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Inversion of parity splitting in alternating parity bands at high angular momenta

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The angular-momentum dependence of parity splitting in ground-state alternating parity bands and especially the sign inversion of parity splitting are considered. It is shown that the complicated odd-even staggering structure of the alternating parity bands can be interpreted as the result of two simultaneously manifesting effects: (1) penetration of the barrier separating two minima with the opposite signs of the reflection asymmetric deformation and (2) alignment of the angular momentum of the intrinsic excitations.

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I. INTRODUCTION

The presence of strong octupole correlations in nuclei is reflected in a significant lowering of the excitation energies of the negative-parity states. This phenomenon was observed in different nuclei [1,2]. In these nuclei low-lying states with positive and negative parities form alternating parity bands with $I^\pi = 0^+, 1^-, 2^+, 3^-, \ldots$, with the negative-parity states shifted up in energy with respect to the positive-parity states. The experimental data show that this shift depends on angular momentum and decreases with increasing angular momentum. The decrease of the parity splitting was interpreted as due to a decrease of the penetration probability through the barrier separating two physically equivalent minima in the nuclear collective potential energy that had different signs of the reflection asymmetric deformation, which can be the octupole [3] or mass-asymmetry [4,5] degree of freedom. Since the moment of inertia of a nucleus depends on the reflection asymmetric deformation and is larger at the minimum than at the top of the barrier, the height of the barrier relative to the minimum increases with angular momentum I, and the barrier-penetration probability decreases. This explains the decrease of parity splitting with I at relatively low values of I.

The picture described above is expressed by the following formula for the excitation energies $E(I)$ of the states belonging to the ground-state alternating parity band:

$$E(I) = E_{\text{average}}(I) - \tfrac{1}{2}(-1)^I \Delta E(I). \tag{1}$$

In Eq. (1) $E_{\text{average}}(I)$ is a smooth function of I describing the angular-momentum dependence of the excitation energies averaged over parity. The quantity $\Delta E(I)$ is the value of the parity splitting, which is positive at low I.

However, in light Ra and Th isotopes it was observed that the parity splitting $\Delta E(I)$, after decreasing to zero, inverts its sign and increases again in absolute value, i.e., in some angular-momentum interval at higher I, negative parity states are shifted down with respect to the positive-parity states in contrast to the situation at low I. Moreover, in the light Ra isotopes, parity splitting $\Delta E(I)$ changes sign for the second time. However, the size of this inverted parity splitting is 5–10 times smaller than at low I. In Ref. [6] it was indicated that the Coriolis interaction can be the reason for the sign inversion of parity splitting. The role of rotational motion in the sign inversion of the parity splitting at high I has been investigated in Ref. [7], in which it was shown that the band-crossing phenomenon can play an important role. In Ref. [8] it was shown that the effect of the quadrupole-octupole correlations can be important for the description of parity splitting at high I.

It is the aim of this work to formulate a unified model based on a Hamiltonian that explains both observed parity-splitting effects: parity splitting at low I and an inversion of the sign of the parity splitting at high I.

II. PARITY SPLITTING AT LOW ANGULAR MOMENTA

We start with the standard Hamiltonian,

$$\hat{H} = \hat{H}_p + \hat{H}_{\text{rot}} + \hat{H}_c + \hat{H}_{\text{oct}}, \tag{2}$$

containing the term $\hat{H}_p$ that describes the intrinsic motion of particles or quasiparticles in a mean field and their coupling to collective octupole motion, the rotational-energy term $\hat{H}_{\text{rot}}$,

$$\hat{H}_{\text{rot}} = \frac{\hbar^2}{2\Im} I(I+1), \tag{3}$$

and the Coriolis interaction $\hat{H}_c$,

$$\hat{H}_c = -\frac{\hbar^2}{2\Im}(\hat{j}_+\hat{I}_- + \hat{j}_-\hat{I}_+), \tag{4}$$

where $\hat{j}_\pm$ are rising and lowering components of the intrinsic angular momentum and $\hat{I}_\mp$ are rising and lowering components of the total angular-momentum operator expressed through the Cartesian projections of the angular-momentum operators on the body-fixed axis:

$$\hat{j}_\pm = \hat{j}_1 \pm \imath \hat{j}_2, \quad \hat{I}_\pm = \hat{I}_1 \pm \imath \hat{I}_2. \tag{5}$$

In this section we consider the octupole deformation only, keeping in mind, however, that the other odd-multipolarity degrees of freedom can be important also.

To treat a parity splitting at low I, the Hamiltonian $\hat{H}$ contains the term $\hat{H}_{\text{oct}}$ describing an octupole motion of a nuclear shape. We assume that $\hat{H}_{\text{oct}}$ conserves axial symmetry. The octupole part of the total Hamiltonian $\hat{H}_{\text{oct}}$ can be written as a sum of the kinetic and potential energies of the octupole motion:

$$\hat{H}_{\rm oct} = -\frac{\hbar^2}{2B}\frac{d^2}{d\beta_{30}^2} + U(\beta_{30}), \tag{6}$$

where $U(\beta_{30})$ is an even function of β_{30} with two symmetrically located minima at $\beta_{30} = \pm\beta_{\rm min}$. The inertia coefficient B and potential U are assumed not to depend on I. Below we treat simultaneously $\hat{H}_{\rm rot}$ and $\hat{H}_{\rm oct}$ since the moment of inertia $\Im$ depends on β_{30}. For simplicity we neglect a noncommutativity of $\hat{H}_{\rm oct}$ with $\hat{H}_p$ and $\hat{H}_c$. This is of course an approximation since the particles are coupled to the octupole motion, and, in the general case, the Coriolis interaction can disturb the octupole part of the total wave function. However, the first effect produces an anharmonicity in the collective octupole motion that is included, in principle, in the potential $U(\beta_{30})$, and the last effect becomes more important at high I when the octupole deformation is already stabilized.

The total wave vector $|\Psi_{\rm total}\rangle$ that depends on all degrees of freedom of the Hamiltonian can be presented as [9]

$$|\Psi_{\rm total}\rangle = \frac{1}{\sqrt{2}}[1 + (-1)^I \hat{P}\hat{\pi}]\phi(\beta_{30})|\Psi\rangle, \tag{7}$$

where $\hat{P}$ is the parity operator acting on only the octupole part of the wave function, $\hat{\pi}$ is the parity operator of particles, the total parity p of the states under consideration is $(-1)^I$, and $|\Psi\rangle$ describes the rotational motion and the intrinsic particle motion.

It was shown in Ref. [10] that a good approximation for $\phi(\beta_{30})$ is a Gaussian function,

$$\phi(\beta_{30}) = \left(\frac{\pi\hbar}{B\omega}\right)^{-1/4} \exp\left[-\frac{B\omega}{2\hbar}(\beta_{30} - \beta_{\rm min})^2\right], \tag{8}$$

where the parameter ω characterizes the width of the maximum at $\beta_{30} = \beta_{\rm min}$. The second term in the wave vector of Eq. (7), which is multiplied by the space reflection operators and by the phase factor $(-1)^I$, appears because of the existence of the second minimum in the potential located at $\beta_{30} = -\beta_{\rm min}$. A tunneling through the barrier separating the minima produces the parity splitting $\Delta E(I)$ in the total excitation energy $E(I)$ that shifts the energy of the state, described by the wave function that is symmetric or antisymmetric with respect to reflection $\beta_{30} \to -\beta_{30}$. It is known [11] that this splitting is determined by the behavior of the wave function of Eq. (8) at the top of the barrier at $\beta_{30} = 0$:

$$\Delta E(I) \simeq \frac{\hbar^2}{B}\phi(\beta_{30} = 0)\phi'(\beta_{30} = 0). \tag{9}$$

Substituting Eq. (8) into relation, (9) we obtain

$$\Delta E(I) = \hbar\omega\sqrt{\frac{B\omega\beta_{\rm min}^2}{\pi\hbar}} \exp\left(-\frac{B\omega\beta_{\rm min}^2}{\hbar}\right). \tag{10}$$

In Ref. [3] it was shown that $\Delta E(I)$ can be presented as

$$\Delta E(I) = \Delta E(0) \exp\left[-\frac{I(I+1)}{a}\right]. \tag{11}$$

This phenomenological result is well reproduced in the cluster-model calculations for many actinides and rare earth-nuclei [4,5]. Comparing Eqs. (10) and (11), we obtain the angular-momentum dependence of ω, which is illustrated in Table I by the results we obtained by using the experimental data [12] for ^{220}Ra. In addition, the ratio of the magnitude of the collective wave function taken at the barrier and at the minimum of the potential $\phi(\beta_{30} = 0)/\phi(\beta_{30} = \beta_{\rm min})$ is presented for different I. We have used $B = 163\hbar^2\,{\rm MeV}^{-1}$, which corresponds to the value of the inertia parameter used in the cluster-model calculations of the parity splitting in Ref. [5].

TABLE I. Angular-momentum dependence of the parameter $\hbar\omega$ characterizing the collective wave function $\phi(\beta_{30})$ [see Eq. (8)] and of the ratio $\phi(\beta_{30} = 0)/\phi(\beta_{30} = \beta_{\rm min})$ of the magnitude of the collective wave function at the barrier to its magnitude at the minimum of the potential. The experimental data on the parity splitting in ^{220}Ra that are used in the calculations of $\hbar\omega$ are taken from Ref. [12].

I	$\hbar\omega$	$\phi(\beta_{30} = 0)\phi(\beta_{30} = \beta_{\rm min})$
2	1.807	0.30
3	2.291	0.22
4	2.737	0.16
5	3.181	0.12
6	3.689	0.09
7	4.309	0.06
8	5.277	0.03

III. PARITY SPLITTING AT HIGH ANGULAR MOMENTA

Consider now a situation in which the angular momentum I is large enough to stabilize an octupole deformation. It happens because of the dependence of the moment of inertia $\Im$ on β_{30}. Stabilization of the octupole deformation means that we can neglect the barrier penetration and an oscillation of β_{30} around the minimum. Therefore, at these values of I, the parity splitting described by expression (11) disappears. We can neglect the effect of $\hat{H}_{\rm oct}$ on $\hat{H}_p$ and $\hat{H}_c$ and put $\beta_{30} = \beta_{\rm min}$ in $\hat{H}_p$.

Consider the rest part of the total Hamiltonian, namely $\hat{h} \equiv (\hat{H}_p + \hat{H}_c)$. Let us diagonalize $\hat{h}$ by some unitary transformation up to terms containing rising and lowering components of the total angular-momentum operator $I_\pm$ in a degree not higher than some fixed value. This task can be solved by the transformation [13,14]

$$\hat{h}' \equiv \exp(\hat{T})\hat{h}\exp(-\hat{T}), \tag{12}$$

where the anti-hermitian operator $\hat{T}$ is given by an expansion $\hat{T} = \sum_n \hat{T}_n$ with

$$\begin{aligned}
\hat{T}_1 &= (\hat{\epsilon}_+ I_- - \hat{\epsilon}_- I_+) + (\hat{u}_+^{(11)}\{I_3, I_-\}_+ - \hat{u}_-^{(11)}\{I_3, I_+\}_+),\\
\hat{T}_2 &= (\hat{u}_+^{(20)} I_-^2 - \hat{u}_-^{(20)} I_+^2) + (\hat{u}_+^{(30)} I_-^3 - \hat{u}_-^{(30)} I_+^3),\\
\hat{T}_3 &= (\hat{u}_+^{(21)}\{I_3, I_-^2\}_+ - \hat{u}_-^{(21)}\{I_3, I_+^2\}_+)\\
&\quad + \left(\hat{u}_+^{(12)}\left\{I_3^2, I_-\right\}_+ - \hat{u}_-^{(12)}\left\{I_3^2, I_+\right\}_+\right).
\end{aligned} \tag{13}$$

The index n corresponds to the degree of the operators I_i in $\hat{T}_n$. The curly brackets $\{A, B\}_+$ in Eq. (13) denote the anticommutator. In Eq. (13) the expansion operators $\epsilon_\pm$ and $u_\pm^{(ij)}$ are functions of the intrinsic degrees of freedom.

They are fixed by the condition that they diagonalize $\hat{h}'$ up to terms containing components of the total angular-momentum operator in degrees not higher than some fixed value. Thus we obtain

$$\begin{aligned}\hat{h}' &\equiv \exp(\hat{T})\hat{h}\exp(-\hat{T}) = \exp(\hat{T})(\hat{H}_p + \hat{H}_c)\exp(-\hat{T}) \\ &= \hat{H}_p + [\hat{T}_1, \hat{H}_p] + \hat{H}_c + [\hat{T}_2, \hat{H}_p] + [\hat{T}_1, \hat{H}_c] \\ &\quad + \tfrac{1}{2}[\hat{T}_1, [\hat{T}_1, \hat{H}_p]] + [\hat{T}_3, \hat{H}_p] + [\hat{T}_2, \hat{H}_c] \\ &\quad + \tfrac{1}{2}[\hat{T}_2, [\hat{T}_1, \hat{H}_p]] + \tfrac{1}{2}[\hat{T}_1, [\hat{T}_2, \hat{H}_p]] \\ &\quad + \tfrac{1}{2}[\hat{T}_1, [\hat{T}_1, \hat{H}_c]] + \tfrac{1}{6}[\hat{T}_1, [\hat{T}_1, [\hat{T}_1, \hat{H}_p]]] + \cdots. \end{aligned} \tag{14}$$

Since the Coriolis term $\hat{H}_c$ is nondiagonal with respect to the third projection K of $\vec{I}$, it is convenient to determine $\hat{T}_1$ by the relation

$$[\hat{T}_1, \hat{H}_p] = -\hat{H}_c, \tag{15}$$

from which it follows that $\hat{T}_1$ is also nondiagonal in K. The operator $\hat{T}_2$ is determined as

$$[\hat{T}_2, \hat{H}_p] = -\tfrac{1}{2}([\hat{T}_1, \hat{H}_c])_{\text{nondiag.part}}. \tag{16}$$

For the operator $\hat{T}_3$ we set

$$\begin{aligned}[\hat{T}_3, \hat{H}_p] = &-\big(\tfrac{1}{2}[\hat{T}_2, \hat{H}_c] - \tfrac{1}{4}[\hat{T}_1, ([\hat{T}_1, \hat{H}_c])_{\text{nondiag.part}}] \\ &+ \tfrac{1}{3}[\hat{T}_1, [\hat{T}_1, \hat{H}_c]]\big)_{\text{nondiag.part}}. \end{aligned} \tag{17}$$

Finally, $\hat{h}'$ can be written as

$$\begin{aligned}\hat{h}' \equiv \exp(\hat{T})\hat{h}\exp(-\hat{T}) = \hat{H}_p + \hat{t}_1 I_3 + \hat{t}_2\big(I^2 - I_3^2\big) \\ + \hat{t}_2' I_z^2 + \hat{t}_3 I_3^3 + \hat{t}_3' I_3\big(I^2 - I_3^2\big) + \cdots. \end{aligned} \tag{18}$$

In Eq. (18) $\hat{t}_i$ and $\hat{t}_i'$ are intrinsic operators whose matrix elements are expressed in terms of the matrix elements of the intrinsic angular-momentum operators $j_\pm$ and the matrix elements of H_p. For instance,

$$\begin{aligned}\hat{t}_1 &= -\frac{\hbar^2}{4\Im}(\{\hat{\epsilon}_+, \hat{j}_-\}_+ + \{\hat{\epsilon}_-, \hat{j}_+\}_+) \\ \hat{t}_2 &= -\frac{\hbar^2}{4\Im}([\hat{\epsilon}_+, \hat{j}_-] + [\hat{\epsilon}_-, \hat{j}_+]), \end{aligned} \tag{19}$$

where $\hat{\epsilon}_\pm$ are determined by the equation

$$[\hat{H}_p, \hat{\epsilon}_\pm] = \pm\frac{\hbar^2}{2\Im}\hat{j}_\pm. \tag{20}$$

Assume, for simplicity, that there is only one intrinsic state for every value of K. Then the operators $\hat{t}_i$ and $\hat{t}_i'$ will have only diagonal matrix elements because they do not change K, as it is seen, for instance, from Eqs. (19) and (20). Therefore, for the Hamiltonian $\hat{h}'$, K will be a good quantum number. The eigenvector $|\Psi'\rangle$ of the Hamiltonian $\hat{h}'$ is connected to the eigenvector $|\Psi\rangle$ of $\hat{h}$ by the relation

$$|\Psi\rangle = \exp(-\hat{T})|\Psi'\rangle. \tag{21}$$

As a consequence of factorization (7), the eigenvalue $\mathcal{E}_I$ of H is the sum of the eigenvalue of $(\hat{H}_{\text{oct}} + \hat{H}_{\text{rot}})$ denoted by ϵ_I and the eigenvalue of $\hat{h}'$. The expression for $\langle K|\hat{h}'|K\rangle$ can be obtained from Eq. (18) by substitution of their eigenvalues instead of operators. Up to the terms of the second order in $\hbar^2/\Im$, this can be done with Eqs. (19) and (20) and is given by

$$\begin{aligned}\langle K|\hat{h}'|K\rangle \approx E_K + \frac{1}{4}\left(\frac{\hbar^2}{\Im}\right)^2 \\ \times \Bigg\{[I(I+1) - K^2 + K]\frac{\langle K|j_+|K-1\rangle^2}{E_K - E_{K-1}} \\ - [I(I+1) - K^2 - K]\frac{\langle K+1|j_+|K\rangle^2}{E_{K+1} - E_K}\Bigg\}, \end{aligned} \tag{22}$$

where

$$E_K = \langle K|\hat{H}_p|K\rangle. \tag{23}$$

It is seen that expression (22) is even with respect to the reflection $K \to -K$ since E_K is an even function of K. The lowest eigenvalue of the system is determined by the value $|K| = K_{\min}$ that minimizes $\langle K|\hat{h}'|K\rangle$. In principle, the values of $|K|$ corresponding to the lowest energy sequence changes with angular momentum I. This is easily seen if one considers $|K|$ a continuous variable [7]. In general $\langle K|\hat{h}'|K\rangle$ has more than one minimum, in K. The deepest minimum corresponds to the yrast level sequence, while the higher minima correspond to excited bands. For a given "continuous" minimum, the discrete quantum number K is determined as the closest integer number. At low I the ground-state band has the component with $K = 0$ as the main one. The minimum of the energy at $K = 0$ when I is low is due to the energy of the two-quasiparticle states that should be excited to a increase K from zero to a finite value. The shift of the position of the minima from $K = 0$ to nonzero value of K is due to the Coriolis interaction, which becomes more important with increasing I.

The matrix element $\langle K|\hat{h}'|K\rangle$ determines, together with the rotational and octupole terms, the energy of the nuclear state. In the general case its dependence on K is quite complicated and cannot be treated analytically. For this reason, to understand the problem qualitatively, let us come back to the part of the total Hamiltonian denoted by $\hat{h}$.

The nonzero matrix elements of $\hat{h}$ have the following expressions:

$$\begin{aligned}\langle IK'|\hat{h}|IK'\rangle &= E(K'), \\ \langle IK'|\hat{h}|IK'\pm1\rangle &= -\frac{\hbar^2}{2\Im}\sqrt{(I\pm K'+1)(I\mp K')}\langle K'|j_\mp|K'\pm1\rangle, \end{aligned} \tag{24}$$

where we use the quantum number K' to characterize the basis with the aim of distinguishing it from the quantum number K discussed above as the characteristic of the eigenstate of $\hat{h}'$.

To get a qualitative understanding of the problem let us consider the case of $I \gg K'$ for which the results can be obtained analytically. If $I \gg K'$, nondiagonal matrix elements increase proportionally to I. In this limit we can neglect the diagonal matrix elements because they are independent of I and use the following approximate expression for the nondiagonal matrix elements:

$$\langle IK'|\hat{h}|IK' \pm 1\rangle = -\frac{\hbar^2 I}{2\Im}\langle K'|j_\mp|K' \pm 1\rangle. \tag{25}$$

If we take into account in the expansion of Eqs. (24) the correction of the order of K'^2/I^2, we obtain the following result:

$$\langle IK'|\hat{h}|IK'\pm 1\rangle = -\frac{\hbar^2 I}{2\Im}\langle IK'|\left(1+\frac{1}{8I^2}\right)\hat{j}_{\mp} + \frac{1}{4I^2}\{\hat{j}_z^2, \hat{j}_{\mp}\}_+|IK'\pm 1\rangle. \quad (26)$$

In fact, by the approximation used to derive Eqs. (25) we fix axis 1 as the axis along which the total angular momentum is directed. This can be seen easily if we use the boson representation of the operators $I_\pm$ and I_3.

The Holstein-Primakoff boson representation of the operators $I_\pm$ and I_3 looks like

$$\hat{I}_- = b^+\sqrt{2I - b^+b}, \quad \hat{I}_+ = \sqrt{2I - b^+b}\,b, \quad \hat{I}_3 = b^+b - I, \quad (27)$$

where $[b, b^+] = 1$. It is seen from the representation for I_3 that the assumption $|K'| \ll I$ means that the number of bosons is approximately equal to I and the matrix elements of the boson operators b and b^+ are large. In this case it is convenient to do the following unitary transformation of the boson operators:

$$b^+ = \sqrt{I} + c^+, \quad b = \sqrt{I} + c, \quad (28)$$

where $[c, c^+] = 1$. The new boson operators c and c^+ describe small fluctuations around $\sqrt{I}$, and we can do an expansion in powers of c and c^+. Conserving boson operators up to the first degree only, we obtain

$$\begin{aligned} I_- &\approx I + \tfrac{1}{2}\sqrt{I}(c^+ - c), \\ I_+ &\approx I - \tfrac{1}{2}\sqrt{I}(c^+ - c), \\ I_z &\approx \sqrt{I}(c^+ + c), \end{aligned} \quad (29)$$

or

$$I_1 \approx I, \quad I_2 \approx \frac{\imath}{\sqrt{2}}(c^+ - c). \quad (30)$$

It is seen from approximations (29) and (30) that the matrix elements of I_2 and I_z are small and the total angular momentum is directed along axis 1.

As follows from Eq. (25), in the limit $I \gg K'$ the matrix elements of the Hamiltonian $\hat{h}$ coincide with the matrix elements of the following operator:

$$-\frac{\hbar^2 I}{2\Im}(\hat{j}_+ + \hat{j}_-) \equiv -\frac{\hbar^2 I}{\Im}\hat{j}_1. \quad (31)$$

Since the total angular momentum $\vec{I}$ is directed along axis 1 we can write in Eq. (31), instead of $I\hat{j}_1$, the scalar product $\hat{I}\cdot\hat{j}$. It demonstrates that our Hamiltonian is time-reversal invariant. The corresponding Hamiltonian matrix is diagonalized by the following unitary transformation $\exp(\imath\frac{\pi}{2}\hat{j}_2)$, which plays the role of the operator $\exp(\hat{T})$ from Eq. (12). The transformed Coriolis part of the Hamiltonian takes the form

$$\exp\left(\imath\frac{\pi}{2}\hat{j}_2\right)\left(-\frac{\hbar^2 I}{\Im}\hat{j}_1\right)\exp\left(-\imath\frac{\pi}{2}\hat{j}_2\right) = -\frac{\hbar^2 I}{\Im}\hat{j}_3, \quad (32)$$

and the eigenvalue of $(\hat{h} + \hat{H}_{\rm rot})$ is equal to

$$\begin{aligned} &\frac{\hbar^2}{2\Im(\beta_{30})}I(I+1) - \frac{\hbar^2}{\Im(\beta_{30})}IK + \frac{\hbar^2}{2\Im(\beta_{30})}K^2 \\ &\quad\simeq \frac{\hbar^2}{2\Im(\beta_{30})}(I-K)^2, \end{aligned} \quad (33)$$

where K is the projection of the intrinsic momentum on the direction of the total momentum. The term quadratic in K on the left-hand side of Eq. (33) appears because of the presence of the centrifugal term in $\hat{H}_p$. In fact, we obtain an alignment [15–18] of the angular momentum of the intrinsic excitation along the axis perpendicular to the symmetry axis, i.e., along the axis of the collective rotation. As is seen from Eq. (33), the height of the barrier separating the two octupole minima decreases with the appearance of the nonzero K for larger values of I. If K is large enough, the wave function $\phi(\beta_{30})$ will take again a nonzero value at $\beta_{30} = 0$, recreating nonzero parity splitting.

Thus, because of the alignment of the particle momentum, the value of the collective rotational momentum needed to obtain the same total angular momentum I is decreased by the value of K. If the aligned single-particle configuration has even parity, then the parity splitting $\Delta E(I)$ does not change the sign; however, the interval of the values of I for which $\Delta E(I)$ is not equal to zero becomes larger. If the aligned single-particle configuration has odd parity, then the collective wave function $\phi(\beta_{30})$ in Eq. (7) should be odd for even I and even for odd I with respect to the transformation $\beta_{30} \to -\beta_{30}$ in order to get a state with the total parity equal to $(-1)^I$. As a result the negative-parity states will be shifted down by the parity splitting in contrast to the situation at low I.

The assumption that $I \gg K'$ was necessary only to obtain the results analytically. The key point is the alignment of the intrinsic excitation. This can happens for a less-rigid relation between I and K' than that used to derive Eq. (25).

Two effects, however, can decrease the magnitude of the parity splitting in this region of values of I. First, the aligned configuration has components with different intrinsic parities because of presence of mirror asymmetric deformation, and $|\langle\hat{\pi}\rangle| < 1$, where $\langle\hat{\pi}\rangle$ is the average parity. This effect decreases the parity splitting since we obtain the in-band splitting by multiplying the parity splitting of the states based on the quasiparticle vacuum by the factor $\langle\hat{\pi}\rangle$ [9,19,20]. Thus,

$$\Delta E(I) = \Delta E(0)\exp\left[-\frac{I(I+1)}{a}\right]\langle\hat{\pi}\rangle_I. \quad (34)$$

Second, it is possible that for these values of I the collective rotational momentum is nevertheless larger than zero, in spite of the alignment of the intrinsic momentum. Therefore the height of the barrier separating the two minima will be larger than near the ground state and the value of the parity splitting after sign inversion will be smaller than that at $I = 0$, in qualitative agreement with the experimental data. This can explain the inversion of the sign of the parity splitting that has been observed in light Ra and Th isotopes [1].

The alignment of the intrinsic octupole boson angular momentum has been already considered in Refs. [21,22]. In Refs. [23,24] it was suggested that with the increase of the

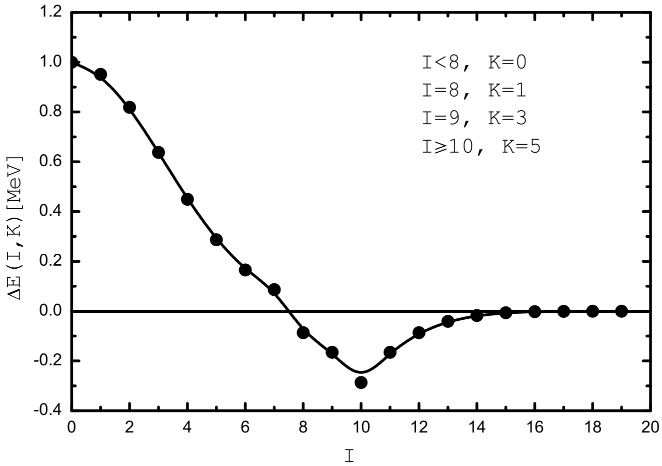

FIG. 1. The parity splitting $\Delta E(I, K)$ as a function of I calculated with the expression $\Delta E(I, K) = \Delta E(0)\exp[-\frac{(I-K)^2}{a}]$, with $\Delta E(0) = 1$ MeV, $a = 20$, and $K = 0$ for $I < 8$, $K = 1$ for $I = 8$, $K = 3$ for $I = 9$, and $K = 5$ for $I \geqslant 10$.

rotation the octupole vibrational angular momentum tends to align, giving rise to distortions in the spectra of the alternating parity bands. Calculations of the alignment of the vibrational and quasiparticle angular momenta within the framework of the microscopic approach have been performed in Ref. [18] and for the octupole vibrational momentum in Ref. [25], however, for nuclei in which parity splitting inversion was not observed. To analyze the possibility of the intrinsic angular-momentum alignment a dependence of the moments of inertia on the rotational frequency was considered in Ref. [26]. For ^{220}Ra and ^{222}Th, in which the parity-splitting inversion has been observed, it was found that the dynamical moment of inertia shows a smooth upbending that can be interpreted as a reflection of the band crossing. For ^{222}Th an alignment of the angular-momentum of the intrinsic excitations has been calculated in Ref. [27].

For an illustration of the effect considered above, let us apply formula (11) to the description of the angular-momentum dependence of the parity splitting-replacing $I(I+1)$ with $(I-K)^2$ in Eq. (11) in agreement with approximation (33). We put $\Delta E(0) = 1$ MeV, $a = 20$, and, rather arbitrarily only for the aim of illustration, we set $K = 0$ for $I < 8$, $K = 1$ for $I = 8$, $K = 3$ for $I = 9$, and $K = 5$ for $I \geqslant 10$. The results are shown in Fig. 1. It is seen that, in agreement with the discussion above, parity splitting changes the sign at $I = 8$. The absolute value of the inverted parity splitting at $I = 10$, where the deviation from zero takes its maximum value, is about four times smaller than the value at $I = 0$.

Consider also the quantity Stg(I):

$$\mathrm{Stg}(I) = \frac{1}{16}[6\Delta F(I) - 4\Delta F(I-1) - 4\Delta F(I+1) + \Delta F(I+2) + \Delta F(I-2)], \quad (35)$$

where

$$\Delta F(I) = E(I+1) - E(I) \quad (36)$$

was introduced in Ref. [7] to characterize the beat patterns of the staggering. The results of calculations of Stg(I) for the

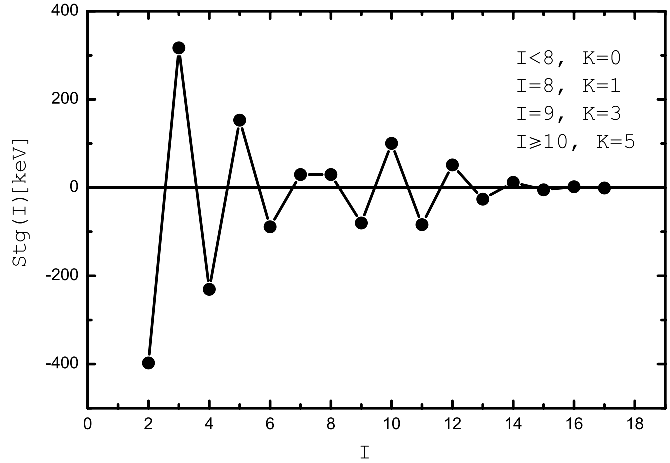

FIG. 2. Odd-even staggering patterns as functions of I obtained with expression (35) with the same parameters as in Fig. 1.

same choice of the parameters as in Fig. 1 are shown in Fig. 2. It is seen that for $I > 8$ the oscilations in Stg(I) are shifted in phase compared with the situation at low I: The maxima are located at even I, whereas at low I they are located at odd I.

After the first inversion of the sign of the parity splitting that is due to the alignment of the negative-parity single-particle configuration, a subsequent alignment at even higher values of I of a positive-parity particle excitation will invert the sign of the parity splitting again.

To see a relative role of the $\langle\hat{\pi}\rangle$ factor and of the alignment of the angular momentum of the intrinsic excitations, we performed calculations for ^{220}Ra in which the observed effect of the parity-splitting inversion is mostly pronounced. We took $\beta_2 = 0.11$, $\beta_3 = 0.09$, and $\beta_4 = 0.08$. The values of β_2 and β_3 are close to those used in the literature. The value of β_4 is somewhat larger. However, the results of

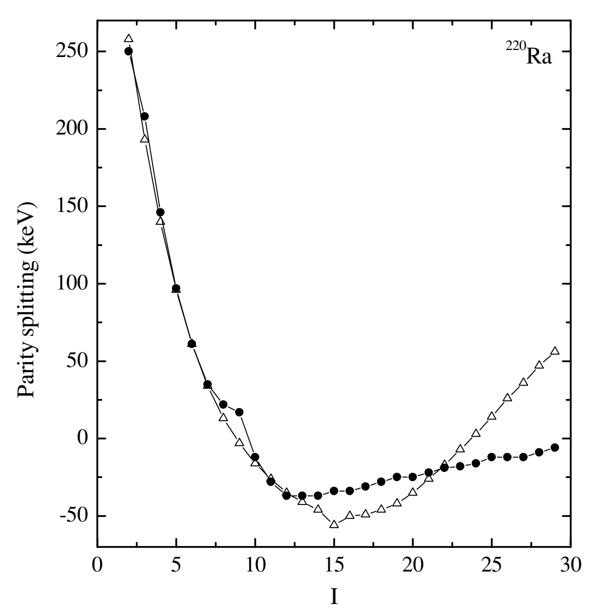

FIG. 3. Comparison of the experimental (curve with open triangles) and calculated (curve with filled dots) values of the parity splitting for ^{220}Ra. The experimental data are taken from Ref. [12].

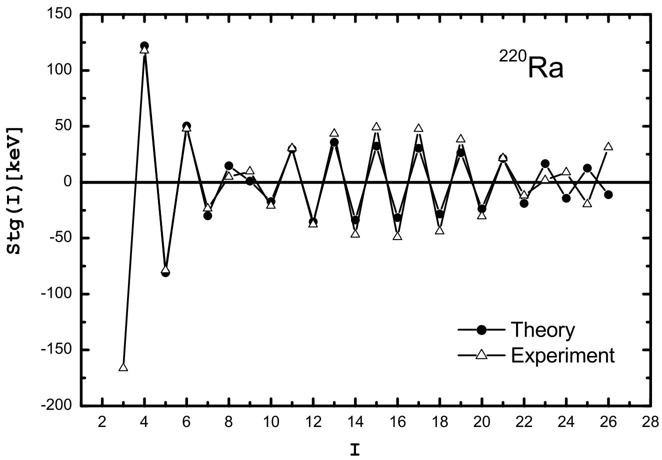

FIG. 4. Odd-even staggering patterns as functions of I obtained with the experimental data for ^{220}Ra taken from [12] and with the results of calculations of parity splitting shown in Fig. 3.

calculations are sensitive to this value. The ordering of the spherical single-particle states and their relative separations were taken in agreement with the single-particle scheme shown in Ref. [28]. The pairing was taken into account in the BCS approximation with $\Delta_p = 0.722$ MeV and $\Delta_n = 0.565$ MeV [29]. Thus, the one-body Hamiltonian, including the deformed single-particle field and Coriolis interaction and pairing in the BCS approximation, has been diagonalized.

Calculations have shown that the projection of the intrinsic angular momentum on the axis of the collective rotation increases smoothly with I up to $I = 10$. At $I = 10$ the ground-state band is crossed by the two-quasiparticle band with negative $\langle \hat{\pi} \rangle$. Above $I = 10$ the total angular momentum is practically exhausted by the aligned intrinsic momentum. The value of $\langle \hat{\pi} \rangle$, which is negative for $I \geqslant 10$, has a minimum at $I = 12$ and then increases again, smoothly approaching zero. The two-quasiparticle state aligned at $I = 10$ is composed of the neutron quasiparticles, in agreement with the results of Ref. [26]. The results of these calculations of the parity splitting are compared with the experimental data in Figs. 3 and 4. It is seen in Fig. 3 that the first inversion of the sign of the parity splitting at $I = 10$ is reproduced by the calculations. Unfortunately, our calculations do not describe the second parity splitting inversion at $I = 23$. A calculation with a decreasing pairing with I does not change the conclusion qualitatively. In Fig. 4 it is demonstrated that the theoretical energy levels accurately reproduce the experimental staggering pattern in ^{220}Ra up to spin $I = 22$.

The above consideration suggests that the complicated odd-even staggering structure of alternating parity bands can be interpreted as the result of two simultaneously manifesting dynamical effects in the nucleus. The one is the penetration of the barrier separating two minima of the potential energy having different signs of the mirror asymmetric deformation, which is described by Eq. (11), and the other is the high-order effect in the angular momentum of the rotating system that is a reason for an alignment of the angular momentum of the intrinsic excitation given by approximation Eq. (33). In the low angular-momentum region of the spectrum, the staggering effect is only because of the penetration of barrier, while in the higher-spin region the beat staggering structure of the band is determined additionally by higher orders of the Coriolis effect. An effect of the mixing of the intrinsic states having different parities because of the mirror asymmetric deformation is very important.

IV. SUMMARY

In conclusion, we have suggested an explanation of the sign inversion of parity splitting in the ground state, alternating parity bands at high values of the angular momentum. At low I the parity splitting shifts the positive-parity states down. This is due to a penetration of the barrier that separates two physically equivalent, symmetrically located minima in the nuclear collective potential, depending on a reflection asymmetric deformation. With an increase in I, the height of the barrier increases also and the barrier-penetration probability decreases. Thus the parity splitting is going to zero. However, at sufficiently high values of I, the Coriolis interaction becomes important, producing an alignment in the angular-momentum of the intrinsic excitation along the axis of the collective rotation. Because of this alignment the same value of the total angular momentum can be obtained with a smaller value of the collective rotational momentum. A decrease of the collective rotational momentum leads to a decrease in the barrier height. Then the barrier-penetration probability increases, thus recreating a parity, splitting. If the aligned single-particle configuration has an even average parity, then the parity splitting does not change the sign; however, the interval of the values of I for which $\Delta E(I)$ is not equal to zero becomes larger. If the aligned single-particle configuration has an odd average parity then for even I the collective wave function $\phi(\beta_{30})$ in Eq. (7) should be odd with respect to the transformation $\beta_{30} \to -\beta_{30}$ and vice versa in order to get the total parity for the state equal to $(-1)^I$. As a result, the negative-parity states will be shifted down by the parity splitting, in contrast to the situation at low I.

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