Φ measures of Bose-Einstein correlations for different multiplicity distributions in high energy heavy-ion collisions

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Φ measures of Bose-Einstein correlations are studied for four different pion multiplicity distributions. It is found that Φ measures depend strongly on the multiplicity distribution if the phase space density becomes large. To exclude the effects of the multiplicity fluctuations, Φ measures for fixed multiplicities should be calculated. The effects of multiplicity fluctuation on the Φ^* measure are also discussed.

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One of the main goals of relativistic heavy-ion collisions is to search for the quark-gluon plasma (QGP) [1]. It has been pointed out that the study of event-by-event fluctuations in high energy heavy-ion collisions may provide important information on collision dynamics [2–18]. The Φ measure was suggested in Ref. [9] and was studied by the NA49 Collaboration [19] at SPS energy. The Φ measure is defined as

$$\Phi_2 = \left(\frac{\langle Z^2\rangle}{\langle N\rangle}\right)^{1/2} - \left(\frac{\langle z^2\rangle}{\langle N\rangle}\right)^{1/2}, \tag{1}$$

with

$$\langle z(y)^2\rangle = \frac{1}{M}\sum_{j=1}^{M}\sum_{i=1}^{N_j} y(\mathbf{p_i})^2 \tag{2}$$

and

$$\langle Z(y)^2\rangle = \frac{1}{M}\sum_{j=1}^{M}\left[\sum_{i=1}^{N_j} y(\mathbf{p_i})\right]^2. \tag{3}$$

Here M is the total number of events, $\langle N\rangle$ is the mean pion multiplicity, and N_j is the multiplicity of the jth event. $y(\mathbf{p_i})$ is defined as

$$y(\mathbf{p}) = x(\mathbf{p}) - \bar{x}, \quad \bar{x} = \frac{1}{\langle N\rangle M}\sum_{j=1}^{M}\sum_{i=1}^{N_j} x(\mathbf{p_i}). \tag{4}$$

If $y(\mathbf{p_i})$ is multiplicity independent [20], we have [21–23]

$$\langle Z(y)^2\rangle = \int y^2(\mathbf{p})P(\mathbf{p})d\mathbf{p} + \int d\mathbf{p_1}d\mathbf{p_2}y(\mathbf{p_1})y(\mathbf{p_2})P(\mathbf{p_1},\mathbf{p_2}) \tag{5}$$

and

$$\langle z(y)^2\rangle = \int y^2(\mathbf{p})P(\mathbf{p})d\mathbf{p}. \tag{6}$$

Here $P(\mathbf{p})$ and $P(\mathbf{p_1},\mathbf{p_2})$ are single- and two-particle inclusive distributions. If

$$P(\mathbf{p_1},\mathbf{p_2}) = P(\mathbf{p_1})P(\mathbf{p_2}), \tag{7}$$

then

$$\Phi_2 = 0. \tag{8}$$

If AA collisions consist of only nn collisions and if there are no correlations among those nn collisions, then the value of the Φ measure should be zero. Thus, a nonzero value of the Φ measure indicates the existence of correlations and non-nn collisions in the AA collisions. The two-particle correlations may come from Bose-Einstein correlation, energy-momentum conservation, resonance decay, and others. It has been been pointed out in Refs. [23,24] that multipion Bose-Einstein correlations greatly affect the Φ_2 measure when the phase space density is high. Thus it should be very interesting to study multipion Bose-Einstein correlations effects on the Φ measure. In this Brief Report, we will calculate the Φ measure of pions by using a pion distribution function in both coordinate and momentum spaces. It has been shown in Ref. [25] that when coordinate space volume goes to infinity, multiparticle Bose-Einstein correlations will force pions to develop the Bose-Einstein distribution. So the results of Ref. [12] can be obtained in our calculations by letting the volume go to infinity.

We will see that there is a contribution to the Φ measure coming from fluctuations of the multiplicity, and to eliminate this contribution, in the second step, we will determine the Φ measure for fixed multiplicity.

The N pion (π^+ or π^-) distribution function for N pion events can be written as [26][1]

$$M_N(\mathbf{p_1},\ldots,\mathbf{p_N}) = \sum_{\sigma}\rho^{(N)}(\mathbf{p_1},\mathbf{p_{\sigma(1)}})\ldots\rho^{(N)}(\mathbf{p_N},\mathbf{p_{\sigma(N)}}), \tag{9}$$

with

$$\rho^{(N)}(\mathbf{p_i},\mathbf{p_j}) = \int g^{(N)}\left(x, \frac{p_i+p_j}{2}\right)\exp[i(p_i-p_j)\cdot x]dx. \tag{10}$$

[1]Here $M_N(\mathbf{p_1},\ldots,\mathbf{p_N})$ is not normalized. Its normalization factor is $1/[\omega(N)N!]$ [See Eq. (13)]. Since Eqs. (11) and (12) contain the ratio between two $\omega(\ldots)s$ thus, Eqs. (11) and (12) will not depend on this normalization factor.

Here $g^{(N)}(\mathbf{x}, \mathbf{K})$ is a Wigner function, which can be interpreted as the probability of finding a pion at point $\mathbf{x}$ with momentum $\mathbf{K}$ [26]. The superscript N denotes that these quantities are calculated for a system with pion multiplicity N. $\sigma(i)$ denotes the ith element of a permutation of the sequence $\{1, 2, \ldots, N\}$, and the sum over σ denotes the sum over all $N!$ permutations of this sequence. In the following we will ignore the fluctuation caused by the source sizes being different for different multiplicities. Thus, $\rho^{(N)}(p_i, p_j)$ is assumed to be multiplicity independent and we will ignore the superscript N in the following.

The single-pion and two-pion inclusive distributions read [25,27–29],

$$P(\mathbf{p}) = \sum_{m_1=1}^{\infty} \sum_{N=m_1}^{\infty} P(N) \frac{\omega(N-m_1)}{\omega(N)} G_{m_1}(\mathbf{p}, \mathbf{p}), \tag{11}$$

$$\begin{aligned} P(\mathbf{p_1}, \mathbf{p_2}) = & \sum_{m_1=1}^{\infty} \sum_{m_2=1}^{\infty} \sum_{N=m_1+m_2}^{\infty} P(N) \frac{\omega(N-m_1-m_2)}{\omega(N)} \\ & \times [G_{m_1}(\mathbf{p_1}, \mathbf{p_2}) G_{m_2}(\mathbf{p_2}, \mathbf{p_1}) \\ & + G_{m_1}(\mathbf{p_1}, \mathbf{p_1}) G_{m_2}(\mathbf{p_2}, \mathbf{p_2})], \end{aligned} \tag{12}$$

with

$$\begin{aligned} \omega(N) &= \frac{1}{N!} \int d\mathbf{p_1} \ldots d\mathbf{p_N} M_N(\mathbf{p_1}, \ldots, \mathbf{p_N}) \\ &= \frac{1}{N} \sum_{i=1}^{N} \omega(N-i) \int d\mathbf{p} G_i(\mathbf{p}, \mathbf{p}), \\ \omega(0) &= 1, \quad G_1 = \rho(\mathbf{p}, \mathbf{p}), \end{aligned} \tag{13}$$

and

$$G_i(\mathbf{p}, \mathbf{q}) = \int \rho(\mathbf{p}, \mathbf{p_1}) d\mathbf{p_1} \rho(\mathbf{p_1}, \mathbf{p_2}) \cdots d\mathbf{p_{i-1}} \rho(\mathbf{p_{i-1}}, \mathbf{q}). \tag{14}$$

Here $P(N)$ is the pion multiplicity distribution.

In the following we will calculate the effect of quantum correlation on the Φ measure. If $g(\mathbf{x}, \mathbf{K})$ is a Gaussian function in both coordinate and momentum spaces, $\rho(\mathbf{p_i}, \mathbf{p_j})$ can be expressed as [25,27]

$$\begin{aligned} \rho(\mathbf{p_i}, \mathbf{p_j}) &= \int g\left(\mathbf{x}, \frac{\mathbf{p_i}+\mathbf{p_j}}{2}\right) \exp[i(\mathbf{p_i}-\mathbf{p_j}) \cdot x] d\mathbf{x} \\ &= \frac{1}{(2\pi\Delta^2)^{\frac{3}{2}}} \exp\left[-\frac{(\mathbf{p_i}-\mathbf{p_j})^2 R^2}{2}\right] \\ &\quad \times \exp\left[-\frac{(\mathbf{p_i}+\mathbf{p_j})^2}{8\Delta^2}\right], \end{aligned} \tag{15}$$

where R and Δ are the source radius and temperature, respectively. In Fig. 1, $\Phi_2(p_t)$ versus mean pion multiplicity is shown for four different multiplicity distributions. In the calculation, we have chosen $x = p_t$(transverse momentum) in Eq. (4). With the help of Eqs. (1), (5), and (6), we have

$$\Phi_2(p_t) = \left(\frac{\langle Z^2 \rangle}{\langle N \rangle}\right)^{1/2} - \left[\frac{1}{\langle N \rangle} \int P(\mathbf{p}) y^2(p_t) d\mathbf{p}\right]^{1/2}, \tag{16}$$

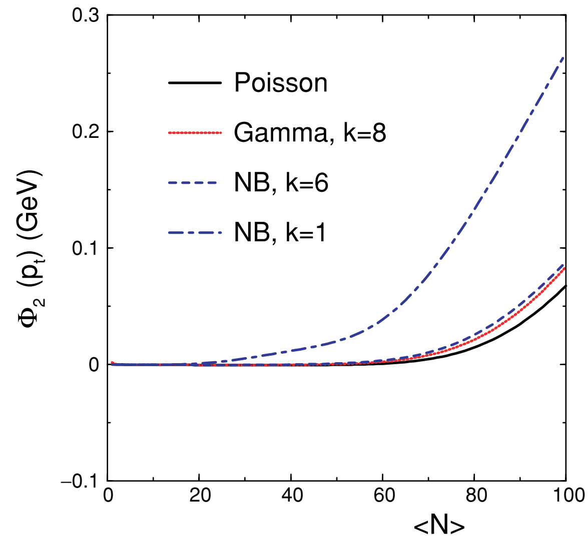

FIG. 1. (Color online) $\Phi_2(p_t)$ vs mean pion multiplicity $\langle N \rangle$. In the calculation we choose $\Delta = 0.175$ GeV, $R = 6.5$ fm. We choose the pion multiplicity distribution $P(N)$ as Poisson form (solid line), gamma distribution for $k = 8$ (dotted line), and negative binomial distributions for $k = 6$ (dashed line) and $k = 1$ (dot-dashed line), respectively.

with

$$y(p_t) = p_t - \langle p_t \rangle, \quad \langle p_t \rangle = \frac{1}{\langle N \rangle} \int P(\mathbf{p}) p_t d\mathbf{p}, \tag{17}$$

where $\langle N \rangle$ is the mean multiplicity. Using the function $\rho(p_i, p_j)$ given in Eq. (15), we can calculate $G_{m_1}(p_1, p_2)$ according to Eq. (14) and $\omega(N)$ according to Eq. (13). Giving a function form of $P(N)$, we can calculate $P(\mathbf{p_1}, \mathbf{p_2})$ and $P(\mathbf{p})$. Substituting all those formulas into Eq. (16), we get $\Phi_2(p_t)$.

Because symmetrization effects of pions depend on the pion multiplicity distributions [25,27,29], the Φ measure will also depend on the pion multiplicity distribution [25,27,29]. We have calculated Φ measures in Fig. 1 for a negative binomial (NB) distribution,

$$P(N, k) = \frac{(N+k-1)!}{N!(k-1)!} \frac{\left(\frac{\langle N \rangle}{k}\right)^N}{\left(1+\frac{\langle N \rangle}{k}\right)^{N+k}}, \tag{18}$$

a γ distribution

$$P_\Gamma(N, k) = \frac{1}{\Gamma(k)} \left(\frac{kN}{\langle N \rangle}\right)^k \exp^{-kN/\langle N \rangle}, \tag{19}$$

and a Poisson distribution

$$P(N) = \frac{\langle N \rangle^N}{N!} \exp(-\langle N \rangle). \tag{20}$$

Because of fluctuations of the multiplicity, the $\Phi_2(p_t)$ are different for the four different multiplicity distributions. The larger fluctuations of pion multiplicity are, the larger the $\Phi_2(p_t)$ measure will be. In our calculations, the multiplicity

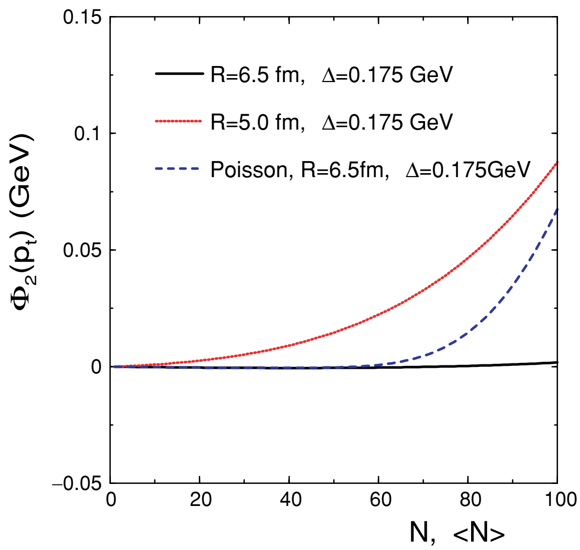

FIG. 2. (Color online) $\Phi_2(p_t)$ vs multiplicity N. In the calculation we choose $\Delta = 0.175$ GeV, $R = 6.5$ fm (solid line) and $R = 5.0$ fm (dotted line), respectively. For comparison, $\Phi_2(p_t)$ vs $\langle N\rangle$ for a Poisson distribution is also shown in the plot (dashed line).

fluctuations of the NB distribution for $k = 1$ (which is a Bose-Einstein distribution) is the largest among the four different multiplicity distributions; accordingly, the $\Phi_2(p_t)$ for the Bose-Einstein distribution is also the largest. For the Poisson distribution, the pion multiplicity fluctuation is the smallest among the four different multiplicity distributions; correspondingly, the $\Phi_2(p_t)$ measure for the Poisson distribution is also the smallest. It is also seen from Fig. 1 that when the mean multiplicity increases, the corresponding phase space density $\langle N\rangle/(R\Delta)^3$ increases and the differences among the Φ measures for the four different multiplicity distributions become large.

In heavy-ion collisions, $\Phi_2(p_t)$ consists of two parts: the fluctuation of the multiplicity for different events and the fluctuation of events with the same multiplicity. To study the fluctuations of events with the same multiplicity, $\Phi_2(p_t)$ will be calculated for events with a fixed multiplicity. In Fig. 2, $\Phi_2(p_t)$ for fixed multiplicities are shown for two sets of different parameters. This calculation of $\Phi_2(p_t)$ is similar to that in Fig. 1 and the only difference is that the multiplicity distribution is $P(N) = \delta_{N,N_0}$. It is clear that when phase space density is large, the $\Phi_2(p_t)$ measure will become large. This observation is similar to the behavior of Φ_2 in Fig. 1. But the multiplicity fluctuation is excluded. For comparison, $\Phi_2(p_t)$ versus $\langle N\rangle$ for a Poisson distribution is also shown in the plot. The $\Phi_2(p_t)$ values for the Poisson distribution are larger than those of fixed multiplicities. This is because we need to take an average over events with different multiplicities for the Poisson case and because Bose-Einstein correlations are large for large multiplicity events.

The Φ^* measure was suggested in Ref. [24] and was defined as

$$\Phi^* = \frac{\langle Z^2\rangle}{\langle N\rangle} - \frac{\langle z^2\rangle}{\langle N\rangle}. \tag{21}$$

The definitions of $\langle Z^2\rangle$ and $\langle z^2\rangle$ are the same as in Eq. (1). Applying Eq. (21) for events with multiplicity N, we have

$$\Phi^*_N = \frac{\langle Z^2\rangle}{N} - \frac{\langle z^2\rangle}{N}. \tag{22}$$

Here the superscript N denotes that this calculation is done for the events with multiplicity N. It is easily checked that for all events with a multiplicity distribution $P(N)$, we have

$$\langle Z^2\rangle = \sum_N P(N)\langle Z^2\rangle_N \tag{23}$$

and

$$\langle z^2\rangle = \sum_N P(N)\langle z^2\rangle_N. \tag{24}$$

Thus

$$\Phi^* = \frac{1}{\langle N\rangle}\sum_N P(N)N\Phi^*_N. \tag{25}$$

Equation (25) shows clearly that the Φ^* measure consists of fluctuations of the multiplicity and fluctuations within the events of the same multiplicity. To exclude the fluctuations of the multiplicity, the Φ^* measure should be calculated for events with fixed multiplicities.

In summary, starting from a pion source distribution in coordinate and momentum spaces, we have calculated Φ measures of Bose-Einstein correlations. It is shown that $\Phi_2(p_t)$ will be less influenced by the Bose-Einstein correlation when phase space density is low [30]. The Φ_2 measure is also affected by the fluctuations of the multiplicity. Because of this, we have calculated $\Phi_2(p_t)$ for fixed multiplicities and the fluctuations of the multiplicity can be excluded. We have shown that $\Phi_2(p_t)$ measures correlations of the two-particle spectrum. The origins of this correlations may be due to QGP, multiparticle symmetrization correlations, energy-momentum conservation, and resonance decay. Therefore, a nonzero value of Φ cannot provide information on the correlation from the initial state if the final-state correlations are large. However, if final-state correlations are small, a comparison of Φ_2 of different energies might indicate some correlations from the initial state. Finally, because we need to take an average of many events to calculate the Φ measure, the Φ measure will not be a good measurement if there are only rare events undergoing phase transition [31]. Therefore it is still a challenge for physicists in this field to find a measurement that will be defined for a single event.

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