

Shell-model structure of exotic ^{135}Sb

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(Received 5 August 2005; published 21 November 2005)

Recent studies have provided new experimental information on the very neutron-rich nucleus ^{135}Sb . We have performed a shell-model calculation for this nucleus by using a realistic effective interaction derived from the CD-Bonn nucleon-nucleon potential. This gives a very good description of the observed properties of ^{135}Sb . We show that the anomalously low position of the first excited state, $J^\pi = 5/2^+$, and the strongly hindered $M1$ transition to the ground state have their origin in the effective neutron-proton interaction.

DOI: [10.1103/PhysRevC.72.057302](https://doi.org/10.1103/PhysRevC.72.057302)

PACS number(s): 21.60.Cs, 21.30.Fe, 27.60.+j

The nucleus ^{135}Sb has been investigated through the β decay of ^{135}Sn in two recent studies [1,2]. The former, performed at OSIRIS/STUDSVIK, produced ^{135}Sn by means of fast neutron fission of a ^{238}U target inside a special ion source, whereas the latter, at ISOLDE/CERN, has made use of $^{135-137}\text{Sn}$ nuclei isolated by selective laser ionization. Motivated by astrophysical interest, these studies aimed at acquiring new data on the decay properties of Sn isotopes beyond doubly magic ^{132}Sn , which are of great interest for r process modeling. It is not of minor interest, however, for one to gain access to exotic nuclei, such as Sb isotopes with $N > 82$, in order to explore for possible changes in nuclear-structure properties when moving toward the neutron drip line. Actually, in Refs. [1,2] the level structure of ^{135}Sb was studied. This nucleus, with an N/Z ratio of 1.65, is at present the most exotic nucleus beyond ^{132}Sn for which information exists on excited states.

A level scheme of ^{135}Sb , whose ground state has $J^\pi = 7/2^+$, was first proposed in Ref. [3], in which, by use of prompt fission from ^{248}Cm , four excited states at energies of 707, 1118, 1343, and 1973 keV were identified and given spin-parity assignments $J^\pi = 11/2^+$, $15/2^+$, $19/2^+$, and $23/2^+$, respectively. A shell-model calculation [3] with an empirical two-body interaction interpreted the first three states as originating from the $\pi g_{7/2}(v f_{7/2})^2$ configuration whereas the $23/2^+$ state was attributed to the $\pi g_{7/2}v f_{7/2}h_{9/2}$ configuration.

In Ref. [2] several new excited states were identified, three of them having also been observed in Ref. [1]. In particular, both these studies found a first excited state at 282 keV and assigned to it spin and parity $5/2^+$. None of the other states received in Refs. [1,2] spin-parity assignment. In the recent work of Ref. [4], tentative spin and parity attribution was proposed for some of these states and the position of new levels was established, including the $J^\pi = 3/2^+$ and $9/2^+$ yrast states at 440 and 798 keV, respectively.

When looking at the systematics of the lowest-lying $5/2^+$ state in odd Sb isotopes, it appears that this state falls down in energy in ^{135}Sb . The authors of Ref. [2] suggest that the $5/2^+$ state retains more single-particle (SP) character than does the $7/2^+$ ground state, based on their estimates of $\log ft$ values. In this context, the low position of the $5/2^+$ state is quite unexpected and is viewed as a downshift of the $d_{5/2}$ proton

level relative to the $g_{7/2}$ one, as a consequence of a more diffuse nuclear surface produced by the two neutrons beyond the 82 shell closure. In Ref. [2] results of a shell-model calculation for ^{135}Sb were also presented. We comment on them later.

In the recent work of Refs. [5,6] the nature of the $5/2^+$ state in ^{135}Sb was further investigated. In particular, at OSIRIS/STUDSVIK the advanced time-delayed $\beta\gamma\gamma(t)$ method was used to measure the lifetime of this state. A very small upper limit for the $B(M1)$ was found, thus evidencing a strongly hindered transition. This was seen as a confirmation of the SP nature of the $5/2^+$ state.

In this paper we report on a shell-model study of ^{135}Sb by employing matrix elements of the two-body effective interaction derived from a modern nucleon-nucleon (NN) potential. It is our main aim to verify to what extent a realistic shell-model calculation can account for the properties of ^{135}Sb , with special attention to the $5/2^+$ state, and to try to understand if there is a real need of shell-structure modifications to explain the experimental data.

We assume that ^{132}Sn is a closed core and let the valence neutrons occupy the six levels $0h_{9/2}$, $1f_{7/2}$, $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$, and $0i_{13/2}$ of the 82–126 shell, whereas for the proton the model space includes the five levels $0g_{7/2}$, $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$, and $0h_{11/2}$ of the 50–82 shell.

The two-body matrix elements of the effective interaction are derived from the CD-Bonn NN potential [7]. The strong short-range repulsion of the latter is renormalized by means of the new approach of Ref. [8], which has proved to be an advantageous alternative to the usual G -matrix method [8,9]. In this approach, one constructs a smooth potential, $V_{\text{low-}k}$, by integrating out the high-momentum components, i.e., above a certain cutoff momentum Λ , of the bare NN potential V_{NN} . The $V_{\text{low-}k}$ preserves the physics of V_{NN} up to Λ and can be used directly in the calculation of shell-model effective interactions. In the present paper, we have used for Λ the value 2.2 fm^{-1} .

Once the $V_{\text{low-}k}$ is obtained, the calculation of the effective interaction is carried out within the framework of the \hat{Q} -box plus folded-diagram method. A description of this method can be found in Ref. [10]. Here we mention only that the \hat{Q} box is calculated including diagrams up to second order in $V_{\text{low-}k}$. The computation of these diagrams is performed within the harmonic-oscillator basis making use of intermediate

TABLE I. Proton and neutron SP energies (in MeV).

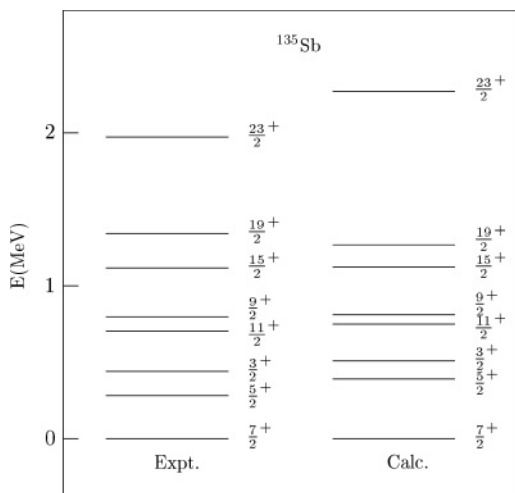
$\pi(n, l, j)$	ϵ	$\nu(n, l, j)$	ϵ
$0g_{7/2}$	-9.66	$1f_{7/2}$	-2.45
$1d_{5/2}$	-8.70	$2p_{3/2}$	-1.60
$1d_{3/2}$	-7.22	$0h_{9/2}$	-0.89
$0h_{11/2}$	-6.87	$2p_{1/2}$	-0.80
$2s_{1/2}$	-6.86	$1f_{5/2}$	-0.45
		$0i_{13/2}$	0.24

states composed of all possible hole states and particle states restricted to the three shells above the Fermi surface. The oscillator parameter used is $\hbar\omega = 7.78$ MeV and for protons the Coulomb force is explicitly added to the $V_{\text{low}-k}$ potential.

As regards the SP energies, they are taken from experiment. In particular, the spectra [11] of ^{133}Sb and ^{133}Sn are used to fix the proton and neutron SP energies, respectively. The only exceptions are the the proton $\epsilon_{s_{1/2}}$ and neutron $\epsilon_{i_{13/2}}$, whose corresponding levels are still missing. Their values are taken from Refs. [12,13], respectively, where it is discussed how they are determined. For the sake of completeness, the adopted SP energies relative to ^{132}Sn are reported in Table I. The needed mass excesses are taken from Ref. [14].

We now present the results of our calculations, which we carried out by using the OXBASH shell-model code [15]. Let us start with the binding energy of the ground state. Our calculated value is 16.411 ± 0.074 MeV, which compares very well with the experimental one, 16.585 ± 0.104 MeV [16]. Note that the error on the calculated value arises from the experimental errors on the proton and neutron separation energies of ^{133}Sb and ^{133}Sn [14].

The experimental [4,11] and calculated spectra of ^{135}Sb are compared in Fig. 1, where we reported the experimental yrast levels that received spin-parity assignment and the calculated yrast states with the same spin and parity. We see that the agreement between theory and experiment is very good, with the largest discrepancy, about 300 keV, occurring for the $23/2^+$ state. It is worth noting that our calculation overestimates the

FIG. 1. Experimental and calculated spectra of ^{135}Sb .

observed $5/2^+$ state by only 100 keV. This result becomes more relevant if we consider the outcome of the previous shell-model calculations of Refs. [2] and [17]. In these two calculations, different two-body matrix elements were used: In [17] an empirical interaction was obtained by modification of the CW5082 interaction of Chou and Warburton [18], whereas in Ref. [2] the effective interaction was derived from the CD-Bonn NN potential by means of a G -matrix folded-diagram method including diagrams up to third order. Both calculations, however, predict the $5/2^+$ state at about 300 keV above the experimental one. To overcome this difficulty, in Ref. [2] a downshift of the proton $d_{5/2}$ energy was proposed.

It is worth mentioning that in the preliminary calculations [5,19] we too could not account for the low-lying $5/2^+$ state in ^{135}Sb , our result coming close to that of Ref. [2]. Actually, the effective interaction used in the present calculation differs from that of Refs. [5,19] in that a larger number of intermediate states are used in its derivation. More precisely, in Refs. [5,19] the computation of the \hat{Q} -box diagrams was performed including intermediate states composed of particle and hole states restricted to two major shells above and below the Fermi surface. Furthermore, the Coulomb interaction was not considered, and a smaller value of the cutoff momentum ($\Lambda = 2.1 \text{ fm}^{-1}$) was used. In a future publication we shall discuss the connection between the dimension of the intermediate-state space and the optimal value of Λ . We verified that although the present effective interaction is generally not very different from the old one, significant changes occur in some neutron-proton matrix elements. In particular, this is the case of those of the $\pi g_{7/2}\nu f_{7/2}$ and $\pi d_{5/2}\nu f_{7/2}$ configurations, which, as we shall see later, play a crucial role in the structure of the $5/2^+$ state of ^{135}Sb . In this connection, it is worth noting that a direct test of these matrix elements is provided by ^{134}Sb . We found that our present effective interaction yields energies of all the members of the $\pi g_{7/2}\nu f_{7/2}$ configuration, which are in very good agreement with those of the eight lowest observed states [20]. Note that with our previous interaction the agreement was not of the same quality [21]. As for the $\pi d_{5/2}\nu f_{7/2}$ multiplet, only two members have been identified, the 1^- and 2^- states at 0.885- and 0.935-MeV excitation energy, respectively. Both of them are very well reproduced by our calculation.

Let us now come back to ^{135}Sb . Our wave functions for the ground state and the first excited $5/2^+$ state are

$$|7/2^+\rangle = 0.87|\pi g_{7/2}(\nu f_{7/2})^2\rangle + \dots, \quad (1)$$

$$|5/2^+\rangle = 0.67|\pi d_{5/2}(\nu f_{7/2})^2\rangle + 0.48|\pi g_{7/2}(\nu f_{7/2})^2\rangle + \dots, \quad (2)$$

where we have omitted components having a weight of less than 5%. We see that the $5/2^+$ state has a large weight of the $\pi d_{5/2}(\nu f_{7/2})^2$ configuration. It is important to point out, however, that this does not arise, as it would be expected, from the pairing correlation between the two like nucleons. Rather, it is a consequence of the neutron-proton interaction. In fact, we have verified that when this interaction is switched off, the $5/2^+$ state is dominated by the $\pi g_{7/2}(\nu f_{7/2})^2$

TABLE II. Diagonal matrix elements of the neutron-proton effective interaction (in MeV) for the $\pi g_{7/2} \nu f_{7/2}$ and $\pi d_{5/2} \nu f_{7/2}$ configurations.

J	$\pi g_{7/2} \nu f_{7/2}$	$\pi d_{5/2} \nu f_{7/2}$
0	-0.58	
1	-0.59	-0.49
2	-0.21	-0.26
3	-0.22	-0.11
4	-0.04	-0.19
5	-0.16	-0.04
6	0.04	-0.59
7	-0.31	

configuration. This is because the neutron-neutron interaction in the $(\nu f_{7/2})_{J^\pi=2^+}^2$ state is very attractive, the difference with the $(\nu f_{7/2})_{J^\pi=0^+}^2$ matrix element being smaller than the $\epsilon_{d_{5/2}} - \epsilon_{g_{7/2}}$ spacing. This is confirmed by the low excitation energy of the 2^+ state observed in ^{134}Sn . The neutron-proton interaction largely increases the weight of the $\pi d_{5/2}(\nu f_{7/2})^2$ configuration in the $5/2^+$ state because it is, on average, more attractive in the $\pi d_{5/2}(\nu f_{7/2})^2$ than in the $\pi g_{7/2}(\nu f_{7/2})^2$ configuration. Of course, for the latter no contribution comes from the states $(\pi g_{7/2} \nu f_{7/2})_{J^\pi=0^+,7^+}$, as they cannot couple to the $f_{7/2}$ neutron to give $J^\pi = 5/2^+$. We show in Table II the diagonal matrix elements of the effective interaction for these two configurations.

It may also be of interest to see how the effective interaction affects the centroids of the $g_{7/2}$ and $d_{5/2}$ proton strengths. Our calculated values are -10.06 and -9.35 MeV for the former and the latter, respectively, with both of them relative to the calculated ground-state energy of ^{134}Sn . In this way, the displacements from the input values -9.66 and -8.70 MeV (see Table I) can be directly related to the neutron-proton effective interaction. We find therefore that the spacing between the two centroids is 0.71 MeV, to be compared with the proton SP splitting, 0.96 MeV. Note that in Ref. [2] the effective interaction produced a spacing of about 1.06 MeV. As already mentioned, a decrease of the $d_{5/2} - g_{7/2}$ splitting of 0.300 MeV was therefore called for in Ref. [2].

We now discuss our prediction for the $5/2^+ \rightarrow 7/2^+$ $M1$ transition. As shown in Eq. (2), our wave function for the $5/2^+$ state contains a sizable component of the $\pi g_{7/2}(\nu f_{7/2})^2$ configuration, which on the other hand dominates the $7/2^+$ ground state. This makes the theoretical $B(M1; 5/2^+ \rightarrow 7/2^+)$

larger than the experimental value when the free $M1$ operator is used. From the measured half-life of the $5/2^+$ level, $T_{1/2} = 6.0(7)$ ns, an upper limit of $0.29 \times 10^{-3} \mu_N^2$ was deduced for the $B(M1)$ transition rate [5,19]. Our calculated value, $25 \times 10^{-3} \mu_N^2$, is about 100 times larger. However, as is well known, the magnetic operator in the nucleus may be significantly different from the free operator owing to core-polarization effects and mesonic-exchange currents. Here, we calculated the matrix elements of effective $M1$ operator by taking into account first-order diagrams in $V_{\text{low}-k}$. Using these matrix elements, we find that the transition rate for the $5/2^+$ state turns out to be $4 \times 10^{-3} \mu_N^2$, which is only an order of magnitude larger than the experimental value. The reason for this decrease is that we now have a nonzero off-diagonal matrix element between the $d_{5/2}$ and $g_{7/2}$ proton levels that is opposite in sign to the diagonal $g_{7/2}$ matrix element. In other words, both components of the $5/2^+$ state contribute to the $B(M1)$ value and their contributions partially compensate for each other. The balance between these two components is a delicate one, being very sensitive to small changes in the wave functions of the involved states or in the effective $M1$ operator. In this connection, it should be pointed out that in our calculation we do not consider any meson-exchange correction. We may therefore consider that the agreement between the experimental and calculated $B(M1)$ values is quite satisfactory. It is important to note that the experimental magnetic moment of the $7/2^+$ ground state is $3.0 \mu_N$, to be compared with the calculated value $1.7 \mu_N$ when the free operator is used and with $2.5 \mu_N$ with the effective $M1$ operator.

In summary, our realistic shell-model calculation gives a very good description of the observed spectroscopic properties of exotic ^{135}Sb . We obtain this by employing an effective interaction derived from the CD-Bonn NN potential within a folded-diagram method including diagrams up to second order in $V_{\text{low}-k}$. A crucial role is played by the neutron-proton interaction, whose effects were tested on the neutron-proton nucleus ^{134}Sb . In particular, we found that this interaction makes the $5/2^+$ state in ^{135}Sb of admixed nature, which can explain its low excitation energy as well as the highly hindered $5/2^+ \rightarrow 7/2^+$ $M1$ transition. We showed that in our shell-model calculation there is no need to introduce an *ad hoc* decrease of the proton $d_{5/2}$ SP energy, the effective interaction properly accounting for the observed effects.

This work was supported in part by the Italian Ministero dell'Istruzione, dell'Università e della Ricerca.

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