Incoherent pion photoproduction on the deuteron with polarization observables. I. Formal expressions

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Formal expressions are developed for the general fivefold differential cross section of incoherent π-photoproduction on the deuteron, including beam and target polarization. The polarization observables of the cross section are described by various beam, target, and beam-target asymmetries for polarized photons and/or polarized deuterons. They are given as bilinear Hermitean forms in the reaction matrix elements divided by the unpolarized cross section. In addition, the corresponding observables for the semiexclusive reaction $\vec{d}(\vec{\gamma},\pi)NN$ are also given.

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I. INTRODUCTION

Photoproduction of pions on light nuclei is an important topic in medium energy nuclear physics. It is motivated by different and complementary aspects. On the one hand one wants to study the elementary reaction on the neutron to which otherwise one has no access. On the other hand one is interested in the influence of a nuclear environment on the elementary production amplitude, and last but not least, one hopes to obtain information on nuclear structure.

In addition to the study of unpolarized total and differential cross sections, polarization observables provide very often further insight into details of the underlying reaction mechanisms and possible structure effects. In this case, such observables will serve as additional critical tests or check points for theoretical models. The considerable progress in experimental techniques for studying polarization phenomena has brought into focus also the question regarding the role polarization effects play in pion photoproduction on nuclei. Of particular interest is photoproduction of pions on the deuteron in view of its simple structure. Indeed, it has been studied quite extensively over the past 50 years (see Ref. [1] and references therein). Although in earlier work mainly total and semiexclusive differential cross sections of incoherent pion production have been studied, polarization observables were considered more recently, both in experiment [2,3] as well as in theory. For example, the spin asymmetry of the total cross section with respect to circular photon polarization, which determines the Gerasimov-Drell-Hearn sum rule, was investigated theoretically in Refs. [4–6] and target asymmetries were considered in Ref. [7].

Subsequently, various polarization asymmetries of the semiexclusive differential cross section $\vec{d}(\vec{\gamma},\pi)NN$ were studied theoretically in a series of papers [8–12]. Unfortunately, many of the results presented there are based on incorrect expressions for polarization observables, because the formal expressions for them were taken in analogy from the corresponding expressions of deuteron photodisintegration [13]. This is in principle possible, because the spin degrees of freedom are the same in both reactions, provided one takes care to check where certain symmetry properties of the reaction amplitude have been used in the derivation of the polarization observables in photodisintegration, because they are not identical in both reactions. This caveat refers in particular to those observables that are related to linearly polarized photons. It appears that this fact was not taken into account so that the results in Refs. [9,10] for them cannot be trusted. But also the results for circularly polarized photons are incorrect, namely the claim in Ref. [8] that all of them vanish identically is wrong. Moreover, this statement is in contradiction to Ref. [10], where a nonvanishing differential spin asymmetry for circularly polarized photons is reported, because this asymmetry is proportional to the beam-target asymmetry T^c_{10} for circularly polarized photons and a vector polarized deuteron, which means that the latter does not vanish. Thus, it is obvious that the importance of polarization effects requires a more careful and thorough treatment as done in Refs. [8–12].

With the present work we want to provide a solid basis for the formal expressions of the various polarization observables that determine the differential cross section, for incoherent pion production on the deuteron with polarized photons and/or polarized deuterons by deriving the general form of the differential cross section, including all possible polarization asymmetries. It complements the work of Blaazer *et al.* [14], who have formally derived all possible polarization observables for coherent pion photoproduction on the deuteron.

II. KINEMATICS

As a starting point, we first consider the kinematics of the photoproduction reaction

$$\gamma(k,\vec{\varepsilon}_\mu) + d(p_d) \rightarrow \pi(q) + N_1(p_1) + N_2(p_2), \qquad (1)$$

where we have defined the notation of the four-momenta of the participating particles. The circular polarization vector of the photon is denoted by $\vec{\varepsilon}_\mu$ with $\mu = \pm 1$. The following formal developments do not depend on the reference frame, laboratory, or center-of-momentum (c.m.) frame. However, in view of our explicit application [15] in which the reaction is evaluated in the laboratory frame, we refer sometimes to this frame for a definition. We choose as independent variables for the description of the final state the outgoing pion momentum

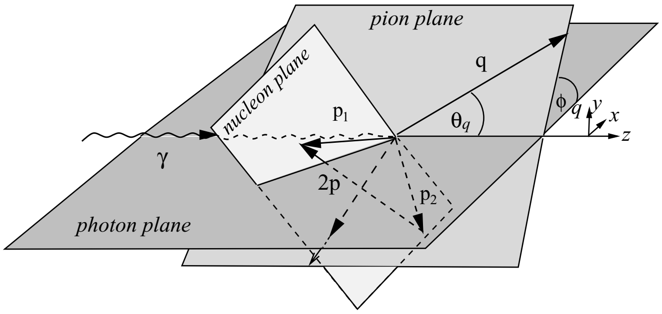

FIG. 1. Kinematics of pion photoproduction on the deuteron in the laboratory system.

$\vec q = (q, \theta_q, \phi_q)$ and the spherical angles $\Omega_p = (\theta_p, \phi_p)$ of the relative momentum $\vec p = (\vec p_1 - \vec p_2)/2 = (p, \Omega_p)$ of the two outgoing nucleons. Together with the incoming photon energy $\omega = k_0$, the momenta of the outgoing nucleons are fixed, i.e., $\vec p_{1/2} = (\vec k + \vec p_d - \vec q)/2 \pm \vec p$. The coordinate system is chosen to have a right-handed orientation with the z axis along the photon momentum $\vec k$. We distinguish in general three planes: (i) the photon plane spanned by the photon momentum and the direction of maximal linear photon polarization, which defines the direction of the x axis, (ii) the pion plane, spanned by the photon and pion momenta, which intersects the photon plane along the z axis with an angle ϕ_q, and (iii) the nucleon plane spanned by the momenta of the two outgoing nucleons intersecting the pion plane along the total momentum of the two nucleons. This is illustrated in Fig. 1 for the laboratory frame. In the case where the linear photon polarization vanishes, one can choose $\phi_q = 0$ and then the photon and pion planes coincide.

III. THE *T* MATRIX

All observables are determined by the T-matrix elements of the electromagnetic pion production current $\vec J_{\gamma\pi}$ between the initial deuteron and the final πNN states. In a general frame, it is given by the following:

$$T_{sm_s,\mu m_d} = -{}^{(-)}\langle \vec p_1\, \vec p_2\, sm_s,\ \vec q\,|\vec\varepsilon_\mu \cdot \vec J_{\gamma\pi}(0)|\vec p_d\, 1m_d\rangle, \qquad (2)$$

where s and m_s denote the total spin and its projection on the relative momentum $\vec p$ of the outgoing two nucleons and m_d is correspondingly the deuteron spin projection on the z axis as quantization axis. Furthermore, transverse gauge has been chosen. The knowledge of the specific form of $\vec J_{\gamma\pi}$ is not needed for the following formal considerations.

The general form of the T matrix after separation of the overall c.m. motion is given by the following:

$$\begin{aligned} T_{sm_s\mu m_d}(q,\ \Omega_q,\ \Omega_p) &= -{}^{(-)}\langle \vec p sm_s,\ \vec q\,|J_{\gamma\pi,\,\mu}(\vec k\,)|1m_d\rangle \\ &= \sqrt{2\pi}\sum_L i^L \hat L\, {}^{(-)}\langle \vec p sm_s,\ \vec q\,|\mathcal{O}^{\mu L}_{\mu}|1m_d\rangle \end{aligned} \qquad (3)$$

with $\mu = \pm 1$ and transverse electric and magnetic multipoles

$$\mathcal{O}^{\mu L}_M = E^L_M + \mu M^L_M. \qquad (4)$$

Furthermore, we use throughout the notation $\hat L = \sqrt{2L+1}$. It is convenient to introduce a partial-wave decomposition of the final states by the following:

$$\begin{aligned} {}^{(-)}\langle \vec p sm_s| &= \frac{1}{\sqrt{4\pi}}\sum_{l_p j_p m_p} \hat l_p (l_p 0 sm_s|j_p m_s) D^{j_p}_{m_s,m_p} \\ &\quad \times (\phi_p, -\theta_p, -\phi_p)\, {}^{(-)}\langle p(l_p s) j_p m_p|, \end{aligned} \qquad (5)$$

$${}^{(-)}\langle \vec q\,| = \frac{1}{\sqrt{4\pi}}\sum_{l_q m_q} \hat l_q\, D^{l_q}_{0,m_q}(\phi_q, -\theta_q, -\phi_q)\, {}^{(-)}\langle q l_q m_q|, \qquad (6)$$

where m_p and m_q like m_d refer to the photon momentum $\vec k$ as quantization axis. Here, the rotation matrices $D^j_{m'm}$ are taken in the convention of Rose [16]. Using the multipole decomposition and applying the Wigner-Eckart theorem yields the following:

$$\begin{aligned} &{}^{(-)}\langle p(l_p s) j_p m_p,\, q\, l_q m_q|\mathcal{O}^{\mu L}_M|1m_d\rangle \\ &\quad = \sum_{J M_J} (-)^{j_p - l_q + J}\, \hat J \begin{pmatrix} j_p & l_q & J \\ m_p & m_q & -M_J \end{pmatrix} \begin{pmatrix} J & L & 1 \\ -M_J & M & m_d \end{pmatrix} \\ &\qquad \times \langle pq((l_p s) j_p l_q) J||\mathcal{O}^{\mu L}||1\rangle, \end{aligned} \qquad (7)$$

with the selection rule $m_p + m_q = M_J = M + m_d$. Rewriting the angular dependence

$$\begin{aligned} &D^{j_p}_{m_s,m_p}(\phi_p, -\theta_p, -\phi_p) D^{l_q}_{0,m_q}(0, -\theta_q, -\phi_q) \\ &\quad = d^{j_p}_{m_s,m_p}(-\theta_p)\, d^{l_q}_{0,m_q}(-\theta_q) e^{i[(m_p - m_s)\phi_p + m_q \phi_q]}, \end{aligned} \qquad (8)$$

and rearranging, using the foregoing selection rule for $M = \mu$,

$$(m_p - m_s)\phi_p + m_q\phi_q = (m_p - m_s)\phi_{pq} + (\mu + m_d - m_s)\phi_q \qquad (9)$$

with $\phi_{pq} = \phi_p - \phi_q$, one finds that the T matrix can be written as follows:

$$T_{sm_s\mu m_d}(\Omega_p, \Omega_q) = e^{i(\mu + m_d - m_s)\phi_q}\, t_{sm_s\mu m_d}(\theta_p,\ \theta_q,\ \phi_{pq}), \qquad (10)$$

where the small t matrix depends only on θ_p, θ_q, and the relative azimuthal angle ϕ_{pq}. Explicitly one has

$$\begin{aligned} &t_{sm_s\mu m_d}(\theta_p,\ \theta_q,\ \phi_{pq}) \\ &\quad = \frac{1}{2\sqrt{2\pi}} \sum_{L l_p j_p m_p l_q m_q J J M_J} i^L\, \hat L\, \hat J\, \hat l_q\, \hat l_p\, \hat j_p\, (-)^{J + l_p + j_p - s + m_s - l_q} \\ &\qquad \times \begin{pmatrix} l_p & s & j_p \\ 0 & m_s & -m_s \end{pmatrix} \begin{pmatrix} j_p & l_q & J \\ m_p & m_q & -M_J \end{pmatrix} \begin{pmatrix} J & L & 1 \\ -M_J & \mu & m_d \end{pmatrix} \\ &\qquad \times \langle pq[(l_p s) j_p l_q] J||\mathcal{O}^{\mu L}||1\rangle d^{j_p}_{m_s,m_p} \\ &\qquad \times (-\theta_p)\, d^{l_q}_{0,m_q}(-\theta_q) e^{i(m_p - m_s)\phi_{pq}}. \end{aligned} \qquad (11)$$

Using this explicit form for the small t matrix, it is quite straightforward to show that if parity is conserved, the following symmetry relation holds for the inverted spin projections:

$$\begin{aligned} &t_{s-m_s-\mu-m_d}(\theta_p,\ \theta_q,\ \phi_{pq}) \\ &\quad = (-)^{s + m_s + \mu + m_d}\, t_{sm_s\mu m_d}(\theta_p,\ \theta_q,\ -\phi_{pq}). \end{aligned} \qquad (12)$$

In the derivation of this relation one has made use of the parity selection rules for the multipole transitions to a final partial wave $|pq[(l_p s)j_p l_q]J\rangle$ with parity $\pi_{J(l_p,l_q)} = (-)^{l_p+l_q+1}$

$$\left\{\begin{array}{l} E^L \ \ \pi_d \pi_{J(l_p,l_q)} (-)^L = 1 \quad \to (-)^{l_p+l_q+L} = -1 \\ M^L \ \ \pi_d \pi_{J(l_p,l_q)} (-)^L = -1 \to (-)^{l_p+l_q+L} = 1 \end{array}\right\}. \tag{13}$$

Therefore, invariance under a parity transformation results in the following property of the reduced matrix element

$$\begin{aligned} &(-)^{l_p+l_q+L} \langle pq[(l_p s)j_p l_q]J||\mathcal{O}^{-\mu L}||1\rangle \\ &\quad = -\langle pq[(l_p s)j_p l_q]J||\mathcal{O}^{\mu L}||1\rangle. \end{aligned} \tag{14}$$

The symmetry property [Eq. (12)] leads to a corresponding relation for the T matrix

$$\begin{aligned} &T_{s-m_s-\mu-m_d}(\theta_p, \phi_p, \theta_q, \phi_q) \\ &\quad = (-)^{s+m_s+\mu+m_d} T_{sm_s\mu m_d}(\theta_p, -\phi_p, \theta_q, -\phi_q). \end{aligned} \tag{15}$$

For an uncoupled spin representation, one finds accordingly, using the transformation

$$\begin{aligned} T_{m_1 m_2 \mu m_d}(\theta_p, \phi_p, \theta_q, \phi_q) = &\sum_{sm_s} \left(\frac{1}{2}m_1 \frac{1}{2}m_2 | sm_s\right) \\ &\times T_{sm_s\mu m_d}(\theta_p, \phi_p, \theta_q, \phi_q), \end{aligned} \tag{16}$$

where m_j denotes the spin projection of the "jth" nucleon on the quantization axis, as symmetry relation

$$\begin{aligned} &T_{-m_1-m_2-\mu-m_d}(\theta_p, \phi_p, \theta_q, \phi_q) \\ &\quad = (-)^{1+m_1+m_2+\mu+m_d} T_{m_1 m_2 \mu m_d}(\theta_p, -\phi_p, \theta_q, -\phi_q). \end{aligned} \tag{17}$$

The small t matrix elements are the basic quantities that determine differential cross section and asymmetries. The latter are given as ratios of bilinear hermitean forms in terms of the t matrix elements [see Eqs. (37) and (38) below].

IV. THE DIFFERENTIAL CROSS SECTION, INCLUDING POLARIZATION ASYMMETRIES

The usual starting point is the general expression for the differential cross section

$$\frac{d^5\sigma}{dq d\Omega_q d\Omega_p} = c(\omega, q, \Omega_q, \Omega_p) tr(T^\dagger T \rho_i), \tag{18}$$

where T denotes the reaction matrix and ρ_i the density matrix for the spin degrees of the initial system. The trace refers to all initial- and final-state spin degrees of freedom comprising incoming photon, target deuteron, and final nucleons. Furthermore, $c(\omega, q, \Omega_q, \Omega_p)$ denotes a kinematic factor that comprises the final-state phase space and the incoming flux. In an arbitrary frame one has

$$c(\omega, q, \Omega_q, \Omega_p) = \frac{1}{(2\pi)^5} \frac{\bar{E}_d}{E_d + p_d} \frac{m_N^2}{4\omega\omega_\pi} \frac{p^* q^2}{W_{NN}}, \tag{19}$$

with

$$p^2 = \frac{(p^* E_{NN})^2}{E_{NN}^2 - P_{NN}^2} \cos(\widehat{p} \cdot \widehat{P}_{NN}), \tag{20}$$

as the relative momentum of the final two nucleons, and

$$\begin{aligned} &\omega = k_0, \quad E_d = \sqrt{p_d^2 + m_d^2}, \quad \omega_\pi = \sqrt{q^2 + m_\pi^2}, \\ &E_{NN} = \omega + E_d - \omega_\pi, \quad \vec{P}_{NN} = \vec{k} + \vec{p}_d - \vec{q}, \\ &p^* = \frac{1}{2}\sqrt{E_{NN}^2 - P_{NN}^2 - 4m_N^2}, \end{aligned} \tag{21}$$

and the symbol "$\widehat{v}$" denotes a unit vector along the vector $\vec{v}$. The density matrix ρ_i in Eq. (18) is a direct product of the density matrices ρ^γ of the photon and ρ^d of the deuteron

$$\rho_i = \rho^\gamma \otimes \rho^d. \tag{22}$$

The photon density matrix has the form

$$\rho^\gamma_{\mu\mu'} = \tfrac{1}{2}(\delta_{\mu\mu'} + \vec{P}^\gamma \cdot \vec{\sigma}_{\mu\mu'}) \tag{23}$$

with respect to circular polarization $\mu = \pm 1$. Here, $|\vec{P}^\gamma|$ describes the total degree of polarization, $P^\gamma_z = P^\gamma_c$ the degree of circular polarization, and $P^\gamma_l = \sqrt{(P^\gamma_x)^2 + (P^\gamma_y)^2}$ the degree of linear polarization. By a proper rotation around the photon momentum, one can choose the x axis in the direction of maximum linear polarization, i.e., $P^\gamma_x = -P^\gamma_l$ and $P^\gamma_y = 0$. Then one has explicitly

$$\rho^\gamma_{\mu\mu'} = \left(1 + \mu\, P^\gamma_c\right) \delta_{\mu\mu'} - P^\gamma_l\, \delta_{\mu,-\mu'}\, e^{2i\mu\phi_q}. \tag{24}$$

Furthermore, the deuteron density matrix ρ^d can be expressed in terms of irreducible spin operators $\tau^{[I]}$ with respect to the deuteron spin space

$$\rho^d_{m_d m_d'} = \frac{1}{3} \sum_{IM} (-)^M \hat{I}\, \langle 1m_d|\tau^{[I]}_M|1m'_d\rangle\, P^d_{I-M}, \tag{25}$$

where $P^d_{00} = 1$ and P^d_{1M} and P^d_{2M} describe vector and tensor polarization components of the deuteron, respectively. The spin operators are defined by their reduced matrix elements

$$\langle 1||\tau^{[I]}||1\rangle = \sqrt{3}\, \hat{I} \quad \text{for} \quad I = 0, 1, 2. \tag{26}$$

From now on we assume that the deuteron density matrix is diagonal with respect to an orientation axis $\vec{d}$ having spherical angles (θ_d, ϕ_d) with respect to the coordinate system associated with the photon plane in the lab frame. Then one has with respect to $\vec{d}$ as quantization axis

$$\rho^d_{m\,m'} = p_m\, \delta_{m\,m'}, \tag{27}$$

where p_m denotes the probability for finding a deuteron spin projection m on the orientation axis. With respect to this axis one finds from Eq. (25) $P^d_{IM}(\vec{d}) = P^d_I\, \delta_{M,0}$, where the orientation parameters P^d_I are related to the probabilities $\{p_m\}$ by the following:

$$\begin{aligned} P^d_I &= \sqrt{3}\, \hat{I} \sum_m (-)^{1-m} \begin{pmatrix} 1 & 1 & I \\ m & -m & 0 \end{pmatrix} p_m \\ &= \delta_{I0} + \sqrt{\frac{3}{2}} (p_1 - p_{-1})\, \delta_{I1} + \frac{1}{\sqrt{2}} (1 - 3\, p_0)\, \delta_{I2}. \end{aligned} \tag{28}$$

The polarization components in the chosen lab frame are obtained from the P^d_I by a rotation, transforming the quantization axis along the orientation axis into the direction of the photon momentum, i.e.,

$$P^d_{IM}(\vec{z}\,) = P^d_I\, e^{iM\phi_d} d^I_{M0}(\theta_d), \tag{29}$$

where $d^j_{mm'}$ denotes a small rotation matrix [16]. Thus the deuteron density matrix becomes finally

$$\rho^d_{m_d m_{d'}} = \frac{1}{\sqrt{3}}(-)^{1-m_d} \times \sum_{IM} \hat{I} \begin{pmatrix} 1 & 1 & I \\ m'_d & -m_d & M \end{pmatrix} P^d_I e^{-iM\phi_d} d^I_{M0}(\theta_d). \quad (30)$$

This means, the deuteron target is characterized by four parameters, namely the vector and tensor polarization parameters P^d_1 and P^d_2, respectively, and by the orientation angles θ_d and ϕ_d. If one chooses the c.m. frame as the reference frame, one should note that the deuteron density matrix undergoes no change in the transformation from the lab to the c.m. system, because the boost to the c.m. system is collinear with the deuteron quantization axis [17].

The evaluation of the general expression of the differential cross section in Eq. (18) can be done analogously to deuteron photodisintegration as described in detail in Ref. [13]. In fact, one can follow the same steps except for the use of the symmetry relation of Eq. (2) in Ref. [13], which is different in case of pion production [see Eq. (12)] because of the additional pion degree of freedom in the final state, in particular its pseudovector character. In terms of the small t matrices as defined in Eq. (10), one finds, inserting the density matrices of photon and deuteron for the general fivefold differential cross section,

$$\frac{d^5\sigma}{dq d\Omega_q d\Omega_p} = \frac{1}{2} \sum_{\mu'\mu IM} P^d_I\, e^{iM(\phi_q-\phi_d)} d^I_{M0}(\theta_d) u^{\mu'\mu}_{IM} \times \left[\left(1 + \mu\, P^\gamma_c\right)\delta_{\mu\mu'} - P^\gamma_l\, \delta_{\mu,-\mu'} e^{2i\mu\phi_q} \right], \quad (31)$$

where we have introduced the quantities

$$u^{\mu'\mu}_{IM}(q,\theta_q,\theta_p,\phi_{pq}) = c(\omega, q, \Omega_q, \Omega_p)\, \frac{\hat{I}}{\sqrt{3}} \times \sum_{m_d m'_d} (-)^{1-m_d} \begin{pmatrix} 1 & 1 & I \\ m'_d & -m_d & M \end{pmatrix} \times \sum_{sm_s} t^*_{sm_s\mu' m'_d}(q,\theta_q,\theta_p,\phi_{pq}) \times t_{sm_s\mu m_d}(q,\theta_q,\theta_p,\, \phi_{pq}). \quad (32)$$

It is straightforward to prove that they behave under complex conjugation as

$$u^{\mu'\mu}_{IM}(q,\theta_q,\theta_p,\phi_{pq})^* = (-)^M\, u^{\mu\mu'}_{I-M}(q,\theta_q,\theta_p,\phi_{pq}). \quad (33)$$

Furthermore, with the help of the symmetry in Eq. (12) one finds

$$u^{-\mu'-\mu}_{IM}(q,\theta_q,\theta_p,\phi_{pq}) = (-)^{I+M+\mu'+\mu} u^{\mu'\mu}_{I-M}(q,\theta_q,\theta_p,-\phi_{pq}), \quad (34)$$

which yields in combination with Eq. (33)

$$u^{-\mu'-\mu}_{IM}(q,\theta_q,\theta_p,\phi_{pq}) = (-)^{I+\mu'+\mu}\, u^{\mu\mu'}_{IM}(q,\theta_q,\theta_p,-\phi_{pq})^*. \quad (35)$$

This relation is quite useful for a further simplification of the semiexclusive differential cross section later on.

Separating the polarization parameters of photon (P^γ_l and P^γ_c) and deuteron (P^d_I), it is then straightforward to show that the differential cross section can be brought into the form

$$\frac{d^5\sigma}{dq d\Omega_q d\Omega_p} = \frac{1}{2} \sum_I P^d_I \sum_{M=-I}^{I} e^{iM\phi_{qd}} d^I_{M0}(\theta_d) \times \left[v^1_{IM} + v^{-1}_{IM} + P^\gamma_c \left(v^1_{IM} - v^{-1}_{IM} \right) + P^\gamma_l \left(w^1_{IM}\, e^{-2i\phi_q} + w^{-1}_{IM}\, e^{2i\phi_q} \right) \right], \quad (36)$$

with $\phi_{qd} = \phi_q - \phi_d$, where we have introduced for convenience the quantities

$$v^\mu_{IM}(q,\theta_q,\theta_p,\phi_{pq}) = u^{\mu\mu}_{IM}(q,\theta_q,\theta_p,\phi_{pq}), \quad (37)$$

$$w^\mu_{IM}(q,\theta_q,\theta_p,\phi_{pq}) = -u^{\mu-\mu}_{IM}(q,\theta_q,\theta_p,\phi_{pq}). \quad (38)$$

According to Eqs. (33) and (35), they have the following properties under complex conjugation:

$$v/w^\mu_{IM}(q,\theta_q,\theta_p,\phi_{pq})^* = (-)^M v/w^\mu_{I-M}(q,\theta_q,\theta_p,\phi_{pq}), \quad (39)$$

$$v^\mu_{IM}(q,\theta_q,\theta_p,\phi_{pq})^* = (-)^I v^{-\mu}_{IM}(q,\theta_q,\theta_p,-\phi_{pq}), \quad (40)$$

$$w^\mu_{IM}(q,\theta_q,\theta_p,\phi_{pq})^* = (-)^I w^\mu_{IM}(q,\,\theta_q,\theta_p,-\phi_{pq}). \quad (41)$$

From Eq. (39) it follows that v^μ_{I0} and w^μ_{I0} are real. The sum over M in Eq. (36) can be rearranged with the help of the Eq. (39) and $d^I_{-M0}(\theta_d) = (-)^M d^I_{M0}(\theta_d)$

$$\begin{aligned} \sum_{M=-I}^{I} e^{iM\phi_{qd}} d^I_{M0}(\theta_d) \left(v^1_{IM} \pm v^{-1}_{IM}\right) &= \sum_{M=0}^{I} \frac{d^I_{M0}(\theta_d)}{1+\delta_{M0}} \left[e^{iM\phi_{qd}} \left(v^1_{IM} \pm v^{-1}_{IM}\right) + e^{-iM\phi_{qd}} (-)^M \left(v^1_{I-M} \pm v^{-1}_{I-M}\right) \right] \\ &= \sum_{M=0}^{I} \frac{d^I_{M0}(\theta_d)}{1+\delta_{M0}} \left[e^{iM\phi_{qd}} \left(v^1_{IM} \pm v^{-1}_{IM}\right) + \text{c.c.} \right], \end{aligned} \quad (42)$$

and furthermore with $\psi_M = M\phi_{qd} - 2\,\phi_q$

$$\begin{aligned} \sum_{M=-I}^{I} e^{iM\phi_{qd}} d^I_{M0}(\theta_d) \left(w^1_{IM}\, e^{-2i\phi_q} + w^{-1}_{IM}\, e^{2i\phi_q}\right) &= \sum_{M=-I}^{I} d^I_{M0}(\theta_d) \left[e^{i\psi_M} w^1_{IM} + e^{-i\psi_M} (-)^M\, w^{-1}_{I-M} \right] \\ &= \sum_{M=-I}^{I} d^I_{M0}(\theta_d) \left(e^{i\psi_M}\, w^1_{IM} + \text{c.c.} \right). \end{aligned} \quad (43)$$

This then yields for the differential cross section

$$\frac{d^5\sigma}{dq d\Omega_q d\Omega_p} = \sum_I P^d_I \left\{ \sum_{M=0}^{I} \frac{1}{1+\delta_{M0}}\, d^I_{M0}(\theta_d) \times \Re e\left[e^{iM\phi_{qd}} \left(v^+_{IM} + P^\gamma_c\, v^-_{IM} \right) \right] + P^\gamma_l \sum_{M=-I}^{I} d^I_{M0}(\theta_d)\, \Re e\left[e^{i\psi_M} w^1_{IM} \right] \right\}, \quad (44)$$

where we have defined

$$v^{\pm}_{IM} = v^{1}_{IM} \pm v^{-1}_{IM}. \qquad (45)$$

Now, introducing various beam, target and beam-target asymmetries by the following:

$$\tau^{0/c}_{IM}(q,\theta_q,\theta_p,\phi_{pq}) = \frac{1}{1+\delta_{M0}}\Re e\, v^{\pm}_{IM}(q,\theta_q,\theta_p,\phi_{pq}), \quad M \geqslant 0, \qquad (46)$$

$$\sigma^{0/c}_{IM}(q,\theta_q,\theta_p,\phi_{pq}) = -\Im m\, v^{\pm}_{IM}(q,\theta_q,\theta_p,\phi_{pq}), \quad M > 0, \qquad (47)$$

$$\tau^{l}_{IM}(q,\theta_q,\theta_p,\phi_{pq}) = \Re e\, w^{1}_{IM}(q,\theta_q,\theta_p,\phi_{pq}), \qquad (48)$$

$$\sigma^{l}_{IM}(q,\theta_q,\theta_p,\phi_{pq}) = -\Im m\, w^{1}_{IM}(q,\theta_q,\theta_p,\phi_{pq}), \quad M \neq 0, \qquad (49)$$

where we took into account that v^{μ}_{I0} and w^{μ}_{I0} are real, one obtains as final expression for the general fivefold differential cross section with beam and target polarization

$$\frac{d^5\sigma}{dq\,d\Omega_q\,d\Omega_p} = \sum_I P^d_I \Bigg(\sum_{M=0}^{I} d^I_{M0}(\theta_d)\big\{\tau^0_{IM}\cos(M\phi_{qd}) + \sigma^0_{IM}\sin(M\phi_{qd}) + P^\gamma_c\big[\tau^c_{IM}\cos(M\phi_{qd}) + \sigma^c_{IM}\sin(M\phi_{qd})\big]\big\} + P^\gamma_l \sum_{M=-I}^{I} d^I_{M0}(\theta_d) \times \big(\tau^l_{IM}\cos\psi_M + \sigma^l_{IM}\sin\psi_M\big)\Bigg). \qquad (50)$$

This constitutes our central result.

We now turn to the semiexclusive reaction $\vec{d}(\vec{\gamma},\pi)NN$ where only the produced pion is detected, which means integration of the fivefold differential cross section $d^5\sigma/dq\,d\Omega_q\,d\Omega_p$ over Ω_p. The resulting cross section will then be governed by the integrated asymmetries $\int d\,\Omega_p\,\tau^\alpha_{IM}$ and $\int d\,\Omega_p\,\sigma^\alpha_{IM}$ ($\alpha \in \{0,c,l\}$), of which quite a few will vanish, either $\int d\,\Omega_p\,\tau^\alpha_{IM}$ or $\int d\,\Omega_p\,\sigma^\alpha_{IM}$. To show this, we first introduce the quantities

$$\begin{aligned} W_{IM}(q,\theta_q) &= \int d\,\Omega_p\, w^1_{IM}(q,\theta_q,\theta_p,\phi_{pq}) \\ &= -\frac{\hat{I}}{\sqrt{3}}\int d\,\Omega_p\, c(\omega,q,\Omega_q,\Omega_p) \times \sum_{m_d m'_d}(-)^{1-m_d}\begin{pmatrix}1 & 1 & I\\ m'_d & -m_d & M\end{pmatrix} \\ &\quad\times \sum_{s m_s} t^*_{s m_s 1 m'_d}(q,\theta_q,\theta_p,\phi_{pq}) \times t_{s m_s -1 m_d}(q,\theta_q,\theta_p,\phi_{pq}), \end{aligned} \qquad (51)$$

$$V^{\pm}_{IM}(q,\theta_q) = V^1_{IM}(q,\theta_q) \pm V^{-1}_{IM}(q,\theta_q), \qquad (52)$$

with

$$\begin{aligned} V^{\mu}_{IM}(q,\theta_q) &= \int d\,\Omega_p\, v^{\mu}_{IM}(q,\theta_q,\theta_p,\phi_{pq}) \\ &= \frac{\hat{I}}{\sqrt{3}}\int d\,\Omega_p\, c(\omega,q,\Omega_q,\Omega_p) \times \sum_{m_d m'_d}(-)^{1-m_d}\begin{pmatrix}1 & 1 & I\\ m'_d & -m_d & M\end{pmatrix} \\ &\quad\times \sum_{s m_s} t^*_{s m_s \mu m'_d}(q,\theta_q,\theta_p,\phi_{pq}) \times t_{s m_s \mu m_d}(q,\theta_q,\theta_p,\phi_{pq}). \end{aligned} \qquad (53)$$

Using now Eq. (40), one finds with the help of

$$\int_0^{2\pi} d\phi_p\, f(-\phi_{pq}) = \int_0^{2\pi} d\phi_p\, f(\phi_{pq}) \qquad (54)$$

for a periodic function $f(\phi_{pq}+2\pi) = f(\phi_{pq})$, the relation

$$\begin{aligned} V^{-1}_{IM}(q,\theta_q) &= \int d\,\Omega_p\, v^{-1}_{IM}(q,\theta_q,\theta_p,\phi_{pq}) \\ &= (-)^I \int d\,\Omega_p\, v^1_{IM}(q,\theta_q,\theta_p,-\phi_{pq})^* \\ &= (-)^I\, V^1_{IM}(q,\theta_q)^*, \end{aligned} \qquad (55)$$

and thus

$$V^{\pm}_{IM}(q,\theta_q) = V^1_{IM}(q,\theta_q) \pm (-)^I\, V^1_{IM}(q,\theta_q)^*. \qquad (56)$$

Correspondingly, using Eq. (41) one obtains

$$\begin{aligned} W_{IM}(q,\theta_q)^* &= (-)^I \int d\,\Omega_p\, w^1_{IM}(q,\theta_q,\theta_p,-\phi_{pq}) \\ &= (-)^I\, W_{IM}(q,\theta_q). \end{aligned} \qquad (57)$$

From the two foregoing equations we can conclude that V^+_{IM} and W_{IM} are real for $I = 0$ and 2 and imaginary for $I = 1$, whereas V^-_{IM} is imaginary for $I = 0$ and 2 and real for $I = 1$. Therefore, according to Eqs. (46) through (49) the following integrated asymmetries vanish

$$\int d\,\Omega_p\,\tau^\alpha_{IM} = 0 \quad \text{for} \quad \begin{Bmatrix}\alpha \in \{0,l\}, & \text{and } I = 1\\ \alpha \in \{c\}, & \text{and } I = 0,2\end{Bmatrix}, \qquad (58)$$

$$\int d\,\Omega_p\,\sigma^\alpha_{IM} = 0 \quad \text{for} \quad \begin{Bmatrix}\alpha \in \{0,l\}, & \text{and } I = 0,2\\ \alpha \in \{c\}, & \text{and } I = 1\end{Bmatrix}. \qquad (59)$$

Instead of using these results for deriving from Eq. (50) the threefold semiexclusive differential cross section, we prefer to start from the expression in Eq. (44) and obtain the following:

$$\frac{d^3\sigma}{dq\,d\Omega_q} = \sum_I P^d_I \Bigg\{\sum_{M=0}^{I}\frac{1}{1+\delta_{M0}}\, d^I_{M0}(\theta_d) \times \Re e\big[e^{iM\phi_{qd}}\big(V^+_{IM} + P^\gamma_c\, V^-_{IM}\big)\big] + P^\gamma_l \times \sum_{M=-I}^{I} d^I_{M0}(\theta_d)\,\Re e\,[e^{i\psi_M} W_{IM}]\Bigg\}. \qquad (60)$$

This expression can be simplified using the fact that $i^{\delta_{I1}}\,W_{IM}$, $i^{\delta_{I1}}\,V^+_{IM}$ and $i^{1-\delta_{I1}}\,V^-_{IM}$ are real according to Eqs. (56) and (57). The latter two quantities can be written as follows:

$$i^{\delta_{I1}}\,V^+_{IM} = 2\,\Re e\,\big(i^{\delta_{I1}}\,V^1_{IM}\big), \qquad (61)$$

$$i^{1-\delta_{I1}}\,V^-_{IM} = 2\,\Re e\,\big(i^{1-\delta_{I1}}\,V^1_{IM}\big) = -2\,\Im m\,\big(i^{-\delta_{I1}}\,V^1_{IM}\big). \qquad (62)$$

Using now

$$\Re e\,[e^{iM\phi_{qd}}\,V^{+}_{IM}] = \Re e\,[e^{i(M\phi_{qd}-\delta_{I1}\pi/2)}\,i^{\delta_{I1}}\,V^{+}_{IM}] = 2\,\Re e\left(i^{\delta_{I1}}\,V^{1}_{IM}\right)\cos[M\phi_{qd}-\delta_{I1}\pi/2], \tag{63}$$

$$\Re e\,[e^{iM\phi_{qd}}\,V^{-}_{IM}] = \Re e\left[\frac{1}{i}\,e^{i(M\phi_{qd}+\delta_{I1}\pi/2)}i^{1-\delta_{I1}}\,V^{-}_{IM}\right] = -2\,\Im m\left(i^{-\delta_{I1}}\,V^{1}_{IM}\right)\sin[M\phi_{qd}+\delta_{I1}\pi/2], \tag{64}$$

$$\Re e\,[e^{i\psi_M}\,W_{IM}] = \Re e\,[e^{i(\psi_M-\delta_{I1}\pi/2)}\,i^{\delta_{I1}}\,W_{IM}] = i^{\delta_{I1}}\,W_{IM}\,\cos[\psi_M-\delta_{I1}\pi/2], \tag{65}$$

we find as final form for the threefold semiexclusive differential cross section

$$\frac{d^3\sigma}{dq d\Omega_q} = \frac{d^3\sigma_0}{dq d\Omega_q}\Bigg[1 + P^{\gamma}_{l}\left\{\widetilde{\Sigma}^{l}\cos 2\phi_q + \sum_{I=1}^{2}P^{d}_{I}\sum_{M=-I}^{I}\widetilde{T}^{l}_{IM}\cos[\psi_M-\delta_{I1}\pi/2]\,d^{I}_{M0}(\theta_d)\right\} + \sum_{I=1}^{2}P^{d}_{I}\sum_{M=0}^{I}\left(\widetilde{T}^{0}_{IM}\cos[M\phi_{qd}-\delta_{I1}\pi/2] + P^{\gamma}_{c}\,\widetilde{T}^{c}_{IM}\sin[M\phi_{qd}+\delta_{I1}\pi/2]\right)d^{I}_{M0}(\theta_d)\Bigg]. \tag{66}$$

Here the unpolarized cross section and the asymmetries are given by the following:

$$\frac{d^3\sigma_0}{dq d\Omega_q} = V^{1}_{00}(q,\theta_q), \tag{67}$$

$$\widetilde{\Sigma}^{l}(q,\theta_q)\,\frac{d^3\sigma_0}{dq d\Omega_q} = W_{00}(q,\theta_q), \tag{68}$$

$$\widetilde{T}^{0}_{IM}(q,\theta_q)\,\frac{d^3\sigma_0}{dq d\Omega_q} = (2-\delta_{M0})\Re e\left[i^{\delta_{I1}}V^{1}_{IM}(q,\theta_q)\right],\quad \text{for } 0 \leqslant M \leqslant I, \tag{69}$$

$$\widetilde{T}^{c}_{IM}(q,\theta_q)\,\frac{d^3\sigma_0}{dq d\Omega_q} = -(2-\delta_{M0})\Im m\left[i^{-\delta_{I1}}V^{1}_{IM}(q,\theta_q)\right],\quad \text{for } 0 \leqslant M \leqslant I, \tag{70}$$

$$\widetilde{T}^{l}_{IM}(q,\theta_q)\,\frac{d^3\sigma_0}{dq d\Omega_q} = i^{\delta_{I1}}\,W_{IM}(q,\theta_q),\quad \text{for } -I \leqslant M \leqslant I. \tag{71}$$

Because V^{1}_{I0} is real according to Eq. (39), the asymmetries $\widetilde{T}_{10}$ and $\widetilde{T}^{c}_{20}$ vanish identically. We point out that in forward and backward pion emission, i.e., for $\theta_q = 0$ and π, the following asymmetries have to vanish:

$$\widetilde{\Sigma}^{l} = 0,\quad \widetilde{T}^{0,c}_{IM} = 0 \text{ for } M \neq 0,\quad \text{and}\quad T^{l}_{IM} = 0 \text{ for } M \neq 2, \tag{72}$$

because in that case the differential cross section cannot depend on ϕ_q, because at $\theta_q = 0$ or π the azimuthal angle ϕ_q is undefined or arbitrary. This feature can also be shown by straightforward evaluation of V^{μ}_{IM} and W_{IM} using the explicit representation of the t matrix in Eq. (11). One finds

$$V^{\mu}_{IM}(q,\theta_q=0/\pi,\theta_p,\phi_{pq}) = 0 \quad \text{for } M \neq 0 \quad \text{and} \\ W_{IM}(q,\theta_q=0/\pi,\theta_p,\phi_{pq}) = 0 \quad \text{for } M \neq 2. \tag{73}$$

The authors of Ref. [12] were not aware of this general kinematic property because they evaluate the asymmetries numerically for $\theta_q = 0$ and π and find that the obtained values are of the order of 10^{-3}. They conclude in the case of T_{11} that it vanishes there but point out that Σ^l does not vanish. For completeness and also in view of the numerous errors in [8–10,12], we list in the appendix the explicit expressions of the asymmetries in terms of the t-matrix elements.

In the case where only the direction of the outgoing pion is measured and not its momentum, the corresponding differential cross section $d^2\sigma/d\Omega_q$ is given by an expression formally analogous to Eq. (66), where only the above asymmetries are integrated over the pion momentum, i.e., by the replacements

$$\frac{d^3\sigma_0}{dq d\Omega_q} \to \frac{d^2\sigma_0}{d\Omega_q} = \int_{q_{\min}(\theta_q)}^{q_{\max}(\theta_q)} dq\,\frac{d^3\sigma_0}{dq d\Omega_q}, \tag{74}$$

$$\frac{d^3\sigma_0}{dq d\Omega_q}\,\widetilde{\Sigma}^{l}(q,\theta_q) \to \frac{d^2\sigma_0}{d\Omega_q}\,\Sigma^{l}(\theta_q) = \int_{q_{\min}(\theta_q)}^{q_{\max}(\theta_q)} dq\,\frac{d^3\sigma_0}{dq d\Omega_q}\,\widetilde{\Sigma}^{l}(q,\theta_q), \tag{75}$$

$$\frac{d^3\sigma_0}{dq d\Omega_q}\,\widetilde{T}^{\alpha}_{IM}(q,\theta_q) \to \frac{d^2\sigma_0}{d\Omega_q}\,T^{\alpha}_{IM}(\theta_q) = \int_{q_{\min}(\theta_q)}^{q_{\max}(\theta_q)} dq\,\frac{d^3\sigma_0}{dq d\Omega_q}\widetilde{T}^{\alpha}_{IM}(q,\theta_q),\quad \alpha \in \{0,l,c\}. \tag{76}$$

The upper and lower integration limits are given by the following:

$$q_{\max}(\theta_q) = \frac{1}{2b}\left(a\,\omega\cos\theta_q + E_{\gamma d}\sqrt{a^2-4b\,m_\pi^2}\right), \tag{77}$$

$$q_{\min}(\theta_q) = \max\left\{0,\,\frac{1}{2b}\left(a\,\omega\cos\theta_q - E_{\gamma d}\sqrt{a^2-4b\,m_\pi^2}\right)\right\}, \tag{78}$$

where

$$a = W^2_{\gamma d} + m^2_\pi - 4\,m^2_N, \tag{79}$$

$$b = W^2_{\gamma d} + \omega^2\sin^2\theta_q, \tag{80}$$

$$W^2_{\gamma d} = m_d(m_d + 2\,\omega), \tag{81}$$

$$E_{\gamma d} = m_d + \omega. \tag{82}$$

The general total cross section is obtained from Eq. (66) by integrating over q and Ω_q, resulting in the following:

$$\sigma\left(P_l^\gamma, P_c^\gamma, P_1^d, P_2^d\right) = \sigma_0 \left[1 + P_2^d\, \overline{T}_{20}^{\,0}\, \frac{1}{2}(3\cos^2\theta_d - 1) + P_c^\gamma\, P_1^d\, \overline{T}_{10}^{\,c}\cos\theta_d + P_l^\gamma\, P_2^d\, \overline{T}_{22}^{\,l}\cos(2\phi_d)\, \frac{\sqrt{6}}{4}\sin^2\theta_d\right], \quad (83)$$

where the unpolarized total cross section and the corresponding asymmetries are given by the following:

$$\sigma_0 = \int d\Omega_q \int_{q_{\min}(\theta_q)}^{q_{\max}(\theta_q)} dq\, \frac{d^3\sigma_0}{dq d\Omega_q}, \quad (84)$$

$$\sigma_0\, \overline{T}_{IM}^{\,\alpha} = \int d\Omega_q \int_{q_{\min}(\theta_q)}^{q_{\max}(\theta_q)} dq\, \frac{d^3\sigma_0}{dq d\Omega_q}\, \widetilde{T}_{IM}^{\alpha}, \quad (85)$$

with $\alpha \in \{0, l, c\}$.

Finally, we point out that for coherent photoproduction of π^0 on the deuteron formally the same expression as in Eq. (66) holds with unpolarized differential cross section and asymmetries $\Sigma^l(\theta_q)$, $T_{IM}(\theta_q)$, and $T_{IM}^{c/l}(\theta_q)$, which are defined in analogy to Eqs. (67) through (71) with the following replacements:

$$V_{IM}^1 \to c(\omega, \Omega_q)\, \frac{\hat{I}}{\sqrt{3}} \sum_{m_d m_d'} (-)^{1-m_d} \begin{pmatrix} 1 & 1 & I \\ m_d' & -m_d & M \end{pmatrix} \times \sum_{m_d''} t^*_{m_d'' 1 m_d'}(\theta_q)\, t_{m_d'' 1 m_d}(\theta_q), \quad (86)$$

$$W_{IM} \to -c(\omega, \Omega_q)\, \frac{\hat{I}}{\sqrt{3}} \sum_{m_d m_d'} (-)^{1-m_d} \begin{pmatrix} 1 & 1 & I \\ m_d' & -m_d & M \end{pmatrix} \times \sum_{m_d''} t^*_{m_d'' 1 m_d'}(\theta_q)\, t_{m_d'' -1 m_d}(\theta_q). \quad (87)$$

Here, $c(\omega, \Omega_q)$ denotes a kinematic factor. A complete listing of all polarization observables including recoil polarization of the final deuteron can be found in Ref. [14].

V. CONCLUSIONS

In this work we have derived formal expressions for the differential cross section of incoherent pion photoproduction on the deuteron, including various polarization asymmetries with respect to polarized photons and deuterons. Obviously, these expressions are generally valid for pseudoscalar meson production. We did not consider polarization analysis of the final state, i.e., spin analysis of one or both outgoing nucleons. In this case one has to evaluate the following:

$$P_\alpha(j)\frac{d^5\sigma}{dq d\Omega_q d\Omega_p} = c(\omega, q, \Omega_q, \Omega_p) tr[T^\dagger \sigma_\alpha(j) T \rho_i], \quad (88)$$

instead of Eq. (18) for the polarization of the "jth" outgoing nucleon or

$$P_{\alpha_1\alpha_2}\frac{d^5\sigma}{dq d\Omega_q d\Omega_p} = c(\omega, q, \Omega_q, \Omega_p) tr[T^\dagger \sigma_{\alpha_1}(1)\sigma_{\alpha_2}(2) T \rho_i], \quad (89)$$

for the polarization of both outgoing nucleons. For the evaluation of these expressions one can proceed straightforwardly as has been done Ref. in [13]. In a subsequent article [15], we investigate the influence of NN- and πN-rescattering on the various asymmetries of the semiexclusive differential cross section of incoherent pion photoproduction on the deuteron.

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APPENDIX: EXPLICIT EXPRESSIONS FOR THE VARIOUS POLARIZATION ASYMMETRIES

We list here the explicit Hermitean, bilinear forms in terms of the t matrix elements for cross section and the various asymmetries.

(i) The semiexclusive differential cross section:

$$\frac{d^3\sigma_0}{dq d\Omega_q} = \frac{1}{3} \int d\,\Omega_p c(\omega, q, \Omega_q, \Omega_p) \sum_{s m_s m_d} |t_{s m_s 1 m_d}|^2. \quad (A1)$$

(ii) The photon asymmetry for linearly polarized photons and unpolarized deuterons:

$$\widetilde{\Sigma}^l\, \frac{d^3\sigma_0}{dq d\Omega_q} = -\frac{1}{3} \int d\,\Omega_p c(\omega, q, \Omega_q, \Omega_p) \times \sum_{s m_s m_d} t^*_{s m_s 1 m_d}\, t_{s m_s -1 m_d}. \quad (A2)$$

(iii) The target asymmetry for vector polarized deuterons and unpolarized photons:

$$\widetilde{T}_{11}^{0}\, \frac{d^3\sigma_0}{dq d\Omega_q} = \sqrt{\frac{2}{3}} \int d\,\Omega_p c(\omega, q, \Omega_q, \Omega_p)\, \Im m \times \sum_{s m_s} (t^*_{s m_s 1 -1}\, t_{s m_s 10} + t^*_{s m_s 10}\, t_{s m_s 11}). \quad (A3)$$

(iv) The target asymmetries for tensor polarized deuterons and unpolarized photons:

$$\widetilde{T}_{20}^{0}\, \frac{d^3\sigma_0}{dq d\Omega_q} = \frac{1}{3\sqrt{2}} \int d\,\Omega_p c(\omega, q, \Omega_q, \Omega_p) \times \sum_{s m_s} \left(|t_{s m_s 11}|^2 + |t_{s m_s 1 -1}|^2 - 2\,|t_{s m_s 10}|^2\right), \quad (A4)$$

$$\widetilde{T}^0_{21}\,\frac{d^3\sigma_0}{dqd\Omega_q}=\sqrt{\frac{2}{3}}\int d\,\Omega_p c(\omega,q,\Omega_q,\Omega_p)\,\Re e\times\sum_{sm_s}\left(t^*_{sm_s1-1}\,t_{sm_s10}-t^*_{sm_s10}\,t_{sm_s11}\right),\tag{A5}$$

$$\widetilde{T}^0_{22}\,\frac{d^3\sigma_0}{dqd\Omega_q}=\frac{2}{\sqrt{3}}\int d\,\Omega_p c(\omega,q,\Omega_q,\Omega_p)\,\Re e\times\sum_{sm_s}t^*_{sm_s1-1}\,t_{sm_s11}.\tag{A6}$$

(v) The beam-target asymmetries for circularly polarized photons and vector polarized deuterons:

$$\widetilde{T}^c_{10}\,\frac{d^3\sigma_0}{dqd\Omega_q}=\frac{1}{\sqrt{6}}\int d\,\Omega_p c(\omega,q,\Omega_q,\Omega_p)\times\sum_{sm_s}\left(|t_{sm_s11}|^2-|t_{sm_s1-1}|^2\right),\tag{A7}$$

$$\widetilde{T}^c_{11}\,\frac{d^3\sigma_0}{dqd\Omega_q}=-\sqrt{\frac{2}{3}}\int d\,\Omega_p c(\omega,q,\Omega_q,\Omega_p)\,\Re e\times\sum_{sm_s}\left(t^*_{sm_s1-1}\,t_{sm_s10}+t^*_{sm_s10}\,t_{sm_s11}\right).\tag{A8}$$

(vi) The beam-target asymmetries for circularly polarized photons and tensor polarized deuterons:

$$\widetilde{T}^c_{21}\,\frac{d^3\sigma_0}{dqd\Omega_q}=\sqrt{\frac{2}{3}}\int d\,\Omega_p c(\omega,q,\Omega_q,\Omega_p)\,\Im m\times\sum_{sm_s}\left(t^*_{sm_s10}\,t_{sm_s11}-t^*_{sm_s1-1}\,t_{sm_s10}\right),\tag{A9}$$

$$\widetilde{T}^c_{22}\,\frac{d^3\sigma_0}{dqd\Omega_q}=-\frac{2}{\sqrt{3}}\int d\,\Omega_p c(\omega,q,\Omega_q,\Omega_p)\,\Im m\times\sum_{sm_s}t^*_{sm_s1-1}\,t_{sm_s11}.\tag{A10}$$

(vii) The beam-target asymmetries for linearly polarized photons and vector polarized deuterons:

$$\widetilde{T}^l_{10}\,\frac{d^3\sigma_0}{dqd\Omega_q}=\sqrt{\frac{2}{3}}\int d\,\Omega_p c(\omega,q,\Omega_q,\Omega_p)\,\Im m\times\sum_{sm_s}\left(t^*_{sm_s11}\,t_{sm_s-11}\right),\tag{A11}$$

$$\widetilde{T}^l_{11}\,\frac{d^3\sigma_0}{dqd\Omega_q}=-\sqrt{\frac{2}{3}}\int d\,\Omega_p c(\omega,q,\Omega_q,\Omega_p)\,\Im m\times\sum_{sm_s}\left(t^*_{sm_s1-1}\,t_{sm_s-10}\right),\tag{A12}$$

$$\widetilde{T}^l_{1-1}\,\frac{d^3\sigma_0}{dqd\Omega_q}=\sqrt{\frac{2}{3}}\int d\,\Omega_p c(\omega,q,\Omega_q,\Omega_p)\,\Im m\times\sum_{sm_s}\left(t^*_{sm_s11}\,t_{sm_s-10}\right).\tag{A13}$$

(viii) The beam-target asymmetries for linearly polarized photons and tensor polarized deuterons:

$$\widetilde{T}^l_{20}\,\frac{d^3\sigma_0}{dqd\Omega_q}=\frac{\sqrt{2}}{3}\int d\,\Omega_p c(\omega,q,\Omega_q,\Omega_p)\,\Re e\times\sum_{sm_s}\left(t^*_{sm_s10}\,t_{sm_s-10}-t^*_{sm_s11}\,t_{sm_s-11}\right),\tag{A14}$$

$$\widetilde{T}^l_{21}\,\frac{d^3\sigma_0}{dqd\Omega_q}=\sqrt{\frac{2}{3}}\int d\,\Omega_p c(\omega,q,\Omega_q,\Omega_p)\,\Re e\times\sum_{sm_s}\left(t^*_{sm_s10}\,t_{sm_s-11}\right),\tag{A15}$$

$$\widetilde{T}^l_{2-1}\,\frac{d^3\sigma_0}{dqd\Omega_q}=\sqrt{\frac{2}{3}}\int d\,\Omega_p c(\omega,q,\Omega_q,\Omega_p)\,\Re e\times\sum_{sm_s}\left(t^*_{sm_s10}\,t_{sm_s-1-1}\right),\tag{A16}$$

$$\widetilde{T}^l_{22}\,\frac{d^3\sigma_0}{dqd\Omega_q}=-\frac{1}{\sqrt{3}}\int d\,\Omega_p c(\omega,q,\Omega_q,\Omega_p)\times\sum_{sm_s}t^*_{sm_s1-1}\,t_{sm_s-11},\tag{A17}$$

$$\widetilde{T}^l_{2-2}\,\frac{d^3\sigma_0}{dqd\Omega_q}=-\frac{1}{\sqrt{3}}\int d\,\Omega_p c(\omega,q,\Omega_q,\Omega_p)\times\sum_{sm_s}t^*_{sm_s11}\,t_{sm_s-1-1}.\tag{A18}$$

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