# Isospin symmetry breaking in an algebraic pairing Sp(4) model

K. D. Sviratcheva,[1] A. I. Georgieva,[1,2] and J. P. Draayer[1]

[1]*Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803, USA*

[2]*Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, Sofia BG-1784, Bulgaria*

(Received 23 August 2004; published 14 November 2005)

An exactly solvable $\mathfrak{sp}(4)$ algebraic approach extends beyond the traditional isospin-conserving nuclear interaction to bring forward effects of isospin symmetry breaking and isospin mixing resulting from a two-body nuclear interaction that includes proton-neutron (*pn*) and like-particle isovector pairing correlations plus significant isoscalar *pn* interactions. The model yields an estimate for the extent to which isobaric analog $0^+$ states in light and medium-mass nuclei may mix with one another and reveals possible, but still extremely weak, nonanalog $\beta$-decay transitions.

DOI: 10.1103/PhysRevC.72.054302 PACS number(s): 21.60.Fw, 23.40.Hc, 21.30.Fe, 21.60.Cs

## I. INTRODUCTION

A fundamental feature of nuclear structure is the basic symmetry between neutrons and protons, namely, the *charge independence* of the nuclear force, which is evident in the striking similarity in the energy spectra of nuclear isobars [1]. This implies that the proton-proton (*pp*) interaction and the neutron-neutron (*nn*) interaction are equal to the isospin $T = 1$ *pn* interaction and leads to "rotational" invariance in isotopic space. However, the isospin invariance is violated by the electromagnetic interaction, mainly the Coulomb repulsion between nucleons, which has become the focus of many phenomenological and microscopic studies [1–20].

The primary effect of the Coulomb force is to introduce into the theory a dependence on the third isospin projection, $T_0$, resulting in energy splitting of isobaric analog states (a $T$ multiplet) without coupling different isospin-multiplets. At the same time, the isospin-violating part of the Coulomb interaction leads to small isospin mixing in nuclear ground states, increasing with $Z$ and largest for $N = Z$. The ground-state isospin impurity is theoretically estimated to be as small as a percent for nuclei in the $1f_{7/2}$ level [1] and up to 4–5% toward the *fpg* shell closure [9]. Another source of mixing probability is the isospin-nonconserving part of the nuclear Hamiltonian, which includes effects resulting from the proton-neutron mass difference and small charge-dependent components in the strong nucleonic interaction [4]. Experimental results clearly reveal the existence of isospin mixing [21,22]. The increase in isospin mixing toward medium-mass nuclei has been detected in novel high-precision experiments [23–27], which continue to push the exploration of unstable nuclei with the advent of advanced radioactive beam facilities.

The violation of the charge independence of the nuclear interaction is well established. The purely nuclear parts of the *pp* force and the $T = 1$ *pn* force differ from each other, which appears to be associated with the electromagnetic structure of the nucleons [4]. Analyses of $^1S$ scattering in the *pn* system and low-energy *pp* scattering lead to the estimate that the nuclear interaction between protons and neutrons ($V_{pn}^{T=1}$) in $T = 1$ states is more attractive than the force between the protons ($V_{pp}$) by 2%, that is, $|V_{pn}^{T=1} - V_{pp}|/V_{pp} \sim 2\%$ [28]. In addition, the charge asymmetry between the *pp* and *nn* interactions was found to be smaller (less than 1%) [29]. More recent investigations confirm isospin violation in light nuclei [13,30–33]. These studies start with modern charge-dependent realistic interactions [34–37] and include valuable input information from particle physics (see, e.g. [38,39]). Furthermore, after the Coulomb energy is taken into account the discrepancy in the isobaric-multiplet energies is bigger for the seniority-zero levels as compared to higher seniority states, indicating the presence of a short-range, charge-dependent interaction [7]. Indeed, the $J = 0$ pairing correlations have been recently shown to have an overwhelming dominance in the isotensor energy difference within isobaric multiplets [40], which manifests itself in the charge-dependent $T = 2$ nature of the pairing interaction.

These findings point out the need for a charge-dependent microscopic description of $J = 0$ pairing correlations. An algebraic $\mathfrak{sp}(4)$ approach is ideally suited for this purpose [41,42] for it combines, on the one hand, a microscopic modeling of the pairing interaction and its charge dependence, and on the other hand, a straightforward scheme for estimating the significance of the isospin mixing resulting from pairing correlations without the need for carrying out large-dimensional matrix diagonalizations. Strong isospin breaking in pair formation, if found, implies a significant presence of isospin admixture among the seniority-zero isobaric analog $0^+$ states including $0^+$ ground states. This in turn will affect the predictive power of precise studies of superallowed $0^+ \rightarrow 0^+$ Fermi $\beta$-decay transitions. This is because the latter provide reliable tests of isospin mixing (see [43] for a review) and as well furnish a precise test of the unitary condition of the Cabbibo-Kobayashi-Maskawa matrix [44] (for a review of this subject, see [45]).

Our objective is to explore isospin mixing beyond that resulting from the Coulomb interaction, which is isolated with the help of an advanced Coulomb correction formula [46]. Specifically, we focus on the isospin-nonconserving part of the *pure* nuclear interaction, which recently has been found to be at least as important as the Coulomb repulsion [40]. The outcome of this study shows the significance of the pairing charge dependence and its role in mixing isospin multiplets of pairing-governed isobaric analog $0^+$ states.

 ©2005 The American Physical Society

## II. THEORETICAL FRAMEWORK: THE REASONABLE APPROXIMATION

We employ a simple but powerful group-theoretical model, which is based on the $\mathfrak{sp}(4)$ algebra (I which is isomorphic to $\mathfrak{so}(5)$ [47–49]). The Sp(4) microscopic model is precisely suitable for the qualitative study of isospin violation in isobaric analog $0^+$ states because it naturally extends the isospin-invariant nuclear interaction to incorporate isospin-nonconserving forces, while it retains the Sp(4) dynamical symmetry of the Hamiltonian (see [50] for a review on dynamical symmetries).

A comparison with experimental data demonstrates that the Sp(4) model provides a reasonable description of the pairing-governed isobaric analog $0^+$ states[1] in light and medium-mass nuclei, where protons and neutrons occupy the same shell [41,42,51]. The two-body model interaction includes proton-neutron and like-particle pairing plus symmetry terms and contains a non-negligible implicit portion of the quadrupole-quadrupole interaction [52]. Moreover, the Sp(4) model interaction itself, which relates to the whole energy spectrum rather than to a single $J^\pi = 0^+, T = 1$ state, was found to be quite strongly correlated (0.85) with the realistic CD-Bonn + 3 terms interaction [53] in the $T = 1$ channel and with an overall correlation of 0.76 with the realistic GXPF1 interaction [54] for the $1f_{7/2}$ orbit [52]. In short, the relatively simple Sp(4) model seems to be a reasonable approximation that reproduces especially that part of the interaction responsible for shaping pairing-governed isobaric analog $0^+$ states.

The Sp(4) model reflects the symplectic dynamical symmetry of isobaric analog $0^+$ states [42] determined by the strong nuclear interaction. The weaker Coulomb interaction breaks this symmetry and significantly complicates the nuclear pairing problem. This is why, in our investigation, we adopt a sophisticated phenomenological Coulomb correction to the experimental energies such that a nuclear system can be regarded as if there is no Coulomb interaction among its constituents. The *Coulomb-corrected* experimental energy, $E_{\mathrm{exp}}$, for given valence protons $N_{+1}$ and neutrons $N_{-1}$ is adjusted to be

$$E_{\mathrm{exp}}(N_{+1}, N_{-1}) = E^C_{\mathrm{exp}}(N_{+1}, N_{-1}) - E^C_{\mathrm{exp}}(0, 0) + V_{\mathrm{Coul}}(N_{+1}, N_{-1}), \tag{1}$$

where $E^C_{\mathrm{exp}}$ is the total measured energy including the Coulomb energy, $E^C_{\mathrm{exp}}(0, 0)$ is the binding energy of the core, and $V_{\mathrm{Coul}}(N_{+1}, N_{-1})$ is the Coulomb correction for a nucleus with mass $A$ and $Z$ protons taken relative to the core: $V_{\mathrm{Coul}}(N_{+1}, N_{-1}) = V_{\mathrm{Coul}}(A, Z) - V_{\mathrm{Coul}}(A_{\mathrm{core}}, Z_{\mathrm{core}})$. The recursion formula for the $V_{\mathrm{Coul}}(A, Z)$ Coulomb energy is derived in [46] with the use of the Pape and Antony formula [55]. The Coulomb-corrected energies (1) should reflect solely the nuclear properties of the many-nucleon systems.

[1]The lowest among these states include ground states for even-even nuclei and only some ($N \approx Z$) odd-odd nuclei, as well as, for example, low-lying $0^+$ states in odd-odd nuclei that have the same isospin as the ground state of a semimagic even-even isobaric neighbor with fully paired protons (or neutrons).

Assuming charge independence of the nuclear force, the general isoscalar Hamiltonian with Sp(4) dynamical symmetry, which consists of one- and two-body terms and conserves the number of particles, can be expressed through the Sp(4) group generators,

$$H_0 = -G \sum_{i=-1}^{1} \hat{A}^\dagger_i \hat{A}_i - \frac{E}{2\Omega}\left(\hat{T}^2 - \frac{3\hat{N}}{4}\right) - C\frac{\hat{N}(\hat{N}-1)}{2} - \epsilon \hat{N}, \tag{2}$$

where $\hat{T}^2 = \Omega\{\hat{T}_+, \hat{T}_-\} + \hat{T}_0^2$ and $2\Omega$ is the shell dimension for a given nucleon type. The generators $\hat{T}_\pm$ and $\hat{T}_0$ are the valence isospin operators, $\hat{A}^{(\dagger)}_{0,+1,-1}$ create (annihilate), respectively, a proton-neutron ($pn$) pair, a proton-proton ($pp$) pair, or a neutron-neutron ($nn$) pair of total angular momentum $J^\pi = 0^+$ and isospin $T = 1$, and $\hat{N} = \hat{N}_{+1} + \hat{N}_{-1}$ is the total number of valence particles with an eigenvalue $n$. The $G$, $E$, and $C$ are interaction strength parameters and $\epsilon > 0$ is the Fermi level energy (see Table I in [42] for estimates). The isospin-conserving Hamiltonian (2) includes an isovector ($T = 1$) pairing interaction ($G \geqslant 0$ for attraction) and a diagonal isoscalar ($T = 0$) force, which is related to a symmetry term ($E$).

Charge-dependent but charge-symmetric nucleon-nucleon interaction ($V_{pp} = V_{nn} \neq V_{pn}$) brings into the nuclear Hamiltonian a small isotensor component (with zero third isospin projection so that the Hamiltonian commutes with $T_0$). This is achieved in the framework of the Sp(4) model by introducing two additional terms,

$$H_{\mathrm{IM}} = -F\hat{A}^\dagger_0\hat{A}_0, \qquad H_{\mathrm{split}} = -D\left(\hat{T}_0^2 - \hat{N}/4\right), \tag{3}$$

to the isospin-invariant model Hamiltonian (2) in a way that the Hamiltonian

$$H = H_0 + H_{\mathrm{IM}} + H_{\mathrm{split}} \tag{4}$$

possesses Sp(4) dynamical symmetry. In other words, charge dependence is introduced into the pairing Hamiltonian (2) by allowing the strength of two of the underlying interactions to vary. The interaction strength parameters $F$ and $D$ (3) determined in an optimum fit over a significant number of nuclei (149) [41] are given in Table I and yield nonzero values. These parameters yield quantitative results that are superior to the ones with $F = 0$ and $D = 0$; for example, in the case of the $1f_{7/2}$ level the variance between the model and experimental energies of the lowest isobaric analog $0^+$ states increases by 85% when the $D$ and $F$ interactions are

TABLE I. Interaction strength parameters related to the isospin problem for three regions of nuclei specified by the valence model space. $F$, $D$, and $E$ are in MeV.

| Strength parameters | Model space | | |
|---|---|---|---|
| | $(1d_{3/2})$ | $(1f_{7/2})$ | $(1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2})$ |
| $F/\Omega$ | 0.007 | 0.072 | 0.056 |
| $D$ | 0.127 | 0.149 | −0.307 |
| $\lvert D/\frac{E}{2\Omega}\rvert$ | 0.090 | 0.133 | 0.628 |

turned off. For the present investigation the parameters in (2) along with $F$ and $D$ (3) are not varied as their values were fixed to be physically valid and to yield reasonable energy [41,42] and fine-structure [51] reproduction for light and medium-mass nuclei with valence protons and neutrons occupying the same shell. For these nuclei in the mass range $32 \leqslant A \leqslant 100$, the pairing-governed isobaric analog $0^+$ states are well described, but still approximately, by the eigenvectors of the effective Hamiltonian (4) in a basis of fully paired $0^+$ states [42].

Whereas the second interaction ($H_{\text{split}}$) in (3) takes into account only the splitting of the isobaric analog energies, the first correction induces small isospin mixing (IM). The IM interaction (3) does not account for the entire interaction that mixes states of same angular momentum and parity but different isospin values. It only describes a possible $\Delta T = 2$ mixing between isobaric analog $0^+$ states owing to a pure nuclear-pairing interaction. Although the extent of such isospin admixing is expected to be smaller than the total mixing resulting from isospin-nonconserving terms [6,10,11, 13,43], it may influence precise model calculations depending on the importance of the charge dependence in pairing correlations.

## III. ISOSPIN-INVARIANCE BREAKING AND ISOSPIN MIXING

The estimate for the model parameters (Table I) can determine the extent to which isospin symmetry is broken while $T$ remains a good quantum number. Breaking of the isospin invariance $|D/\frac{E}{2\Omega}|$ (Table I) is in general negligible for light nuclei ($1d_{3/2}$ and $1f_{7/2}$ levels) in agreement with the experimental data. For medium-mass nuclei in the $1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2}$ major shell isospin breaking is significantly greater. Furthermore, as expected from observations, for the $1d_{3/2}$ level the interaction strengths of all *pn*, *pp*, and *nn* pairing are almost equal ($T$ is a good quantum number), $F \approx 0$ (Table I), and they differ for the $1f_{7/2}$ and for the $1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2}$ shells, with the *pn* isovector strength being more attractive, $F > 0$. Indeed, the $F$ isospin mixing interaction strength is extremely small and hence a charge-independent nuclear interaction (where $F$ is neglected) comprises a quite reasonable approximation. The latter yields major simplifications to the pairing problem and consequently most isovector pairing studies have been done by assuming good isospin.

The question regarding the strength of individual isospin-nonconserving nuclear interactions [such as (3)] still remains open—there are no definitive answers at the present level of experimental results and microscopic theoretical interpretations. Only their overall contribution is revealed by the free nucleon-nucleon data [28] and it is found to be slightly (by 2%) more attractive in the *pn* $T = 1$ system than in the *pp* one. Within the framework of the Sp(4) model, the charge dependence of the pure nuclear interaction can be estimated through the comparison of the $T_0 = 0$ two-body model interaction [(4) with $\varepsilon = 0$] relative to the $T_0 = 1$ in the $T = 1$ multiplets, which, for example in the $1f_{7/2}$ level, is on average $\sim$2.5%. This estimation does not aim to confirm the charge dependence, which is very difficult to pin down at this level of accuracy compared to the broad energy range considered in the model for nuclei with masses $32 \leqslant A \leqslant 100$. Nonetheless, it reflects the fingerprints of the experimental data in the properties of the model interaction (4).

In addition, the Sp(4) model reproduces reasonably well the $c$-coefficient in the well-known isobaric multiplet mass equation [2,8,56]

$$a + bT_0 + cT_0^2 \tag{5}$$

for the binding energies of isobaric analogs (of the same mass number $A$, isospin $T$, angular momentum $J$, etc.), where the coefficient $c$ ($b$) depends on the isotensor (isovector) component of the nuclear interaction [i.e., of rank 2 (1) with respect to isospin "rotations"]. The $c$ coefficient is indeed an energy filter; $2c = E(T_0 + 1) + E(T_0 - 1) - 2E(T_0)$, for a given mass number $A$ and isospin $T$. In the framework of our model, this energy function for the lowest isobaric analog $0^+$ states was found to be in a good agreement with observed fine-structure effects (where data were available) [51] and it reproduces the experimental staggering behavior with respect to $A$ (Fig. 1). Both theoretical and experimental results show this finite energy difference, when centered at an $N = Z$ odd-odd nucleus ($T_0 = 0$ and $A/2$ odd), and hence $c$, to be negative and very close to zero for $T = 1$ multiplets in the $1f_{7/2}$ shell (see Fig. 1 for $A/2$ odd). Such an agreement of the Sp(4) model outcome with experimental evidence is a valuable result. The requirement that the coefficients of (5) are well reproduced is essential for isospin-nonconserving models [6,10,40], which has been achieved in [6] by increasing (by approximately 2%) all the $T = 1$ *pn* matrix elements relative to the *nn* ones and which has led to a conclusion in [40] that the isotensor nature of the nuclear interaction is dominated by a $J = 0$ pairing term.

In short, the freedom allowed by introducing additional parameters (such as $F$ and $D$) reflects the symmetries observed in light nuclei (good isospin) and the comparatively larger symmetry breaking as expected in medium-mass nuclei. Hence, the charge dependence of the nuclear force, being a very challenging problem, yields results, based on a simple group-theoretical approach, that are qualitatively consistent with the observations.

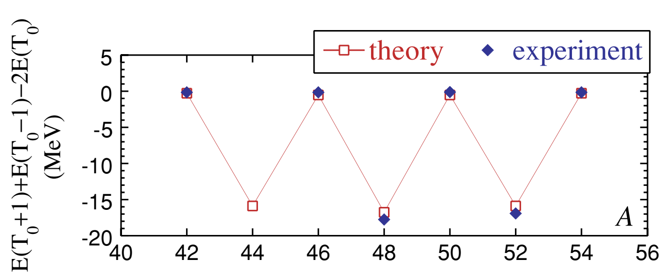


FIG. 1. (Color online) Energy difference, $E(T_0 + 1) + E(T_0 - 1) - 2E(T_0)$ with $T_0 = 0$, for the lowest isobaric analog $0^+$ states in nuclei in the $1f_{7/2}$ level [or twice the $c$ coefficient of (5) for the $A/2$-odd $T = 1$ multiplets] according to the Sp(4) model (red solid line with open squares) in comparison to the experiment (blue diamonds).

TABLE II. Sp(4) model estimate for the overlap (%) of isobaric analog $0^+$ states of almost good isospin $\tilde{T}$ with the states of definite isospin for $^{36}$Ar in the $1d_{3/2}$ level and the nuclei in the $1f_{7/2}$ level. The table is symmetric with respect to the sign of $n - 2\Omega$.

| $^{A}X^{(\tilde{T})}$ | $(N_{+1}, N_{-1})$ | $T=0$ | $T=1$ | $T=2$ | $T=3$ | $T=4$ |
|---|---|---|---|---|---|---|
| $^{36}$Ar$^{(0)}$ | (2, 2) | 99.9999 | — | 0.0001 | — | — |
| $^{44}$Ti$^{(0)}$ | (2, 2) | 99.90 | — | 0.10 | — | — |
| $^{46}$Ti$^{(1)}$ | (2, 4) | — | 99.98 | — | 0.02 | — |
| $^{46}$V$^{(1)}$ | (3, 3) | — | 99.98 | — | 0.02 | — |
| $^{46}$Cr$^{(1)}$ | (4, 2) | — | 99.98 | — | 0.02 | — |
| $^{48}$Ti$^{(2)}$ | (2, 6) | — | — | 99.997 | — | 0.003 |
| $^{48}$V$^{(2)}$ | (3, 5) | — | — | 99.994 | — | 0.006 |
| $^{48}$Cr$^{(0)}$ | (4, 4) | 99.83534 | — | 0.16465 | — | $10^{-5}$ |
| $^{48}$Cr$^{(2)}$ | (4, 4) | 0.143 | — | 99.849 | — | 0.008 |
| $^{48}$Mn$^{(2)}$ | (5, 3) | — | — | 99.994 | — | 0.006 |
| $^{48}$Fe$^{(2)}$ | (6, 2) | — | — | 99.997 | — | 0.003 |

### A. Near isospin symmetry of the isobaric analog $0^+$ states

Neither empirical evidence (such as scattering analysis and finite energy differences) nor the comparison of the model to experimental data (Table I) yield equal pairing strengths ($F \gtrsim 0$), resulting in a coupling of isospin eigenstates $|n, T, T_0\rangle$ from different isospin multiplets with a degree of mixing expected to be very small. Therefore, the eigenvectors of the total Hamiltonian (4), $|n(\tilde{T})T_0\rangle$, have an almost good isospin $\tilde{T}$ quantum number. Their overlap with the states of definite isospin values yields an estimate for the magnitude of the isospin admixture (see Table II for the $1d_{3/2}$ and $1f_{7/2}$ orbits):

$$\delta_{\tilde{T},T} = |\langle n, T, T_0 | n(\tilde{T})T_0\rangle|^2 \times 100[\%]. \tag{6}$$

The overlap percentages in Table II confirm that the nuclear lowest isobaric analog $0^+$ states have primarily isospin $T = |T_0|$ for even-even and $T = |T_0| + 1$ for odd-odd nuclei, with a very small mixture of the higher possible isospin values. As expected, the $\delta_{\tilde{T},T}$ isospin mixing increases as $Z$ and $N$ approach one another and toward the middle of the shell. For nuclei occupying a single-$j$ shell, mixing of isospin states is less than 0.17%. Although isospin mixing is negligible for light nuclei in the $j = 3/2$ orbit, it is clearly greater for the $j = 7/2$ level. The mixing is expected to be even stronger in multishell configurations.

### B. Nonanalog $\beta$-decay transitions

For a superallowed Fermi $\beta$-decay transition ($0^+ \to 0^+$) the $ft$ comparative lifetime is nucleus-independent according to the conserved-vector-current (CVC) hypothesis and is given by

$$ft = \frac{K}{G_V^2 |M_F|^2}, \quad K = 2\pi^3 \hbar \ln 2 \frac{(\hbar c)^6}{(m_e c^2)^5}, \tag{7}$$

where $K/(\hbar c)^6 = 8.120270(12) \times 10^{-7}$ GeV$^{-4}$ s, $m_e$ is the mass of the electron, and $G_V$ is the vector coupling constant for nuclear $\beta$ decay (see, e.g. [10]). $M_F$ is the Fermi matrix element $\langle \mathrm{F}|\sqrt{2\Omega}T_{\pm}|\mathrm{I}\rangle$ between a final (F) state with isospin projection $T_0^{\mathrm{F}}$ and an initial (I) states with $T_0^{\mathrm{I}}$ in a decay generated by the raising (for $\beta^-$ decay) and lowering ($\beta^+$) isospin transition operator[2] $\sqrt{2\Omega}T_{\pm}$, which in the framework of our model is given as

$$|M_F|^2 = 2\Omega |\langle \mathrm{F}; n(\tilde{T})T_0 \pm 1 | T_{\pm} | \mathrm{I}; n(\tilde{T})T_0\rangle|^2. \tag{8}$$

Typically, the isospin impurity caused by isospin-nonconserving forces in nuclei is estimated as a correction to the Fermi matrix element $|M_F|^2$ of the superallowed $\tilde{T}$ analog $0^+ \to 0^+$ transition: $\delta_C = 1 - |M_F|^2/\{\tilde{T}(\tilde{T}+1) - T_0^{\mathrm{F}}T_0^{\mathrm{I}}\}$. For more than two-state mixing, the degree of isospin admixture among isobaric analog $0^+$ states should be estimated using the normalized transition matrix element between nonanalog (NA) states (e.g. [43]),

$$\delta_{\mathrm{IAS}} = \frac{\left|M_F^{\mathrm{NA}}\right|^2}{\left\{\tilde{T}(\tilde{T}+1) - T_0^{\mathrm{F}}T_0^{\mathrm{I}}\right\}}, \tag{9}$$

where $\tilde{T}$ is the almost good isospin of the parent nucleus (see Table III for $1f_{7/2}$). In general, the $\delta_{\mathrm{IAS}}$ correction may be very different than the order of the $\delta_{\tilde{T},T}$ overlap quantity (6) presented in Table II because in decays the degrees of isospin mixing among nonanalog states within both the parent and daughter nuclei are significant.

Analysis of the results shows that the mixing among isobaric analog $0^+$ states (which is at least $\Delta T = 2$ mixing) is on average 0.006%, excluding even-even $N = Z$ nuclei. This is on the order of a magnitude less than the mixing of the first excited $0^+$ nonanalog state owing to isospin-nonconserving interaction, which is typically about 0.04% for the $1f_{7/2}$ level [21,43]. In addition, this yields nonanalog $\beta$ decays weaker than possible Gamow-Teller transitions; the strength of the latter is found to be less than 0.02% of the total beta decay strength for the nuclei in the $1f_{7/2}$ shell [21] and to substantially increase with increasing mass number $A$ [14,25,57]. This makes $\delta_{\mathrm{IAS}}$ mixing very difficult to detect especially when the isospin symmetry breaking correction ($\delta_C$)

[2]The factor of $\sqrt{2\Omega}$ appears because of the normalization of the basis operators adopted in the $\mathfrak{sp}(4)$ algebraic model.

TABLE III. Nonanalog $\beta$-decay transitions to energetically accessible $0^+$ states under consideration and the corresponding isospin mixing estimates $\delta_{\rm IAS}$ (9) according to the Sp(4) model for nuclei in the $1f_{7/2}$ level.

| $\beta$ decay | $\delta_{\rm IAS}$ |
|---|---|
| ${}^{A}_{Z}X^{(\tilde{T}_X)} \rightarrow {}_{Z-1}^{A}Y^{(\tilde{T}_Y)}$ | (%) |
| ${}^{44}_{23}V^{(2)} \rightarrow {}^{44}_{22}Ti^{(0)}$ | 0.098 |
| ${}^{46}_{25}Mn^{(3)} \rightarrow {}^{46}_{24}Cr^{(1)}$ | 0.0169 |
| ${}^{46}_{24}Cr^{(3)} \rightarrow {}^{46}_{23}V^{(1)}$ | 0.0104 |
| ${}^{46}_{23}V^{(3)} \rightarrow {}^{46}_{22}Ti^{(1)}$ | 0.00447 |
| ${}^{48}_{27}Co^{(4)} \rightarrow {}^{48}_{26}Fe^{(2)}$ | 0.00327 |
| ${}^{48}_{26}Fe^{(4)} \rightarrow {}^{48}_{25}Mn^{(2)}$ | 0.00280 |
| ${}^{48}_{25}Mn^{(4)} \rightarrow {}^{48}_{24}Cr^{(2)}$ | 0.00189 |
| ${}^{48}_{24}Cr^{(4)} \rightarrow {}^{48}_{23}V^{(2)}$ | 0.00103 |
| ${}^{48}_{23}V^{(4)} \rightarrow {}^{48}_{22}Ti^{(2)}$ | 0.00038 |
| ${}^{48}_{25}Mn^{(4)} \rightarrow {}^{48}_{24}Cr^{(0)}$ | $4.5 \times 10^{-7}$ |
| ${}^{48}_{25}Mn^{(2)} \rightarrow {}^{48}_{24}Cr^{(0)}$ | 0.14328 |
| ${}^{50}_{27}Co^{(3)} \rightarrow {}^{50}_{26}Fe^{(1)}$ | 0.0169 |
| ${}^{50}_{26}Fe^{(3)} \rightarrow {}^{50}_{25}Mn^{(1)}$ | 0.0104 |
| ${}^{50}_{25}Mn^{(3)} \rightarrow {}^{50}_{24}Cr^{(1)}$ | 0.00447 |
| ${}^{52}_{27}Co^{(2)} \rightarrow {}^{52}_{26}Fe^{(0)}$ | 0.098 |

to analog Fermi matrix elements in this level is on the order of a percent [9,14].

Not surprising, the largest values for the $\delta_{\rm IAS}$ correction are observed for $\Delta T = 2$ $\beta^{\pm}$ decays to energetically accessible $0^+$ ground states of even-even $N = Z$ nuclei (Table III). Although for these decays $\delta_{\rm IAS}$ is extremely small, (<0.14%), as expected for the contribution of the higher lying $0^+$ states [43], it is comparable to the order of isospin symmetry breaking corrections for the $1f_{7/2}$ orbit that are typically taken into account [43]. The reason may be that for the even-even $N = Z$ nuclei the second-lying isobaric analog $0^+$ states are situated relatively low owing to a significant *pn* interaction (Fig. 2).

Above all, the $\delta_{\rm IAS}$ results in Table III clearly show the overall pattern and the order of significance of the isospin mixing under consideration. This is evident within the first-order approximation in terms of the $F$ parameter ($F \ll 1$) of $\delta_{\rm IAS}$, which for $1f_{7/2}$ deviates on average by only 2% from its exact calculations in Table III. The $\delta_{\rm IAS}$ isospin mixing correction is then proportional to $F^2$ and one finds out that its order of magnitude remains the same for large variations of the $F$ parameter of more than 60%. In addition, greater $F$ values are not very likely because the $\delta_{\rm IAS}$ estimates (Table III) all fall below an upper limit, which does not contradict experimental and theoretical results for other types of isospin mixing.

Moreover, in this first-order approximation the ratio of any two isospin corrections for $1f_{7/2}$ is independent of the parameters of the model interaction. This implies that such a ratio does not reflect the uncertainties of the interaction strength parameters but rather is characteristic of the relative strength of both decays. It identifies the decay for which the maximum isospin mixing correction is expected in the $1f_{7/2}$ orbit, namely ${}^{48}_{25}Mn^{(2)} \rightarrow {}^{48}_{24}Cr^{(0)}$, as well as the amount by which $\delta_{\rm IAS}$ of the other possible nonanalog decays is relatively suppressed. For example, the $\delta_{\rm IAS}$ correction for the ${}^{44}_{23}V^{(2)} \rightarrow {}^{44}_{22}Ti^{(0)}$ decay is around two-thirds the maximum one and that for the ${}^{46}_{25}Mn^{(3)} \rightarrow {}^{46}_{24}Cr^{(3)}$ decay is around one-eighth the maximum one. Such ratios exhibit a general trend of increasing $\delta_{\rm IAS}$ isospin mixing with $Z$ within the same isospin multiplets and reveals enhanced $\Delta T = 2$ decays to the ground state of even-even $N = Z$ nuclei with increasing $\delta_{\rm IAS}$ toward the middle of the shell. Furthermore, this behavior continues for the nonanalog $\beta$ decays between nuclei with the same valence proton and neutron numbers as in Table III but occupying the $1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2}$ major shell. Therefore, among the nonanalog $\beta$ decays for the $A = 60$–64 isobars with valence protons and neutrons in the $1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2}$ shell the $\delta_{\rm IAS}$ isospin mixing of the ${}^{64}_{33}As^{(2)} \rightarrow {}^{64}_{32}Ge^{(0)}$ decay is expected to be the largest with a tendency of a further increase toward the middle of shell. In short, the significance of isospin mixing caused by charge-dependent $J = 0$ pairing correlations is evident from Table III for the $1f_{7/2}$ level and this trend continues for the upper *fp* shell.

The limited mixing of the $0^+$ isospin eigenstates from different isospin multiplets yields very small but nonzero $|M_F^{\rm NA}|^2$ matrix elements for nonanalog $\beta^{\pm}$ decay transitions, as indicated by $\delta_{\rm IAS}$ in Table III. For nuclei in the $1f_{7/2}$ shell, such nonanalog transitions to energetically accessible states are shown in Fig. 2 along with the *ft* values (where we use $K/G_V^2 = 6200$ s [60]). In the framework of the Sp(4) model, these values are symmetric with respect to the sign of $T_0$ (possible $\beta^-$ decays are not shown in Fig. 2) and of $n - 2\Omega$ (the $A = 50$ and $A = 52$ multiplets are analogous to the $A = 46$ and $A = 44$ ones, respectively). The results yield that 10 of the transitions are classified as forbidden ($\log_{10} ft \geqslant 7$); The other four are suppressed ($\log_{10} ft \approx 7$) and the four decays to the ground state of an even-even $N = Z$ nucleus appear to have comparatively larger decay rates ($\log_{10} ft \approx 6$). In Fig. 2 the theoretically calculated isobaric analog $0^+$ state a energies are shown together with the available experimental ones. It is worth mentioning that while the energies of the lowest isobaric analog $0^+$ states directly determined the parameters of the model interaction, a quite good reproduction of the experimental higher lying isobaric analog $0^+$ state energies followed without any parameter adjustment [42]. This outcome is important because the energy difference between two isobaric analog $0^+$ states within a nucleus directly affects the degree of their mixing. In summary, the theoretical Sp(4) model suggests the possible existence, albeit highly hindered, of $\Delta T = 2$ nonanalog $\beta$-decay transitions.

## IV. CONCLUSIONS

We employed a group-theoretical approach based on the Sp(4) dynamical symmetry to describe microscopically possible isospin mixing induced by a short-range charge-dependent nuclear interaction. The Sp(4) model interaction incorporates

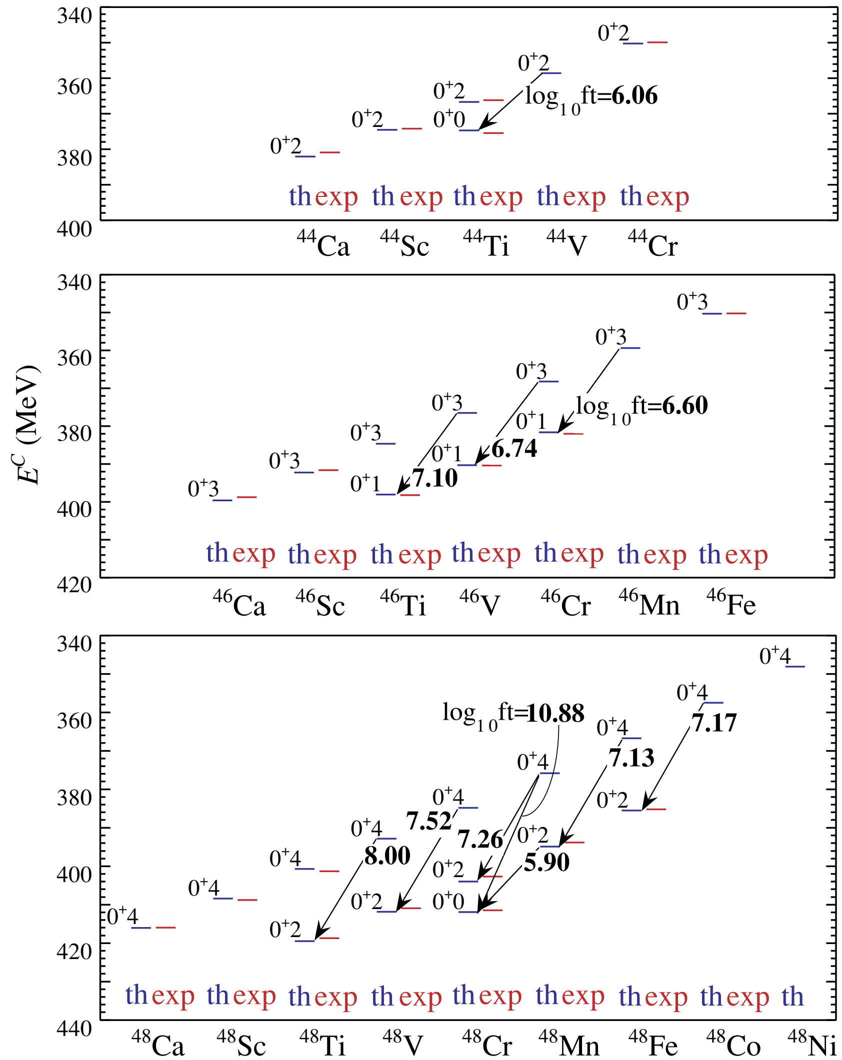


FIG. 2. (Color online) Nonanalog $0^+ \to 0^+$ $\beta^+$-decay transitions to energetically accessible $J^\pi \tilde{T}$ states under consideration indicated by arrows and by the corresponding theoretically calculated $\log_{10} ft$ values (which lack available experimental $ft$ values for comparison). The theoretical (blue, "th") and experimental [58,59] (red, "exp") binding energies $E^C$ in MeV (including the Coulomb potential energy) are shown for the isobar sequences with $A = 44$ to $A = 48$ in the $1f_{7/2}$ level.

the main driving forces, including $J = 0$ pairing correlations and an implicit quadrupole-quadrupole term, that shape the nuclear pairing-governed isobaric analog $0^+$ states in the $1f_{7/2}$ level where the Sp(4) Hamiltonian correlates strongly with realistic interactions. This approach provides a reasonable reproduction of the energies of the lowest isobaric analog $0^+$ states in a total of 149 nuclei. It also reproduces the available experimental energies of the higher lying isobaric analog $0^+$ states in the $1f_{7/2}$ level and fine structure effects without any variation of the parameters of the model interaction. In this respect, as predicted by our model, the coefficient related to the isotensor part of a general nonconserving force, $c$, which has been recently found to be dominated by a charge-dependent $J = 0$ pairing interaction [40], agrees quite well with the experimental values.

The breaking of isospin symmetry resulting from the coupling of isobaric analog $0^+$ states in nuclei was estimated to be extremely small for nuclei in the $1d_{3/2}$ and $1f_{7/2}$ orbitals, with the $N = Z$ even-even nuclei being an exception. For these nuclei, strong pairing correlations, including a significant $pn$ interaction, are responsible for the existence of comparatively greater isospin mixing, although the latter is still at least an order of a magnitude smaller than the overall isospin admixture in the ground state. The results also show that a variation of more than 60% in the $F$ isospin mixing parameter is required to reduce the present $\delta_{\rm IAS}$ results by an order of a magnitude.

The analysis also shows that there is a trend of increasing isospin mixing among isobaric analog $0^+$ states owing to a charge-dependent $J = 0$ pairing interaction toward the middle of the shell and for $\Delta T = 2$ decays to the ground state of an even-even $N = Z$ daughter nucleus. Such behavior is free of the uncertainties in the strength parameters of the interaction and is adequate for larger multi-$j$ shell domains such as $1f_{5/2}2p_{1/2}2p_{3/2}1g_{9/2}$. For nuclei with valence protons and neutrons occupying the $1f_{7/2}$ level the strongest nonanalog decay is identified to be $^{48}_{25}\mathrm{Mn}^{(2)} \to {}^{48}_{24}\mathrm{Cr}^{(0)}$, while the $\delta_{\rm IAS}$ isospin mixing correction for the rest of the decays that is 2/3 to 1/300 the maximum one.

In short, the $\mathfrak{sp}(4)$ algebraic model yields an estimate for the decay rates of possible nonanalog $\beta$-decay transitions resulting from a pure strong interaction, which, although a few of them may slightly affect precise calculations, are not expected to comprise the dominant contribution to the isospin symmetry breaking correction tested in studies of superallowed Fermi $\beta$-decay transitions.

## ACKNOWLEDGMENTS

The authors thank Dr. Vesselin G. Gueorguiev for his computational MATHEMATICA programs for noncommutative algebras. This work was supported by the U.S. National Science Foundation, Grant No. 0140300.

[1] A. Bohr and B. R. Mottelson, *Nuclear Structure*, Vol. I (Benjamin, New York, 1969); Vol. II (Benjamin, New York 1975).
[2] E. P. Wigner, *Proceedings of the Robert A. Welch Foundation Conference on Chemical Research* (R. A. Welch Foundation, Houston, Texas, 1957), Vol. 1, p. 67.
[3] H. T. Chen and A. Goswami, Nucl. Phys. **88**, 208 (1966).
[4] N. Auerbach, J. Hufner, A. K. Kerman, and C. M. Shakin, Rev. Mod. Phys. **44**, 48 (1972).
[5] G. F. Bertsch and B. H. Wildenthal, Phys. Rev. C **8**, 1023 (1973).
[6] I. S. Towner, J. C. Hardy, and M. Harvey, Nucl. Phys. **A284**, 269 (1977).
[7] R. D. Lawson, Phys. Rev. C **19**, 2359 (1979).
[8] W. Benenson and E. Kashy, Rev. Mod. Phys. **51**, 527 (1979).
[9] J. Dobaczewski and I. Hamamoto, Phys. Lett. **B345**, 181 (1995).
[10] W. E. Ormand and B. A. Brown, Nucl. Phys. **A440**, 274 (1985); Phys. Rev. C **52**, 2455 (1995).
[11] H. Sagawa, N. Van Giai, and T. Suzuki, Phys. Rev. C **53**, 2163 (1996).
[12] O. Civitarese, M. Reboiro, and P. Vogel, Phys. Rev. C **56**, 1840 (1997).
[13] P. Navratil, B. R. Barrett, and W. E. Ormand, Phys. Rev. C **56**, 2542 (1997).
[14] J. C. Hardy and I. S. Towner, Phys. Rev. Lett. **88**, 252501 (2002).
[15] A. F. Lisetskiy *et al.*, Phys. Rev. Lett. **89**, 012502 (2002).
[16] D. R. Bes and O. Civitarese, Nucl. Phys. **A741**, 60 (2004).
[17] S. Aberg, A. Heine, G. E. Mitchell, and A. Richter, Phys. Lett. **B598**, 42 (2004).
[18] N. Michel, W. Nazarewicz, and M. Ploszajczak, Phys. Rev. C **70**, 064313 (2004).
[19] A. Petrovici, K. W. Schmid, O. Radu, and A. Faessler, Nucl. Phys. **A747**, 44 (2005).
[20] R. Álvarez-Rodríguez, E. Moya de Guerra, and P. Sarriguren, Phys. Rev. C **71**, 044308 (2005).
[21] E. Hagberg *et al.*, Phys. Rev. Lett. **73**, 396 (1994).
[22] G. Savard *et al.*, Phys. Rev. Lett. **74**, 1521 (1995).
[23] P. D. Cottle *et al.*, Phys. Rev. C **60**, 031301(R) (1999).
[24] P. E. Garrett *et al.*, Phys. Rev. Lett. **87**, 132502 (2001).
[25] A. Piechaczek *et al.*, Phys. Rev. C **67**, 051305(R) (2003).
[26] E. Farnea *et al.*, Phys. Lett. **B551**, 56 (2003).
[27] J. Ekman, D. Rudolph, C. Fahlander, A. P. Zuker, M. A. Bentley, S. M. Lenzi, C. Andreoiu, M. Axiotis, G. de Angelis, E. Farnea, A. Gadea, Th. Kroll, N. Marginean, T. Martinez, M. N. Mineva, C. Rossi-Alvarez, and C. A. Ur, Phys. Rev. Lett. **92**, 132502 (2004).
[28] E. M. Henley, in *Isobaric Spin in Nuclear Physics*, edited by J. D. Fox and D. Robson (Academic, New York, 1966), p. 3.
[29] E. Baumgartner, H. E. Conzett, E. Shield, and R. J. Slobodrian, Phys. Rev. Lett. **16**, 105 (1966).
[30] P. Navratil, J. P. Vary, and B. R. Barrett, Phys. Rev. C **62**, 054311 (2000).
[31] R. Machleidt and H. Müther, Phys. Rev. C **63**, 034005 (2001).
[32] S. C. Pieper, V. R. Pandharipande, R. B. Wiringa, and J. Carlson, Phys. Rev. C **64**, 014001 (2001); S. C. Pieper and R. B. Wiringa, Annu. Rev. Nucl. Part. Sci. **51**, 53 (2001).
[33] P. Navratil and E. Caurier, Phys. Rev. C **69**, 014311 (2004).
[34] V. G. J. Stoks, R. A. M. Klomp, C. P. F. Terheggen, and J. J. de Swart, Phys. Rev. C **49**, 2950 (1994).
[35] R. B. Wiringa, V. G. J. Stoks, and R. Schiavilla, Phys. Rev. C **51**, 38 (1995).
[36] R. Machleidt, F. Sammarruca, and Y. Song, Phys. Rev. C **53**, 1483(R) (1996); R. Machleidt, Phys. Rev. C **63**, 024001 (2001).
[37] D. R. Entem and R. Machleidt, Phys. Rev. C **68**, 041001(R) (2003).
[38] S. A. Coon and M. D. Scadron, Phys. Rev. C **26**, 562 (1982); P. G. Blunden and M. J. Iqbal, Phys. Lett. **B198**, 14 (1987); S. A. Coon and R. C. Barrett, Phys. Rev. C **36**, 2189 (1987).
[39] T. Barnes, F. E. Close, and H. J. Lipkin Phys. Rev. D **68**, 054006 (2003).
[40] A. P. Zuker, S. M. Lenzi, G. Martinez-Pinedo, and A. Poves, Phys. Rev. Lett. **89**, 142502 (2002).
[41] K. D. Sviratcheva, A. I. Georgieva, and J. P. Draayer, J. Phys. G **29**, 1281 (2003).
[42] K. D. Sviratcheva, A. I. Georgieva, and J. P. Draayer, Phys. Rev. C **70**, 064302 (2004).
[43] I. S. Towner and J. C. Hardy, Phys. Rev. C **66**, 035501 (2002).
[44] N. Cabbibo, Phys. Rev. Lett. **10**, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
[45] I. S. Towner and J. C. Hardy, J. Phys. G **29**, 197 (2003).
[46] J. Retamosa, E. Caurier, F. Nowacki, and A. Poves, Phys. Rev. C **55**, 1266 (1997).
[47] K. T. Hecht, Nucl. Phys. **63**, 177 (1965); Phys. Rev. **139**, B794 (1965); Nucl. Phys. **A102**, 11 (1967).
[48] J. N. Ginocchio, Nucl. Phys. **74**, 321 (1965).
[49] J. Engel, K. Langanke, and P. Vogel, Phys. Lett. **B389**, 211 (1996).
[50] P. Van Isacker, Rep. Prog. Phys. **62**, 1661 (1999).
[51] K. D. Sviratcheva, A. I. Georgieva, and J. P. Draayer, Phys. Rev. C **69**, 024313 (2004).
[52] K. D. Sviratcheva, J. P. Draayer, and J. P. Vary (in preparation).
[53] S. Popescu, S. Stoica, J. P. Vary, and P. Navratil (to be published).
[54] M. Honma, T. Otsuka, B. A. Brown, and T. Mizusaki, Phys. Rev. C **69**, 034335 (2004).
[55] A. Pape and M. S. Antony, At. Data Nucl. Data Tables **39**, 201 (1988).
[56] W. E. Ormand, Phys. Rev. C **55**, 2407 (1997).
[57] I. Hamamoto and H. Sagawa, Phys. Rev. C **48**, R960 (1993).
[58] G. Audi and A. H. Wapstra, Nucl. Phys. **A595**, 409 (1995).
[59] R. B. Firestone and C. M. Baglin, *Table of Isotopes*, 8th ed. (Wiley, New York, 1998).
[60] J. B. Gerhart, Phys. Rev. **95**, 288 (1954); **109**, 897 (1958).