$^{16}O+^{16}O$ elastic scattering in an α-folding model

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An α folding potential for the $^{16}O+^{16}O$ system is presented based on an α-particle model of the nucleus ^{16}O. The angular distributions of the $^{16}O+^{16}O$ elastic scattering at incident energies ranging from 75 to 350 MeV have been calculated by using the α folding potential with an imaginary potential of standard Woods-Saxon type. The main features of the measured angular distributions can be satisfactorily described.

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I. INTRODUCTION

In recent years, there has been considerable interest in the study of light heavy-ion reactions such as $^{12}C+^{12}C$, $^{12}C+^{16}O$, and $^{16}O+^{16}O$. The refractive effects in the elastic scattering angular distributions of these systems are expected to provide information on the manifestation of the major features of the interaction between light heavy ions [1]. A large number of investigations have been devoted to the $^{16}O+^{16}O$ scattering. Since the publication of the measured 350 MeV elastic scattering cross section of ^{16}O on ^{16}O with high accuracy over a large angular range [2], many theoretical analyses have been performed [3–5]. Recently, detailed and complete angular distributions of the elastic $^{16}O+^{16}O$ scattering at nine energies between 75 and 124 MeV have been measured [6], and theoretical analyses of the data have been carried out [6,7]. In these analyses, either phenomenological potentials or folding model potentials based on an effective nucleon-nucleon interaction have been used, the experimental angular distributions can be well reproduced by the obtained potentials which share the common feature of deep real and shallow imaginary parts.

On the other hand, from the point of view of the nuclear cluster structure, ^{16}O is a typical α structure nucleus and is assumed to consist of four α-particles [8,9]. Then, the $^{16}O+^{16}O$ scattering can be regarded as the scattering between two 4α systems. In the framework of the folding model, we can obtain the double folding potential for the description of the $^{16}O+^{16}O$ scattering by folding the α-particle density in the ^{16}O nucleus with the α-α interaction. The available precise $^{16}O+^{16}O$ scattering experimental data over a wide range of energies and angles provide an opportunity to test different models for the optical potential. Previously, basing on the four α-particle model for the ^{16}O nucleus, we have presented an α folding potential for the $\alpha+^{16}O$ system [10]. The obtained potential can satisfactorily describe the experimental angular distributions of the elastic $\alpha+^{16}O$ scattering at incident energies between 25 to 54 MeV. In the present work, we will consider further the application of the 4α structure model of ^{16}O to the $^{16}O+^{16}O$ elastic scattering to examine the predictive ability of the model.

II. α-FOLDING MODEL POTENTIAL

From the viewpoint of the nuclear cluster structure, ^{16}O is a nucleus with four α-particle structure. Then, according to the folding model, the real part of the optical potential for the $^{16}O+^{16}O$ scattering can be represented by the double-folding potential:

$$V(R)=\int\int d\vec{r}_1 d\vec{r}_2 V_{\alpha\alpha}(\vec{R}+\vec{r}_1-\vec{r}_2)\rho_\alpha(\vec{r}_1)\rho_\alpha(\vec{r}_2), \quad (1)$$

where $\rho_\alpha(\vec{r})$ is the α-particle density distribution in the ^{16}O nucleus and $V_{\alpha\alpha}$ is the interaction between the α-particle in the projectile and the α-particle in the target.

For the α-α interaction $V_{\alpha\alpha}$, Buck *et al.* [11] have given a potential with the simple form

$$V_{\alpha\alpha}(r)=-122.6225\exp(-0.22r^2)\,\mathrm{MeV}. \quad (2)$$

This potential can accurately reproduce the measured α-α scattering phase shifts for center-of-mass (c.m.) energies up to 20 MeV and approximately reproduce the experimental data for energies up to 40 MeV.

From the α-particle structure wave function for the ^{16}O nucleus given in Refs. [10,12], the form factor describing the α-particle distribution in the ^{16}O nucleus can be obtained as

$$\eta_\alpha(q)=\Theta(q)\left[1-\left(\frac{1}{6}+\frac{\sqrt{6}}{12}\right)(aq)^2+\frac{1}{48}(aq)^4\right]e^{-a^2q^2/4}, \quad (3)$$

where a is the harmonic-oscillator constant (for ^{16}O, $a=1.2$ fm), and $\Theta(q)=e^{a^2r^2/16}$ is the c.m. correction factor for an independent particle wave function. By multiplying the internal charge form factor of the α particle itself to the form factor, one obtains the total charge form factor for ^{16}O and can fit the experimental data of electron scattering on ^{16}O very

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well. The form factor (3) can be directly used in the calculation of the α folding potential (1) performed in momentum space.

For the imaginary part of the interaction, we take the standard Woods-Saxon form

$$W(R) = -W_0 \left[1 + \exp\left(\frac{R - R_W}{a_W}\right)\right]^{-1}, \quad (4)$$

for the $^{16}O+^{16}O$ system $R_W = r_W(16^{1/3} + 16^{1/3})$.

Then the complex interaction potential used to describe the $^{16}O+^{16}O$ scattering is

$$U(R) = NV(R) + iW(R) + V_C(R), \quad (5)$$

where N is the renormalization factor. The Coulomb potential V_C used in our calculation is generated as the interaction between two uniformly charged spheres of radii equal to 3.54 fm as in Ref. [6].

III. COMPARISON WITH EXPERIMENTAL DATA AND DISCUSSION

The experimental data chosen for our study are the nine sets at incident energies between 75 and 124 MeV measured by the Strasbourg group [6]. In this incident energy interval, refractive effects in the angular distributions are observed. Analyses have shown [6,7] that in this incident energy interval the strength of the real part of the optical potential, indicated by J_R, is very weakly dependent on energy. From the α-folding model presented in Sec. II, the α-folding potential obtained from Eq. (1) by using the Buck's α-α potential is an energy-independent one. The best working energy range for the Buck's potential is $E_{\text{lab}} \leqslant 40$ MeV for the α incident energy. Then, for the $^{16}O+^{16}O$ scattering, the best applicable energy range of the Buck's potential is the ^{16}O incident energy $E_{\text{lab}} \leqslant 160$ MeV, that is the energy range of the Strasbourg data.

The calculated results of the nine elastic scattering angular distributions of the $^{16}O+^{16}O$ system at incident energies $E_{\text{lab}} = 75.0$, 80.6, 87.2, 92.4, 94.8, 98.6, 103.1, 115.9, and 124.0 MeV are shown in Fig. 1 in comparison with the experimental data. The corresponding values of N, parameters of the imaginary potential and the volume integrals of the real and the imaginary potentials are listed in Table I. One can see that the α-folding potential with a standard Woods-Saxon imaginary part can well describe the main features of the $^{16}O+^{16}O$ elastic scattering angular distributions, although detailed fits to the experimental data at some angles are not very satisfying. The renormalization factors $N \approx 0.85 \pm 0.04$ are required to get fit to experimental data, which show very weakly dependent on energy in this incident energy range. The obtained volume integrals of the real potential J_R are form 339 to 368 MeV fm^3 and the imaginary potentials J_I are from 34 to 62 MeV fm^3 (see Table I). The values of the volume integrals J_R and J_I are coincident with that obtained by the detailed analyses in Ref. [6]. This is also in agreement with the general feature of the optical potentials for the $^{16}O+^{16}O$ scattering: A relatively deep real part together with a rather weak absorption.

In our previous investigation for the $\alpha+^{16}O$ elastic scattering at incident energies between 25 and 54 MeV [10], we obtained that $J_R \approx 340$ MeV fm^3 and $N \approx 0.84$. In this work for the $^{16}O+^{16}O$ elastic scattering at $E_{\text{lab}} = 75$–124 MeV, $J_R \approx 350$ MeV fm^3, and $N \approx 0.85$ are obtained. It shows that the α-folding model is consistent for the $\alpha+^{16}O$ and the $^{16}O+^{16}O$ scattering at energies E/A between 5 and 8 MeV.

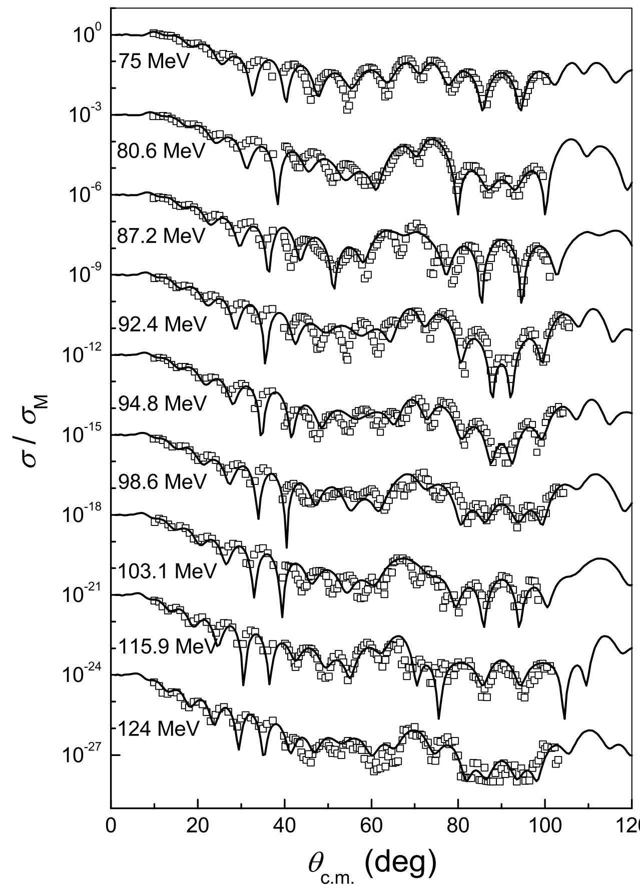

FIG. 1. Comparison of the α-folding model calculations with the experimental $^{16}O+^{16}O$ elastic scattering angular distributions between 75 and 124 MeV. The data are taken from Ref. [6]. The potential parameters are given in Table I. For clarity, with increasing energy, each data set has been multiplied by a factor of 10^{-3} relative to the preceding one.

TABLE I. Values of the renormaliztion factor, the parameters of the imaginary potential, and the volume integrals for $E_{\text{lab}} = 75$–124 MeV.

E_{lab} (MeV)	N	W_0 (MeV)	r_W (fm)	a_W (fm)	J_R (MeV fm^3)	J_I (MeV fm^3)
75.0	0.87	15.0	1.00	0.50	360	34
80.6	0.85	11.0	1.10	0.70	352	35
87.2	0.86	11.5	1.10	0.70	356	37
92.4	0.82	14.0	1.10	0.65	339	44
94.8	0.85	16.0	1.12	0.65	352	53
98.6	0.84	13.0	1.15	0.70	348	47
103.1	0.85	14.0	1.15	0.70	352	51
115.9	0.89	13.5	1.20	0.65	368	54
124.0	0.86	15.5	1.20	0.65	356	62

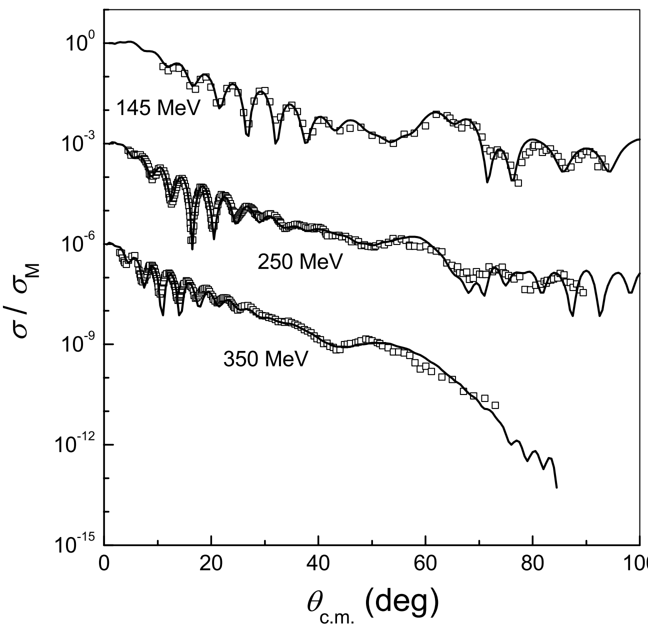

FIG. 2. The same as Fig. 1 but for $E_{\rm lab} = 145$, 250, and 350 MeV. The data are taken from Refs. [2,5,13,14]. The potential parameters are given in Table II. For clarity, with increasing energy, each data set has been multiplied by a factor of 10^{-3} relative to the preceding one.

There are higher energy experimental data of the ^{16}O+^{16}O elastic scattering angular distributions. Among those, the 145, 250, and 350 MeV data are in the best working energy range ($E_{\rm lab} \leqslant 160$ MeV) or in the approximately applicable energy range ($E_{\rm lab} \approx 160-320$ MeV) for the Buck's potential, as mentioned above. Figure 2 shows the α-folding model calculated results of the 145, 250, and 350 MeV ^{16}O+^{16}O elastic scattering angular distributions. The corresponding parameters are listed in Table II.

From Fig. 2, we see that the α-folding model can well reproduce the experimental data of the angular distributions of $E_{\rm lab} = 145$, 250, and 350 MeV. From Table II one notices that, in the higher energy region of 145−350 MeV, the renormalization factors $N \simeq 0.9-0.7$ are required to get fit to the experimental data. The magnitude of the deviation of the renormalization factor from unity is considered to be a measurement of the successfulness of the folding model. However, the larger difference of N from unity for higher energies of 250 and 350 MeV does not mean the failure of the α-folding model. As the α-α interaction used in the α-folding model denoted by Eq. (2) is energy independent and approximately applicable for the α incident energy range above $E_{\rm lab} = 40$ MeV. In fact, the α-α interaction will be reduced with the further increase in energy. This can be expected from the fact that the effective nucleon-nucleon interaction is reduced with the increase in energy. We can expect that, if an α-α interaction suitable for the higher energy region or an effective α-α interaction can be used instead of Eq. (2) in the α-folding model, the values of N in the higher energy region should be closer to 1 than those in Table II. Unfortunately, there is no α-α interaction available for the higher energy region at present. Therefore, we might well regard the decrease in N with increasing energy as the effect of the reduction in the strength of the α-α interaction with the increase in energy.

TABLE II. Values of the renormaliztion factor, the parameters of the imaginary potential and the volume integrals for $E_{\rm lab} = 145$–350 MeV.

$E_{\rm lab}$ (MeV)	N	W_0 (MeV)	r_W (fm)	a_W (fm)	J_R (MeV fm^3)	J_I (MeV fm^3)
145.0	0.89	16.0	1.22	0.65	368	68
250.0	0.72	26.0	1.12	0.75	298	90
350.0	0.66	31.0	1.08	0.70	273	95

There is a significant difference between the α-folding model and the usual folding model, i.e., the elementary interaction is a bare interaction for the former model, but an effective interaction for the latter one. It has been shown [1,15] that, the bare nucleon-nucleon potential, obtained from analysis of nucleon-nucleon scattering measurements, is too strong to be used directly, an effective nucleon-nucleon interaction must be used to describe the interaction of two nucleons immersed in nuclear medium. In the α-folding model, the used bare α-α potential is directly obtained by fitting the α-α scattering experimental data, the effects from the nuclear medium which require the use of an effective nucleon-nucleon interaction have been "automatically" included to a certain extent in the bare α-α potential. Therefore, this bare α-α potential could be used successfully to construct the α-folding model potential. The success of this model for the description of the elastic ^{16}O+^{16}O scattering suggests that the α-particle in the ^{16}O nucleus is slightly distorted. This is in agreement with the fact that an α-particle is bound much more weakly than a nucleon in the ^{16}O nucleus.

As is mentioned above, the intention of the present work is to examine the predictive ability of the α-folding model but not to devote to finding the optimum fit to the experimental data. For this purpose, to outstand the role of the real potential obtained from the α-folding model, we have simplified the phenomenological imaginary potential as standard Woods-Saxon type of three parameters. The obtained potential, which is coincident with the common accepted feature of deep real and shallow imaginary parts, can reproduce the main features of the ^{16}O+^{16}O elastic scattering angular distributions. We think, the α-folding model has been correctly examined. As can be observed in Figs. 1 and 2, the quality of the fits in this work is as good as that obtained by the phenomenological potential with the same simple Woods-Saxon imaginary potential of three parameters in Refs. [7,16]. One may hope to get better fits to the experimental data than that shown in Figs. 1 and 2 by adding the surface-peaked derivative term in the absorption, as pointed out in Ref. [6] that adding the surface term resulted in a good agreement with the experimental data. In this work we do not go further into calculations by adding the WSD term, as these do not give any more essential information to examine the model.

In summary, we have constructed an α-folding model to describe the ^{16}O+^{16}O scattering based on the α-particle structure of the ^{16}O nucleus, the angular distributions of the ^{16}O+^{16}O elastic scattering at incident energies between

75 and 350 MeV have been calculated by using the α-folding potential with a imaginary potential of standard Woods-Saxon type. Although, at some incident energies and angles, these calculations do not fit the data in detail, they succeed remarkably well in reproducing the main features of the measured angular distributions for all the considered incident energies. The success of the predictions once again provides a support to the α-folding model and the α-particle structure of the ^{16}O nucleus.

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