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Reappearance of the pairing correlations at finite temperature

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Rotational and deformation dependence of isovector and isoscalar pairing correlations at finite temperature are studied in an exactly solvable cranked deformed shell model Hamiltonian. It is shown that isovector pairing correlations, as expected, decrease with increasing deformation and the isoscalar pairing correlations remain constant at temperature, $T = 0$. However, it is observed that at finite temperature both isovector and isoscalar pairing correlations are enhanced with increasing deformation. It is also demonstrated that the pair correlations, which are quenched at $T = 0$ and high rotational frequency reappear at finite temperature. The changes in the individual multipole pairing fields as a function of rotation and deformation are analyzed in detail.

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The interplay between the deformation forces and the pairing correlations plays a fundamental role in our understanding of rotational nuclei [1,2]. The deformation leads to regular band structures and can be measured from the observed transition probabilities. Most of the nuclei with more than a few valance neutrons and protons are known to be deformed. The importance of the pairing correlations is inferred, for instance, from the odd-even mass differences and the moments of inertia of the rotational states. These correlations are known to decrease with increasing rotational frequency along the yrast line. The deformation and the pairing forces are understood to have opposite tendencies.

The dependence of the pairing correlations on temperature is mainly derived from the mean-field grand-canonical ensemble theory. The experimental data at finite temperature (excitation energy) are difficult to analyze because of the quasicontinuum γ-ray spectrum [3]. The mean-field models demonstrate that the pairing correlations drop with increasing temperature and depict a phase transition from the paired to the unpaired configuration, where the pairing has completely vanished. The mean-field based approach is appropriate for a macroscopic system, for example, a metallic superconductor. The experimental data for a bulk superconductor quite clearly depict a sharp transition as a function of temperature as predicted by the mean-field models. The application of the mean-field theory to finite mesoscopic systems also predict a sharp phase transition [4–6] as in the case of macroscopic systems. However, the experimental data and the exact solutions of some toy models do not depict any sharp transition and show a smooth phase transition from the paired to the unpaired phase [7,8]. The application of the mean-field theory to finite mesoscopic systems is quite inadequate as it does not contain the fluctuations. The fluctuations, as is known from statistical mechanics [9], are inversely proportional to the square root of the particle number and become important for a finite system. In particular, at finite temperature, the mean-field Slater determinant is a mixture of both even and odd particle numbers and the fluctuations become exceedingly important. At zero temperature, the mean-field wavefunction is mixture of only even or odd particle numbers. There has been some effort to investigate the effects of thermal fluctuations on the pairing energy and the quadrupole shape [10,11].

The projected mean-field theory, which incorporates fluctuations, is now available at zero temperature [12–14]. Recently, there has been an attempt to formulate the projection theory at finite temperature [15]. A partial particle number projection at finite temperature in which the particle number parity projection (even or odd particle number is projected out) has been recently studied in Ref. [16]. It has been shown, for a system with odd particle number, that the pairing correlations which are quenched at zero temperature and finite magnetic field (rotational frequency) reappear at finite temperature. This interesting and quite unexpected observation was further investigated in an exactly solvable model [8]. It was found that the pair correlations which were quenched at zero temperature either by an external magnetic field in a small superconductor or by rapid rotation in a nucleus, reappear at finite temperature. These "temperature induced pair correlations" were noted both for even and odd particle numbers. In Refs. [8,16] the temperature dependence of only the monopole pairing field has been investigated. It is quite interesting to study the temperature dependence of the higher multipole pair fields for nuclei, where these higher fields are known to be important. In particular, it is quite instructive to investigate the temperature and rotational dependence of the isoscalar pairing field, which has attracted considerable interest in recent years [17,18].

The purpose of the present work is to study the rotational and deformation dependence of isovector and isoscalar pairing correlations at finite temperature in an exactly solvable model. It is demonstrated that the isovector pairing correlations decrease with increasing deformation at $T = 0$ and the isoscalar pairing field remains constant with increasing deformation. However, at finite temperature the pairing correlations depict an increase with increasing deformation, which is totally unexpected and contradict the mean-field predictions for the specific model investigated in this paper.

The exactly solvable model Hamiltonian employed in the present study consists of a deformed one-body term ($h_{\rm def}$) and a scalar two-body delta-interaction (V_2) [19]. The one-body term is the familiar Nilsson mean-field potential which takes into account of the long-range part of the nucleon-nucleon interaction. The residual short-range interaction is specified by the delta-interaction. The model Hamiltonian is given by

$$\hat{H} = \hat{h}_{\rm def} + \hat{V}_2, \tag{1}$$

where,

$$\hat{h}_{\rm def} = -4\kappa\sqrt{\frac{4\pi}{5}}\sum_{i,j}\langle j|Y_{20}|i\rangle\delta_{\tau_i\tau_j}\delta_{m_im_j}c_j^\dagger c_i, \tag{2}$$

and

$$\hat{V}_2 = \frac{1}{4}\sum_{ijkl}\langle ij|v_a|kl\rangle c_i^\dagger c_j^\dagger c_l c_k = \frac{1}{2}\sum_{JMtt_z} v_{Jt}A^\dagger_{JM;tt_z}A_{JM;tt_z}, \tag{3}$$

with $A^\dagger_{JM;tt_z} = (c^\dagger_{j1/2}c^\dagger_{j1/2})_{JM;tt_z}$ and $A_{JM;tt_z} = (A^\dagger_{JM;tt_z})^\dagger$. The labels $i, j, \ldots$ in the above equations denote the magnetic quantum number and the isospin projection quantum number τ ($\tau = 1/2$ for neutrons and $\tau = -1/2$ for protons). For the antisymmetric-normalized two-body matrix-element (v_{Jt}), we use the delta-interaction, which for a single j-shell is given by [20]

$$v_{Jt} = -G\frac{(2j+1)^2}{2(2J+1)}\left\{\begin{bmatrix} j & j & J\\ \frac{1}{2} & -\frac{1}{2} & 0\end{bmatrix}^2 + \frac{1}{2}[1+(-1)^t]\begin{bmatrix} j & j & J\\ \frac{1}{2} & -\frac{1}{2} & 0\end{bmatrix}^2\right\}, \tag{4}$$

where the symbol [ ] denotes the Clebsch-Gordon coefficient. The deformation energy κ in Eq. (2) is related to the deformation parameter β [19]. For the case of $f_{7/2}$ shell, $\kappa = 2.4$ approximately corresponds to $\beta = 0.25$. It is to be noted that the maximum angular-momentum possible for valance particles in this shell is $16\hbar$ with four-protons and four-neutrons. For states with large angular frequencies studied in the present work, the total angular-momentum has predominantly core (rotor) component. The present model is, therefore, applicable to rotational bands with low-K. For high-K bands, the valance particle contribution is quite significant.

In the present work, the pairing correlations have been calculated using canonical ensemble since the exact solutions have well-defined particle number. The average value of a physical quantity "F" in canonical ensemble is given by [9]

$$\langle\langle F\rangle\rangle = \sum_i F_i e^{-E_i/T}/Z, \tag{5}$$

where

$$Z = \sum_i e^{-E_i/T}$$
$$\hat{H}|i\rangle = E_i|i\rangle \tag{6}$$
$$F_i = \langle i|\hat{F}|i\rangle.$$

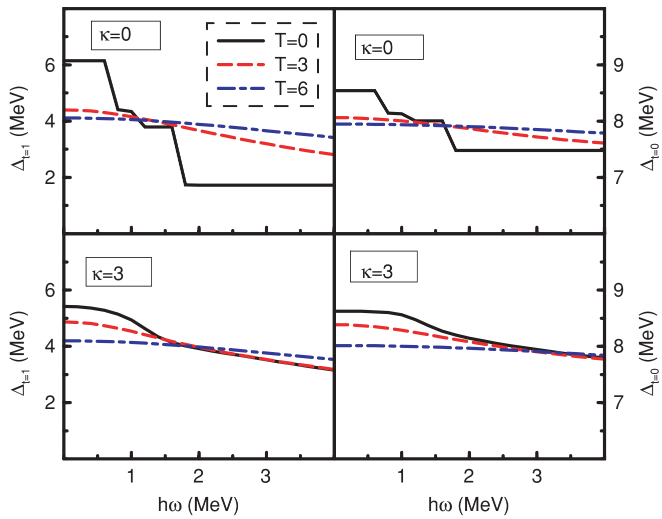

FIG. 1. (Color online) Results of the total isovector ($\Delta_{t=1}$) and isoscalar ($\Delta_{t=0}$) pair gaps are plotted as a function of rotational frequency for three different temperatures of $T = 0, 3$, and 6 MeV. The results for spherical ($\kappa = 0$ MeV) and deformed ($\kappa = 3$ MeV) nuclei are shown in upper and lower panels, respectively.

The pairing correlations for the (Jt) multipole field is calculated in the canonical ensemble as

$$E_{Jt}(\text{pair}) = v_{Jt}\sqrt{(2J+1)(2t+1)}\big(\langle\langle(A^\dagger_{Jt}\times\tilde{A}_{Jt})_{00;00}\rangle\rangle - {}_0\langle\langle(A^\dagger_{Jt}\times\tilde{A}_{Jt})_{00;00}\rangle\rangle_0\big), \tag{7}$$

where the uncorrelated contribution denoted by $_0\ll\gg_0$ has been subtracted [8]. The pair gap for the multipole field (Jt) is then calculated through the expression

$$E_{Jt}(\text{pair}) = \frac{\Delta_{Jt}\Delta^*_{Jt}}{v_{Jt}}. \tag{8}$$

The model Hamiltonian has been solved exactly for protons and neutrons in $f_{7/2}$ subshell. We have considered this subshell as in our earlier studies [19], since the dimensions of the matrices to be diagonalized are tractable. The results of the total $t = 1$ and 0 pair gaps as a function of rotational frequency for four-protons and four-neutrons are presented in Fig. 1. For temperature, $T = 0$, both $\Delta_{t=1}$ and $\Delta_{t=0}$ depict changes as a function of rotational frequency. The decrease in the pair gaps occur in steps for $\kappa = 0$ and for $\kappa = 3$ the drop occurs in a smoother manner. This decrease in the pairing correlations is due to crossing of the aligned configurations with the ground-state band. The yrast band at low frequencies is a paired state and this band is crossed by the aligned bands which have reduced pairing correlations. These aligned bands become favored with increasing rotational frequency. For higher temperatures, it is noted that the pairing drops smoothly. The reason for the smooth decrease is that at higher temperatures there are crossings of many bands that occur at slightly different frequencies. The average of these crossings then gives rise to a smoother drop [8].

The drop in the $t = 1$ channel in Fig. 1 is similar to what was found in Ref. [8] for the monopole pairing among identical particles. As discussed below, the decrease of the total $t = 1$

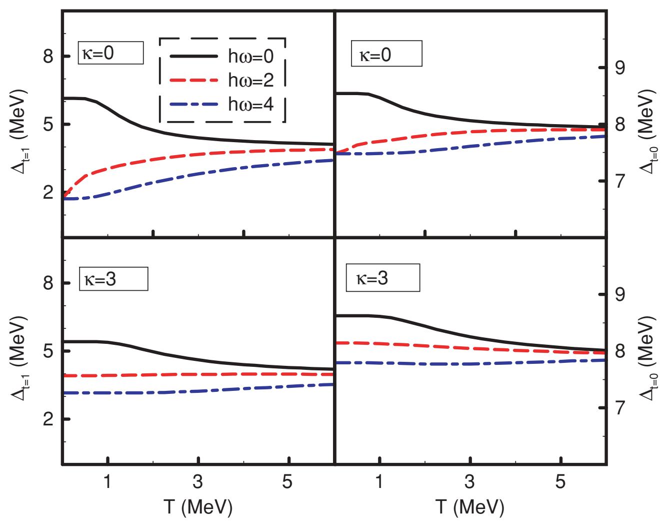

FIG. 2. (Color online) Results of the total isovector ($\Delta_{t=1}$) and isoscalar ($\Delta_{t=0}$) pair gaps are plotted as a function of temperature for three different rotational frequencies of $\hbar\omega = 0$, 2, and 4 MeV. The upper panel shown the results for $\kappa = 0$ and the lower panel depicts the results for $\kappa = 3$ MeV.

gap reflects decrease of the monopole term, which is the dominating part. The pair correlations in $t = 0$ channel, on the other hand, are only moderately reduced. This has been discussed before [18,21,22], and is expected as the $t = 0$ pairing should be favored by rotation (the isoscalar pairs carry angular momenta). However, we shall demonstrate below that the dipole field ($J = 1$) of $t = 0$ pairing channel is reduced by the rotational alignment as the monopole field ($J = 0$) of $t = 1$ pairing, which results in the moderate decrease of the total $t = 0$ pair correlation. For the $J = 1$ pair field, the angular momenta of the nucleons are nearly anti-parallel and, therefore, rotational alignment of a pair of nucleons breaks both the dipole $J = 1$ and the monopole $J = 0$ pairs.

It is also evident from Fig. 1 that with increasing temperature, the pairing correlations at higher frequencies become stronger. This is quite unexpected since mean-field theory predicts vanishing of pair correlations both with increasing rotational frequency and temperature. In order to demonstrate it more clearly, the pair gaps are plotted in Fig. 2 as a function of temperature for three different values of the rotational frequency. The upper panel of Fig. 2 gives the results of pairing correlations for the spherical case with $\kappa = 0$. For $\hbar\omega = 0$, the pair correlations drop with increasing temperature, but it is to be noticed that this drop is very smooth and they do not tend to zero with increasing temperature. In the mean-field BCS theory, the pair correlations depict a sudden transition from finite Δ to zero Δ. The nonvanishing of the total pair gap will be shown later due to nonzero values of higher-multipole pairing fields. For the $\hbar\omega = 2$ and 4, it is seen from Fig. 2 that the pair gaps increase with increasing temperature. The pairing correlations, which are quenched for these cases at low temperatures, reappear at finite temperature. This result contradicts the notion based on the mean-field models that pair correlations are quenched with increasing temperature for a similar model Hamiltonian [5] considered in the present work.

It has first been reported in Ref. [16] that the number-parity projection leads to reappearance of the pairing correlations for an odd-particle system at finite temperature. In the case of even-particle system, however, no such reappearance was found. This result appears difficult to comprehend as it is expected that the pair correlations should be stronger for an even system as compared to an odd-system and, therefore, the reappearance should be more pronounced in an even system. The reason for this inconsistency is due to the approximate partial particle-number projection (number-parity) performed in Ref. [16]. Studying the exact solutions of a system of fermions in a single j-shell interacting via a monopole pair force, Ref. [8] found that the pair correlations that are quenched at zero temperature by rotation or a magnetic field reappear at finite temperature *both* for even and odd particle numbers.

The reason for the above reappearance of the pairing correlations at finite temperature can be understood by first noting that only the occupations of the time-reversed states close to the Fermi level contribute to the pairing correlations. At higher rotational frequencies, the particles near the Fermi surface occupy aligning states, which have large angular-momenta along the rotational axis. The occupation of these aligning states close to the Fermi surface block the pairing correlations. However, with increasing temperature these aligning particles are promoted to higher excited states and the states close to the Fermi surface can now be occupied in time-reversed form and consequently increasing the pairing correlations with temperature.

In order to critically investigate the behavior of the pair gaps as a function of rotational frequency and temperature, the results of the individual multipole pair-fields are presented in Figs. 3 and 4. In Fig. 3, the results are shown for the individual pairing fields as a function of the rotational frequency at zero temperature, $T = 0$. It is evident from this figure that for $\kappa = 0$, the changes in the total isovector pair gaps ($t = 1$) noted in Fig. 1 are primarily due to the changes in the

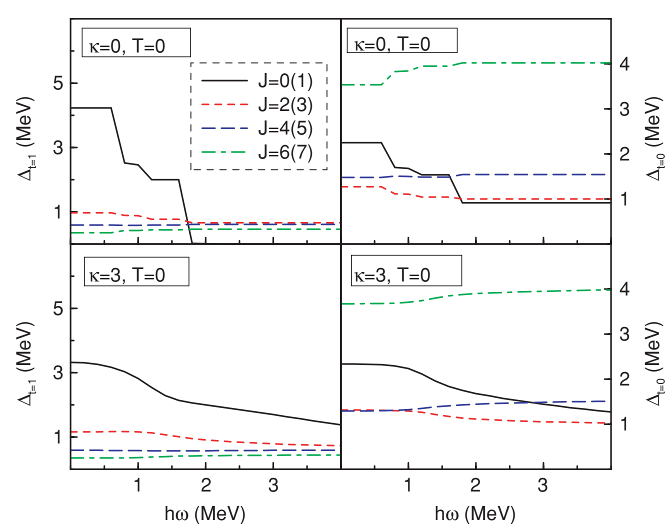

FIG. 3. (Color online) Results of the individual contributing pair fields of $J = 0(1)$, 2(3), 4(5), and 6(7) for the isovector (isoscalar) are plotted as a function of the rotational frequency for temperature, $T = 0$.

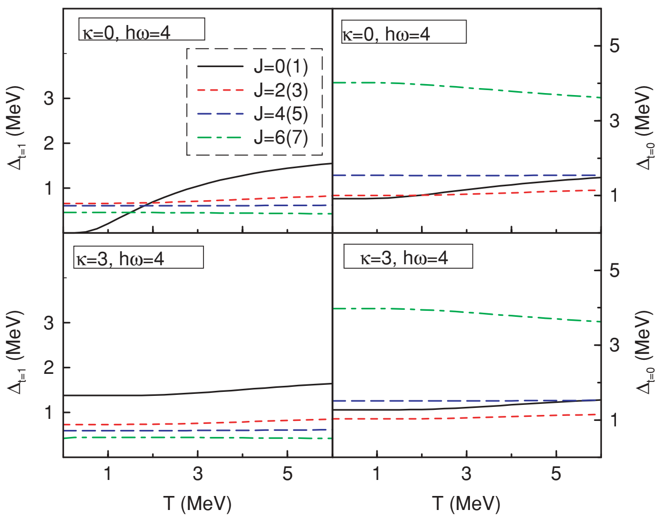

FIG. 4. (Color online) Results of the individual contributing pair fields of $J = 0(1), 2(3), 4(5)$, and $6(7)$ for the isovector (isoscalar) are plotted as a function of temperature for the rotational frequency of $\hbar\omega = 4$ MeV.

monopole pairing-field ($\Delta_{t=1}(J = 0)$). The other contributing pair multipoles of $J = 2, 4$, and 6 are quite small and do not depict any changes as a function of rotational frequency. For the higher multipoles the two nucleons are not in nearly or in fully antiparallel states and, therefore, these pairs are comparatively less affected by rotational alignment. For the case of isoscalar pairing gap, it is seen that the dipole field drops with rotational frequency. This drop is quite similar to that seen on the left hand side of the figure for the monopole case. However, the magnitude of the changes for the dipole field are lower than the monopole field. It is also noted from the figure that the $J = 7$ multipole field of the isoscalar pair gap increases with rotational frequency which is expected, because the angular momenta of the two nucleons are parallel. The net result is that the total $\Delta_{t=0}$ shows lesser variation with rotational frequency as compared to the $t = 1$ pair field. This strong reduction of the $t = 1$ correlations and the weak change of the $t = 0$ correlations has also been found in a realistic cranked shell model Monte Carlo (CSMMC) study for ^{74}Rb [17]. In this study, the pair correlations were studied at zero temperature and it would be interesting to perform CSMMC calculation at finite temperature to confirm the reappearance of the pairing correlations as obtained in the present model study.

For the deformed case, shown in the lower panel of Fig. 3, there is a smoother drop with increasing rotational frequency. The reason for this smoother drop is that the wave functions in the deformed case do not have well defined angular-momentum, which leads to a smoothing out of the band crossings. In the spherical case, the band crossings are sharp, since the wave function has well defined angular momentum. As a consequence, the pairing field depicts sudden changes with rotational frequency. It is also noted from Fig. 3 that the $t = 1$ pair field is smaller in the deformed case as compared to the spherical case. However, the $t = 0$ pairing field is quite similar in the two cases.

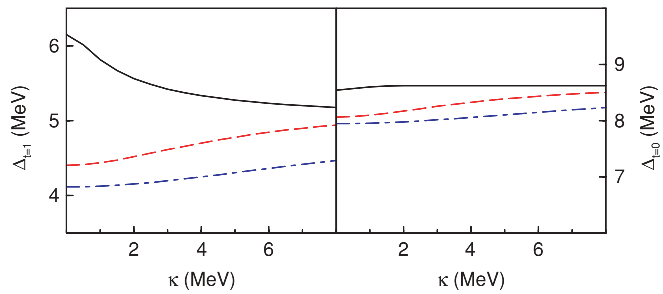

FIG. 5. (Color online) Deformation dependence of the total isovector and isoscalar pair correlations for three temperatures of $T = 0, 3$, and 6 MeV, shown by solid, long dashed, and dot-dashed lines, respectively.

The results for $\hbar\omega = 4$ are shown in Fig. 4. It is now clearly evident from this figure, that the reappearance of the pairing correlations apparent in Fig. 2 are due to the reemergence of the monopole (dipole) pair-correlations in the $t = 1(0)$ channel. The monopole field is zero at low temperatures and becomes nonzero at about $T = 0.6$ MeV and then shows a smooth increase with temperature. The dipole field on the right hand panel of Fig. 4 is almost constant at lower temperatures, but starts increasing at around $T = 3$ MeV. For finite deformation, the monopole pair-field is constant till $T = 3$ MeV and then shows an increase.

In our single j-shell model, the single particle levels spread out with increasing deformation. For a given interaction strength, the pair correlations are expected to decrease with increasing distance between the time-reversed states among which the pairs can scatter. In order to investigate it, the pair correlations are presented in Figs. 5 and 6 as a function of the deformation parameter, κ. The pair gaps in Fig. 5, show

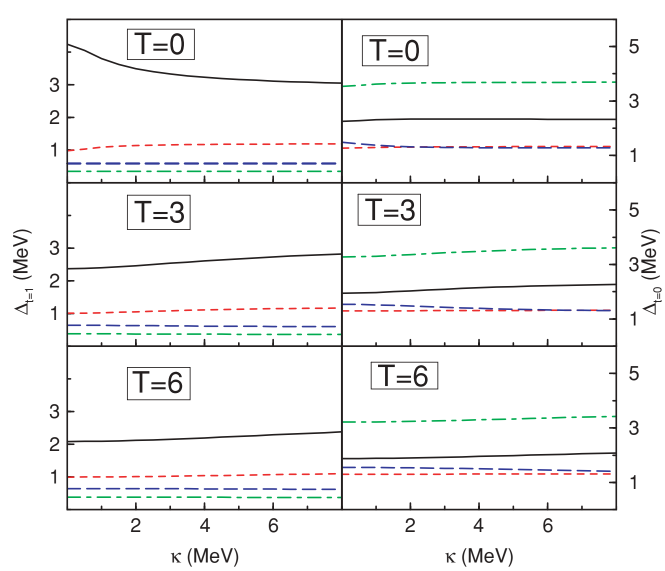

FIG. 6. (Color online) Deformation dependence of the individual contributing pair fields of $J = 0(1), 2(3), 4(5)$, and $6(7)$ for the total isovector (isoscalar) correlations at temperatures of $T = 0, 3$, and 6 MeV. $J = 0(1)$ is shown by solid line, $J = 2(3)$ by dashed line, $J = 4(5)$ by long dashed line, and $J = 6(7)$ by dot-dashed line.

the expected deformation dependence at temperature $T = 0$. The isovector correlations drop with increasing deformation and the isoscalar pair correlations remain constant. However, for finite temperature, both the isovector and the isoscalar pair correlations slightly increase with deformation. For example, the isovector, $\Delta_{t=1}$ increases from 4.4 MeV at $\kappa = 0$ to a value of 5.0 MeV at $\kappa = 8$ MeV. This increase in the pair correlations with deformation is quite unexpected and contradicts the mean-field analysis for the specific model studied in the present work. The mean-field studies predict vanishing of pair correlations both with temperature [5] and deformation [23] for the single j-shell model.

The dependence of individual pair fields as a function of deformation is presented in Fig. 6. It is evident from this figure that the increase and decrease in the isovector pair field is essentially determined by the monopole component. The monopole pair field decreases with deformation at temperature, $T = 0$ and shows an increasing trend for $T = 3$ and 6 MeV. The other multipole components $J = 2, 4$, and 6 are quite small and do not depict any significant changes with deformation. For the isoscalar pair field, it is noted that all the components are constant with deformation at temperature, $T = 0$. At higher temperatures of $T = 3$ and 6, the isoscalar pair fields of $J = 1$ and 7 slightly increase with deformation.

It is to be noted that the real deformation dependence of the pair correlations may be different from our simple model, for which the the deformation leads only to an increase of the distance between the single particle levels. In a realistic potential, the level density is an oscillating function of the deformation and the strength of the interaction matrix elements also depends on the deformation. The important result of the present analysis that the pair correlations may be enhanced at finite temperature as compared to zero temperature is thus applicable to specific models for which the level density is low and deformation leads only to an increase of the distance between the levels. Furthermore, it is emphasized that at higher temperatures, the particles get excited to higher single-particle states. This in the present model would correspond to the core polarization, which is not considered in the present work. Thus, the present results for higher temperatures may be unrealistic. As a matter of fact, the results from a realistic Hamiltonian and model space indicate that the particle evaporation occurs at about 3 MeV [24,25]. Nevertheless, the main inference of the present work that the pairing correlations reappear occurs even at very low temperature for which the present model is applicable.

In conclusion, we have investigated in an exactly solvable model the rotational and the deformation dependence of the isovector and isoscalar pair correlations at finite temperature. The results at higher temperatures have been shown to be quite surprising. It has been noted that the pair correlations reappear at finite temperature after they have been quenched at zero temperature and high rotational frequency. It has shown that the monopole and the dipole pair fields are responsible for this reappearance. It has been also observed that the pair correlations increase with deformation at finite temperature.

[1] A. Bohr and B. Mottelson, *Nuclear Structure II* (W. A. Benjamin, Reading, MA, 1975).
[2] P. Ring and P. Schuck, *The Nuclear Many-Body Problem* (Springer Verlag, New York, 1980).
[3] T. Dossing, B. Herskind, S. Leoni, A. Bracco, R. A. Broglia, M. Matsuo, and E. Vigezzi, Phys. Rep. **268**, 1 (1996), and references therein.
[4] A. L. Goodman, Nucl. Phys. **A352**, 30 (1981).
[5] A. L. Goodman, Phys. Rev. C **29**, 1887 (1984).
[6] J. L. Egido and P. Ring, J. Phys. G **19**, 1 (1993).
[7] N. D. Dang, P. Ring, and R. Rossignoli, Phys. Rev. C **47**, 606 (1993).
[8] S. Frauendorf, N. K. Kuzmenko, V. M. Mikhajlov, and J. A. Sheikh, Phys. Rev. B **68**, 024518 (2003).
[9] L. D. Landau and E. M. Lifshitz, *Statistical Physics* (Butterworth-Heinemann, 1999).
[10] J. L. Egido, P. Ring, S. Iwasaki, and H. J. Mang, Phys. Lett. **B154**, 1 (1985).
[11] V. Martin, J. L. Egido, and L. M. Robledo, Phys. Rev. C **68**, 034327 (2003).
[12] M. Anguiano, J. L. Egido, and L. M. Robledo, Nucl. Phys. **A696**, 467 (2001).
[13] H. Flocard and N. Onishi, Ann. Phys. (NY) **254**, 275 (1996).
[14] J. A. Sheikh and P. Ring, Nucl. Phys. **A665**, 71 (2000).
[15] R. Rossignoli, N. Canosa, and P. Ring, Phys. Rev. Lett. **80**, 1853 (1998).
[16] R. Balian, H. Flocard, and M. Veneroni, Phys. Rep. **317**, 251 (1999).
[17] D. J. Dean, S. L. Koonin, K. Langanke, and P. B. Radha, Phys. Lett. **B399**, 1 (1997).
[18] A. Satula and R. Wyss, Nucl. Phys. **A676**, 120 (2000).
[19] J. A. Sheikh, D. D. Warner, and P. Van Isacker, Phys. Lett. **B443**, 16 (1998).
[20] P. J. Brussaard and P. W. M. Glaudemans, *Shell Model Applications in Nuclear Spectroscopy* (North-Holland, Amsterdam, 1977).
[21] A. L. Goodman, Phys. Rev. C **60**, 014311 (1999).
[22] S. Frauendorf and J. A. Sheikh, Phys. Scr. **T88**, 162 (2000).
[23] J. A. Sheikh, P. Ring, E. Lopes, and R. Rossignoli, Phys. Rev. C **66**, 044318 (2002).
[24] P. Bonche, S. Levit, and D. Vautherin Nucl. Phys. **A427**, 278 (1984).
[25] P. Bonche, S. Levit, and D. Vautherin, Nucl. Phys. **A436**, 265 (1985).