PHYSICAL REVIEW C **72**, 054608 (2005)

Dynamical interpretation of average fission-fragment kinetic energy systematics and nuclear scission

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(Received 29 April 2005; published 29 November 2005)

A dynamical interpretation of the well-known systematics for average total kinetic energy of fission fragments $\langle E_K \rangle$ over a wide range of the Coulomb parameter ($600 < Z^2/A^{1/3} < 2200$) is given. Different scission criteria traditionally employed in fission theory—at zero neck radius and at finite neck radius—have been applied in dynamical calculations. Both have resulted in a fairly good description of the dependence of $\langle E_K \rangle$ on the Coulomb parameter. The results of dynamical calculations of $\langle E_K \rangle$ within three-dimensional Langevin dynamics show that the mean distance between the centers of mass of nascent fragments at the scission configuration increases linearly with the parameter $Z^2/A^{1/3}$. This distance changes approximately from $2.35R_0$ for ^{119}Xe to $2.6R_0$ for ^{256}Fm. In spite of this increase in mean distance between future fragments at scission, the linear dependence of $\langle E_K \rangle$ on the parameter $Z^2/A^{1/3}$ remains approximately valid over a wide range of the Coulomb parameter $Z^2/A^{1/3}$.

DOI: 10.1103/PhysRevC.72.054608 PACS number(s): 25.85.−w, 05.10.Gg, 24.75.+i

I. INTRODUCTION

Nuclear scission is a process through which the initial compound nucleus divides predominantly into two fragments. Any theoretical description or simulation of nuclear scission inevitably requires one to consider the condition under which scission occurs. By scission, one means here a transition from a continuous nuclear configuration (which becomes unstable for a number of reasons) to a configuration in which the nuclear system consists of two separated fragments. The problem of the rupture of the neck between nascent fragments has been addressed many times (see, e.g., Refs. [1–10], but it has not been solved completely yet. So far, the problem of neck rupture and closely related questions concerning the parameters of the fission fragment mass-energy distributions still remain an intricate and unsolved puzzle of fission physics.

For the first time the problem of rupture of the neck in nuclear fission has been solved quantitatively by Strutinsky and co-workers [1–4]. They have solved an integro-differential equation for the determination of the equilibrium shapes in a liquid drop model (LDM) for a given deformation (a fixed elongation) of the fissioning nucleus. From these calculations they have revealed the existence of the so-called critical [1–4] or exit [5] deformation that has been interpreted as the scission configuration. As defined in Refs. [1–4] the critical deformation is almost independent of the fissility parameter. It was found that the critical deformation is characterized by a relatively thick neck with a radius $R_N^{\rm crit} \simeq 0.24R_0$ for $Z^2/A \simeq 0$ and $R_N^{\rm crit} \simeq 0.27R_0$ for $Z^2/A \simeq 36$ (with R_0 the radius of the initial spherical nucleus) and the following distances between the centers of mass of nascent fragments: $D^{\rm crit} \simeq 2.3R_0$ for $Z^2/A \simeq 0$ and $D^{\rm crit} \simeq 2.38R_0$ for $Z^2/A \simeq 36$. The origin of the critical deformation can be understood if one considers the structure of the energy surface in the LDM. The main feature of the energy surface in the LDM for highly deformed nuclei revealed in Refs. [1–4] is the presence of three conditional extremes found under the condition that the elongation parameter has a given value. Two of them correspond to minima of the deformation energy—one for usual continuous shapes (the bottom of the fission valley) and another for separated two-fragment shapes (the bottom of the fragment valley). The third extremum corresponds to the maximum (ridge) of the potential barrier that separates the two minima at the same elongation. We note that the presence of two valleys and a ridge between them that vanishes at large nuclear elongations was confirmed by Hartree-Fock calculations of the potential energy [11]. It is attractive from the physical point of view to define the scission condition on the basis of the instability criterion of a nucleus with respect to variations of the neck thickness [1–5], in which case the ridge between the fission valley and the valley of the separated fragments disappears. This scission condition corresponds to scission configurations of a fissioning nucleus that are characterized by a neck of finite radius, on average equal to $0.3R_0$ [5].

Another acceptable and physically reasonable scission criterion is based on the equality of the Coulomb repulsion and nuclear attraction forces between future fragments. It was shown in Ref. [12] that this scission criterion leads to scission configurations that have a finite neck radius $R_N \simeq 0.2R_0$ for nuclei with $Z^2/A^{1/3} \simeq 500$ and a neck radius $R_N \simeq 0.3R_0$ for nuclei with $Z^2/A^{1/3} \simeq 2000$. In a random neck rupture model Brosa with co-workers [6,7] have used the criterion of hydrodynamic instability of the neck against rupture. It leads to scission configurations with a finite neck radius $R_N \simeq (0.3–0.4)R_0$. In contrast, the scission condition of zero neck radius, $R_N = 0$, has been used very often [13–23] and successfully in the theory of the fission process. Although this scission condition is consistent with the model representing a nucleus as a liquid drop with a sharp surface [1,2,24], it is unsatisfactory since a description of the nucleus based on LDM loses significance [5] when the neck radius becomes comparable with the distance between nucleons. Summing up, one can note that at the present time there is no unambiguous scission criterion, and the fission theory most often employs two criteria (conditions) that are obvious limiting cases with respect to each other. One of them is the zero-neck-radius ($R_N = 0$) scission criterion. Another one is the scission criterion at finite neck radius. All scission conditions using of a finite neck radius lead to scission configuration with a rather thick neck, $R_N \simeq 0.3R_0$ on average, that weakly depends on the fissility parameter.

The average total kinetic energy $\langle E_K \rangle$ has been used quite often in two-dimensional Langevin calculations for the determination of the nuclear viscosity value [20–23,25–28]. It was shown in Refs. [29,30] that the $\langle E_K \rangle$ values in three-dimensional Langevin calculations become less dependent on the value of the viscosity coefficient than in the two-dimensional case owing to the inclusion of the mass-asymmetry degree of freedom in the dynamical consideration. From the other side, the $\langle E_K \rangle$ values depend on the scission deformation because Coulomb repulsion and nuclear attractive energies strongly depend on the distance between the mass centers of nascent fragments as well as on the deformation of the fragments. Therefore, we chose the quantity $\langle E_K \rangle$ for investigating the nuclear scission process in the present study based on the three-dimensional Langevin approach.

The major part of the average total kinetic energy does come from Coulomb repulsion at scission configuration. Since the elongation D^{crit} of scission configuration is approximately identical for all values of the fissility parameter, the Coulomb repulsion between fission fragments will be proportional to $Z^2/A^{1/3}$. Consequently, observed values of the average total kinetic energy can be described by the simple linear dependence of $Z^2/A^{1/3}$. This is the essence of systematics established by Terrell [31] from the experimental data on $\langle E_K \rangle$. All conclusions about existence of the exit scission configuration [5] and its independence on the fissility parameter have been made by Strutinsky and co-workers [1–4] on the basis of the performed static variational calculations within the LDM. Therefore, the identical scission configurations can be considered as a static interpretation of a linear dependence, $\langle E_K \rangle (Z^2/A^{1/3})$.

At the present time all well-known systematics of the experimental data on $\langle E_K \rangle$ use the linear dependence of $\langle E_K \rangle$ on $Z^2/A^{1/3}$, but with different coefficients. The values of these coefficients depend particularly on the distance between the centers of mass of nascent fragments at scission deformation. Therefore, the aim of our study is twofold. First, we would like to analyze and elucidate correlations between the calculated mean kinetic energy of fragments, $\langle E_K \rangle$, and the exit scission configurations in fission of excited compound nuclei. The second one concerns the systematic investigation of fission fragment energy distribution parameters, especially $\langle E_K \rangle$, in a broad range of the Coulomb parameter $Z^2/A^{1/3}$ by using different scission criteria.

The main purpose of the present study was to explain the $\langle E_K \rangle$ systematics on the basis of multidimensional Langevin calculations. The experimentally observed two-dimensional mass-energy distribution of fission fragments cannot be obtained in terms of two-dimensional Langevin calculations [20–23,27,28]. In this case, it is necessary to consider at least three collective coordinates, since $\langle E_K \rangle$ is obtained from the mass-energy distribution of fission fragments by integrating it over mass. The $\langle E_K \rangle$ value obtained for a symmetric mass split in the two-dimensional Langevin approach is 2–3 MeV larger than $\langle E_K \rangle$ obtained by integrating the mass-energy distribution of fission fragments over mass in the case of light fissioning nuclei. This difference increases to 6–7 MeV for heavy fissioning nuclei. Therefore, a calculation of $\langle E_K \rangle$ on the base of the three-dimensional Langevin equations is more reliable for making a more careful comparison with the experimental data. To the best of our knowledge such systematic three-dimensional Langevin calculations of $\langle E_K \rangle$ over a wide range of fissility parameter have not yet been performed. A dynamical interpretation of the $\langle E_K \rangle$ systematics within the three-dimensional Langevin approach has also not been given. In the present study, we have performed systematic three-dimensional Langevin calculations of $\langle E_K \rangle$ for a large number of fissioning nuclei that are close to the line of beta stability. The equation for the line of beta stability is given in Ref. [32].

The paper is organized as follows. The dynamical model with necessary input will be described in the next section. Section III is devoted to the results obtained from this study and a discussion. A summary of the results with conclusions will be given in Sec. IV.

II. THE DYNAMICAL MODEL AND BASIC EQUATIONS

In our dynamical calculations we used a well-known $\{c, h, \alpha\}$ parametrization. This conveniently provides a three-parametric family of shapes that have been employed in numerous studies of static [5] as well as dynamical [25–28] characteristics of fissioning nuclei. It was shown [5] that this simple parametrization describes with rather good quantitative accuracy the properties of the saddle point shapes obtained in the LDM calculations, where practically no restrictions were imposed on nuclear shapes [1,24].

In cylindrical coordinates the surface of the nucleus is given by

$$\rho_s^2(z) = \begin{cases} (c^2 - z^2)\left(A_s + Bz^2/c^2 + \frac{\alpha z}{c}\right), & B \geqslant 0, \\ (c^2 - z^2)\left(A_s + \frac{\alpha z}{c}\right)\exp(Bcz^2), & B < 0, \end{cases} \quad (1)$$

where z is the coordinate along the symmetry axis and ρ_s is the radial coordinate of the nuclear surface. In Eq. (1) the quantities A_s and B are defined by means of the shape parameters as

$$B = 2h + \frac{c-1}{2}, \quad A_s = \begin{cases} c^{-3} - \frac{B}{5}, & B \geqslant 0, \\ -\frac{4}{3}\frac{B}{\exp(Bc^3) + \left(1 + \frac{1}{2Bc^3}\right)\sqrt{-\pi Bc^3}\,\mathrm{erf}(\sqrt{-Bc^3})}, & B < 0. \end{cases} \quad (2)$$

In Eqs. (1) and (3) c denotes the elongation parameter (with the length of the nucleus equal to $2c$ in units of the spherical nucleus radius R_0), the parameter h describes a variation in the thickness of the neck for a given elongation of the nucleus, and the parameter of the "mirror" (mass) asymmetry α determines the ratio of the volumes (masses) of future fragments. In the symmetrical case $\alpha = 0$ a family of symmetric shapes is obtained, ranging from the spherical shape ($c = 1, h = 0$) to the two-fragment shapes ($A_s < 0$). For the case of $\alpha \neq 0$ different asymmetric shapes are obtained.

The appearance of a neck in the nuclear shape is associated with the instant at which the profile function $\rho_s(z)$ starts to have three extrema, two maxima corresponding to nascent fragments and a minimum between them, which corresponds to the minimum neck thickness. The minimum appears at the point

$$z_N = 2\sqrt{\frac{p}{3}}\cos\left(4\pi + \arccos\left(\frac{q\sqrt{27}}{6(p)^{3/2}}\right)\right) - \frac{\alpha c}{4B}, \quad (3)$$

where

$$p = -\frac{c^2}{4B}\left(\frac{2}{c^3} - \frac{12}{5}B - \frac{3\alpha^2}{4B}\right), \quad (4)$$

$$q = -\frac{\alpha c^3}{4B}\left(\frac{\alpha^2}{8B^2} - \frac{2}{5} - \frac{1}{2Bc^3}\right). \quad (5)$$

The condition of the existence of a neck in the nuclear shape can be written in the form

$$\frac{q^2}{4} - \frac{p^3}{27} < 0. \quad (6)$$

Before the appearance of the neck, the profile function has only one extremum, a maximum; the corresponding nuclear shapes are monoshapes. The coordinates of the extrema of the profile function and the values of $\rho_s(z)$ at these extrema vary in response to variations in the shape parameters. The equation for the scission surface can be written in the form

$$\rho_s(z_N) = R_N, \quad (7)$$

where R_N is the neck radius corresponding to the prescission shape. The condition of zero neck radius corresponds to the case of $\rho_s(z) = 0$ and could be written in the following way:

$$4BA_s - \alpha^2 = 0. \quad (8)$$

The hydrodynamic scission criterion proposed in Refs. [6,7] has the form $l = 11R_N$, where l is the length of the nucleus. In the case of $\{c, h, \alpha\}$ parametrization it can be written as

$$\rho_s(z_N) = 2c/11. \quad (9)$$

The shape of the nucleus lies between the limits $z_{\min} = -c$ and $z_{\max} = c$. The continuous shapes of the nucleus are determined from the condition that ρ_s^2 is equal to zero only at the two end points, at $z = \pm c$ of the interval $[z_{\min}, z_{\max}]$. If another two roots of the equation $\rho_s^2 = 0$ lie within the interval $[z_{\min}, z_{\max}]$ then ρ_s^2 will describe separated shapes of the nucleus. All other cases correspond to shapes that have no physical interpretation (the nonphysical shapes). A detailed discussion of such shapes is presented in Ref. [33].

We would like to discuss the problem with nonphysical shapes that arises in dynamical calculations. The problem of excluding such shapes from the dynamical treatment arises inevitably, and then one must use the $\{c, h, \alpha\}$ parametrization over a wide range of changes of the parameters c, h, and α. Usually, the restriction of a rectangular grid along the shape parameters h, α is the only way to avoid the appearance of the nonphysical shapes in dynamical calculations. But this simple restriction of the parameters h, α by some limit values will exclude from dynamical consideration not only the nonphysical shapes but also the possible shapes of the nucleus. To exclude from dynamical consideration only the nonphysical shapes and keep the rectangular grid with respect to the shape parameters, we have introduced in Ref. [34] a new set of collective coordinates. As a new mass-asymmetry parameter we used

$$q_3 = \begin{cases} \alpha/(A_s + B), & B \geqslant 0, \\ \alpha/A_s, & B < 0. \end{cases} \quad (10)$$

In this case all possible mass-asymmetric shapes of the nucleus for any values of c and h can be generated by the parameter $|q_3| \leqslant 1$. The value $q_3 = 0$ corresponds to the symmetrical shapes, and the values $q_3 = \pm 1$ correspond to the limit cases, so then the mass of one fragment is equal to zero.

It is attractive to choose a new neck parameter in such a way that the condition of the zero neck radius ($R_N = 0$) will be satisfied at the one value of a new neck parameter. In the symmetrical case $\alpha = 0$ the condition $\rho_s(z_N) = 0$ will be satisfied if h will be equal to h_{sc} which is given by the following expression

$$h_{sc} = \frac{5}{2c^3} + \frac{1 - c}{4}. \quad (11)$$

It is seen from Eq. (11) that h_{sc} is substantially dependent on the parameter c. As a new neck parameter we chose

$$q_2 = \frac{h + 3/2}{h_{sc} + 3/2}. \quad (12)$$

If the neck parameter q_2 equals zero then h will be equal to $-3/2$. This value of h guarantees high potential energy values of continuous nuclear shapes, so the value $q_2 = 0$ could be a lower limit for the new neck parameter q_2. In the symmetrical case ($q_3 = 0$) the value $q_2 = 1$ corresponds to the nuclear shapes with zero neck radius for any values of the parameter c. In the asymmetric case ($q_3 \neq 0$) the condition $R_N = 0$ will be satisfied at values of q_2 close to 1. So, in our opinion, the collective coordinates $\{q_1 = c, q_2, q_3\}$ are the most appropriate for dynamical calculations on the basis of the $\{c, h, \alpha\}$ parametrization because they help to avoid the nonphysical shapes in dynamical calculations and keep all possible shapes given by the $\{c, h, \alpha\}$ parametrization in the rectangular grid. In the present calculations we used $\{q_1 = c, q_2, q_3\}$ parameters within the following limits: $q_1 \in [0.5, 4.5]$, $q_2 \in [0, 1]$, and $q_3 \in [-1, 1]$.

In the present study a stochastic approach [29,35–37] has been used to describe the dynamics of the fission process. In the stochastic approach the evolution of the collective coordinates is considered as a motion of Brownian particles, which interact stochastically with a large number of internal degrees of freedom, constituting the surrounding "heat bath." The hydrodynamic friction force is assumed to be derived from the random force averaged over a time larger than the collisional time scale between collective and internal degrees of freedom. The random part is modeled as a Gaussian white noise, which causes fluctuations of the collective variables, and, as a final result, fluctuations of the physical observables in the fission process. The coupled Langevin equations used in the dynamical calculations have the form

$$\dot{q}_i = \mu_{ij} p_j,$$
$$\dot{p}_i = -\frac{1}{2} p_i p_k \frac{\partial \mu_{jk}}{\partial q_i} - \frac{\partial F}{\partial q_i} - \gamma_{ij}\mu_{jk} p_k + \theta_{ij}\xi_j, \quad (13)$$

where $\mathbf{q} = (q_1, q_2, q_3)$ are the collective coordinates, $\mathbf{p} = (p_{q_1}, p_{q_2}, p_{q_3})$ are the conjugate momenta, $F(\mathbf{q}) = V(\mathbf{q}) - a(\mathbf{q})T^2$ is the Helmholtz free energy, $V(\mathbf{q})$ is the zero-temperature potential energy, m_{ij} ($\|\mu_{ij}\| = \|m_{ij}\|^{-1}$) is the tensor of inertia, γ_{ij} is the friction tensor, $\theta_{ij}\xi_j$ is a random force, and ξ_j is a random variable satisfying the relations

$$\langle \xi_i \rangle = 0,$$
$$\langle \xi_i(t_1)\xi_j(t_2)\rangle = 2\delta_{ij}\delta(t_1 - t_2). \quad (14)$$

Thus, the Markovian approximation is assumed to be valid. In these equations, and further in this paper, we use the convention that repeated indices are to be summed over from 1 to 3, and the angular brackets denote averaging over an ensemble.

Eigenvalues and eigenvectors of the diffusion matrix D_{ij}, which are used [37] for calculation of the strength of the random force, have been calculated by the Jacoby method [38]. The strength of the random force is related to the diffusion tensor D_{ij} by the equation

$$D_{ij} = \theta_{ik}\theta_{kj}, \quad (15)$$

which, in turn, satisfies the Einstein relation

$$D_{ij} = T\gamma_{ij}. \quad (16)$$

Here T is the temperature of the "heat bath," which is determined by the Fermi-gas model [39] formula

$$T = (E_{\text{int}}/a(\mathbf{q}))^{1/2}, \quad (17)$$

where E_{int} is the internal excitation energy of the nucleus, and $a(\mathbf{q})$ is the level density parameter, which has been taken from the work of Ignatyuk *et al.* [40].

We start modeling fission dynamics from a spherical nucleus that is, $\mathbf{q}(t = 0) = (q_1 = 1, q_2 = 0.375, q_3 = 0)$. The initial state is assumed to be characterized by the thermal equilibrium momentum distribution, and by a spin distribution $F(L)$ for heavy-ion complete fusion. We have parametrized the compound nuclei spin distribution $F(L)$ according to the scaled prescription proposed in Ref. [37]. During a random walk along the Langevin trajectory in space of the collective coordinates, we used the energy conservation law in the form

$$E^* = E_{\text{int}} + E_{\text{coll}} + V(\mathbf{q}) + E_{\text{evap}}(t), \quad (18)$$

where E^* is the total excitation energy of the nucleus, E_{coll} is the kinetic energy of the collective degrees of freedom, and $E_{\text{evap}}(t)$ is the energy carried away by evaporated particles by time t.

The inertia tensor was calculated by means of the Werner-Wheeler approximation for incompressible irrotational flow. A description of the method is given, for example, in Refs. [15,41]. The potential energy of the nucleus was calculated within the framework of a macroscopic model with a finite range of nuclear forces [32,42]. A modified one-body mechanism of nuclear dissipation [43,44] with a reduction factor k_s from a wall formula has been used for determination of the dissipative part of the driving forces. We used the value $k_s = 0.25$ in the present calculations. The experimental parameters of mass-energy distributions as well as multiplicities of prescission particles can be described quite well in theoretical calculations with $k_s = 0.25$ [45,46].

As has been noted before, many theoretical models predict that scission of the nucleus into fragments takes place at the finite neck radius $R_N \simeq 0.3R_0$. Therefore, in the present calculations we used three different scission criteria: The scission criterion at the zero neck radius $R_N = 0$, that at the finite neck radius $R_N = 0.3R_0$, and the scission criterion of hydrodynamic instability of the neck. A scission criterion of the nucleus determines a scission surface in the space of collective coordinates. Equations (7) and (8) determine scission surfaces in the case of scission criterion at finite and zero neck radius, whereas Eq. (9) specifies the scission surface in the case of the hydrodynamic scission criterion. The scission surface determines the possible scission deformations of the nucleus. As a result of modeling the fission process on the basis of the Langevin equations one will obtain an ensemble of stochastic Langevin trajectories. Each trajectory describes one fission event that in particular is characterized by the scission configuration. The scission configuration is determined by the intersection points of the stochastic Langevin trajectories of the fissioning system with the scission surface in the coordinate subspace. Thus it is possible to introduce the notion of a mean trajectory and mean scission deformation. The mean dynamical trajectory and mean scission deformation are obtained by averaging over an ensemble of Langevin trajectories. The mean trajectory will correspond to the symmetrical shapes. Examples of mean trajectories obtained in the three-dimensional Langevin calculations are presented in Ref. [45].

III. RESULTS AND DISCUSSION

The possible deformations, which determine the mass distribution of fission fragments, continue to be the subjects of scientific discussion [9,29,45,47]. At the same time a generally accepted statement asserts that the kinetic energy of fission fragments is mainly determined by the scission configuration of the fissioning nucleus (scission point). Therefore, we will mainly be concerned with the parameters of energy distribution of fission fragments during the discussion of the results of our calculations with different scission conditions.

It is interesting to consider the mean scission configurations typical for different scission criteria. In this case we would like to demonstrate the difference between static and dynamical calculations. The mean fission trajectories for different fissioning nuclei and the bottom of the LDM fission valley are shown in Fig. 1. The mean fission trajectories correspond to the symmetrical case $q_3 = 0$. The bottom of the LDM fission valley lies at $h = q_3 = 0$. Therefore, the two-dimensional grid in the collective coordinates q_1 and q_2 is shown together with the illustration of the possible nuclear shapes. As one can see

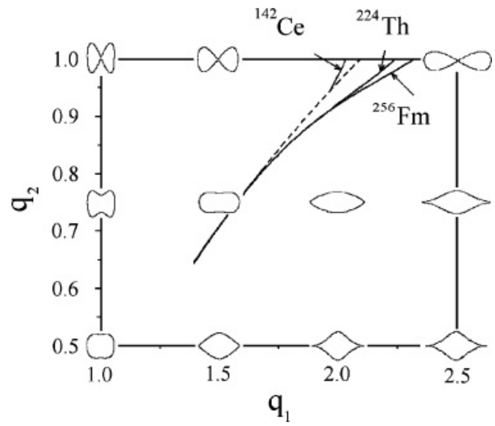

FIG. 1. The mean trajectories for the ^{256}Fm, ^{224}Th, and ^{142}Ce fissioning nuclei in the collective coordinates q_1 and q_2 ($q_3 = 0$). The dashed curve corresponds approximately to the bottom of the LDM fission valley and lies at $h = \alpha = 0$. The examples of nuclear shapes are presented for the appropriate values of the parameters q_1 and q_2 ($q_3 = 0$).

from this figure, the mean trajectory for the nucleus ^{142}Ce passes near the bottom of the LDM fission valley, whereas the mean trajectories for the heavier nuclei deviate from the bottom of the LDM fission valley to the more elongated shapes. This deviation arises from the behavior of inertia and friction tensors on the descent from saddle point to scission [45]. As a result, more elongated mean scission configurations are realized in dynamical calculations with the modified one-body dissipation than in pure static calculations along the bottom of the LDM fission valley.

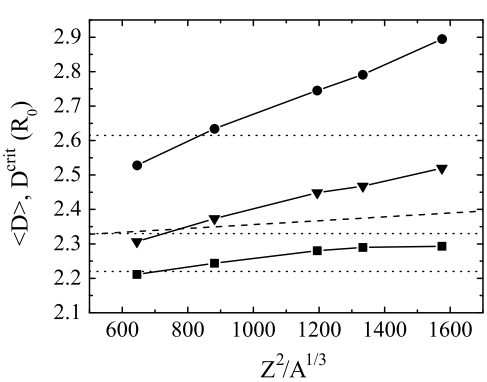

FIG. 2. The distance between mass centers of nascent fragments $\langle D\rangle$ of the mean scission deformations for the different scission criteria. The circles correspond to the zero-neck scission criterion. The triangles correspond to the scission criterion at finite neck radius $R_N = 0.3R_0$. The squares correspond to the hydrodynamic scission criterion [6,7]. The solid lines connect symbols that correspond to the calculations along the mean trajectories. The dotted lines correspond to the calculations along the bottom of the LDM fission valley ($h = \alpha = 0$). The value $\langle D\rangle = 2.62$ is obtained for the zero-neck-radius scission criterion, $\langle D\rangle = 2.33$ for scission criterion $R_N = 0.3R_0$, and $\langle D\rangle = 2.22$ for hydrodynamic scission. The dashed line corresponds to the calculations of the critical deformation D^{crit} performed in Ref. [1].

The comparison of mean scission configurations for different scission criteria is presented in Fig. 2. In this figure circles correspond to the zero-neck scission criterion, triangles to the finite-neck scission criterion ($R_N = 0.3R_0$), and squares to the hydrodynamic scission criterion [6,7]. As one can see, the hydrodynamic scission criterion results in the lowest values of $\langle D\rangle$. The dashed line in Fig. 2 corresponds to the calculations of the critical deformation D^{crit} performed in Ref. [1]. The dotted lines in this figure determine the scission deformations obtained in the static calculations along the bottom of the LDM fission valley. These lines correspond to the following values: $\langle D\rangle = 2.62R_0$ for the zero-neck scission criterion, $\langle D\rangle = 2.33R_0$ for the scission criterion $R_N = 0.3R_0$, and $\langle D\rangle = 2.22R_0$ for hydrodynamic scission. As one would expect the mean scission deformations are approximately the same in static and dynamical calculations for sufficiently light fissioning nuclei. The mean trajectories deviate from the bottom of the fission valley to the more elongated shapes for heavy nuclei. Therefore, the mean scission deformations for heavy nuclei obtained in dynamical calculations are different from the scission deformation obtained in the static calculations along the bottom of the LDM fission valley. The same results of more elongated scission shapes for heavy nuclei have been obtained earlier in dynamical calculations based on the generalized Hamiltonian equations [41].

The average total kinetic energy of fission fragments, $\langle E_K\rangle$, is measured in experimental investigations with very high accuracy at the present time. It was determined in Ref. [8] that $\langle E_K\rangle$ does not depend on angular momentum L nor on the temperature of the fissioning nucleus (a conclusion valid at high excitation energies when shell effects disappear). Usually, $\langle E_K\rangle$ is approximated by the expression

$$\langle E_K\rangle = C_1 Z^2/A^{1/3} + C_2, \tag{19}$$

where C_1 and C_2 are some constants. The most often used systematics of $\langle E_K\rangle$ was presented in Ref. [48], where a large amount of experimental data was used to obtain coefficients C_1 and C_2. The systematics of $\langle E_K\rangle$ from Ref. [48] reads as follows:

$$\langle E_K\rangle = 0.1189 Z^2/A^{1/3} + 7.3. \tag{20}$$

At the same time Itkis and co-workers [8,49] determined that dependence of $\langle E_K\rangle$ on $Z^2/A^{1/3}$ for heavy and light nuclei should be described by different linear dependences with different coefficients C_1 and C_2 if one uses only experimental data on $\langle E_K\rangle$ obtained from fusion-fission reactions. In this connection the following systematics of $\langle E_K\rangle$ was proposed in Refs. [8,49]:

$$\langle E_K\rangle = \begin{cases} 0.131 Z^2/A^{1/3}, & Z^2/A^{1/3} < 900, \\ 0.104 Z^2/A^{1/3} + 24.3, & Z^2/A^{1/3} > 900. \end{cases} \tag{21}$$

The mean kinetic energy of fission fragments in the present paper was calculated as follows:

$$\langle E_K\rangle = \langle V_C\rangle + \langle V_n\rangle + \langle E_{ps}\rangle, \tag{22}$$

where V_C is the repulsive Coulomb energy, V_n is the nuclear attractive energy, and E_{ps} is the kinetic energy of the relative motion of the nascent fragments (prescission kinetic energy). All parts of the sum in expression (22) are calculated at the

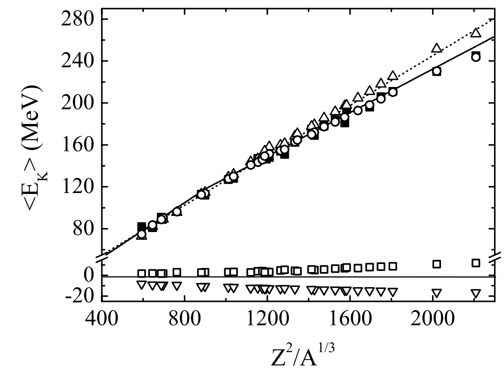

FIG. 3. Plot of $\langle E_K \rangle$, $\langle E_{ps} \rangle$, and $\langle V_n \rangle$ values as a function of the parameter $Z^2/A^{1/3}$ for different fissioning nuclei. The solid line represents the $\langle E_K \rangle$ systematics proposed in Refs. [8,49], Eq. (21); the dashed line represents Viola's systematics [48], Eq. (20). The filled squares are experimental data of $\langle E_K \rangle$. The open triangles are results of $\langle E_K \rangle$ calculations with scission criterion at the finite neck radius $R_N = 0.3R_0$. The open circles are results of calculations with the zero-neck scission criterion. Open squares are results of $\langle E_{ps} \rangle$ calculations and inverted triangles are results of $\langle V_n \rangle$ calculations with scission criterion at the finite neck radius $R_N = 0.3R_0$.

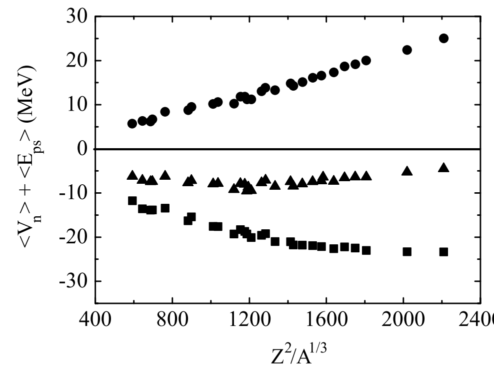

FIG. 4. The sum $\langle E_{ps} \rangle + \langle V_n \rangle$ calculated for different scission criteria as a function of the parameter $Z^2/A^{1/3}$. Filled circles are results of calculations with the zero-neck scission criterion. Filled triangles are results of calculations with scission criterion at the finite neck radius $R_N = 0.3R_0$. Filled squares are results of calculations with the hydrodynamic scission criterion.

moment of scission. The energies V_C and V_n were calculated within the framework of a macroscopic model with a finite range of nuclear forces [32,42]. The expressions for the calculation of these energies may be found in Ref. [50]. In the present paper the errors of the calculated observables mainly arise from the finite number of trajectories in the Langevin calculations. These purely statistical errors are calculated according to the formulas given in Ref. [51] and do not exceed 0.5% for all data presented in this paper.

The experimental data together with the calculated $\langle E_K \rangle$ values are presented in Fig. 3 for different scission criteria. As one can see from this figure the zero-neck scission criterion as well as the finite-neck-radius scission criterion $R_N = 0.3R_0$ result in approximately equal values of $\langle E_K \rangle$, which are in good agreement with experimental data for the light fissioning nuclei with $Z^2/A^{1/3} < 900$. For the scission criterion $R_N = 0.3R_0$ the increasing deviation of calculated $\langle E_K \rangle$ values from experimental data and $\langle E_K \rangle$ systematics suggested in Ref. [8] is seen in Fig. 3 for heavy nuclei. For these nuclei the calculated $\langle E_K \rangle$ values are close to Viola's systematics [48]. At the same time for heavy nuclei the calculated values of $\langle E_K \rangle$ are in a good agreement with experimental data for the zero-neck-radius scission criterion. The hydrodynamic scission criterion for light fissioning nuclei gives approximately the same $\langle E_K \rangle$ values as the scission criterion $R_N = 0.3R_0$. For heavy nuclei the hydrodynamic scission criterion results in $\langle E_K \rangle$ values that are approximately 10 MeV greater than Viola's systematics. But we would like to point out that we employed the hydrodynamic scission criterion in the $\{c, h, \alpha\}$ parametrization, whereas in the original work of Brosa with co-workers [6,7] another parametrization of nuclear shape has been used and this generated elongated nuclear shapes with a thick neck. Thus, in our opinion, this difference could be the reason for such high $\langle E_K \rangle$ values obtained in the present calculations when using the hydrodynamic scission criterion.

The lower part of Fig. 3 presents $\langle V_n \rangle$ and $\langle E_{ps} \rangle$ for the scission criterion $R_N = 0.3R_0$. The energies $\langle V_n \rangle$ and $\langle E_{ps} \rangle$ are approximately equal to each other in magnitude but have opposite signs. In this connection it is useful to investigate the dependence of $\langle V_n \rangle$ and $\langle E_{ps} \rangle$ on the scission criterion. The descent of the fissioning system from the saddle point to scission is a slow process in the case of the one-body dissipation mechanism. Consequently, $\langle E_{ps} \rangle$ has approximately the same values during the descent. Therefore, the $\langle E_{ps} \rangle$ values will be approximately the same and equal to the values indicated in Fig. 3 for all scission criteria. In contrast, the nuclear attractive energy $\langle V_n \rangle$ of nascent fragments strongly depends on the scission configuration of the compound nucleus. It decreases rapidly when the distance between future fragments increases and the neck radius decreases. The dependence of the sum $\langle V_n \rangle + \langle E_{ps} \rangle$ is presented in Fig. 4 for different scission criteria. There is no compensation of $\langle V_n \rangle$ and $\langle E_{ps} \rangle$ for the hydrodynamic scission criterion and scission criterion $R_N = 0.3R_0$. For these criteria the sum $\langle V_n \rangle + \langle E_{ps} \rangle < 0$ for all nuclei considered in the present study. At the same time, the sum $\langle V_n \rangle + \langle E_{ps} \rangle > 0$ for the zero-neck scission criterion. Our estimations show that the compensation of $\langle V_n \rangle$ and $\langle E_{ps} \rangle$ occurs approximately at a finite neck radius $R_N \simeq 0.2R_0$.

A systematic investigation of experimental data on E_K and scission deformation in the fission process was undertaken recently and published in Refs. [52,53]. Information about scission deformation was extracted from experimental data on E_K. The heavy fissioning nuclei with mass numbers $200 < A < 260$ $(1100 < Z^2/A^{1/3} < 1600)$ were investigated in Refs. [52,53]. The distance between the two charge centers of fragments at scission point $D(A_1, A_2)$ was estimated from the following equation:

$$D(A_1, A_2) = Z_1 Z_2 e^2 / E_K(A_1, A_2). \quad (23)$$

To allow a comparison of the degree of scission deformation among various fissioning nuclei, a shape elongation β was defined as

$$\beta = D(A_1, A_2)/D_0(A_1, A_2), \quad (24)$$

where $D_0(A_1, A_2)$ is the distance between the charge centers of two touching spherical fragments.

We used E_K instead of V_C in Eq. (23) because of an assumption made in Refs. [52,53] that $E_K \simeq V_C$ at the moment of scission. Moreover, the expression for Coulomb interactions of two point charges is used in Eq. (23). The main result obtained in Refs. [52,53] is the constancy of scission deformation, $\beta \simeq 1.65$, for symmetric fission at high excitation energies when the shell effects disappear.

In the present paper we slightly modified Eq. (23) to make a comparison between the theoretical and experimental values of β easier. We supposed that $A_1 = A_2 = A/2$. In this case $D_0(A/2; A/2) = 2^{2/3}R_0 \simeq 1.59R_0$. We used the mean kinetic energy of fission fragments, $\langle E_K\rangle$, instead of $E_K(A_1, A_2)$ in Eq. (23) also. We are convinced that such a procedure is relevant as we only deal with fusion-fission reactions at high excitation energies, when the shell effects do not influence the parameters of the fission fragments' mass-energy distribution. In this case the mass and energy distributions of fission fragments are approximated by a Gaussian function with high accuracy. The maximum value of mass distribution corresponds to symmetrical fission, and the maximum value of energy distribution approximately corresponds to $\langle E_K\rangle$. So, we propose to investigate the dependence of the parameter $\langle\beta_{E_K}\rangle$ using the following relation:

$$\langle\beta_{E_K}\rangle = \frac{D(A/2, A/2)}{D_0(A/2, A/2)} = \frac{Z^2e^2}{2^{8/3}r_0A^{1/3}\langle E_K\rangle}. \quad (25)$$

The calculated values of $\langle\beta_{E_K}\rangle$ obtained on the basis of the $\langle E_K\rangle$ systematics (20) and (21) are presented in Fig. 5. It is seen from this figure that the obtained values of $\langle\beta_{E_K}\rangle$ are not the constant over the wide range of $Z^2/A^{1/3}$. The parameter $\langle\beta_{E_K}\rangle$ increases from light to heavy nuclei. Furthermore, if $C_2 = 0$ in the $\langle E_K\rangle$ systematics then the parameter $\langle\beta_{E_K}\rangle$ will not depend on $Z^2/A^{1/3}$. In this case $\langle\beta_{E_K}\rangle$ will be equal to $e^2/(2^{8/3}r_0C_1)$.

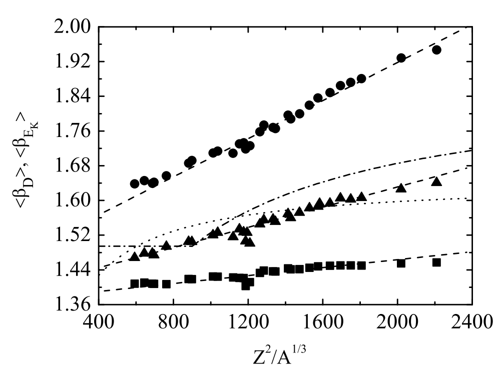

FIG. 5. The mean scission deformation parameters $\langle\beta_{E_K}\rangle$ and $\langle\beta_D\rangle$ vs the parameter $Z^2/A^{1/3}$. The filled circles, triangles, and squares are $\langle\beta_D\rangle$ values obtained from calculations with the zero neck radius $R_N = 0$, finite neck radius $R_N = 0.3R_0$, and hydrodynamic scission criterion, respectively. The dashed lines are obtained by the least-squares method on the basis of circles, triangles, and squares. The dotted curve is the $\langle\beta_{E_K}\rangle$ dependence given by Eq. (25) with the coefficients C_1 and C_2 from the Viola's systematics [48] [Eq. (20)]. The dashed-dotted curve is the $\langle\beta_{E_K}\rangle$ dependence given by Eq. (25) with the coefficients C_1 and C_2 from the $\langle E_K\rangle$ systematics proposed in Refs. [8,49] [Eq. (21)].

The coefficient C_2 is not equal to zero in the generally used $\langle E_K\rangle$ systematics and, as a result, $\langle\beta_{E_K}\rangle$ depends on the parameter $Z^2/A^{1/3}$. This is easy to check if one calculates the derivative

$$\frac{d\langle\beta_{E_K}\rangle}{d(Z^2/A^{1/3})} = \frac{e^2C_2}{2^{8/3}r_0(C_1Z^2/A^{1/3} + C_2)^2}. \quad (26)$$

As one can see from this equation, the sign of the derivative $d\langle\beta_{E_K}\rangle/d(Z^2/A^{1/3})$ is determined by the sign of the coefficient C_2. The parameter $\langle\beta_{E_K}\rangle$ will increase from light to heavy nuclei if the coefficient $C_2 > 0$ and vice versa.

In the present paper we also calculated the parameter $\langle\beta_D\rangle = \langle D\rangle/D_0(A/2; A/2)$ using the mean distance between the mass centers of future fragments, $\langle D\rangle$, obtained in dynamical calculations. The results are presented in Fig. 5 by filled symbols. In our opinion the value $\langle\beta_D\rangle$ is the most direct estimate of scission deformation, as it uses explicitly the distance between the mass centers obtained from the dynamical calculations and does not use the approximation about compensation of $\langle V_n\rangle$ and $\langle E_{ps}\rangle$ and the approximation for the Coulomb interactions of nascent fragments such as two point charges. As one can see from Fig. 5 the parameters $\langle\beta_D\rangle$ and $\langle\beta_{E_K}\rangle$ are close to each other. But the parameter $\langle\beta_D\rangle$ demonstrates a linear increase with increasing $Z^2/A^{1/3}$, whereas the dependence $\langle\beta_{E_K}\rangle(Z^2/A^{1/3})$ is not linear. This qualitative discrepancy between $\langle\beta_D\rangle$ and $\langle\beta_{E_K}\rangle$ arises from approximations used in the calculations of $\langle\beta_{E_K}\rangle$.

If one approximates the dependence $\langle D\rangle(Z^2/A^{1/3})$ by a linear expression of the form $\langle D\rangle = kZ^2/A^{1/3} + b$, then the coefficients k and b will have the following values: $k = 1.8 \times 10^{-4}$ and $b = 2.21$ for the scission criterion at finite neck radius $R_N = 0.3R_0$. By using these values of the coefficients k and b it is easy to demonstrate that the dependence of Coulomb repulsion energy, $\langle V_C\rangle(Z^2/A^{1/3})$, will be linear with high accuracy. For symmetric fission, V_C is given by the following expression:

$$\langle V_C\rangle \simeq \frac{Z^2e^2}{4\langle D\rangle R_0} = \frac{Z^2e^2}{4r_0A^{1/3}(kZ^2/A^{1/3} + b)}. \quad (27)$$

The curves $\langle V_C\rangle(Z^2/A^{1/3})$ obtained from Eq. (27) with the values $e^2 = 1.442$ MeV fm and $r_0 = 1.16$ fm and coefficients k, b for scission criterion $R_N = 0.3R_0$ are presented in Fig. 6. In this figure the solid line demonstrates Viola's systematics for $\langle E_K\rangle$. It should be emphasized that the linear dependence, $\langle V_C\rangle(Z^2/A^{1/3})$ stays valid in spite of a small increase of $\langle D\rangle$ with increasing parameter $Z^2/A^{1/3}$.

It is necessary to mention that $\langle D\rangle$ depends on the scission condition as well as the type and magnitude of nuclear viscosity used in dynamical calculations. As an illustration of this fact we presented in Fig. 5 the dependence of $\langle\beta_D\rangle(Z^2/A^{1/3})$ for the different scission criteria. It is seen from this figure that the calculated dependences $\langle\beta_D\rangle(Z^2/A^{1/3})$ have different slopes for the different scission criteria.

One can see from our results that in dynamical calculations the mean scission deformation increases approximately 10–15% in the interval $600 < Z^2/A^{1/3} < 2200$. This increase

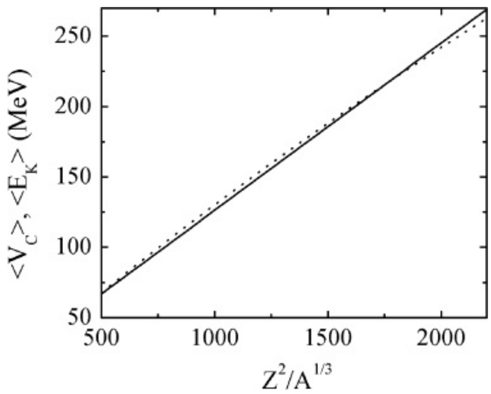

FIG. 6. Comparison of experimental average fission-fragment kinetic energies and the calculated values of the Coulomb interaction energies at scission as a function of the parameter $Z^2/A^{1/3}$. The solid line is Viola's systematics [48] for $\langle E_K \rangle$. The dotted curve is the calculated dependence of $\langle V_C \rangle$ given by Eq. (27) with the coefficients obtained from the criterion at finite neck radius $R_N = 0.3R_0$.

corresponds to the positive values of the coefficient C_2 in the $\langle E_K \rangle$ systematics. This increase maintains the validity of the linear dependence of $\langle V_C \rangle(Z^2/A^{1/3})$ with high accuracy in the case of the scission criterion at a finite neck radius $R_N = 0.3R_0$ and the calculations of $\langle E_K \rangle$ following Eq. (22) do not break the linear dependence of $\langle E_K \rangle(Z^2/A^{1/3})$ as well. The dependences $\langle E_K \rangle(Z^2/A^{1/3})$ obtained in our calculations for different scission criteria deviate from Viola's systematics by less than 5 MeV for the nucleus $^{306}_{122}$X and by less than 3 MeV for other nuclei. Consequently, we can assert that the linear dependence of $\langle E_K \rangle(Z^2/A^{1/3})$ could be described sufficiently well in theoretical calculations performed with the modified one-body dissipation mechanism, and the mean scission deformation is dependent on the parameter $Z^2/A^{1/3}$.

IV. CONCLUSIONS

A systematic study of the mean kinetic energy has been made over a wide range of the Coulomb parameter ($600 < Z^2/A^{1/3} < 2200$) on the basis of three-dimensional Langevin dynamics. The scission criteria at a finite neck radius $R_N = 0.3R_0$ and at zero neck radius and the hydrodynamic scission criterion have been applied in dynamical calculations. The calculations with scission criterion $R_N = 0.3R_0$ as well as zero-neck scission criterion reproduce sufficiently well the experimental data and follow Viola's systematics of experimental $\langle E_K \rangle$ values. The difference in mean scission deformations obtained on the basis of static and dynamical calculations has been demonstrated. The mean scission deformations obtained in the static calculations along the bottom of the LDM fission valley have approximately a constant value over a wide range of the Coulomb parameter. In contrast, the mean scission deformations obtained in dynamical calculations demonstrate a linear increase from light to heavy nuclei. This increase corresponds to the positive values of the coefficient C_2 in many $\langle E_K \rangle$ systematics that have the following form: $\langle E_K \rangle = C_1 Z^2/A^{1/3} + C_2$. However, the increase of the mean scission deformations in dynamical calculations is not considerable, and the linear dependence of the $\langle E_K \rangle$ systematics on the Coulomb parameter $Z^2/A^{1/3}$ remains valid even for very heavy and light fissioning nuclei. The sum of the nuclear attractive energy and prescission kinetic energy, $\langle V_n \rangle + \langle E_{ps} \rangle$, is always negative in the case of the hydrodynamic scission criterion and at a finite neck radius $R_N = 0.3R_0$. The sum $\langle V_n \rangle + \langle E_{ps} \rangle$ is positive at the zero-neck scission condition. The energies $\langle V_n \rangle$ and $\langle E_{ps} \rangle$ approximately compensate each other for the configurations of a fissioning nucleus with neck radius $R_N \simeq 0.2R_0$.

A comparison of calculated results obtained by using the zero-neck scission criterion and the scission criterion at a finite neck radius $R_N = 0.3R_0$ does not allow one to make a choice between the scission criteria used in the present study, because both of them describe experimental values of $\langle E_K \rangle$ quite well. To discriminate among different scission criteria one needs quite novel data about the fission-fragment energy distribution, such as variances and third and fourth moments of the energy distribution [54,55]. Furthermore, an investigation of the center-of-mass distance D distribution of fission fragments is a subject of interest [55,56].

In this connection, one should also mention an interesting attempt [57,58] to describe the total kinetic energies in very asymmetric fission, which have been compared with measurements of intermediate-mass fragments produced in heavy-ion reactions. The upper and lower limits of the total kinetic energy have been calculated from the interaction energy of the two nascent fragments at the conditional saddle point and at the asymmetric scission points, respectively. The asymmetric scission points have been determined by approximate methods without dynamical consideration of the descent from the saddle point to scission. The calculated limit estimates agree fairly well with the data but it was concluded that fully dynamical calculations are necessary. Our dynamical model is suitable for the solution of this problem, and these calculations will be performed in the future.

ACKNOWLEDGMENTS

We would like to thank Drs. K.-H. Schmidt, A. V. Karpov, A. Ya. Rusanov, and S. I. Mulgin for stimulating and useful discussions and correspondence. One of us (P.N.N.) is grateful to the Humboldt Foundation for financing the stay at GSI, Darmstadt. We are grateful to Dr. R. Dreyer for a careful reading of the manuscript and to M. V. Borunov for providing some calculations.

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