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New model of binding energies of heavy nuclei with $Z \geqslant 90$

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A new form of the binding energy formula of heavy nuclei with $Z \geqslant 90$ is proposed where new terms beyond the standard Bethe and Weizsäcker formula are introduced by analytical expressions. This can be considered an interesting development of the Bethe and Weizsäcker mass formula for heavy nuclei with $Z \geqslant 90$. Two versions of the formulae are presented. The first version of the formula can reproduce the 117 known binding energies of nuclei with $Z \geqslant 90$ and $N \geqslant 140$ with an average deviation 0.118 MeV. This is the first time that the binding energies of heavy nuclei with $Z \geqslant 90$ and $N \geqslant 140$ can be calculated very accurately by a formula with only seven parameters. The binding energies, α-decay energies, and α-decay half-lives of unknown superheavy nuclei are predicted. The second version of the formula is obtained by fitting the 181 data of nuclei with $Z \geqslant 90$ with nine parameters and good agreement with experimental binding energies is also reached for all nuclei with $Z \geqslant 90$.

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I. INTRODUCTION

The binding energy of a nucleus or nuclear mass is one of the most important quantities of nuclear ground properties. It plays a crucial role for the stability of a nucleus on β decay, α decay, and spontaneous fission of heavy-mass region with $Z \geqslant 90$. The half-life of an unstable nucleus is directly related to the value of its binding energy or to the difference of the binding energies of neighboring nuclei. For the production of a new superheavy nuclide by nuclear reactions one needs to estimate the production cross section before experiments. In calculations of reaction cross sections the binding energies of unknown nuclei are the key input quantities of nuclear reaction models. Therefore the very accurate prediction on binding energies of unknown superheavy nuclei is important not only for estimating the half-life of unknown nuclei but also for estimating the production cross section of superheavy nuclei.

Since the discovery of the neutron at the beginning of the 1930s nuclear physicists have spent much time developing various nuclear models to calculate accurately the binding energies of nuclei [1–12]. The original studies on nuclear masses are the semiempirical mass formula proposed by Weizsäcker and Bethe in the middle of 1930s [1,2]. This formula successfully gave the experimental average binding energy curve of nuclei (B/A) and led to the successful explanation of large energy release of the ^{235}U fission by Bohr and Wheeler in 1939 [13]. At present, various studies on variations of nuclear masses are still the hot points of nuclear physics [4–12,14–20]. A large number of mass models were available for the whole range of Z and A numbers of present and future interest and a complete review on them was made by Haustein in 1989 [4,5]. New progress on nuclear masses can be found in review articles by Mittig *et al.* [16], Lunney *et al.* [19], and Audi *et al.* [12]. Here we simply discuss a few theoretical models in this field. Swiatecki *et al.* and Möller *et al.* have made pioneering work on reliable calculations of nuclear binding energies [6–8]. Myers and Swiatecki [6] calculated the binding energies of many nuclei by the Thomas-Fermi model. Möller *et al.* [8] calculated the ground-state properties of nuclei by the finite-range droplet model and folded-Yukawa single particle potential (FRDM). Self-consistent mean-field calculations have been also carried out for the ground-state properties of many nuclei [9–11]. These calculations successfully reproduce the experimental binding energies with root-mean-square deviations $\Delta B \approx 0.67$–2.0 MeV where the numbers of the adjusting parameters of models range from 10 to 29. We consider that these models are very successful for the global behavior of nuclear binding energies of the whole mass range. We further consider that it is very difficult to find a new model to replace these successful models for the global behavior of the whole mass range. However, it is strongly hoped that a more refined model of binding energies is used for a very accurate prediction of nuclear binding energies in a local unknown mass range. This is the case of transuranium nuclei where very expensive experiments on superheavy nuclei are being done [21–27]. Usually few events of α decays of a new superheavy nuclide have been observed at the expense of running a big accelerator in weeks or even months [21–27]. For transuranium range the half-lives of nuclei are extremely sensitive to their decay energies that are the differences of binding energies of neighboring nuclei. For example, Möller *et al.* pointed out [8] that an uncertainty of 1 MeV in α-decay energy corresponds to an uncertainty of α-decay half-life ranging from 10^5 to 10^3 times for the heavy-element region. This is further supported by the model calculations of α decay [28–30]. According to the Swiatecki's formula of spontaneous fission half-life and the new formula of spontaneous fission half-life [31–33], an uncertainty of 1 MeV of the binding energy can also lead to the uncertainty of the half-life of spontaneous fission with a factor 10^4 or 10^5. So a new formula of the binding energies with precisions 0.1 or 0.2 MeV is very useful for studies of superheavy elements. This is the motivation of this article.

This article is organized in the following way. Section II is the first version of the new form of binding energy formula

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where the range of nuclei is $Z \geqslant 90$ and $N \geqslant 140$. We choose this special mass range because we think that the nuclei with $Z \geqslant 90$ and $N \geqslant 140$ are the neighboring nuclei of superheavy nuclei. The formula of binding energies obtained from these neighboring nuclei could have good predicting ability for superheavy nuclei. We expect the values of binding energies of nuclei with $A \leqslant 229$ are not closely related to those of superheavy nuclei because the nuclei with $A \leqslant 229$ are far from superheavy nuclei. The region with $Z \geqslant 90$ and $N \geqslant 140$ is also a smooth area that is suitable to establish systematic behavior by an analytical equation with few parameters. This region also covers the long-lifetime nuclei such as ^{230,232}Th and ^{234,235,238}U because we are interested in the possible existence of long-lived superheavy nuclei. In Sec. III we loosen this limit of the mass range by including all known binding energies of nuclei with $Z \geqslant 90$ (181 nuclei). In this way we obtain the second version of the formula where the experimental binding energies of lighter isotopes with $Z = 90$–92 can also be well reproduced and the nuclei around magic number $N = 126$ are included. A short summary is given in Sec. IV.

II. NEW FORM OF BINDING ENERGY FORMULA OF HEAVY NUCLEI WITH $Z \geqslant 90$ AND $N \geqslant 140$

Our starting point is the well-known Bethe and Weizsäcker formula of nuclear binding energies [1–3,34]:

$$B(Z, A) = a_v A - a_s A^{2/3} - a_c Z^2 A^{-1/3} - a_a \left(\frac{A}{2} - Z\right)^2 A^{-1} + a_p \delta A^{-1/2}. \quad (1)$$

In above formula a_v, a_s, a_c, a_a, a_p correspond to the coefficients of the volume energy, the surface energy, the Coulomb energy, the symmetry energy, and the pairing energy [3,34], $\delta = 1, 0, -1$ for even-even, odd-A, and odd-odd nuclei, respectively. In standard textbooks [3,34] the values of a_v, a_s, a_c, a_a, a_p are usually determined by fitting experimental binding energies of nuclei from the light nucleus ^{16}O to the heavy nucleus such as ^{238}U. To obtain a local mass formula of heavy nuclei (with $Z \geqslant 90$) with high precision, here we use a novel way to determine the parameter values by choosing the experimental binding energies of nuclei with $Z \geqslant 90$ and $N \geqslant 140$ as a reference. It is known that for nuclei with $Z \geqslant 90$ and $N \geqslant 140$ there are 117 data of experimental binding energies [12]. At first, we use the 117 experimental data to fit the values of the coefficients a_v, a_s, a_c, a_a, a_p. This is a standard minimization with $\chi^2 = \Sigma_{i=1,117}(B^i_{\rm exp} - B^i_{\rm cal})^2$. In this way, we obtain a new set of parameters in the Bethe and Weizsäcker formula for the nuclei with $Z \geqslant 90$. The new values of parameters are as follows:

$$\begin{cases} a_v = 15.7226\,\text{MeV} \\ a_s = 17.7523\,\text{MeV} \\ a_c = 0.7062\,\text{MeV} \\ a_a = 96.2350\,\text{MeV} \\ a_p = 10.6028\,\text{MeV} \end{cases}$$

The formula with the above parameters is now accurate for the binding energies of nuclei with $Z \geqslant 90$ and $N \geqslant 140$ where no shell effect is included. The average deviation and the root-mean-square deviation of the binding energies are as follows:

$$\langle\sigma\rangle = \Sigma_{i=1,117}\left|B^i_{\rm exp} - B^i_{\rm cal}\right|/117 = 0.227\,\text{MeV} \quad (2)$$

$$\sqrt{\sigma^2} = \left(\Sigma_{i=1,117}\left(B^i_{\rm exp} - B^i_{\rm cal}\right)^2/117\right)^{1/2} = 0.290\,\text{MeV}. \quad (3)$$

This is a much improvement of the agreement between experimental binding energies and calculated ones by the formula. Although the much improvement is reached, the agreement between experimental data and calculated ones is not perfect for some nuclei. The maximum deviation of the binding energy is 0.7 MeV and this happens around $Z = 100$ and $N = 152$. This should be the shell influence around $Z = 100$ and $N = 152$. Consequently, we introduce two new terms to simulate the shell effect on binding energies of heavy nuclei. The new form of the Bethe and Weizsäcker formula of nuclear binding energies is as follows:

$$B(Z, A) = a_v A - a_s A^{2/3} - a_c Z^2 A^{-1/3} - a_a \left(\frac{A}{2} - Z\right)^2 A^{-1} + a_p \delta A^{-1/2} + a_6 |A - 252|/A - a_7 |N - 152|/N. \quad (4)$$

Now we use this new form of Bethe and Weizsäcker formula to fit the binding energies of 117 nuclei. The values of parameters are as follows:

$$\begin{cases} a_v = 15.65636\,\text{MeV} \\ a_s = 17.15717\,\text{MeV} \\ a_c = 0.70887\,\text{MeV} \\ a_a = 97.15094\,\text{MeV} \\ a_p = 10.45136\,\text{MeV} \\ a_6 = 5.9427\,\text{MeV} \\ a_7 = 23.1377\,\text{MeV}. \end{cases}$$

The new fitting is also equivalent to a renormalization of effective parameters and the shell influence can be suitably included by the fitting process of parameters. We expect that the important role of the Coulomb interaction on binding energies of heavy nuclei with $Z \geqslant 90$ has been taken into account during the fitting process. The average deviation and the root-mean-square deviation of the binding energies with new form of the formula are as follows:

$$\langle\sigma\rangle = \Sigma_{i=1,117}\left|B^i_{\rm exp.} - B^i_{\rm cal.}\right|/117 = 0.118\,\text{MeV} \quad (5)$$

$$\sqrt{\sigma^2} = \left[\Sigma_{i=1,117}\left(B^i_{\rm exp.} - B^i_{\rm cal.}\right)^2/117\right]^{1/2} = 0.150\,\text{MeV} \quad (6)$$

It is interesting to note that the deviation has been reduced to the half by including the shell influence [see Eq. (2) and Eq. (6)]. In Fig. 1 we draw the numerical results of the deviations without shell influence and with shell influence for comparison. As shown in Fig. 1, the new form of the Bethe and Weizsäcker formula is very accurate for the 117 data of nuclei with $Z \geqslant 90$ and $N \geqslant 140$. The average deviation of binding energies is only 0.118 MeV (see above equations) and this is almost perfect for the total binding energy with a value around 2000 MeV. This is the first time that the binding energies of heavy nuclei with $Z \geqslant 90$ have been reproduced with such high precision ($\delta B/B \approx 0.118/2000 \approx 0.006\%$).

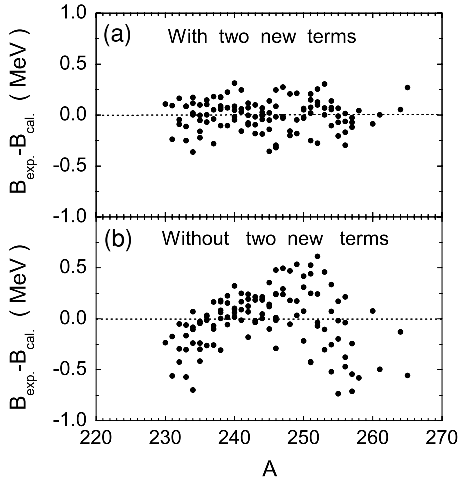

FIG. 1. The variation of the deviations between experimental binding energies and calculated ones for the cases with shell influence and without shell influence in the formula of binding energies of the 117 heavy nuclei with $Z \geqslant 90$ and $N \geqslant 140$.

The maximum deviation of the binding energies is only 0.30 MeV and this appears only for a few nuclei around $N = 152$. In Table I we list experimental binding energies and calculated values with the new form of the formula for 117 nuclei from ^{230}Th to ^{264}Hs. It is seen again from Table I that calculated binding energies are approximately equal to experimental ones and therefore the new form of the binding energy formula is very accurate. This is the first local mass formula that is specially designed for the properties of heavy and superheavy nuclei.

It is stressed that the calculations of binding energies by above formula are very clear. This means that the calculations are completely transparent. Any nuclear physicist can repeat these calculations within an hour, although we spent many months in constructing a very accurate mass formula. Experimental physicists themselves can judge the reliability of numerical results for unknown superheavy nuclei before planing an experiment to synthesize new superheavy nuclei. This can effectively avoid the possible misleading role of the very complicated numerical calculations on experiments where experimental physicists cannot repeat the results of very complicated calculations.

We have proposed the new idea to obtain a local mass formula by fitting the experimental data of the nuclei with $Z \geqslant 90$ and $N \geqslant 140$ and reached a success. This idea to improve the agreement of binding energies can be further developed in future with the accumulation of more and more data of binding energies. This idea can also be used for other models to improve the precisions of agreement and to predict the binding energies of unknown superheavy nuclei reliably. For example, the effective parameters in the Thomas-Fermi model and in the finite-range droplet model can also be determined in this way and this will lead to a new set of parameters for heavy nuclei with $Z \geqslant 90$ in the two models. It is believed that the new parameters obtained in this way will works very well for superheavy nuclei. For the use of the idea in other models the new effective parameters in

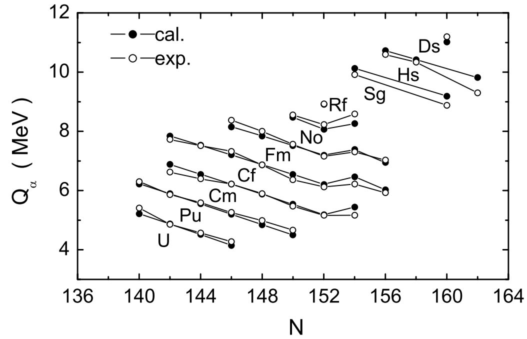

FIG. 2. The variation of the experimental and calculated α-decay energies with neutron number for even-even heavy nuclei with $Z \geqslant 92$.

Skyrme-Hartree-Fock model and in relativistic mean-field model can also be obtained based on the fitting of binding energies of heavy nuclei with $Z \geqslant 90$. So this will pave a new way for the very accurate prediction of the ground-state properties of superheavy nuclei. This will also lead to more accurate prediction of the reaction cross section of superheavy nuclei where binding energies are input quantities.

After comparing the calculated binding energies with experimental ones, we now see the variations of α-decay energies, two-neutron separation energies, one-neutron separation energies, and one-proton separation energies of heavy nuclei with $Z \geqslant 90$. The α-decay energies of even-even nuclei with $Z \geqslant 90$ are drawn in Fig. 2. In Fig. 2 the x axis is the neutron number and the y axis is the decay energy. The experimental data are denoted by hollow circles and calculated ones by solid circles. It is seen that the agreement between experimental data and theoretical ones is very good. This very good agreement is not accidental because the α-decay energies are the differences of the binding energies of parent nuclei, daughter nuclei, and the α particle. The very accurate values of binding energies by the new form of the formula lead to the very accurate values of α-decay energies.

It is also seen from Fig. 2 that calculated α-decay energies follow experimental curve well around the deformed magic number $N = 152$. This clearly shows that our treatment on shell correction in the formula is correct. So the experimental magic number $N = 152$ can be reproduced by the new formula. For the superheavy nuclei ^{270}Ds and ^{266}Hs, the calculated α-decay energies agree well with the experimental data, although the two nuclei have not been included in the fitting parameters because their experimental binding energies are unknown in the 2003 mass table [12]. This shows that the new form of the Bethe and Weizsäcker formula has the predicting ability for superheavy nuclei.

In Fig. 3 we plot the variation of two-neutron separation energies with neutron number for even-even nuclei. One sees again that the very good agreement is reached between the experimental curve and theoretical curve. Especially the sudden decrease of the two-neutron separation energies at $N = 154$ can be reproduced and this is because of the deformed subshell effect of $N = 152$.

In Fig. 4 we show the even-odd effect of one-proton separation energy for the $N = 152$ isotonic chain. Because

TABLE I. The experimental and calculational binding energies of heavy and superheavy nuclei (Z = 90–108) by the new form of the Bethe and Weizsäcker formula.

Elt.	A	B_{exp} (MeV)	B_{cal} (MeV)	Elt.	A	B_{exp} (MeV)	B_{cal} (MeV)
Th	230	1755.130	1755.023	Cm	243	1829.041	1828.943
	231	1760.246	1760.153		244	1835.842	1835.818
	232	1766.686	1766.523		245	1841.363	1841.224
	233	1771.472	1771.387		246	1847.820	1847.837
	234	1777.663	1777.489		247	1852.976	1852.990
	235	1782.092	1782.094		248	1859.189	1859.347
Pa	231	1759.855	1760.092		249	1863.902	1863.950
	232	1765.404	1765.450		250	1869.735	1869.759
	233	1771.933	1772.044		251	1874.147	1874.128
	234	1777.153	1777.134	Bk	243	1826.751	1826.845
	235	1783.236	1783.458		244	1832.799	1832.730
	236	1788.290	1788.287		245	1839.770	1839.820
	237	1794.066	1794.347		246	1845.689	1845.443
	238	1799.011	1798.922		247	1852.237	1852.269
U	232	1765.959	1766.052		249	1864.021	1864.204
	233	1771.721	1771.635		250	1868.990	1869.019
	234	1778.566	1778.450		251	1874.784	1875.036
	235	1783.863	1783.763	Cf	242	1817.251	1817.345
	236	1790.409	1790.306		244	1831.252	1831.293
	237	1795.534	1795.355		245	1837.416	1837.395
	238	1801.689	1801.631		246	1844.782	1844.697
	239	1806.495	1806.424		247	1850.808	1850.533
	240	1812.425	1812.441		248	1857.777	1857.569
Np	233	1769.910	1770.159		249	1863.362	1863.148
	234	1775.973	1775.970		250	1869.987	1869.923
	235	1782.957	1783.009		251	1875.096	1874.947
	236	1788.694	1788.548		252	1881.268	1881.170
	237	1795.271	1795.311		253	1886.072	1886.003
	238	1800.759	1800.583		254	1892.104	1892.032
	239	1806.974	1807.078	Es	251	1873.936	1873.861
	240	1812.043	1812.091		252	1879.224	1879.097
	241	1818.167	1818.325		253	1885.577	1885.575
	242	1823.083	1823.085		254	1890.670	1890.617
Pu	234	1774.798	1775.161		255	1896.644	1896.851
	235	1781.034	1781.196	Fm	246	1837.170	1837.483
	236	1788.387	1788.456		248	1851.546	1851.746
	237	1794.268	1794.216		250	1865.520	1865.469
	238	1801.268	1801.198		251	1871.679	1871.472
	239	1806.914	1806.689		252	1878.920	1878.664
	240	1813.449	1813.400		253	1884.459	1884.156
	241	1818.690	1818.630		254	1890.976	1890.838
	242	1825.000	1825.077		255	1896.152	1896.086
	243	1830.034	1830.052		256	1902.536	1902.522
	244	1836.055	1836.241		257	1907.504	1907.533
	245	1840.827	1840.967	Md	255	1894.326	1894.337
	246	1846.609	1846.905		256	1899.625	1899.794
Am	238	1798.228	1798.055		257	1906.315	1906.435
	239	1805.330	1805.257		258	1911.695	1911.652
	240	1811.282	1810.971	No	252	1871.292	1871.568
	241	1817.929	1817.899		254	1885.592	1885.696
	242	1823.466	1823.349		255	1891.534	1891.605
	243	1829.831	1830.012		256	1898.634	1898.698
	244	1835.197	1835.204		257	1904.288	1904.360
	245	1841.250	1841.606	Rf	256	1890.657	1890.952
	246	1846.228	1846.547		261	1923.935	1923.933
Cm	238	1796.472	1796.574	Sg	260	1909.029	1909.115
	240	1810.285	1810.199		265	1943.152	1942.883
	241	1816.379	1816.132	Hs	264	1926.735	1926.682
	242	1823.348	1823.276				

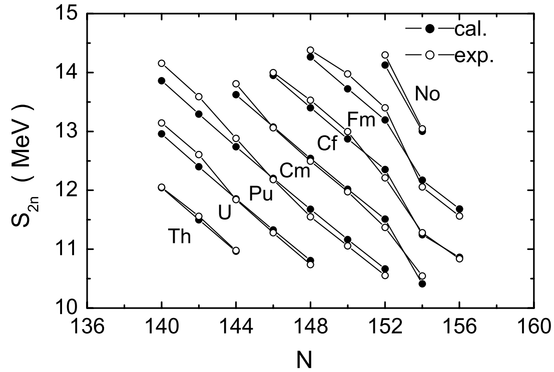

FIG. 3. The variation of the experimental and calculated two-neutron separation energies with neutron number for even-even nuclei with $Z \geqslant 90$.

there are only four experimental data of proton separation energies (denoted by the hollow circles with an arrow in Fig. 4), we have also added the estimated values (denoted by the hollow circles without an arrow) of the 2003 mass table by Audi *et al*. [12]. It is concluded from Fig. 4 that the experimental even-odd effect of $N = 152$ isotonic chain is reproduced by the calculations. This conclusion is also valid for other isotonic chains.

The even-odd effect of one-neutron separation energy of the Pu isotopic chain is plotted in Fig. 5. The experimental even-odd effect can be well described by the calculations. This holds true for the even-odd effect of other isotopic chains. Here we do not repeat them.

Based on the very good agreement of binding energies, of α-decay energies, and of the nucleon separation energies, we consider that the new form of the formula is reliable and it can be generalized into superheavy region. In Table II we predict the binding energies, α-decay energies, and α-decay half-lives of superheavy nuclei with $Z = 108$–112 by the new form of the Bethe and Weizsäcker formula.

In Table II the first column is the nucleus and the second column is the calculated binding energy (in megaelectron volts). The third column is the calculated α-decay energy (in megaelectron volts). The fourth column is the calculated

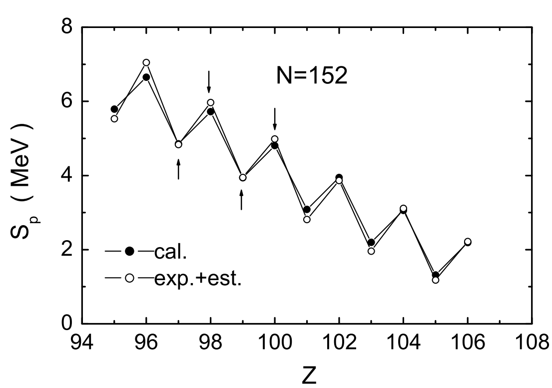

FIG. 4. The variation of the experimental (or estimated) and calculated one-proton separation energies with proton number for $N = 152$ isotonic chain.

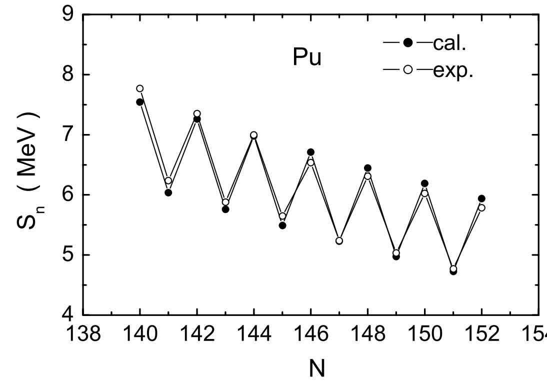

FIG. 5. The variation of the experimental and calculated one-neutron separation energies with neutron number for $Z = 94$ isotopic chain.

α-decay half-life and the fifth column is the experimental α-decay half-life. To calculate the α-decay half-lives from the α-decay energies, we use the Viola-Seaborg formula [35,36] with new parameters for numerical calculations. The Viola-Seaborg formula [35,36] is as follows:

$$\log_{10} T_\alpha = (aZ + b)Q_\alpha^{-1/2} + (cZ + d) + h_{\log}, \qquad (7)$$

where $a = 1.64062$, $b = -8.54399$, $c = -0.19430$, and $d = -33.9054$ and the hindrance factors [36] are as follows:

$$h_{\log} = \begin{cases} 0, & Z \text{ even}, N \text{ even} \\ 0.8937, & Z \text{ even}, N \text{ odd} \\ 0.5720, & Z \text{ odd}, N \text{ even} \\ 0.9380, & Z \text{ odd}, N \text{ odd} \end{cases} \qquad (8)$$

For the calculated binding energies and α-decay energies in Table II they agree well with the available experimental data (see Figs. 1 and 2). These calculated binding energies can be used as the inputs of future calculations of reaction cross sections. The experimental half-lives are listed in the last column for comparison where many data are from Ref. [12]. At present, experiments on superheavy nuclei are very difficult and experimental error bars are large. The appearance of the isomers also leads to the experimental difficulty to identify the branch ratio of the ground-state decay. In some cases only few events of decays are observed. Therefore a few experimental half-lives of the 2003 mass table [12] may be changed in future with the improvement of experimental precision. For ^{269}Hs ($Z = 108$) and ^{271}Ds ($Z = 110$), we consider that it is useful to list the other choice of the values and the related references.

In Table II the calculated decay energies are used as inputs and the calculated half-lives agree with the data within a factor of 40. This is because of the extreme sensitivity of the decay half-lives to the decay energies. Möller *et al*. pointed out [8] that an uncertainty of 1 MeV in α-decay energy corresponds to an uncertainty of α-decay half-life ranged from 10^5 to 10^3 times for the heavy-element region. Here the new form of the Bethe and Weizsäcker formula has on average reduced the uncertainty of decay energy to 0.1–0.2 MeV (see Fig. 2). So the deviation between calculated half-lives with calculated

TABLE II. The calculated binding energies, α-decay energies and α-decay half-lives of superheavy nuclei ($Z = 108-112$). The α-decay half-lives are calculated by the Viola-Seaborg formula. The number in the bracket is the reference where experimental half-lives are from.

Nuclide	$B_{\rm cal}$ (MeV)	$Q_\alpha^{\rm cal}$ (MeV)	$T_{\alpha,\rm cal}^{V-S}$	$T_\alpha^{\rm exp}$ (Ref.)
^{264}Hs	1926.681	10.730	0.39 ms	1.08 ms [12]
^{265}Hs	1933.324	10.573	7.38 ms	2.1 ms [12]
^{266}Hs	1941.124	10.426	2.18 ms	2.7 ms [12]
^{267}Hs	1947.517	10.268	42.9 ms	32 ms [12]
^{268}Hs	1955.067	10.120	13.3 ms	
^{269}Hs	1961.218	9.961	0.27 s	27s [12]/7s [24]
^{270}Hs	1968.523	9.810	89.9 ms	
^{271}Hs	1974.436	9.649	1.97 s	
^{272}Hs	1981.502	9.497	0.68 s	
^{273}Hs	1987.183	9.335	15.9 s	
^{274}Hs	1994.014	9.182	5.82 s	
^{275}Hs	1999.468	9.018	2.42 m	
^{276}Hs	2006.069	8.864	56.8 s	
^{277}Hs	2011.300	8.699	0.422 h	
^{278}Hs	2017.677	8.544	10.63 m	
^{279}Hs	2022.691	8.378	5.14 h	
^{280}Hs	2028.848	8.221	2.35 h	
^{266}Mt	1933.962	11.015	1.45 ms	
^{267}Mt	1941.962	10.871	1.36 ms	
^{268}Mt	1948.557	10.716	7.45 ms	53 ms [12]
^{269}Mt	1956.305	10.570	7.32 ms	
^{270}Mt	1962.656	10.413	42.0 ms	
^{271}Mt	1970.158	10.265	43.3 ms	
^{272}Mt	1976.269	10.107	0.26 s	
^{273}Mt	1983.529	9.957	0.28 s	
^{274}Mt	1989.406	9.797	1.81 s	
^{275}Mt	1996.430	9.646	2.07 s	
^{276}Mt	2002.078	9.485	14.0 s	
^{277}Mt	2008.870	9.333	16.9 s	
^{278}Mt	2014.294	9.172	2.0 m	
^{279}Mt	2020.860	9.018	2.6 m	
^{280}Mt	2026.065	8.854	0.34 h	
^{267}Ds	1935.460	11.461	0.25 ms	0.01 ms [12]
^{268}Ds	1943.658	11.319	16.5 μs	
^{269}Ds	1950.452	11.167	1.16 ms	0.23 ms [12]
^{270}Ds	1958.397	11.023	0.32 ms	0.16 ms [12]
^{271}Ds	1964.944	10.868	5.84 ms	210 ms [12]/1.3 ms [12,23]
^{272}Ds	1972.640	10.723	1.67 ms	
^{273}Ds	1978.946	10.567	1.9 ms	0.36 ms [12]
^{274}Ds	1986.399	10.420	9.59 ms	
^{275}Ds	1992.469	10.263	0.19 s	
^{276}Ds	1999.683	10.114	60.5 ms	
^{277}Ds	2005.523	9.956	1.27 s	
^{278}Ds	2012.504	9.806	0.42 s	
^{279}Ds	2018.118	9.646	9.36 s	
^{280}Ds	2024.870	9.494	3.30 s	
^{272}111	1965.544	11.309	1.19 ms	2.0 ms [12]
^{273}111	1973.435	11.166	1.10 ms	
^{274}111	1979.939	11.013	5.84 ms	
^{275}111	1987.585	10.868	5.60 ms	
^{276}111	1993.852	10.713	31.1 ms	
^{277}111	2001.258	10.567	31.1 ms	
^{278}111	2007.291	10.411	0.18 s	
^{279}111	2014.463	10.263	0.19 s	
^{280}111	2020.268	10.105	1.16 s	
^{272}112	1960.052	11.902	13.1 μs	
^{273}112	1966.995	11.753	0.22 ms	

TABLE II. (*Continued.*)

Nuclide	$B_{\rm cal}$ (MeV)	$Q_\alpha^{\rm cal}$ (MeV)	$T_{\alpha,\rm cal}^{V-S}$	$T_\alpha^{\rm exp}$ (Ref.)
$^{274}112$	1975.080	11.612	56 μs	
$^{275}112$	1981.779	11.461	0.95 ms	
$^{276}112$	1989.616	11.319	0.26 ms	
$^{277}112$	1996.075	11.167	4.54 ms	1.1 ms [12]
$^{278}112$	2003.671	11.023	1.27 ms	
$^{279}112$	2009.896	10.869	23.5 ms	
$^{280}112$	2017.255	10.723	6.88 ms	

decay energies and experimental half-lives has been reduced within a factor of 40. In many cases the deviation is within a factor of 10. Even so, the deviation of decay energies could be as large as 0.3–0.5 MeV for few nuclei (see Fig. 2). The corresponding deviation of the half-lives could be close to 100 times in few cases. At first we focus on the half-lives of $Z = 110$ chains. For superheavy nuclei around ^{270}Ds there are some isomers [12,22]. Different decay chains have been observed from a same nucleus and this makes the situations very complicated [21,22]. It is very difficult to classify the decays from isomeric states and from the ground state by current experiments. Therefore a few assignments on the half-lives of ground states can belong to the half-lives of isomeric states. This is also the cause that the data of half-lives in different publications may be different for superheavy nuclei with $Z \geqslant 108$. For example, it is considered that the half-life of the ground state of ^{270}Ds is 0.16 ms and the half-life of its isomeric state is 10 ms in the 2003 mass table [12,22]. This means that ground-state half-life is shorter than the half-life of its isomer for ^{270}Ds. But for ^{271}Ds the half-life of the ground state is 210 ms and that of its isomeric state is 1.3 ms in the 2003 mass table [12]. This is contrary to the case of ^{270}Ds. For ^{273}Ds the half-life of the ground state is 0.36 ms and that of its isomeric state is 120 ms in the 2003 mass table [12]. It is again similar to the case of ^{270}Ds where the ground-state half-life is shorter than its isomeric one [12]. It is not clear what is new physics on this. We may guess that the half-lives of the ground-state of ^{270,271,273}Ds are shorter than their isomeric states and therefore the half-life of the ground state of ^{271}Ds should be 1.3 ms. With this guess the variation of the half-lives of the ground states in the Ds chain will be smooth. Otherwise, an abnormal increase of half-life (approximately 10^3 times) appears between ^{270}Ds and ^{271}Ds (the blocking effect leads to an increase of several times or tens for α-decay half-life in ordinary cases). It is interesting to note that above discussions on ^{271}Ds are consistent with Hofmann's views [23]. For ^{269}Hs and ^{270}Hs we may guess that there are isomers in them and a similar phenomenon like ^{270}Ds may occur because they are the neighboring nuclei of ^{270}Ds. The further experiments on ^{269}Hs and ^{270}Hs are being done at GSI to check the previous results and to measure the half-lives of ^{270}Hs [37]. If it is finally confirmed that the ground-state half-lives are shorter than the half-lives of isomers around ^{270}Ds, this will be very interesting in physics. It will mean that there is a new inversion island of superheavy nuclei around ^{270}Ds, where the isomers have longer lifetime than the ground states. This will suggest that there exists shape coexistence in superheavy region and it will be very important for future studies of the mechanism of the existence of superheavy islands. For superheavy nuclei ^{268}Mt, $^{272}111$, and $^{277}112$, the calculated half-lives in Table II are in good agreement with experimental data [12]. This shows the good predicting ability of the new form of the binding energy formula. It is concluded from the reasonable agreement of half-lives of Table II that the new form of the formula can be used to predict the binding energies, decay energies, and decay half-lives of unknown superheavy nuclei.

Before ending this section, let us make a short discussion on the shell effect of superheavy nuclei. At present there are different predictions on the possible existence of nuclear shell effects from different models or from a same model with different force parameters. Macroscopic-microscopic models favor the existence of spherical magic numbers with $Z = 114$ and $N = 184$. Skyrme-Hartree-Fock models with some parameters sets predict the nuclei around $Z = 114$ and $N = 184$ are spherical but the shell effect around $Z = 114$ and $N = 184$ is not evident [38]. Skyrme-Hartree-Fock models concludes that there are the spherical magic numbers $Z = 126$ and $N = 184$ [38]. As the number of parameters sets of Skyrme-Hartree-Fock model is more than 40, the predictions are strongly dependent on the inputs of the parameters. For the relativistic mean-field models, many force parameters favor the existence of shape coexistence of superheavy nuclei [10,11,39]. This can be the new mechanism of the existence of superheavy nuclei [10,11,39–41]. The RMF model favors a deformed magic number with $Z = 120$ and $N = 184$ or a deformed subshell with $Z = 114$ and $N = 184$. We consider that these magic numbers are only hypothesis before experimental confirmation of their existence. These predictions of magic numbers are made mainly based on the α-decay properties, the β-decay properties, and the distributions of single particle levels. In many cases there are no detailed calculations on spontaneous fission half-lives when these predictions are made by models. This is because we do not have a reliable model for the calculation of spontaneous fission half-lives even for nuclei with $Z = 90$–104. Although one can make dramatic extrapolations of calculations of spontaneous fission from known nuclei with $Z = 90$–104 to unknown ones around $^{298}114$, the reliability of the extrapolations is completely unknown because we are not very clear about the possible role of the isospin on spontaneous fission (SF). Especially, it is difficult to demonstrate the sudden change of the fission modes around $N = 158$ for known Fm, No, and Rf isotopes where the symmetrical fissions of these isotopes dominate for $N \geqslant 156$–158 [42,43]. This leads to a sudden decrease of SF half-lives when $N \geqslant 156$–158 [42,43]. With more and more knowledge on SF [32,33,42,43], we gradually believe that SF will strongly influence both the properties of superheavy nuclei and the existence of possible magic numbers. This is the essential difference between the magic number of superheavy nuclei and that of heavy nuclei such as ^{132}Sn and ^{208}Pb. This is because SF has never been observed for the ground-state nuclei below $Z = 90$. At the moment, $N = 152$ is the only deformed magic number of ground-state nuclei beyond $N = 126$, which is confirmed by experimental physicists. $N = 146$ is possibly the deformed magic number of the isomeric states of transuranium nuclei. Although $N = 162$ is predicted to be a deformed magic number around $Z = 106 - 110$, its existence will be challenged by the possible existence of symmetric fission around $N = 164$ ($N = 164 = 82 + 82$). At GSI, there are primary indications that SF half-lives of the nuclei with $Z = 106-108$ and $N = 158-162$ may be shorter than previously believed [37]. Because the magic number of superheavy nuclei is an open problem, we pursue it not further in this article. Our formula of binding energies of this section shows the systematic behavior of nuclear binding energies for nuclei with $Z \geqslant 90$ and $Z \geqslant 140$. It should be reliable for predictions of neighboring nuclei such as $Z = 106-110$ and $N = 150-170$. It may be invalid for the magic nuclei in superheavy region such as $Z = 114$ or $Z = 120, 126$ if these numbers are really confirmed to be magic numbers by future experiments. Usually a very sensitive change of nuclear structure and binding energies appears near magic numbers and this belongs to an extreme quantum effect of many-body systems (this holds true for spherical shell effect but the variation of deformed shell effect cannot be very sensitive). It is very difficult to trace the sensitive effect by an analytical formula because the analytical formula is a law of systematic behavior. Even so one can try to simulate this sensitive change in a special case and we make this test in next section. Finally we would like to mention that it is also possible that the nuclear shell effect of superheavy nuclei is not evident as compared with that of ^{208}Pb.

III. NEW FORM OF BINDING ENERGY FORMULA OF HEAVY NUCLEI WITH $Z \geqslant 90$

In previous section we obtain the formula of binding energies of nuclei with $Z \geqslant 90$ and $N \geqslant 140$. This is a special mass range based on the special interest of superheavy nuclei which usually have $Z \geqslant 106$ and $N \geqslant 150$. In this section we demonstrate the second version of the formula that includes all known experimental data of binding energies of nuclei with $Z \geqslant 90$. The number of data used in fit is 181 [12]. Because the mass range is enlarged and the nuclei around magic number $N = 126$ are included, we should introduce a new term to reduce the total deviation between experimental data and calculated values to a similar level as that in previous section. The formula obtained in this section is called as the second version of the formula for convenience. The second version of the formula is as follows:

$$B(Z, A) = a_v A - a_s A^{2/3} - a_c Z^2 A^{-1/3} - a_a \left(\frac{A}{2} - Z\right)^2 A^{-1} + a_p \delta A^{-1/2} + a_6 |A - 252| / A - a_7 |N - 152| / N + \frac{a_8}{(A - 214)^2 + a_9}, \quad (9)$$

where an additional term (the last term of above equation) is introduced as compared with the formula in previous section. This term is the simulation of the spherical shell effect around ^{216}Th. The values of parameters are obtained by fitting the 181 data of nuclei ranged from ^{209}Th to ^{264}Hs [12].

$$\begin{cases} a_v = 15.63284\,\text{MeV} \\ a_s = 17.23767\,\text{MeV} \\ a_c = 0.70479\,\text{MeV} \\ a_a = 96.19350\,\text{MeV} \\ a_p = 11.46055\,\text{MeV} \\ a_6 = 5.92378\,\text{MeV} \\ a_7 = 30.70391\,\text{MeV} \\ a_8 = 296.192\,\text{MeV} \\ a_9 = 43.997. \end{cases}$$

The average deviation and root-mean-square deviation of the 181 nuclei are as follows:

$$\langle\sigma\rangle = \Sigma_{i=1,181} \left|B^i_{\text{exp}} - B^i_{\text{cal}}\right| / 181 = 0.269\,\text{MeV} \quad (10)$$

$$\sqrt{\sigma^2} = \left(\Sigma_{i=1,181} \left(B^i_{\text{exp}} - B^i_{\text{cal}}\right)^2 / 181\right)^{1/2} = 0.370\,\text{MeV}. \quad (11)$$

These deviations of the 181 nuclei are also satisfying although they are slightly larger than the deviations of the 117 nuclei in the previous section. Usually the standard Bethe and Weizsäcker formula leads to anomalously large deviations near the magic number $N = 126$. Here the deviations near $N = 126$ are reduced greatly because of the introduction of additional terms. To see the details of deviations we calculate the deviations for two regions with the second version of the formula. One region corresponds to 117 nuclei with $N \geqslant 140$ and another corresponds to the nuclei with $N \leqslant 139$ ($Z \geqslant 90$). The deviations for the two regions are as follows:

$$\langle\sigma\rangle = \Sigma_{i=1,117} \left|B^i_{\text{exp}} - B^i_{\text{cal}}\right| / 117 = 0.216\,\text{MeV} \quad (12)$$

$$\sqrt{\sigma^2} = \left[\Sigma_{i=1,117} \left(B^i_{\text{exp}} - B^i_{\text{cal}}\right)^2 / 117\right]^{1/2} = 0.268\,\text{MeV} \quad (13)$$

and

$$\langle\sigma\rangle = \Sigma_{i=1,64} \left|B^i_{\text{exp}} - B^i_{\text{cal}}\right| / 64 = 0.367\,\text{MeV} \quad (14)$$

$$\sqrt{\sigma^2} = \left[\Sigma_{i=1,64} \left(B^i_{\text{exp}} - B^i_{\text{cal}}\right)^2 / 64\right]^{1/2} = 0.507\,\text{MeV}. \quad (15)$$

These results of the deviations clearly show that the main deviation between the formula and the data appears for the 64 nuclei with $N \leqslant 139$. The deviation in the region $N \geqslant 140$ is significantly less than the deviation in the region of $N \leqslant 139$. So we expect that the second version of the formula is also accurate for superheavy nuclei which are current interests. This demonstrates that both versions of the formulas can be applied for superheavy nuclei because their results are stable

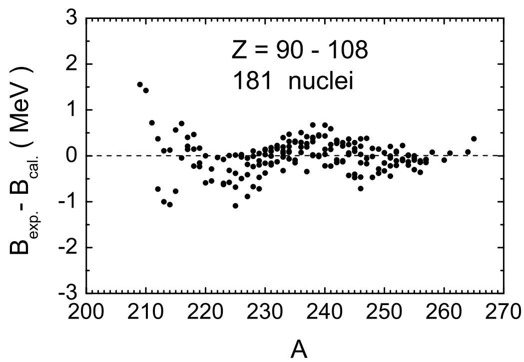

FIG. 6. The variation of the deviations between experimental binding energies and calculated ones from the formula of binding energies of the 181 heavy nuclei.

and close. Of course the second version of the formula could also be used for calculations of the binding energies of lighter isotopes with $Z = 90$–100 although the deviation in this range could be slightly larger than that in superheavy region.

The detailed results from the second version of the formula are also drawn in Figs. 6–9. In Fig. 6 the y axis is the deviation of the 181 nuclei and x axis is the mass number. One sees again that the deviation is very small for superheavy region. The variations of average binding energies of even-even nuclei are plotted in Figs. 7 and 8 for Th, U, Pu, Cm, Cf, and Fm isotopes.

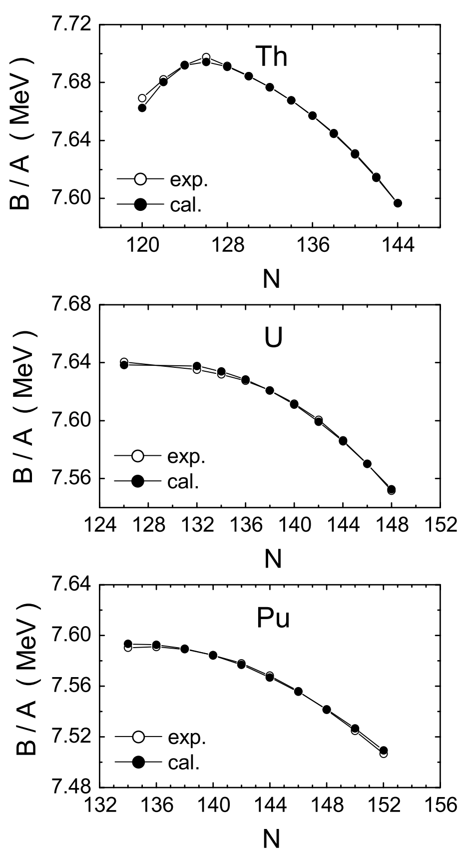

FIG. 7. The variation of the experimental and calculated average binding energies of even-even nuclei with $Z = 90$–94 where the second version of the formula is used.

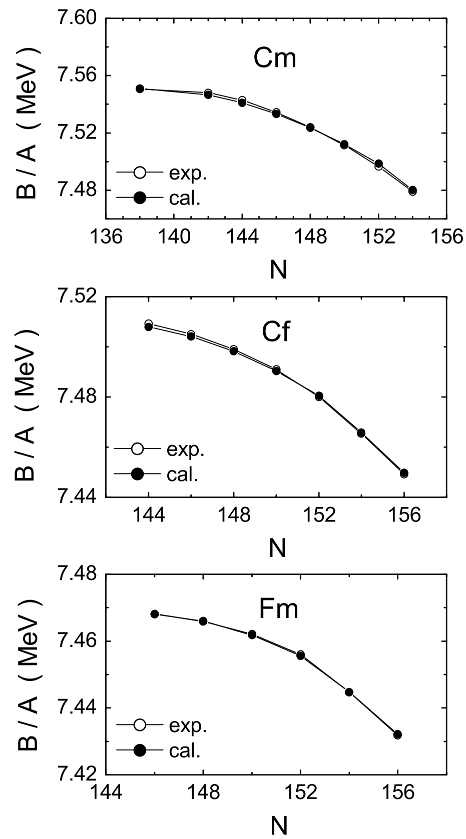

FIG. 8. The variation of the experimental and calculated average binding energies of even-even nuclei with $Z = 96$–100 where the second version of the formula is used.

The theoretical curves are very close to experimental ones. For light and medium isotopes with $Z \leqslant 83$ the maximum of the average binding energies on an isotopic chain usually lies near the β stable line and the nuclei with the maximum are stable or have very long half-lives. It is not clear what will happen for superheavy nuclei because the maximum of the average binding energies on an isotopic chain will not situate on the predicted center of the superheavy island. Because the nuclei with the maximum are tightly bound in quantum many-body problems, we do not know which nucleus will live the longest on an isotopic chain when α decay, β decay, and spontaneous fissions compete each other in superheavy regions. This may also influence the predictions of existence of magic numbers.

In Fig. 9 we draw the variations of the two-neutron separation energies, two-proton separation energies, and α-decay energies for the uranium isotopic chain. The black circles in the figure denote the calculated values from the second version of the formula. The hollow circles denote the experimental data or estimated values by Audi *et al.* [12]. The arrow near the hollow circle denotes that the circle corresponds to an estimated value. One can see that theoretical curves follows the trend of experimental data well. For heavier U isotopes around $N = 148$, the calculated values approximately coincide with experimental ones. So it is concluded again that the results from the second version of the formula agree well with the data.

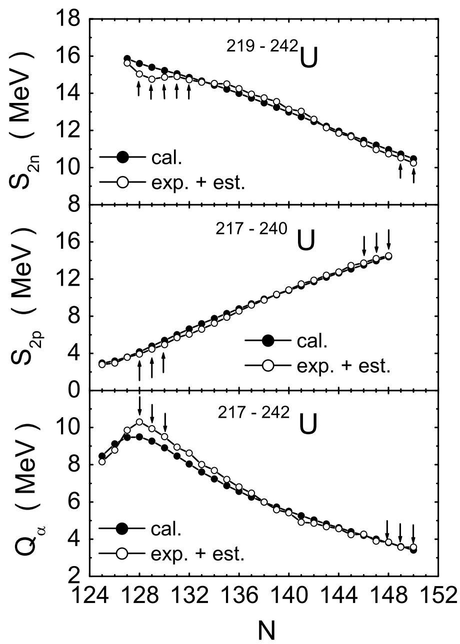

FIG. 9. The upper part of this figure is the variation of the experimental (or estimated) and calculated two-neutron separation energies of the uranium isotopes where the second version of the formula is used. The hollow circles with an arrow are estimated values from Audi *et al.* when experimental data are not available. The middle one of this figure is the variation of two-proton separation energies and the low one is that of α-decay energies of the uranium isotopes.

IV. CONCLUSIONS

In summary, we have proposed new local mass formulas of heavy nuclei with $Z \geqslant 90$ by introducing new terms on the Bethe and Weizsäcker formula. This can be considered a new form of the Bethe and Weizsäcker formula for heavy and superheavy region. The first version of the formula can very accurately reproduce the binding energies of the 117 nuclei with $Z \geqslant 90$ and $N \geqslant 140$. The average deviation between the experimental data and calculated ones is 0.118 MeV where the binding energies themselves are as high as 2000 MeV in this mass range. The calculated α-decay energies and two-neutron separation energies agree well with experimental data of heavy nuclei with $Z \geqslant 90$. Both the proton even-odd effect and neutron even-odd effect of one-nucleon separation energies are also well reproduced by the formula. This is the first time that experimental data of heavy nuclei can be reproduced with a very high precision ($\delta B/B \approx 0.118/2000 \approx 0.006\%$). The formula is useful for accurate estimation of the binding energies of unknown superheavy nuclei. The binding energies, α-decay energies, and α-decay half-lives of unknown superheavy nuclei are predicted. This will be useful for future experiments of superheavy nuclei. We have proposed the idea to determine the effective parameters by fitting the data of heavy nuclei with $Z \geqslant 90$. This idea will be useful for other models to predict the properties of superheavy nuclei very reliably. This will improve the significant deviation between the experimental data and the numerical results that exists in current models. For the second version of the formula the 181 nuclei with known binding energies are included in fitting the parameters. The deviations of the 181 nuclei are also small and the second version of the formula is satisfying although the deviations are slightly larger than those of the first version of the formula. Especially the results from both versions of formulas are very close for superheavy region. This shows that they can give stable and reliable results for superheavy region and this is useful for future experiments of superheavy nuclei.

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