experimental data.

In conclusion, we have shown that the effective momentum approximation (using the Coulomb potential at $\frac{2}{3}R$) with an overall focusing factor of $[p'_i(0)/p_i]^2$ is a very good approximation of the Coulomb distortion effects for the transverse contributions to the quasielastic cross section. However, for electron energies less than about 600 MeV, it is not a good approximation of the longitudinal contributions. At higher electron energies, this approximation does get better for the longitudinal contribution; but for accurate extraction of the longitudinal structure function, it is necessary to use distortion factors calculated from a nuclear model. The procedure we have proposed minimizes the model dependence by only using the model to evaluate the Coulomb distortion effects not included in EMA-f. Based on the cases we have examined, we believe the errors in the extracted structure functions arising from Coulomb distortion effects using our proposed procedure should be less than 5% for incident electron energies above 600 MeV.

ACKNOWLEDGMENT

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