Neutron drip line in odd and even mass calcium and nickel nuclei

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Neutron-rich Ca and Ni nuclei have been studied in a spherical relativistic mean-field formalism in coordinate space. A δ interaction has been adopted to treat the pairing correlations for the neutrons. Odd nuclei have been treated in the blocking approximation. The effect of the positive-energy continuum and the role of pairing in the stability of nuclei have been investigated by use of the resonant-BCS approach. In Ca isotopes, $N = 50$ is no longer a magic number, whereas in Ni nuclei, a new magic number emerges at $N = 70$. There is a remarkable difference in the relative positions of the drip lines for odd and even isotopes. In Ca isotopes, the last bound even and odd nuclei are found to be ^{72}Ca and ^{59}Ca, respectively. In Ni isotopes, the corresponding nuclei are ^{98}Ni and ^{97}Ni, respectively. The origin of this difference in relative positions of the drip line in even and odd isotopes in the two chains is traced to the difference in the single-particle level structures and consequent modification in the magic numbers in the two elements. Pairing interaction is seen to play a major role. The effect of the width of the resonance states on pairing has also been investigated.

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In recent years, it has been possible to populate and study a number of neutron-rich nuclei from fusion evaporation reactions using radioactive ion beams as well as from fission fragment studies. The last bound neutrons in such a nucleus may lie very close to the continuum, and the effect of the positive-energy continuum on the structure of such nuclei should be studied carefully. Another very important aspect of nuclei near the drip line is the additional stability provided by the pairing interaction. It is important to study the effect of pairing by studying the odd mass nuclei along with the even mass ones. Although a number of such studies have been undertaken in lighter mass regions, nuclei in the medium and the heavy mass regions have not yet been studied in sufficient detail. In the present calculation, neutron-rich Ca and Ni nuclei have been studied by use of the relativistic mean-field (RMF) formalism in coordinate space.

RMF theory is a major tool of nuclear structure physics. Very often, the RMF equations are solved by expanding on a harmonic-oscillator basis. However, it is well known that the basis expansion approach on a harmonic-oscillator basis cannot explain the density in halo nuclei near the drip line because of slow convergence in the asymptotic region. Nonrelativistic Hartree-Fock-Bogoliubov (HFB) and relativistic Hartree-Bogoliubov (RHB) methods in coordinate space have emerged as two very accurate approaches of treating the nuclei very close to the drip line.

The effect of the states in the continuum has been incorporated into most of the calculations by the solution of the equations in coordinate space using the box normalization condition, thus replacing the continuum with a set of discrete positive-energy states. However, in this case, the single-particle energy levels depend on the size of the box chosen, leading to a rather unsatisfactory scenario. Nonrelativistic mean-field equations involving continuum states have been solved with exact boundary conditions [1,2] for zero-range and finite-range pairing forces. RMF equations involving continuum states have also been solved [3,4] with exact boundary conditions. All these calculations have also taken into account the effect of the width of the continuum levels. For example, Cao and Ma [4] have also compared their results with calculations that assume the continuum levels are of zero width. We have also used the RMF calculation in coordinate space including the continuum states, to study neutron-rich even-even C and Be nuclei in detail [5]. It has been observed that the results of this method and the more complicated RHB approach are in good agreement [3,5].

In the present work we study the structure of neutron rich Ca and Ni nuclei beyond ^{48}Ca and ^{68}Ni, respectively, using the RMF formalism in coordinate space with exact boundary conditions, paying particular attention to the odd mass nuclei. There have been numerous calculations on even-even neutron-rich Ca and Ni nuclei up to the neutron drip line that use RMF and nonrelativistic mean-field approaches. We list only a few of them that have been published recently. Nonrelativistic Hartree-Fock and HFB approaches have been utilized by Fayans *et al.* [6] and Im and Meng [7] to study Ca isotopes. The relativistic density-dependent Hartree-Fock approach [8] as well as the relativistic continuum Hartree-Bogoliubov (RCHB) method [9] have also been utilized to study neutron-rich isotopes of Ca and Ni [10,11]. Yadav *et al.* [12] have performed RMF+BCS calculations for Ca and Ni isotopes by discretizing the continuum with box normalization. The problem of wrong asymptotic behavior in the harmonic-oscillator basis has been overcome by use of expansion in a Woods-Saxon basis in a RMF calculation [13]. The RCHB approach particularly gives a very good description of the nuclei. Terasaki *et al.* [14] have studied even-even Ni isotopes from the proton to the neutron drip line by using the HFB approach in three dimensions. However, we have come across no mean-field calculations for odd Ni isotopes near the drip line. Even in Ca isotopes, the odd mass nuclei have not been studied in detail. The aim of

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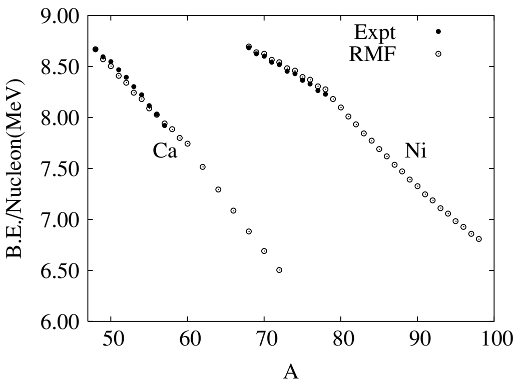

FIG. 1. Experimental and calculated binding energies per nucleon in Ca and Ni nuclei.

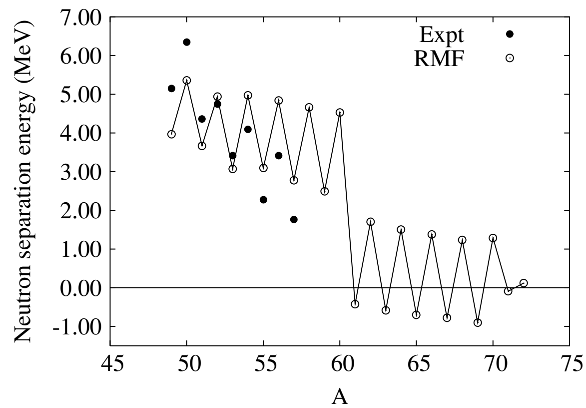

FIG. 2. Neutron separation energy in Ca isotopes.

the present work is to investigate the odd mass neutron-rich Ni and Ca nuclei up to the drip line.

The RMF theory is well known, and readers are referred to Refs. [15] for details. We have used the force NLSH [16]. We have assumed spherical symmetry for all the nuclei. The BCS calculation has been performed with a δ interaction adopted to treat the pairing correlations between neutrons, i.e., $V = V_0\delta(\vec{r}_1 - \vec{r}_2)$. The usual BCS equations now contain contributions from the bound states as well as the resonant continuum. The equations involving these states have already been obtained [1,3] and have been referred to as resonant BCS (rBCS) equations. We have also included the effect of the width of the positive-energy levels. These equations have been solved in the coordinate basis on a grid of size 0.08 fm. The positive-energy resonance solutions are obtained with the scattering approach. All the negative-energy states beyond $N = 20$ and the positive-energy states up to $N = 82$ for which resonance solutions are available have been included in the rBCS calculation for studying the Ca isotopes. For the Ni isotopes, the corresponding numbers are $N = 28$ and $N = 126$, respectively. We have assumed that, beyond 20 fm, the effect of nuclear interaction vanishes. We choose $V_0 = -350$ MeV for the strength of the δ interaction. This value of the strength has been used by Yadav *et al.* [12] in Ca nuclei. In an odd nucleus the last odd nucleon breaks the time-reversal symmetry of the system. However, it is well known that bulk quantities, such as binding energy or radii, are not affected by the breaking of the symmetry. Because we are interested only in these quantities, we have used the blocking approximation to study odd mass nuclei.

We have calculated the binding energy corresponding to the different levels in odd nuclei. Although the ground-state spin-parity is unknown in most of the nuclei, we find that, except for ^{75}Ni, the spin-parity has been correctly predicted wherever the ground state is unambiguously known. In Fig. 1 we plot the binding-energy values of the all the nuclei studied in the present work that are stable against neutron emission. The experimental or empirical values are from Ref. [17]. One can see that the theoretical values agree with experiment. In Figs. 2 and 3, we have plotted the one-neutron separation energies for Ca and Ni isotopes, respectively. The one-neutron separation energy has been defined as $S_n(Z, N) = \mathrm{B.E.}(Z, N) - \mathrm{B.E.}(Z, N-1)$, where B.E. is the binding energy. In Ni nuclei, the theoretical results are very close to experimental measurements. For the Ca isotopes, the agreement is slightly poorer. The calculated results for the two-neutron separation energy for the even mass nuclei are very close to the RCHB results obtained by Zhang *et al.* [11].

One of the most interesting differences between the two chains studied is the difference in the positions of the drip line for even and odd isotopes. For Ca isotopes, the last odd mass nucleus stable against neutron emission is ^{59}Ca, and the corresponding even mass isotope is ^{72}Ca. On the other hand, the corresponding nuclei in Ni are ^{97}Ni and ^{98}Ni, respectively. In other words, all the Ni isotopes are stable against neutron emission up to $N = 70$, beyond which neither the even nor the odd isotopes are stable for this element. For Ca the odd isotopes are stable below $N = 40$ whereas stable even isotopes range up to $N = 52$.

To understand this dramatic difference we next study the single-particle neutron levels. In Fig. 4 we have plotted the single-particle energy levels near the Fermi level for even-even Ca nuclei. The Fermi level is indicated by the solid line. Near the drip line the Fermi level rises slowly and finally becomes positive beyond ^{72}Ca. The states $1g_{9/2}$ and $2d_{5/2}$ start as positive-energy states but as the neutron number increases they become loosely bound states. Because of the absence of

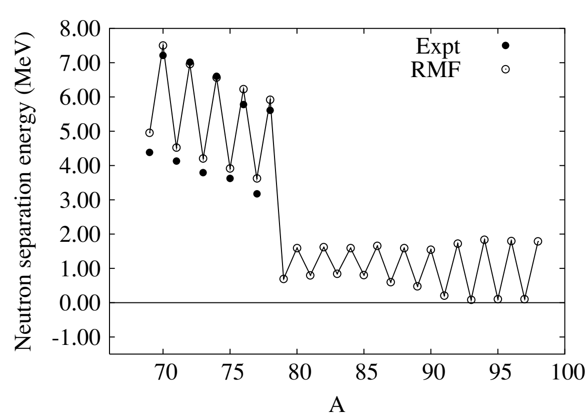

FIG. 3. Neutron separation energy in Ni isotopes.

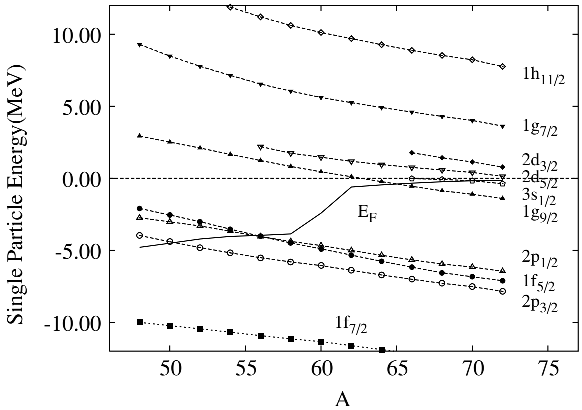

FIG. 4. Single-particle neutron energy levels in even Ca isotopes. The Fermi level is shown by the solid line.

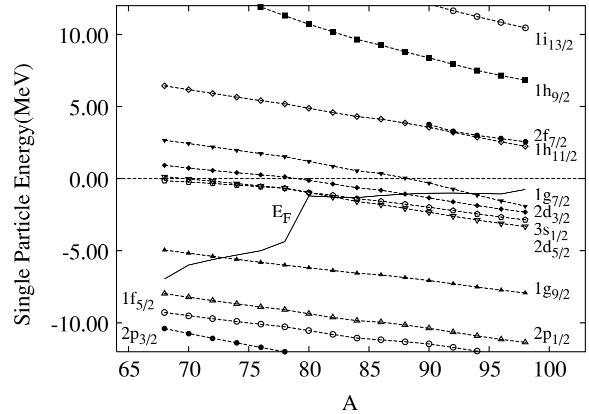

FIG. 6. Single-particle neutron energy levels in even Ni isotopes. The Fermi level is shown by the solid line.

the centrifugal barrier, the level $3s_{1/2}$ does not have a resonant solution. The levels $3s_{1/2}$, $2d_{5/2}$, and $2d_{3/2}$ lie very close to each other and are close to the $1g_{9/2}$ level in $^{66\text{–}72}$Ca. The shell closure is taken to be the neutron number for which the pairing energy vanishes. We find that in Ca isotopes, no shell closure is observed beyond $N = 40$ because of the modification in the energy levels. The Fermi level beyond $N = 40$ increases very slowly and lies very close to the continuum. The single-particle density is large near it. Hence pairing correlations that bind the loosely bound nucleons can develop in even-even nuclei. As pointed out by Meng *et al.* [18] these conditions indicate that a neutron halo should develop in these nuclei. In Fig. 5, we have plotted the neutron radii of the nuclei studied. The sudden increase in radius is clearly observed. Importantly, in Ca isotopes, the level $1g_{9/2}$ lies either in the continuum or just bound around $N = 40$. Hence the stability of the isotopes depends crucially on the pairing interaction and the contribution of the states in the continuum. Thus the odd mass Ca isotopes beyond $N = 40$ are unstable against neutron emission. There is a similar situation in light nuclei. The drip line nuclei ^{11}Li and ^{14}Be are known to have two neutron halos. It is known that neither ^{10}Li nor ^{13}Be are bound. The pairing interaction stabilizes the last neutron pair so that the above two drip line nuclei are stable against neutron emission.

On the other hand, the Ni isotopes present a completely different picture. The single-particle levels are shown in Fig. 6.

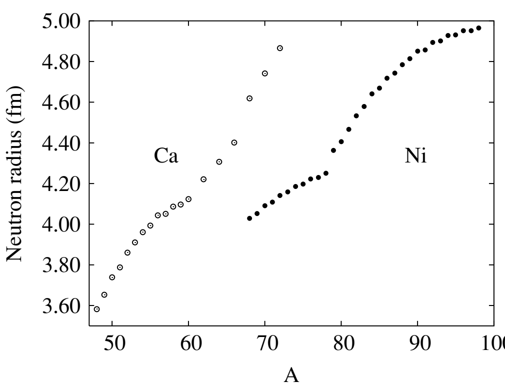

FIG. 5. Neutron radius in neutron-rich Ca and Ni nuclei.

Here, the Fermi level is deeper and abruptly becomes positive beyond $A = 98$. The levels $3s_{1/2}$, $1g_{7/2}$, $2d_{3/2}$, and $2d_{5/2}$ lie very close to each other. All of them except the first start as positive-energy states and become bound states at higher neutron numbers. The intruder orbit $1h_{11/2}$ lies much higher in energy. This is a consequence of the quenching of the spin-orbit splitting in neutron-rich nuclei that has been obtained in many mean-field calculations and also experimentally observed in light nuclei. Thus $N = 70$ becomes a new magic number. Because the occupied levels are much deeper, one does not expect a halolike structure, and the binding of the odd nucleus in this case does not depend overmuch on the pairing. Hence we obtain both even and odd mass nuclei that are stable against particle emission up to the new magic number $N = 70$. The calculated neutron radius values are shown in Fig. 5. One can see that the neutron radius shows a shell closure at $N = 50$ and, more importantly, at $N = 70$.

It is interesting to study the effect of the width of the resonant states in the nuclei very close to the drip line. In Table I, we present the results for some of these nuclei. The width of different levels, the binding energies, and the neutron radii (r_n) for ^{70,72}Ca and ^{96,98}Ni are presented. Pairing correlations usually become stronger if the width of the resonances is neglected [1,3], thus increasing the total binding energy and decreasing the neutron radius. One can see that, as ^{98}Ni is a closed shell nuclei, there is no effect of the resonance states beyond $N = 70$ in the binding energy and the radius.

To summarize, neutron-rich Ca and Ni nuclei have been studied by use of the RMF formalism in the coordinate space. Exact boundary conditions have been used to obtain the wave function of the states in the continuum. A δ interaction has been used for interaction between neutrons. Odd nuclei have been treated in the blocking approximation. The study of odd mass neutron-rich nuclei in this mass region reveals interesting features. There is a remarkable difference in the relative positions of the drip lines for odd and even isotopes in the two elements. In Ca isotopes, the last bound even and odd nuclei are found to be ^{72}Ca and ^{59}Ca, respectively. In Ni isotopes, the corresponding nuclei are ^{98}Ni and ^{97}Ni, respectively. The origin of this difference in the relative positions of the drip

TABLE I. Widths of different resonant levels and binding energies and neutron radii for calculations involving width (RMFW) and ignoring width (RMFN).

Nucleus	Level	Width (MeV)	B.E.(MeV)		r_n(fm)	
			RMFW	RMFN	RMFW	RMFN
^{70}Ca	$2d_{5/2}$	0.102	468.30	468.35	4.728	4.581
	$1g_{7/2}$	0.874				
	$1h_{11/2}$	1.524				
	$2d_{3/2}$	0.593				
^{72}Ca	$2d_{5/2}$	0.013	468.35	468.47	4.994	4.710
	$1g_{7/2}$	0.448				
	$1h_{11/2}$	1.501				
	$2d_{3/2}$	0.478				
^{96}Ni	$1h_{11/2}$	0.033	665.02	665.32	4.952	4.926
	$1h_{9/2}$	1.678				
	$1i_{13/2}$	2.474				
	$2f_{7/2}$	2.289				
^{98}Ni	$1h_{11/2}$	0.018	667.07	667.07	4.965	4.965
	$1h_{9/2}$	1.403				
	$1i_{13/2}$	2.446				
	$2f_{7/2}$	2.000				

line in even and odd isotopes in the two chains is traced to the difference in the single-particle level structures and the modification in the magic numbers in the two elements. In Ca isotopes, $N = 50$ is no longer a magic number, whereas in Ni nuclei, a new magic number emerges at $N = 70$. The level density near the continuum is higher in Ca isotopes, and pairing interaction plays a crucial role in the their stability. In odd mass nuclei, the pairing interaction is weaker and not able to bind the neutrons in the halo orbitals. If one neglects the width of the resonance states, the pairing correlation increases.

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